

Correlation functions for a two-dimensional electron system with bosonic interactions and a square Fermi surface

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We calculate zero-temperature correlation functions for a model of two-dimensional interacting electrons with short-range interactions and a square Fermi surface. The model was arrived at by mapping electronic states near a square Fermi surface with Hubbard-like interactions onto one-dimensional quantum chains, retaining terms that can be written in terms of bosonic density operators. Interactions between orthogonal chains, corresponding to orthogonal faces of the square Fermi surface, are neglected. The correlation functions become sums of Luttinger-type correlation functions due to the bosonic model. However, the correlation function exponents differ in form from those of the Luttinger model. As a consequence, the simple scaling relations found to exist between the Luttinger model exponents, do not carry over to the leading exponents of our model. We find that for repulsive effective interactions, charge-density wave/spin-density wave instabilities are dominant. We do not consider d -wave instabilities here. [S0163-1829(99)00343-4]

I. INTRODUCTION

In recent years, research in theoretical condensed matter physics has focused intensely on the possible breakdown of Landau's Fermi-liquid theory in strongly correlated electron systems with spatial dimension larger than one. This has been motivated by Anderson's observation¹ that the normal state of the high- T_c superconductors has characteristics which seem irreconcilable with the predictions of Fermi liquid theory.² It is well known that one-dimensional (1D) systems of strongly correlated electrons with gapless excitations belong to the so-called Luttinger liquid universality class,^{3,4} but an analogous rigorous theoretical foundation for non-Fermi-liquid physics in two-dimensional (2D) systems is still lacking.

A number of authors have treated the 2D interacting electron gas with a circular Fermi surface using bosonization and/or renormalization-group techniques.⁵⁻⁷ Resummations of perturbation theory have also been carried out,⁸ including the case of a square Fermi surface within the parquet approximation.⁹ It is invariably found that the Luttinger-liquid fixed point is unstable in more than one spatial dimension. The underlying physics is that, due to the restricted kinematics on the Fermi surface in one spatial dimension, forward scattering is singular. In higher dimensions, small-angle scattering appears to avoid such a singularity, unless the interactions are very long ranged.¹⁰ On the other hand, for a nested or nearly nested Fermi surface such as is observed in the high- T_c cuprates,¹¹ the phase space for small-angle scattering is suppressed. Under such circumstances, it appears that a breakdown of Fermi-liquid theory may be feasible even with regular interactions.¹² It should be noted that approaches based on, for instance, resummation of perturbation series, such as the parquet approximation of Ref. 9, are expected to be reliable for not-too-strong correlations. Alternatively, one could consider a regime of interaction parameters where correlation effects give rise to qualitatively phe-

nomena that should be incorporated even in a zeroth order approximation to the problem. This would then complement, for instance, the work of Ref. 9.

Recently, a 2D system of electrons on a square lattice with nearest-neighbor hopping and Hubbard-like interactions was considered, when the Fermi surface is a square even away from half filling.¹² The necessary requirement for this to be a valid starting point is that the interactions of the underlying lattice model must be strong enough. This approach, therefore, does not have a free-fermion limit; the zeroth order Hamiltonian of the problem is rather that of free bosons. In Ref. 12, the problem was mapped onto two sets of one-dimensional chains, one set for each axis of the square Fermi surface. The kinetic energy and the interaction were separated into terms that could be written in terms of bosonic density operators for the chains, and terms of nonbosonic form. The bosonic Hamiltonian was then diagonalized, assuming no interactions between orthogonal chains, and the single-particle Green's function was evaluated, with a resulting Luttinger-liquid form. The nonbosonic terms in the Hubbard-like interactions were classified as relevant or irrelevant with respect to the solution of the bosonic Hamiltonian. Discarding irrelevant operators, the Hamiltonian for a particular case of sufficiently strong repulsive interactions was solved, treating the relevant operators as perturbations on the bosonic solution.

