

## Substitutional carbon in $\text{Si}_{1-x}\text{Ge}_x$

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Local vibrational modes of carbon impurities in relaxed  $\text{Si}_{1-x}\text{Ge}_x$  have been studied with infrared absorption spectroscopy in the composition range  $0.05 \leq x \leq 0.50$ . Carbon modes with frequencies in the range  $512\text{--}600\text{ cm}^{-1}$  are observed in  $^{13}\text{C}^+$ -implanted  $\text{Si}_{1-x}\text{Ge}_x$  after annealing at  $550^\circ\text{C}$ . Measurements on samples coimplanted with  $^{12}\text{C}^+$  and  $^{13}\text{C}^+$  show that these modes originate from defects containing a single carbon atom and from the variation of the mode frequencies with composition  $x$ , the modes are assigned to substitutional carbon in  $\text{Si}_{1-x}\text{Ge}_x$ . Based on the frequencies obtained from a simple vibrational model, the observed modes are assigned to specific combinations of the four Si and Ge neighbors to the carbon. The intensities of the modes indicate that the combination of the four neighbors deviates from a random distribution. *Ab initio* local-density-functional cluster theory has been applied to calculate the structure and the local mode frequencies of substitutional carbon with  $n$  Ge and  $4-n$  Si neighbors in a Si and a Ge cluster. The calculated frequencies are  $\sim 9\%$  higher than those observed, but the ordering and the splitting of the mode frequencies agree with our assignments. [S0163-1829(99)01243-6]

### I. INTRODUCTION

Layers of  $\text{Si}_{1-x}\text{Ge}_x$  grown on Si substrates have been studied extensively due to the potential applications of band-gap engineering and production of ultrafast electronic devices.<sup>1</sup> A basic limitation arises from the 4% mismatch between the Si and Ge lattice constants, which results in formation of strain-relieving misfit dislocations, when the layer thickness exceeds a critical value, e.g.,  $\sim 200\text{ \AA}$  in  $\text{Si}_{0.80}\text{Ge}_{0.20}$  (Ref. 2). The strain in a  $\text{Si}_{1-x}\text{Ge}_x$  layer can be compensated, if C atoms are added to the alloy.<sup>3</sup> High-quality  $\text{Si}_{1-x-y}\text{Ge}_x\text{C}_y$  layers, several hundred nanometers thick, have been obtained by nonequilibrium techniques, such as molecular-beam epitaxy (MBE),<sup>4,5</sup> combined ion and molecular-beam deposition,<sup>6</sup> chemical vapor deposition,<sup>7-8</sup> and ion implantation followed by solid-phase regrowth.<sup>9</sup> In these alloys, most of the C atoms are believed to be located at substitutional sites.<sup>3-9</sup> Below, we use the label  $\text{C}_s$  for substitutional C. Detailed knowledge about the physical properties of  $\text{C}_s$  in  $\text{Si}_{1-x}\text{Ge}_x$  may elucidate important aspects of the  $\text{Si}_{1-x-y}\text{Ge}_x\text{C}_y$  alloy formation. Moreover, C atoms are common impurities in  $\text{Si}_{1-x}\text{Ge}_x$  compounds. Therefore,  $\text{C}_s$  in  $\text{Si}_{1-x}\text{Ge}_x$  is an interesting defect. We expect that for  $\text{C}_s$ , the C atom forms covalent bonds with its four neighbor atoms but the preferential composition of Si and Ge neighbors is unknown. In pure Si and Ge,  $\text{C}_s$  has a single three-dimensional local mode ( $T_2$ ) with frequency at  $607$  and  $531\text{ cm}^{-1}$ , respectively.<sup>10,11</sup> This demonstrates that the frequencies associated with stretching of Si-C and Ge-C bonds are well separated. Hence, the frequencies of the  $\text{C}_s$  modes in  $\text{Si}_{1-x}\text{Ge}_x$  may distinguish the different combinations of Si

and Ge neighbors. So far, only the Si-C mode at  $607\text{ cm}^{-1}$  has been observed in  $\text{Si}_{1-x-y}\text{Ge}_x\text{C}_y$  alloys and it remains unknown whether Ge-C bonds are formed.<sup>3,5,6,8,9</sup>

The present paper addresses this problem. Strain-relaxed  $\text{Si}_{1-x}\text{Ge}_x$  samples implanted with C ions have been investigated by infrared absorption spectroscopy. Strong evidence is presented that  $\text{C}_s$  in  $\text{Si}_{1-x}\text{Ge}_x$  may bond to Si as well as Ge neighbors and several different neighbor combinations are identified.

A short report on preliminary results of this paper has been published previously by our group.<sup>12</sup> Recently, Kulik *et al.* also reported on the vibrational properties of  $\text{C}_s$  in SiGe alloys.<sup>13</sup> The sample production and the theoretical analysis applied by these authors differ from the methods used in this paper.

### II. EXPERIMENTAL DETAILS

Relaxed, epitaxial  $\text{Si}_{1-x}\text{Ge}_x$  layers with compositions  $x = 0.05, 0.15, 0.25, 0.35,$  and  $0.50$  were grown on Si substrates by MBE. Samples with  $x > 0.50$  were not studied due to the unsatisfactory crystalline quality obtained in this composition range. The growth technique and the characterization of the  $\text{Si}_{1-x}\text{Ge}_x$  layers were described previously,<sup>14,15</sup> and only a brief account is given here. First, a  $1\text{-}\mu\text{m}$  Si layer was grown on a high-resistivity, float-zone, single-crystalline,  $\langle 100 \rangle$  Si wafer. On top of this layer, a graded SiGe buffer layer was grown with a grading rate of  $10\text{ at. \% Ge}/\mu\text{m}$ . Finally, a  $4\text{-}\mu\text{m}$ -thick  $\text{Si}_{1-x}\text{Ge}_x$  top layer of uniform

composition  $x$  was grown. Transmission electron microscopy, photoluminescence, atomic force microscopy, and ion channeling measurements confirmed the high-crystalline quality of the  $\text{Si}_{1-x}\text{Ge}_x$  top layer.<sup>14,15</sup> Samples with typical dimensions of  $\sim 8 \times 8 \times 0.5 \text{ mm}^3$  were cut from the wafers, and the  $8 \times 8 \text{ mm}^2$  backside surface of the Si substrate was mechanically polished to ensure large transmission of infrared light.