The N -component one-dimensional electron gas has very recently been treated using bosonization,¹³ obtaining the operator dimensions for a number of nonbosonic terms that could appear in the Hamiltonian.¹⁴ One particular realization of such a system is the N -chain Hubbard model. Given the fact that we neglect interactions between orthogonal chains in our model, the system we effectively study has much in common with that of Ref. 13. However, our work differs in one important respect, namely that we consider correlation

functions of fermion operators that are nonlocal in chain indices.

The minimal microscopic model appropriate for the description of high- T_c superconductivity is not yet agreed upon. Much work has been done for Hubbard-like models, also for the square Fermi surface case.^{15–19} Recently, claims were however made that nonexistence of $d_{x^2-y^2}$ -wave superconductivity in the 2D Hubbard model had been rigorously proved at any finite temperature, and under certain circumstances even at zero temperature.²⁰ If that were to be true, the single-band Hubbard model is not likely to be a fruitful microscopic starting point for explaining the phenomenology of the high- T_c cuprates, where there is now broad consensus on a dominant $d_{x^2-y^2}$ -component of the order parameter. In this situation, the examination of the properties of various hypothetical effective theories may give useful information about what directions one should take in solving this exceedingly difficult problem.

In this paper, we investigate the ground-state properties of the bosonic Hamiltonian in more detail, through the calculation and analysis of explicit expressions for various ground-state correlation functions, which enables us to identify the dominant divergent susceptibilities. We calculate the correlation functions for $2k_F$ charge-density wave (CDW), $2k_F$ spin-density wave (SDW), s -wave singlet (SS), and triplet (TS) superconductivity fluctuations. The similarities to the results obtained for the Luttinger model are: (i) the separate terms making up the correlation functions have Luttinger-model form, due to the bosonic form of our model, (ii) there is a degeneracy between CDW and SDW fluctuations, and between SS and TS fluctuations, (iii) for effective repulsive interactions, the CDW/SDW instabilities are dominant. However, because the expressions for our correlation function *exponents* differ in form from those of the Luttinger model, the simple scaling relations valid for the exponents of the Luttinger model do not hold for our model.

This paper is organized as follows. In Sec. II, we describe the fundamentals of the model under consideration. In Sec. III, various ground state correlation functions for the model are calculated, and their exponents are discussed. Section IV summarizes the obtained results.

II. THE MODEL

In this section, we give an introduction to the model under consideration, and establish the notation that will be used in the rest of the paper. For a more detailed discussion we refer the reader to Ref. 12.

A. The kinetic energy. Mapping to chains

We consider a 2D electron system on a square lattice with lattice constant a and N lattice points in each direction (although we will not indicate it explicitly, the thermodynamic limit $N \rightarrow \infty$ will always be implied at the end of our calculations). The kinetic-energy operator is

$$H_{\text{kin}} = -t \sum_{\langle ij \rangle s} c_{is}^\dagger c_{js}, \quad (1)$$

where c_{is}^\dagger and c_{is} are creation and annihilation operators for an electron with spin s at site i . The sums over i and j are

restricted to nearest neighbors, where t is the hopping matrix element. We introduce coordinates (x, y) in real space and (k_x, k_y) in reciprocal space, where the coordinate axes are rotated $+45^\circ$ with respect to the primitive vectors connecting nearest-neighbor sites. The kinetic-energy operator can then be written

$$H_{\text{kin}} = -4t \sum_{\vec{k}s} \cos(k_x a / \sqrt{2}) \cos(k_y a / \sqrt{2}) c_{\vec{k}s}^\dagger c_{\vec{k}s}, \quad (2)$$

where the \vec{k} summation is over the first Brillouin zone. For a half-filled band, the Fermi surface is a square, with faces defined by k_x or k_y equal to $\pm k_F$ where $k_F \equiv \pi / \sqrt{2}a$, giving a vanishing Fermi energy.