In most cases, the  $\text{Si}_{1-x}\text{Ge}_x$  layers were implanted with  $^{13}\text{C}^+$  ions. For the  $^{12}\text{C}$  isotope, the relevant infrared absorption lines fall in the same frequency band as the most dominant multiphonon absorption from the Si substrate. For the  $^{13}\text{C}$  isotope, the corresponding lines fall at lower frequencies, where the background absorption is reduced significantly. Therefore, the  $^{13}\text{C}$  isotope has been applied in this paper. The implantation was carried out at 17 different energies, in a sequence starting at 450 keV and going down to 50 keV in steps of 25 keV. The dose implanted at each energy was adjusted to yield a nearly uniform C concentration of  $1 \times 10^{20} \text{ cm}^{-3}$  in the range from 0.1 to 1.0  $\mu\text{m}$  below the surface of the sample. A few samples were coimplanted with  $^{12}\text{C}^+$  and  $^{13}\text{C}^+$  into overlapping profiles with  $1 \times 10^{20} \text{ cm}^{-3}$  of each isotope. The implantation was carried out at room temperature and at a background pressure of  $5 \times 10^{-7}$  torr. The normal of the  $8 \times 8 \text{ mm}^2$  surfaces was tilted  $7^\circ$  off the direction of the  $\text{C}^+$  beam to reduce channeling effects during implantation. Furthermore, the beam was swept horizontally and vertically across the sample to ensure a homogeneous lateral distribution of implants. The sample holder was electrically insulated from the vacuum chamber, and the implanted dose was determined directly from integration of the current to the sample. Emission of secondary electrons was suppressed by a negative potential supplied to a shield surrounding the sample.

The infrared absorption spectrum of each sample was recorded after each step in a series of isochronal heat treatments (annealings). Each annealing was performed in a furnace continuously flowed with nitrogen gas. The duration of each annealing was 20 min and during this time the temperature was stable to within  $\pm 2^\circ\text{C}$ . The first annealing was carried out at  $300^\circ\text{C}$ , and in each subsequent step the temperature was increased by  $50^\circ\text{C}$ . Prior to each annealing and before each measurement, all samples were etched in 1% hydrofluoric acid to remove surface oxides.

The infrared absorbance spectra were recorded with a Nicolet, System 800, Fourier-transform spectrometer, equipped with a closed cycle helium cryostat with CsI windows. The measurements were carried out at 9 K with an apodized resolution of  $3.4 \text{ cm}^{-1}$  in the spectral range from 400 to  $4000 \text{ cm}^{-1}$ . We used a Ge-KBr beamsplitter, a global light source, and a Mercury-Cadmium-Telluride detector. Absorbance spectra measured on nonimplanted  $\text{Si}_{1-x}\text{Ge}_x$  samples were subtracted from all spectra recorded on the implanted samples.

### III. VIBRATIONAL PROPERTIES OF $\text{C}_s$ IN $\text{Si}_{1-x}\text{Ge}_x$

#### A. Frequencies of local modes

In order to guide the assignment of the observed local vibrational modes to  $\text{C}_s$ , we now discuss the expected vibrational properties of this defect in  $\text{Si}_{1-x}\text{Ge}_x$ . Because we are

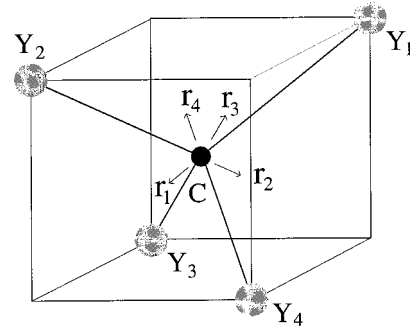


FIG. 1. Molecular model of  $\text{C}_s$  bound to four lattice atoms  $Y_i$ , where  $Y_i$  is Si or Ge. The displacement coordinates  $r_i$  associated with stretching of the  $Y_i$ -C bonds are sketched.

interested only in local modes, the  $\text{C}_s$  defect is described as a five atom molecule, with a central C atom and four tetrahedrally coordinated Si and Ge atoms, labeled  $Y_1$ ,  $Y_2$ ,  $Y_3$ , and  $Y_4$  in Fig. 1. It is assumed that the positions of the four neighbor atoms do not depend on the detailed distribution of Si and Ge atoms in the surrounding lattice. However, we expect that the many different configurations of the surrounding lattice show up as a broadening of the absorption lines associated with the local vibrational modes of  $\text{C}_s$ .

The bend modes of the five-atom molecule involve mainly Si and Ge atoms. Consequently, these modes will have low frequencies and be delocalized due to strong coupling with the lattice phonons. The stretching modes will have higher frequencies and be approximately decoupled from the other modes. We assume that the valence-force approximation<sup>16</sup> is valid and express the potential energy term for the stretch mode vibrations as

$$V = 1/2 \sum_{i=1}^4 f_{Y_i} r_i^2, \quad (1)$$

where  $r_i$  is the extension coordinate of the  $Y_i$ -C bond indicated in Fig. 1 and  $f_{Y_i}$  is the effective force constant of the  $Y_i$ -C bond. There are five different configurations of nearest neighbors, which we denote  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  ( $0 \leq n \leq 4$ ). For a given configuration, the  $4-n$  Si-C bonds are equivalent and the same applies for the  $n$  Ge-C bonds. Therefore, only two force constants  $f_s$  (Si-C bonds) and  $f_g$  (Ge-C bonds) are needed for a given configuration. With standard techniques<sup>16</sup> the frequencies of  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  can be calculated and expressed as functions of  $f_s$ ,  $f_g$ , and the atomic masses. The results are summarized in Table I. As can be seen from the table, the five configurations give rise to a total of nine C modes. The number of modes for each configuration is easily found from group theory.  $\text{Si}_4\text{Ge}_0:\text{C}$  and  $\text{Si}_0\text{Ge}_4:\text{C}$  have  $T_d$ -point group and possess a single three-dimensional C mode, denoted  $T_2$  in accordance with the irreducible representation describing its symmetry properties. The configurations with  $C_{3v}$  symmetry,  $\text{Si}_3\text{Ge}_1:\text{C}$  and  $\text{Si}_1\text{Ge}_3:\text{C}$ , have a two-dimensional  $E$  mode and a nondegenerate  $A_1$  mode. Finally,  $\text{Si}_2\text{Ge}_2:\text{C}$  has  $C_{2v}$  symmetry, which results in three nondegenerate modes, denoted  $B_1$ ,  $B_2$ , and  $A_1$ .

At a first glance, some of the frequencies in Table I appear identical. However,  $f_s$  and  $f_g$  depend on the length of the Si-C and Ge-C bonds, which depend on the configuration as well as the composition  $x$  of the  $\text{Si}_{1-x}\text{Ge}_x$  sample. In the

TABLE I. The square of the local mode frequencies of  $\text{C}_s$  for all configurations of  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$ .