Since we are interested in the low-energy physics, the kinetic energy is linearized in the direction perpendicular to each of the four faces of the square Fermi surface. Consider the two parallel faces defined by $k_x = \alpha k_F$, $-k_F \leq k_y \leq k_F$, where $\alpha = \pm 1$. These faces will from now on be referred to as the “+” and “-” faces, respectively. The linearized kinetic energy dispersion near face α is

$$E_{\text{kin},\alpha}(\vec{k}) = v_F(k_y)(\alpha k_x - k_F), \quad (3)$$

where $v_F(k_y) = 2\sqrt{2}ta \cos(k_y a / \sqrt{2})$. This form is reminiscent of the dispersion for the Luttinger model, except that in our case, the dispersion is only valid within a finite region ($-k_F \leq k_y \leq k_F$), and the Fermi velocity $v_F(k_y)$ depends on the momentum parallel to the face, vanishing on the edges of the face. We also define the field operator for excitations near face α as

$$\psi_{\alpha s}(x, y) = \frac{1}{N} \sum_{k_x, k_y} e^{ik_x x + ik_y y} c_{\alpha s}(k_x), \quad (4)$$

where the summation over k_y is between $-k_F$ and k_F , and the summation over k_x should be cut off for $\alpha k_x \leq k_F$. Similar considerations and definitions can be made for the two other faces of the square Fermi surface.

We now consider a two-point function like $\langle \psi^\dagger(x, y, t) \psi(0, 0, 0) \rangle$, where the average is taken with respect to the ground state of the kinetic-energy operator. The total field operator is written as a sum of the four face-field operators of the type given in Eq. (4), making the two-point function above a sum of four terms, one from each face. The contribution from face α will be $\langle \psi_\alpha^\dagger(x, y, t) \psi_\alpha(0, 0, 0) \rangle$. It can be shown that the result for this correlator is exactly reproduced by the following mapping of the field operator,

$$\psi_{\alpha s}(x, y) \rightarrow \sqrt{\frac{\pi}{k_F}} \sum_{l'=0}^{2N-1} g(l-l') \psi_{\alpha l' s}(x) \quad (5)$$

with the resulting mapping of the unperturbed Hamiltonian,

$$H_{\text{kin},\alpha} \rightarrow \sum_s \sum_{l,l'} t_{l,l'} \int_0^L dx \psi_{\alpha l s}^\dagger(x) \left(\frac{\alpha}{i} \frac{\partial}{\partial x} - k_F \right) \psi_{\alpha l' s}(x), \quad (6)$$

where

$$t_{l,l'} = \frac{1}{2N} \sum_{n_y=-N/2}^{N/2-1} v_F(k_y) e^{ip_y(l-l')}. \quad (7)$$

Here, the integer $l = \sqrt{2}y/a \in [0, 2N-1]$, $k_y = 2\pi n_y / \sqrt{2}Na$, $p_y \equiv \pi n_y / N$, so that $k_y y = p_y l$. Furthermore,

$$g(l) = \frac{1}{2N} \sum_{n_y = -N/2}^{N/2-1} e^{ip_y l} = \frac{(-i)^l}{2N} \frac{1 - (-1)^l}{1 - e^{i\pi l/N}} \quad (8)$$

satisfying $\sum_{l_1} g(l-l_1)g(l_1-l') = g(l-l')$. Letting $N \rightarrow \infty$ for fixed l , we obtain

$$g(l) = \frac{a}{\sqrt{2}} \int_{-k_F}^{k_F} \frac{dk_y}{2\pi} e^{ik_y y} = \frac{1}{2} \delta_{l,0} + \frac{\sin[\pi l/2]}{\pi l} (1 - \delta_{l,0}). \quad (9)$$

The operators $\psi_{\alpha l s}(x)$ describe right-moving ($\alpha=1$) and left-moving ($\alpha=-1$) electrons living on fictitious one-dimensional chains (labeled by l) of length $L=Na$. In terms of operators creating electrons with definite momenta, these chain-field operators are given by

$$\psi_{\alpha l s}(x) = \frac{1}{\sqrt{L}} \sum_{k_x} e^{ik_x x} c_{\alpha l s}(k_x), \quad (10)$$

where the normalization indicates that x is now taken to be a continuous variable. The discrete wave vectors are given by $k_x = 2\pi n_x / L$, with integer n_x . Note that since $g(l-l')$ is generally nonzero for $l \neq l'$, the mapping (5) is nonlocal.