Config.	Mode	$\omega^2$
$\text{Si}_4\text{Ge}_0:\text{C}$	$T_2$	$\left(\frac{4}{3m_c} + \frac{1}{m_s}\right)f_s$
$\text{Si}_3\text{Ge}_1:\text{C}$	$A_1$	$\frac{1}{2}\left\{\left(\frac{1}{3m_c} + \frac{1}{m_s}\right)f_s + \left(\frac{1}{m_c} + \frac{1}{m_g}\right)f_g\right.$ $\left. + \sqrt{\left[\left(\frac{1}{3m_c} + \frac{1}{m_s}\right)f_s - \left(\frac{1}{m_c} + \frac{1}{m_g}\right)f_g\right]^2 + \frac{4}{3m_c^2}f_s f_g}\right\}$
	$E$	$\left(\frac{4}{3m_c} + \frac{1}{m_s}\right)f_s$
$\text{Si}_2\text{Ge}_2:\text{C}$	$A_1$	$\frac{1}{2}\left\{\left(\frac{2}{3m_c} + \frac{1}{m_s}\right)f_s + \left(\frac{2}{3m_c} + \frac{1}{m_g}\right)f_g\right.$ $\left. + \sqrt{\left[\left(\frac{2}{3m_c} + \frac{1}{m_s}\right)f_s - \left(\frac{2}{3m_c} + \frac{1}{m_g}\right)f_g\right]^2 + \frac{16}{9m_c^2}f_s f_g}\right\}$
	$B_1$	$\left(\frac{4}{3m_c} + \frac{1}{m_s}\right)f_s$
	$B_2$	$\left(\frac{4}{3m_c} + \frac{1}{m_g}\right)f_g$
$\text{Si}_1\text{Ge}_3:\text{C}$	$A_1$	$\frac{1}{2}\left\{\left(\frac{1}{3m_c} + \frac{1}{m_g}\right)f_g + \left(\frac{1}{m_c} + \frac{1}{m_s}\right)f_s\right.$ $\left. + \sqrt{\left[\left(\frac{1}{3m_c} + \frac{1}{m_g}\right)f_g - \left(\frac{1}{m_c} + \frac{1}{m_s}\right)f_s\right]^2 + \frac{4}{3m_c^2}f_s f_g}\right\}$
	$E$	$\left(\frac{4}{3m_c} + \frac{1}{m_g}\right)f_g$
$\text{Si}_0\text{Ge}_4:\text{C}$	$T_2$	$\left(\frac{4}{3m_c} + \frac{1}{m_g}\right)f_g$

appendix, it is described how the values of  $f_s$  and  $f_g$  may be estimated. For the configuration  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  the result is

$$f_s = f_s^0 \left[ 1 - \frac{2\alpha\Delta b}{R_{\text{Si-C}}^0} \frac{f_g^0(4x-n)}{(4-n)f_g^0 + nf_s^0} \right] \quad (2)$$

$$f_g = f_g^0 \left[ 1 - \frac{2\alpha\Delta b}{R_{\text{Ge-C}}^0} \frac{f_s^0(4x-n)}{(4-n)f_g^0 + nf_s^0} \right] \quad (3)$$

where  $f_s^0 = 9.235 \text{ eV } \text{\AA}^{-2}$  is the value of  $f_s$  for  $\text{Si}_4\text{Ge}_0:\text{C}$  in pure Si and  $R_{\text{Si-C}}^0 = 1.962 \text{ \AA}$  is the corresponding bondlength (see Sec. V). Likewise,  $f_g^0 = 8.302 \text{ eV } \text{\AA}^{-2}$  is the value of  $f_g$  for  $\text{Si}_0\text{Ge}_4:\text{C}$  in pure Ge and  $R_{\text{Ge-C}}^0 = 2.035 \text{ \AA}$  is the corresponding bondlength. Finally,  $\alpha\Delta b$  is a factor with dimension of length. The value of  $\alpha\Delta b$  has been adjusted to obtain frequency variations with  $x$  in reasonable agreement with the observations described below. The best agreement is obtained with  $\alpha\Delta b = 0.16 \text{ \AA}$ , and this value is used throughout. With the expression given in Table I and Eqs. (2)–(3), the frequencies for all configurations  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  can be estimated at any value of  $x$ .

At this point we mention that Kulik *et al.*<sup>13</sup> recently calculated local mode frequencies of  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  in  $\text{Si}_{1-x}\text{Ge}_x$ ,

using an anharmonic Keating model. The results obtained by these authors qualitatively agree with our findings.

### B. Intensities of local modes

To estimate the population of each configuration  $\text{Si}_{4-n}\text{Ge}_n:\text{C}$  from the observed spectra, we need to elaborate further on the model described above. We assume that the dipole moment induced by the stretching of a bond is parallel to the bond and that the total dipole moment of  $\text{C}_s$  can be calculated as the sum of the dipole moments of the four bonds. To first order in the displacement coordinates  $r_i$  ( $i = 1, 2, 3, 4$ ), the dipole moment  $\mathbf{d}$  is then given by

$$\mathbf{d} = \mathbf{d}_0 + \sum_{i=1}^4 e_i r_i \hat{\mathbf{n}}_i, \quad (4)$$

where  $\mathbf{d}_0$  is the permanent dipole moment,  $e_i$  is the derivative of the dipole moment with respect to  $r_i$ , which we denote the effective charge of the  $Y$ -C bond, and  $\hat{\mathbf{n}}_i$  is a unit vector pointing from the  $i$ th neighbor atom  $Y_i$  towards C (see Fig. 1). For a given configuration the effective charge  $e_s(n)$  [or  $e_g(n)$ ] is the same for all Si-C (or Ge-C) bonds since they are equivalent. The integrated absorption coefficient (in-

tensity) of the local mode  $\Gamma$  associated with the configuration  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  may be expressed as<sup>16</sup>

$$I_{\Gamma}(n) = \frac{2\pi^2 N_{\text{tot}}}{3n_R c} p(n) \mu_{\Gamma}(n)^2. \quad (5)$$

$N_{\text{tot}}$  is the total concentration of  $\text{C}_s$ ,  $p(n)$  is the fraction of complexes with configuration  $\text{Si}_{4-n}\text{Ge}_n$ ,  $n_R$  is the refractive index, and  $c$  is the velocity of light. The parameter  $\mu_{\Gamma}(n)$  is given by

$$\mu_{\Gamma}(n)^2 = \sum_{a=1}^{m_{\Gamma}} \left( \frac{\partial \mathbf{d}}{\partial Q_{\Gamma a}} \right)^2, \quad (6)$$

where  $\{Q_{\Gamma a}\}$  is the set of normal coordinates associated with the normal mode  $\Gamma$ , and  $m_{\Gamma}$  is the dimension of the mode. The parameter  $\mu_{\Gamma}(n)$  is a function (quadratic form) of the effective charges  $e_s(n)$  and  $e_g(n)$ , which are unknown. In addition,  $N_{\text{tot}}$  is not known in our samples. Hence, we cannot obtain  $p(n)$  by inserting the measured value of  $I_{\Gamma}(n)$  into Eqs. (5)–(6). However, for a given composition  $x$ , the value of  $p(n)e_s(n)^2$  normalized to  $p(0)e_s(0)^2$  can for  $n \in \{0,1,2,3\}$  be determined from the formula

$$[p(n)e_s(n)^2]_{\text{rel}} \equiv \frac{p(n)e_s(n)^2}{p(0)e_s(0)^2} = \frac{I_{\Gamma}(n)}{I_{\Gamma}(0)} \left[ \frac{\mu_{T_2}(0)e_s(n)}{e_s(0)\mu_{\Gamma}(n)} \right]^2. \quad (7)$$

For  $n \in \{1,2,3\}$  the value of  $\mu_{\Gamma}(n)/e_s(n)$  depends for some modes on the ratio  $e_g(n)/e_s(n)$ , which can be obtained from the intensity ratio between two different modes of the same configuration  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ .

As discussed above, the Si-C and Ge-C bondlengths vary with changes in  $x$  and from one configuration to another. Because the first derivative of the bond-dipole moment may depend on the length of the bond,  $e_s(n)$  and  $e_g(n)$  may depend on  $x$  as well as on the specific configuration of the neighboring atoms. However, for a given composition  $x$ , the bondlengths differ by less than 5% between the different configurations. Thus, we suspect that  $e_s(n)$  does not change dramatically with  $n$  at a given  $x$ .

Therefore, we expect that  $[p(n)e_s(n)^2]_{\text{rel}}$  approximately equals the relative population,  $[p(n)]_{\text{rel}} \equiv p(n)/p(0)$ , of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ . If the distribution of the four Si and Ge neighbors is random,  $[p(n)]_{\text{rel}}$  is easily calculated for all compositions

$$[p(n)]_{\text{rel}} = \binom{4}{n} \left( \frac{x}{1-x} \right)^n. \quad (8)$$

#### IV. EXPERIMENTAL RESULTS

##### A. Observation of local vibrational modes of $\text{C}_s$

Two broad absorption bands at  $\sim 542$  and  $\sim 584$   $\text{cm}^{-1}$  are observed in  $\text{Si}_{0.50}\text{Ge}_{0.50}$  after implantation of  $^{12}\text{C}^+$  and subsequent annealing at  $550$   $^{\circ}\text{C}$ , as can be seen from curve *a* in Fig. 2. A similar pair of bands is observed when  $^{12}\text{C}^+$  is substituted by  $^{13}\text{C}^+$  (see curve *b*) but the frequencies are shifted downwards by a factor of  $\sim 1.03$ , which is close to  $13/12$ . Therefore, the bands represent local vibrational modes of C bound to a heavier element.

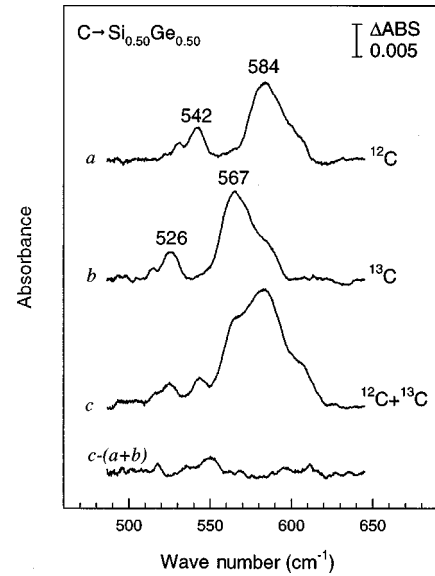


FIG. 2. Absorbance spectra measured at 9 K on  $\text{Si}_{0.50}\text{Ge}_{0.50}$  samples annealed at  $550$   $^{\circ}\text{C}$  after implantation with (a)  $^{12}\text{C}^+$ , (b)  $^{13}\text{C}^+$ , and (c) equal doses of both isotopes. The spectrum at the bottom is the result of subtracting spectra of the isotopic pure samples (curves *a* and *b*) from the spectrum of the coimplanted sample (curve *c*).

Coimplantation of  $^{12}\text{C}^+$  and  $^{13}\text{C}^+$  produces a spectrum with an even broader absorption band, depicted as curve *c* in the figure. When the coimplanted spectrum is subtracted by the single isotope spectra, the resulting spectrum is basically flat (see Fig. 2). Hence, the bands at about  $542$  and  $584$   $\text{cm}^{-1}$  originate from defects, which contain a single C atom. Similar bands have been observed for all compositions  $x$  and in all cases they are found to represent local modes of defects, which contain a single C atom.

The absorbance spectra recorded after annealing at  $550$   $^{\circ}\text{C}$  on  $^{13}\text{C}^+$ -implanted  $\text{Si}_{1-x}\text{Ge}_x$  samples with different compositions are shown in Fig. 3. Each spectrum is depicted so that its base line corresponds to the composition of the sample. For pure Si, a single absorption line reflecting the  $T_2$  mode of  $^{13}\text{C}_s$  is observed at  $589$   $\text{cm}^{-1}$  (Ref. 10). Likewise, a single line at  $512$   $\text{cm}^{-1}$  observed in pure Ge represents the  $T_2$  mode of  $^{13}\text{C}_s$  in that material.<sup>11</sup> As the Ge content is increased from  $x=0$ , the  $T_2$  line broadens into a band of lines and shifts downwards in frequency. When the Ge content exceeds  $\sim 15$  at. % another broad band appears at lower frequencies, which also shifts downwards in frequency with increasing Ge content. As can be seen from the figure, the two bands are for all configurations largely confined to the frequency interval defined by the  $T_2$  lines of the pure materials. Moreover, the frequency shifts with composition  $x$  indicate that in the limits  $x \rightarrow 0$  and  $x \rightarrow 1$ , the high- and low-frequency bands transform into the  $T_2$  lines of pure Si and Ge, respectively. This strongly suggests that the bands originate from different configurations of  $\text{C}_s$ . This assignment is consistent with the involvement of a single C atom in each of the underlying defects.

The frequencies calculated for all configurations of  $^{13}\text{C}_s$  from the formulas in Sec. III are also depicted in Fig. 3 as straight lines. A line is shown solid in the composition range, where the intensity of the corresponding mode is expected to