We take the noninteracting system at half filling with linearized dispersion as our unperturbed problem. Perturbations, in the form of interactions or operators taking the system away from half filling, can in principle be treated within the framework of many-body perturbation theory, where their effect is expressed in terms of integrals over time-ordered multiple-point correlation functions. Using Wick's theorem, such correlation functions can be expressed as sums of products of two-point functions. Since the latter are correctly reproduced by the mapping (5) [with Eq. (6) governing the dynamics], the entire perturbation series will be correctly generated. This, together with the fact that standard 1D bosonization can be applied to the chains, is what makes the mapping useful.

Electron creation and annihilation operators belonging to different faces and/or chains anticommute, by definition. Thus the associated density operators $\rho_{\alpha l s}(n_x) = \sum_{k'_x} c_{\alpha l s}^\dagger(k'_x + k_x) c_{\alpha l s}(k'_x)$ ($:: \dots ::$ means normal-ordering) commute. For the “+/-” faces this is expressed by the commutator

$$[\rho_{\alpha l s}(n_x), \rho_{\alpha' l' s'}(-n'_x)] = -\alpha n_x \delta_{\alpha, \alpha'} \delta_{l, l'} \delta_{s, s'} \delta_{n_x, n'_x}. \quad (11)$$

This is simply the result for the usual single-chain case generalized to the case of many chains, the only effect of the generalization being the introduction of the factor $\delta_{l, l'}$. The Fourier transformed operators

$$\rho_{\alpha s}(\vec{n}) = \sum_{l=0}^{2N-1} \rho_{\alpha l s}(n_x) e^{-ip_y l}, \quad (12)$$

where $\vec{n} = (n_x, n_y)$, then obey the commutation relations

$$[\rho_{\alpha s}(\vec{n}), \rho_{\alpha' s'}(-\vec{n}')] = -2N \alpha n_x \delta_{\alpha \alpha'} \delta_{s s'} \delta_{\vec{n}, \vec{n}'}. \quad (13)$$

It was shown in Ref. 12 that in the presence of interactions, the terms with $l' \neq l$ in Eq. (6) are irrelevant in the renormalization-group sense. Therefore we only keep the diagonal part, which we define as $H_{\text{kin}, \alpha}^d$. This will be a sum of independent chains. Bosonization is performed for each chain, giving

$$H_{\text{kin}, \alpha}^d = \frac{\pi v_0}{2NL} \sum_{n, s} : \rho_{\alpha s}(\vec{n}) \rho_{\alpha s}(-\vec{n}) :, \quad (14)$$

where $v_0 \equiv t_{ll} = 2\sqrt{2}ta/\pi$, and the sum over n_y is between $-N$ and $N-1$ (this will be the summation range for n_y in the rest of the article as well).

Completely analogous considerations can be made for the two faces with $k_y = \pm k_F$, the only difference being that the associated chains will then lie along the y direction, taken to be continuous, while the x direction is kept discrete. Boson density operators for perpendicular faces commute.

B. Interactions. Solution of the bosonic Hamiltonian

The form of the interactions that are included in the bosonic Hamiltonian can be extracted from the Hubbard model interaction, given by $U \sum_i n_{i\uparrow} n_{i\downarrow}$, where U is the on-site repulsion and $n_{i s}$ is the number operator at site i with spin s . Writing the field operator as a sum of the field operators for the four faces of the square Fermi surface, the Hubbard interaction will contain terms that couple perpendicular faces and terms that only couple parallel faces. The former terms are not included in the bosonic Hamiltonian, while from the latter terms, which for the “+/-” faces can be written