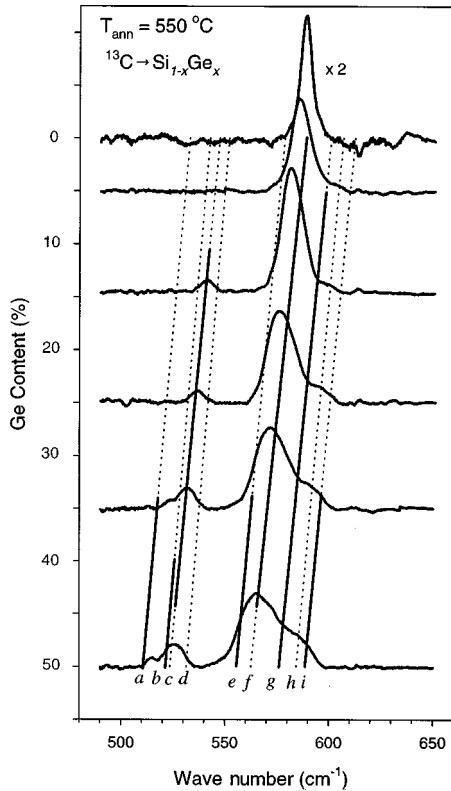


FIG. 3. Absorbance spectra recorded at 9 K on  $^{13}\text{C}^+$ -implanted  $\text{Si}_{1-x}\text{Ge}_x$  samples with different compositions ( $0 \leq x \leq 1$ ) after annealing at 550 °C. The baseline of each spectrum is displaced vertically to correspond to the composition of the corresponding sample. The straight lines are calculated from the model described Sec. III A. Each line shows the variation in frequency for one specific mode as a function of Ge concentration. A line is shown solid in the composition range, where the intensity of the corresponding mode is expected to exceed 10% of the total intensity of all  $\text{C}_s$  modes, and the dotted part of the lines corresponds to the range where the modes are expected to be very weak. The labels correspond to individual modes: (a)  $B_2$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$ , (b)  $E$  mode of  $\text{Si}_1\text{Ge}_3:\text{C}$ , (c)  $A_1$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$ , (d)  $T_2$  mode of  $\text{Si}_0\text{Ge}_4:\text{C}$ , (e)  $A_1$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$ , (f)  $T_2$  mode of  $\text{Si}_4\text{Ge}_0:\text{C}$ , (g)  $E$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$ , (h)  $A_1$  mode of  $\text{Si}_1\text{Ge}_3:\text{C}$ , and (i)  $B_1$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$ .

exceed 10% of the total intensity of all  $\text{C}_s$  modes. The dotted part of the lines indicates the composition range where the modes are expected to be very weak. To estimate the composition ranges, it has been assumed that the distribution of the four neighbors to the C is random.

The spectra recorded on the  $^{13}\text{C}$ -implanted  $\text{Si}_{0.65}\text{Ge}_{0.35}$  sample after annealing at 550 and 750 °C are shown in Fig. 4. Four lines denoted  $L_1$ ,  $L_2$ ,  $L_4+L_5$ , and  $L_6$  are resolved. According to Fig. 3, the modes of the configurations  $\text{Si}_4\text{Ge}_0:\text{C}$ ,  $\text{Si}_3\text{Ge}_1:\text{C}$ , and  $\text{Si}_2\text{Ge}_2:\text{C}$  are expected to dominate the spectrum for this composition  $x=0.35$ . Based on the close similarity with the model frequencies, we ascribe the  $L_1$  line to the  $B_2$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$  and the  $L_2$  line to the  $A_1$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$ . The intensity of the  $L_6$  line increases somewhat when the annealing temperature is increased from 550 to 750 °C. A similar behavior is observed for the  $L_1$  line, whereas the  $L_2$  line is constant. Therefore, we ascribe the  $L_6$  line to the  $B_1$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$ . The  $E$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$  and the  $T_2$  mode of  $\text{Si}_4\text{Ge}_0:\text{C}$  should correspond to the two

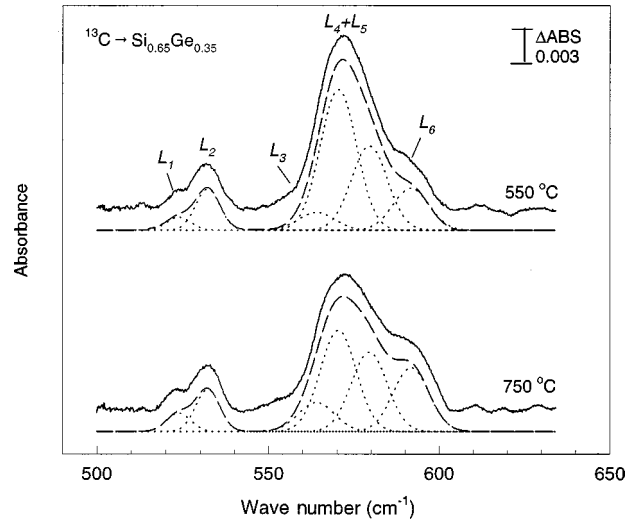


FIG. 4. Spectra recorded at 9 K on a  $^{13}\text{C}$ -implanted  $\text{Si}_{0.65}\text{Ge}_{0.35}$  sample annealed at 550 and 750 °C. The individual lines are denoted  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$ , and  $L_6$ . The best overall fit to the spectra and the individual components are shown as dashed and dotted curves.

most prominent lines in the spectrum, according to our intensity estimates. This strongly suggests that the line  $L_4+L_5$  consists of two overlapping lines  $L_4$  and  $L_5$ , which represent the  $T_2$  mode of  $\text{Si}_4\text{Ge}_0:\text{C}$  ( $L_4$  line) and the  $E$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$  ( $L_5$  line). Additional evidence for this is provided by the spectrum recorded after annealing at 750 °C. In this case, the line profile of  $L_4+L_5$  is clearly asymmetric. The low-frequency tail of the line  $L_4+L_5$  is rather broad, which indicates the presence of yet a line, labeled  $L_3$ . From comparison with Fig. 3, we ascribe this line to the  $A_1$  mode of  $\text{Si}_2\text{Ge}_2:\text{C}$ . Hence, the existence of the  $L_3$  line is consistent with the assignment of the  $L_1$  and  $L_6$  lines to the  $B_2$  and  $B_1$  modes of the same complex.

The spectra in Fig. 4 have been fitted with a sum of six Gaussians<sup>17</sup> corresponding to the lines  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$ , and  $L_6$ . The best fit and the individual components are also shown in the figure, and the peak positions and intensities are presented in Table II. The widths of the Gaussians corresponding to  $L_3$ ,  $L_4$ ,  $L_5$ , and  $L_6$  were forced to be equal during the fit, as were the widths of the Gaussians representing  $L_1$  and  $L_2$ .

A similar procedure has been applied to fit the observed spectra for all compositions covered in this study. The best-fit parameters are also given in Table II together with the assignment of the individual components.

## B. Analysis of local mode intensities

On basis of the local mode intensities given in Table II,  $[p(n)e_S(n)^2]_{\text{rel}}$  can be calculated for  $x = 0.15, 0.25, 0.35, 0.50$ . As mentioned in Sec. III, we expect that  $[p(n)e_S(n)^2]_{\text{rel}}$  approximately equals the relative population distribution  $[p(n)]_{\text{rel}}$  of the four Si and Ge neighbors. In Table III, the values obtained for  $[p(n)e_S(n)^2]_{\text{rel}}$  are compared with those of  $[p(n)]_{\text{rel}}$  corresponding to a random distribution of neighbors [see Eq. (8)]. When  $x$  is small, the random-distribution values of  $[p(n)]_{\text{rel}}$  agree with the values of  $[p(n)e_S(n)^2]_{\text{rel}}$ . However, when  $x$  exceeds 0.25, the two

TABLE II. Peak positions  $\omega$  ( $\text{cm}^{-1}$ ) and intensities  $I$  ( $\text{cm}^{-1}$ ) of the individual modes determined from the best fit to the spectra recorded on  $\text{Si}_{1-x}\text{Ge}_x$  after implantation of  $^{13}\text{C}$  and subsequent annealing at  $550^\circ\text{C}$ . The frequencies and the intensities marked with an \* are uncertain, as these absorption lines may mix with lines from the configuration  $\text{Si}_1\text{Ge}_3:\text{C}$ .

Configuration and mode	$x$	0.00	0.05	0.15	0.25	0.35	0.50
$\text{Si}_4\text{Ge}_0:\text{C}$							
$T_2$ mode	$\omega$	589.0	586.5	580.6	575.4	570.4	563.0
( $L_4$ line)	$I$	0.075	0.194	0.209	0.195	0.166	0.144
$\text{Si}_3\text{Ge}_1:\text{C}$							
$A_1$ mode	$\omega$			541.4	536.6	532.2	525.7*
( $L_2$ line)	$I$			0.017	0.018	0.032	0.043*
$E$ mode	$\omega$			586.6	583.9	579.5	573.2
( $L_5$ line)	$I$			0.097	0.082	0.099	0.114
$\text{Si}_2\text{Ge}_2:\text{C}$							
$B_2$ mode	$\omega$					523.8	515.5
( $L_1$ line)	$I$					0.011	0.014
$A_1$ mode	$\omega$			571.0	569.0	564.0	556.0
( $L_3$ line)	$I$			0.008	0.019	0.021	0.031
$B_1$ mode	$\omega$			600.0	596.1	591.8	585.4*
( $L_6$ line)	$I$			0.013	0.030	0.050	0.086*

sets of values deviate significantly, which indicates a nonrandom distribution of nearest neighbors. Instead, the values of  $[p(n)e_s(n)^2]_{\text{rel}}$  suggest a substantial preference for formation of complexes with many (3 or 4) Si neighbors.

Next, we discuss the thermal stability of the individual modes. Absorption spectra recorded on  $^{13}\text{C}$ -implanted  $\text{Si}_{0.65}\text{Ge}_{0.35}$  after annealing at different temperatures have been fitted with a sum of six Gaussians, with peak positions and widths fixed at the values obtained from the spectrum recorded after annealing at  $550^\circ\text{C}$ , as described in Sec. IV A. The resulting intensities of the individual components are shown against annealing temperature in Fig. 5. All modes appear at about  $525^\circ\text{C}$  and anneal at about  $875^\circ\text{C}$ . The detailed annealing behaviors of the modes differ, but modes assigned to the same configuration display the same behavior. This provides additional support for the assignments made above. The intensity of the  $T_2$  mode of  $\text{Si}_4\text{Ge}_0:\text{C}$

TABLE III. Calculated values of  $[p(n)e_s(n)^2]_{\text{rel}}$ , and the random-distribution values of  $[p(n)]_{\text{rel}}$  as a function of Ge content  $x$ . All intensities were determined from spectra measured after annealing at  $550^\circ\text{C}$ .

$x$	0.15	0.25	0.35	0.50
Model				
$[p(0)e_s(0)^2]_{\text{rel}}$	1.00	1.00	1.00	1.00
$[p(1)e_s(1)^2]_{\text{rel}}$	0.70	0.64	0.90	1.20
$[p(2)e_s(2)^2]_{\text{rel}}$	0.18	0.45	0.58	1.04
$[p(3)e_s(3)^2]_{\text{rel}}$				
Random				
$[p(0)]_{\text{rel}}$	1.00	1.00	1.00	1.00
$[p(1)]_{\text{rel}}$	0.71	1.33	2.15	4.00
$[p(2)]_{\text{rel}}$	0.19	0.67	1.74	6.00
$[p(3)]_{\text{rel}}$	0.02	0.15	0.62	4.00
$[p(4)]_{\text{rel}}$	0.00	0.01	0.08	1.00

reaches maximum at  $\sim 550^\circ\text{C}$  and decreases gradually as the annealing temperature is increased further up to  $850^\circ\text{C}$ . In contrast, the  $A_1$  and  $E$  modes of  $\text{Si}_3\text{Ge}_1:\text{C}$  have nearly constant intensities from  $550^\circ\text{C}$  up to  $850^\circ\text{C}$ , and the modes of  $\text{Si}_2\text{Ge}_2:\text{C}$  display a gradual intensity increase in this temperature range. Hence, the relative populations of the different configurations change from  $550$  to  $850^\circ\text{C}$ . From the intensities obtained after annealing at  $850^\circ\text{C}$ , we find that  $[p(0)e_s(0)^2]_{\text{rel}}=1.0$ ,  $[p(1)e_s(1)^2]_{\text{rel}}=1.1$ , and  $[p(2)e_s(2)^2]_{\text{rel}}=1.0$ . Comparison with the ( $550^\circ\text{C}$ ) values in Table II suggests that after  $850^\circ\text{C}$  annealing the distribution of neighbor atoms is closer to, but remains significantly different from, a random distribution. We note that the thermodynamical equilibrium distribution will differ from a random distribution, unless the total energies of all configura-

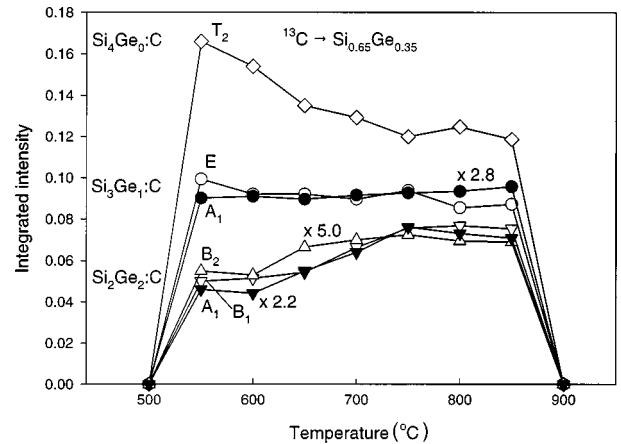


FIG. 5. Intensities of the individual modes of  $^{13}\text{C}_s$  in  $\text{Si}_{0.65}\text{Ge}_{0.35}$  shown against annealing temperature. The intensity of the  $A_1$  mode of  $\text{Si}_3\text{Ge}_1:\text{C}$  has been scaled by a factor of 2.8. The intensities of the  $A_1$  and  $B_2$  modes of  $\text{Si}_2\text{Ge}_2:\text{C}$  have been scaled by 2.2 and 5.0, respectively.

tions  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  are identical. Thus, deviations from a random distribution may not be surprising.