$$U \left(\frac{\pi}{k_F} \right)^2 \sum_{x, y} \sum_{\alpha_1 \alpha_2} \sum_{l_1 l_2} \sum_{\alpha_3 \alpha_4} \sum_{l_3 l_4} g(l-l_1) g(l-l_2) g(l-l_3) g(l-l_4) \\ \times : \psi_{\alpha_1 l_1 \uparrow}^\dagger(x) \psi_{\alpha_2 l_2 \uparrow}(x) \psi_{\alpha_3 l_3 \downarrow}^\dagger(x) \psi_{\alpha_4 l_4 \downarrow}(x) :, \quad (15)$$

we retain the bosonic contributions, which have the form of density-density interactions, obtained when $(\alpha_1, l_1) = (\alpha_2, l_2)$ and $(\alpha_3, l_3) = (\alpha_4, l_4)$. (Note that since perpendicular faces are decoupled in the bosonic Hamiltonian, it is from now on sufficient to focus on one of the two sets of parallel faces, and we choose the “+/-” faces.) The bosonic contributions can be written

$$\frac{Ua}{NL} \sum_{\alpha \alpha'} \sum_n f^2(n_y) : \rho_{\alpha \uparrow}(\vec{n}) \rho_{\alpha \downarrow}(-\vec{n}) :, \quad (16)$$

where $f(n_y) = (1 - |n_y|/N)/2$ is the Fourier transform of $g^2(l)$. Introducing charge ($\nu = \rho$) and spin ($\nu = \sigma$) operators as ($\eta_\rho = 1$, $\eta_\sigma = -1$)

$$\sqrt{2} \nu_\alpha(\vec{n}) = \rho_{\alpha \uparrow}(\vec{n}) + \eta_\nu \rho_{\alpha \downarrow}(\vec{n}), \quad (17)$$

with commutation relations ($\nu, \nu' = \rho, \sigma$)

$$[\nu_\alpha(\vec{n}), \nu_{\alpha'}(-\vec{n}')] = -2N \alpha n_x \delta_{\nu \nu'} \delta_{\alpha \alpha'} \delta_{\vec{n}, \vec{n}'}, \quad (18)$$

the most general interaction of form (16) can be written on bosonized form as follows:

$$H_B \equiv \frac{a}{2NL} \sum_{\nu=\rho,\sigma} \sum_{\alpha\alpha'} \sum_{\vec{n}} \times f^2(n_y) \eta_\nu U_{\alpha\alpha'}^{(\nu)}(n_x) : \nu_\alpha(\vec{n}) \nu_{\alpha'}(-\vec{n}) : \quad (19)$$

where we have allowed for different coupling functions $U_{\alpha\alpha'}^{(\nu)}(n_x)$ for the different scattering processes. Hence, this bosonic interaction is a bit more general than that extracted directly from the Hubbard model. In analogy with the 1D case, a cutoff on the momentum transfer in the x direction will be needed in order to get a well-defined solution; this makes the couplings n_x dependent. Symmetry considerations dictate that there can be at most four different coupling functions: $U_{++}^{(\rho)}$, $U_{+-}^{(\rho)}$, $U_{++}^{(\sigma)}$, and $U_{+-}^{(\sigma)}$.

The kinetic energy $H_0 \equiv \sum_{\alpha} H_{\text{kin},\alpha}^d$ can also be written in terms of charge and spin operators,

$$H_0 = \frac{\pi v_0}{2NL} \sum_{\nu=\rho,\sigma} \sum_{\alpha} \sum_{\vec{n}} : \nu_\alpha(\vec{n}) \nu_\alpha(-\vec{n}) :. \quad (20)$$

The Hamiltonian $H_0 + H_B$, which shows spin-charge separation, will be referred to as the bosonic Hamiltonian. It may be diagonalized separately in the spin and charge sectors, using the canonical transformation $\exp(\sum_{\nu} S_{\nu})(H_0 + H_B)\exp(-\sum_{\nu} S_{\nu}) \equiv H_D$, where

$$S_{\nu} = \frac{1}{2N} \sum_{\vec{n}} \frac{\xi_{\nu}(\vec{n})}{n_x} \nu_+(\vec{n}) \nu_-(-\vec{n}), \quad (21)$$

and $\xi_{\nu}(\vec{n})$ is chosen so that the off-diagonal terms in the transformed Hamiltonian H_D vanish. This gives