## V. THEORETICAL CALCULATIONS

### A. Method

In order to further investigate the credibility of the assignments made in Sec. IV A, the structure and local vibrational modes of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  embedded in a Si and a Ge cluster have been calculated using *ab initio* local density-functional theory.<sup>18</sup> The calculations were made on 71-atom tetrahedral clusters,  $\text{Si}_{35}\text{H}_{36}$  and  $\text{Ge}_{35}\text{H}_{36}$ , where the central Si or Ge atom was replaced by a C atom and the dangling bonds on the surface of the cluster were terminated by infinitely massive H atoms. The wave functions were represented by a basis set of Gaussian orbitals: Eight Gaussian orbitals of different exponents were centered on the C atom and four orbitals on the inner four Si and Ge neighbor atoms. A fixed linear combination of four orbitals was used for the remaining Si or Ge atoms, and a fixed linear combination of three orbitals was placed on each of the H atoms on the surface. Finally, three *s*- and *p*-Gaussian orbitals were placed at the center of every Si-Si or Ge-Ge bond in the cluster. The self-consistent energy and the forces on the atoms were calculated, and all 34 host atoms together with the C atom were allowed to relax until the minimum-energy configuration was obtained. During this calculation, full symmetry constraints were applied, and the H atoms at the surface were fixed at their starting positions. To calculate the local vibrational modes of the defect, the second derivatives of the energy with respect to atomic positions were calculated directly for the C atom and its four neighbors while the derivatives for the remaining atoms were calculated from a Musgrave-Pople potential.<sup>19,20</sup>

The  $\text{C}_s$  atom was first placed at the center of the Si cluster and the optimized positions and local vibrational modes were found. The four Si neighbors of the C atom were then replaced one at a time by Ge atoms, the structure was reoptimized, and the new local modes of the  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  configurations in a bulk Si cluster were calculated. The above process was then carried out in a Ge cluster with the nearest neighbors to C being replaced in turn by Si atoms to obtain the modes in bulk Ge.

### B. Results

The optimized bondlengths of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  are given in Table IV and the calculated modes are given in Table V for the  $^{12}\text{C}$  and  $^{13}\text{C}$  isotopes. We note that the Si-C bondlength of  $\text{Si}_4\text{Ge}_0\text{:C}$  in pure Si is 1.962 Å in good agreement with x-ray studies on C-doped Si, which gave a value of 1.95 Å.<sup>21</sup>

As can be seen from Table IV, the Si-C and Ge-C bondlengths decrease as the number of Ge neighbors are increased in both materials. The reason is that the Si-C bonds have a larger force constant than the Ge-C bonds. Hence, the C atom is displaced from its lattice site in a direction, which causes the Si-C bonds to shorten and the Ge-C bonds to lengthen compared to the ideal ‘‘unrelaxed’’ structures. Due to the near-tetrahedral coordination of the four neighbors, this effect is larger the more Ge neighbors there are.

TABLE IV. Calculated Si-C and Ge-C bondlengths (Å) of  $\text{C}_s$  in Si and Ge clusters for all configurations of nearest neighbors  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ .

Configuration	Point group	Si-C	Ge-C
Si cluster			
$\text{Si}_4\text{Ge}_0\text{:C}$	$T_d$	1.962	
$\text{Si}_3\text{Ge}_1\text{:C}$	$C_{3v}$	1.954	2.043
$\text{Si}_2\text{Ge}_2\text{:C}$	$C_{2v}$	1.947	2.033
$\text{Si}_1\text{Ge}_3\text{:C}$	$C_{3v}$	1.938	2.027
$\text{Si}_0\text{Ge}_4\text{:C}$	$T_d$		2.018
Ge cluster			
$\text{Si}_4\text{Ge}_0\text{:C}$	$T_d$	1.977	
$\text{Si}_3\text{Ge}_1\text{:C}$	$C_{3v}$	1.966	2.065
$\text{Si}_2\text{Ge}_2\text{:C}$	$C_{2v}$	1.957	2.053
$\text{Si}_1\text{Ge}_3\text{:C}$	$C_{3v}$	1.951	2.044
$\text{Si}_0\text{Ge}_4\text{:C}$	$T_d$		2.035

If we take the trigonal ( $C_{3v}$ ) defect  $\text{Si}_3\text{Ge}_1\text{:C}$  in the Si matrix as an example: The Si-C bond length is 0.008 Å shorter than that of  $\text{Si}_4\text{Ge}_0\text{:C}$  and the Ge-C bondlength is 0.025 Å longer than that of  $\text{Si}_0\text{Ge}_4\text{:C}$ . Due to the short Si-C bonds, the  $E$  mode of  $\text{Si}_3\text{Ge}_1\text{:C}$  has higher frequency than the  $T_2$  mode of  $\text{Si}_4\text{Ge}_0\text{:C}$ . This is in violation of the Saxon Hutter theorem.<sup>22</sup> However, this theorem assumes that the force constants are identical so that the shifts in the frequencies are entirely due to the difference in the masses of the constituents. Clearly, this assumption is not fulfilled in the present case since the force constants largely reflect the bond lengths.

The first-principle frequencies are often overestimated and to bring them in closer agreement with experiment we scale the theoretical values by the factor 0.917. This gives the  $T_2$  modes for  $^{13}\text{C}$  at 589  $\text{cm}^{-1}$  in Si (Ref. 10) and at 512  $\text{cm}^{-1}$  in Ge (Ref. 11) in perfect agreement with the observed modes. That the same scaling factor is required for both Si and Ge suggests that this factor is appropriate for all the modes of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  in the alloy.

It is clear that for low  $x$ , the matrix is closer to Si than Ge, whereas it is the reverse for large  $x$ . Thus, the effect of in-

TABLE V. Calculated frequencies ( $\text{cm}^{-1}$ ) of local vibrational modes for  $^{12}\text{C}_s$  and  $^{13}\text{C}_s$  in Si and Ge clusters for all configurations of nearest neighbors  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ . Furthermore, the scaled  $^{13}\text{C}_s$  mode frequencies of  $\text{Si}_{0.65}\text{Ge}_{0.35}$  are given.

Configuration	Mode	$^{12}\text{C}$		$^{13}\text{C}$		$x=0.35$
		Si	Ge	Si	Ge	
$\text{Si}_4\text{Ge}_0\text{:C}$	$T_2$	662	638	642	618	580
$\text{Si}_3\text{Ge}_1\text{:C}$	$A_1$	612	588	593	569	536
	$E$	673	651	653	631	592
$\text{Si}_2\text{Ge}_2\text{:C}$	$B_2$	594	568	574	548	518
	$A_1$	642	617	623	598	563
	$B_1$	684	663	664	642	602
$\text{Si}_1\text{Ge}_3\text{:C}$	$E$	598	573	578	552	522
	$A_1$	675	649	655	629	592
$\text{Si}_0\text{Ge}_4\text{:C}$	$T_2$	605	579	583	558	526

creasing  $x$  is to cause an expansion of the Si-C and Ge-C bonds as can be inferred from Table IV. This expansion causes a drop in each frequency and we assume that the frequencies of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  in the alloy can be obtained from linear interpolation of the corresponding scaled frequencies in the Si and Ge matrices. The frequencies thus obtained for  $^{13}\text{C}_s$  in an alloy with  $x=0.35$  are also given in Table V. As can be seen from comparison with Table II, there is a close correspondence between the calculated and assigned frequencies for the  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  defects ( $n=0, 1, \text{ and } 2$ ). The numerical ordering of the individual modes agrees and the maximum deviation between the two sets of frequencies is only  $12 \text{ cm}^{-1}$ . Hence, we conclude that the *ab initio* calculations provide additional support for the assignments made in Sec. IV A.

The calculations show that there is an energy preference for C to form bonds with Si rather than Ge. By comparing the energies of two relaxed clusters  $\text{CGeSi}_{69}\text{H}_{60}$ , where the Ge atom is a nearest or second-nearest neighbor to C, the Si-C bond was favored by 0.2 eV. Similarly, in the clusters  $\text{CSiGe}_{69}\text{H}_{60}$ , the C-Si bond was favored by 0.25 eV. These values suggest that the equilibrium probability of C being surrounded by four Ge neighbors in the alloy is negligible even at  $850 \text{ }^\circ\text{C}$ . Thus, the deviation from a random distribution of neighbors discussed in Sec. IV B is in accordance with the *ab initio* results.

## VI. CONCLUSION

Infrared absorption spectroscopy has been applied to study  $\text{C}_s$  in  $^{13}\text{C}$ -implanted  $\text{Si}_{1-x}\text{Ge}_x$  as a function of composition  $x$ . It has been shown that  $\text{C}_s$  possesses local vibrational modes with frequencies in the range from  $512$  to  $600 \text{ cm}^{-1}$ . The frequencies were calculated from a harmonic model, and the results are used to assign the observed absorption bands. According to our assignments,  $\text{C}_s$  binds to both Si and Ge neighbor atoms. The intensities of the local modes strongly suggest that the distribution of the four neighbor atoms is not random, but there is a preference for the configuration with four Si neighbors. *Ab initio* local density functional cluster theory has been used to calculate the structure of  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$  and the frequencies of the associated local modes. The results are in good agreement with our observations and provide additional support for our assignments.

## ACKNOWLEDGMENTS

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## APPENDIX

As mentioned in Sec. III the effective force constants  $f_s$  and  $f_g$  depend on the length of the Si-C and Ge-C bonds. For a diatomic molecule  $Y\text{-}Z$  with bondlength  $R_{Y-Z}$  and stretch frequency  $\omega_{Y-Z}$ , Morse<sup>23</sup> proposed the empirical relation

$$\omega_{Y-Z} R_{Y-Z}^\alpha = \text{Constant}, \quad (\text{A1})$$

where  $\alpha \sim 3$  as a rough estimate. We assume that this relation also holds for the  $Y_i\text{-}C$  bonds of the molecules in Fig. 1. Equation (A1) implies that for small changes in the bondlength the effective force constant  $f_Y$  for a  $Y\text{-}C$  bond is given by

$$f_Y = f_Y^0 \left( 1 - 2\alpha \frac{dR_{Y-C}}{R_{Y-C}^0} \right), \quad (\text{A2})$$

where  $R_Y^0$  and  $f_Y^0$  are the bond length and the effective force constant for a reference state. Moreover,  $dR_{Y-C} = R_{Y-C} - R_{Y-C}^0$ . As a reference state, we choose  $\text{Si}_4\text{Ge}_0\text{:C}$  in pure Si for Si-C bonds and  $\text{Si}_0\text{Ge}_4\text{:C}$  in pure Ge for Ge-C bonds. From the frequencies of the  $T_2$  modes,  $607 \text{ cm}^{-1}$  in Si (Ref. 10) and  $531 \text{ cm}^{-1}$  in Ge,<sup>11</sup> and with the expressions in Table I, we calculate  $f_s^0 = 9.235 \text{ eV } \text{Å}^{-2}$  and  $f_g^0 = 8.302 \text{ eV } \text{Å}^{-2}$ . Since reasonable estimates of  $R_{\text{Si-C}}^0$  and  $R_{\text{Ge-C}}^0$  are sufficient we use the theoretical values,  $R_{\text{Si-C}}^0 = 1.962 \text{ Å}$  and  $R_{\text{Ge-C}}^0 = 2.035 \text{ Å}$ , given in Sec. V.

Now we denote the Si-C bondlength for  $\text{Si}_4\text{Ge}_0\text{:C}$  by  $b_s(x)$  and the Ge-C bondlength for  $\text{Si}_0\text{Ge}_4\text{:C}$  by  $b_g(x)$ . With this definition,  $b_s(0) = R_{\text{Si-C}}^0$  and  $b_g(1) = R_{\text{Ge-C}}^0$ . Since the variations of  $b_s(x)$  and  $b_g(x)$  as  $x$  is changed are less than a few percent, we expect that linear expansions are valid

$$b_s(x) = R_{\text{Si-C}}^0 + x\Delta b_s \quad 0 \leq x \leq 1$$

$$b_g(x) = R_{\text{Ge-C}}^0 + (1-x)\Delta b_g \quad (\text{A3})$$

where  $\Delta b_Y = b_Y(1) - b_Y(0)$ . The values of  $\Delta b_s$  and  $\Delta b_g$  are unknown. According to the *ab initio* calculations described in Sec. V,  $\Delta b_s$  and  $\Delta b_g$  differ only by  $0.02 \text{ Å}$ . Therefore, we set  $\Delta b \equiv \Delta b_s = \Delta b_g$ .

To estimate  $R_{\text{Si-C}}(x)$  and  $R_{\text{Ge-C}}(x)$  for the configuration  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ , the  $4-n$  Si and  $n$  Ge atoms are fixed at sites displaced along  $\langle 111 \rangle$  directions from the origo (see Fig. 1). The distance from each neighbor atom to the origo is chosen equal to  $b_s(x)$  for the  $4-n$  Si and  $b_g(x)$  for the  $n$  Ge neighbors. Thus, if the C atom is placed at the origo, the bondlengths are identical to those of  $\text{Si}_4\text{Ge}_0\text{:C}$  and  $\text{Si}_0\text{Ge}_4\text{:C}$ . However, for complexes with both Si and Ge neighbors, the origo is not the equilibrium site for C, because  $f_s$  and  $f_g$  differ. It is a simple task to find the equilibrium site of C, and thus, get the values of  $dR_{\text{Si-C}}$  and  $dR_{\text{Ge-C}}$  for the configuration  $\text{Si}_{4-n}\text{Ge}_n\text{:C}$ . Then, the force constants  $f_s$  and  $f_g$  can be obtained from Eq. (A2) and the result is

$$f_s = f_s^0 \left[ 1 - \frac{2\alpha\Delta b}{R_{\text{Si-C}}^0} \frac{f_g^0(4x-n)}{(4-n)f_g^0 + nf_s^0} \right]$$

$$f_g = f_g^0 \left[ 1 - \frac{2\alpha\Delta b}{R_{\text{Ge-C}}^0} \frac{f_s^0(4x-n)}{(4-n)f_g^0 + nf_s^0} \right]. \quad (\text{A4})$$

The factor  $\alpha\Delta b$  is not known, and as described in Sec. III, its value has been adjusted to reproduce the observed variations of the frequencies with  $x$ . The best value was found to be  $\alpha\Delta b = 0.16 \text{ Å}$ .



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