$$H_D = \frac{\pi}{2NL} \sum_{\nu} \sum_{\alpha} \sum_{\vec{n}} v_{\nu}(\vec{n}) : \nu_\alpha(\vec{n}) \nu_\alpha(-\vec{n}) :, \quad (22)$$

where the parameters $\xi_{\nu}(\vec{n})$ and the renormalized velocities $v_{\nu}(\vec{n})$ are given by

$$\exp[2\xi_{\nu}(\vec{n})] = \sqrt{\frac{1 + a\eta_{\nu}[U_{++}^{(\nu)}(n_x) + U_{+-}^{(\nu)}(n_x)]f^2(n_y)/\pi v_0}{1 + a\eta_{\nu}[U_{++}^{(\nu)}(n_x) - U_{+-}^{(\nu)}(n_x)]f^2(n_y)/\pi v_0}}, \quad (23)$$

$$\frac{v_{\nu}(\vec{n})}{v_0} = \sqrt{(1 + a\eta_{\nu}U_{++}^{(\nu)}(n_x)f^2(n_y)/\pi v_0)^2 - (a\eta_{\nu}U_{+-}^{(\nu)}(n_x)f^2(n_y)/\pi v_0)^2}. \quad (24)$$

Terms in the microscopic Hamiltonian that are not of the type included in H_D will, if they are irrelevant, only lead to modifications in the numerical values of the coupling functions, leaving the structure of the low-energy theory unaffected. In this case, it is these ‘‘effective’’ function values that should enter in the formulas above. As already pointed out below Eq. (13), the nondiagonal terms (i.e., $l \neq l'$) in the kinetic energy (6) are irrelevant perturbations with respect to H_D .¹² We emphasize that for the case of the 2D repulsive Hubbard model, Umklapp scattering, which is of nonbosonic form, is a relevant interaction.¹² The general philosophy of the approach presented here, is to view H_D as a zero-order Hamiltonian for a perturbation treatment of the relevant nonbosonic interactions generated by the underlying microscopic lattice Hamiltonian (which may be different from the Hubbard model).

From the high- T_c point of view, the interesting case is doping away from half filling. In Ref. 12, it was shown that for sufficiently strong interactions, the only effect of the operator causing deviations from half filling was to shift the value of the Fermi wave vector, not changing the square shape of the Fermi surface. In the remainder of the paper, we will assume that we are in this strong-interaction regime with a square Fermi surface.

III. CORRELATION FUNCTIONS

We now turn to the calculation of correlation functions for various fluctuations of interest in the ground state $|\rangle$ of the bosonic Hamiltonian $H_0 + H_B$. The dominant instability is identified by the correlation function with the slowest space-time decay, thus having a generalized susceptibility with the strongest divergence as $T \rightarrow 0$.

A. Definitions of correlation functions. Calculation of their y dependence

The generic form of the correlation functions we consider is

$$R^>(\vec{r}, t) = -i \langle |O(\vec{r}, t) O^\dagger(0, 0)| \rangle, \quad (25)$$

where $O(\vec{r}, t)$ is the operator for the fluctuation under consideration. We will consider the following fluctuations: $2k_F$ charge-density wave (CDW), $2k_F$ spin-density wave (SDW), s -wave singlet superconductivity (SS), and triplet superconductivity (TS). The SDW operator has 3 spatial components, and the TS operator has 3 components corresponding to the total spin in the z direction. Assuming spin-rotation invariance, the 3 associated correlation functions should be equal, although the expressions will be formally different due to the

abelian bosonization used here.⁴ The definitions of the operators O are simple generalizations of the corresponding 1D definitions:^{21,4}

$$O_{CDW}(\vec{r}) = \sum_s \psi_{+,s}^\dagger(\vec{r}) \psi_{-,s}(\vec{r}), \quad (26)$$

$$O_{SDW,x}(\vec{r}) = \sum_s \psi_{+,s}^\dagger(\vec{r}) \psi_{-,-s}(\vec{r}), \quad (27)$$

$$O_{SDW,z}(\vec{r}) = \sum_s s \psi_{+,s}^\dagger(\vec{r}) \psi_{-,s}(\vec{r}), \quad (28)$$

$$O_{SS}(\vec{r}) = \frac{1}{\sqrt{2}} \sum_s \psi_{+,s}^\dagger(\vec{r}) \psi_{-,-s}^\dagger(\vec{r}), \quad (29)$$

$$O_{TS,+1}(\vec{r}) = \psi_{+,\uparrow}^\dagger(\vec{r}) \psi_{-,\uparrow}^\dagger(\vec{r}). \quad (30)$$

We have included the definitions of two of the components of the SDW operator, since the equality of the associated correlation functions will give a condition for determining the parameter $\xi_\sigma(\vec{n})$.

Introducing the mapping (5), and observing that the bosonic Hamiltonian conserves the number of electrons with a given spin on a given face and a given chain, it is clear that each nonzero term in the correlation functions has the form (discarding prefactors)

$$F_{\beta,s_1s_2}(l_1-l_2,x,t) \equiv \langle |\psi_{+,l_1,s_1}^\dagger(x,t) \psi_{-,l_2,s_2}^\beta(x,t) \times \psi_{-,l_2,s_2}^{-\beta}(0,0) \psi_{+,l_1,s_1}(0,0)| \rangle, \quad (31)$$

where $\beta = +1$ (-1) indicates an annihilation (creation) operator. Here we have anticipated that F is a function only of the product of the spins, and of the separation between the chains. In terms of F , the correlation functions are given by

$$R_{CDW}^>(\vec{r},t) \propto \sum_{l_1=0}^{2N-1} \sum_{l_2=0}^{2N-1} H_l(l_1,l_2) F_{++}(l_1-l_2,x,t), \quad (32)$$

$$R_{SDW,x}^>(\vec{r},t) \propto \sum_{l_1=0}^{2N-1} \sum_{l_2=0}^{2N-1} H_l(l_1,l_2) F_{+-}(l_1-l_2,x,t), \quad (33)$$

$$R_{SDW,z}^>(\vec{r},t) \propto \sum_{l_1=0}^{2N-1} \sum_{l_2=0}^{2N-1} H_l(l_1,l_2) F_{++}(l_1-l_2,x,t), \quad (34)$$

$$R_{SS}^>(\vec{r},t) \propto \sum_{l_1=0}^{2N-1} \sum_{l_2=0}^{2N-1} H_l(l_1,l_2) F_{--}(l_1-l_2,x,t), \quad (35)$$

$$R_{TS,+1}^>(\vec{r},t) \propto \sum_{l_1=0}^{2N-1} \sum_{l_2=0}^{2N-1} H_l(l_1,l_2) F_{-+}(l_1-l_2,x,t), \quad (36)$$

where $H_l(l_1,l_2) \equiv g(-l_1)g(l-l_1)g(-l_2)g(l-l_2)$. It is seen already at this point that in this model, the CDW and SDW correlation functions are equal.

In Eqs. (32)–(36) we first sum over all terms with a fixed value of $l_0 \equiv l_1 - l_2$, and then sum over the appropriate values of l_0 . Using $H_l(l_1,l_2) = H_l(l_2,l_1)$ and $F(l_0,x,t) = F(-l_0,x,t)$ (the latter property is established in the next subsection) we obtain

$$R(\vec{r},t) \propto K(l,0)F(0,x,t) + 2 \sum_{l_0=1}^{2N-1} K(l,l_0)F(l_0,x,t), \quad (37)$$

where we have defined

$$K(l,l_0) = \sum_{l_1=l_0}^{2N-1} H_l(l_1,l_1-l_0). \quad (38)$$

For $l_0 \geq 1$ it is convenient to write $K(l,l_0) = K_+(l,l_0) - K_-(l,l_0)$, where

$$K_+(l,l_0) = \sum_{l_1=0}^{2N-1} H_l(l_1,l_1-l_0), \quad (39)$$

$$K_-(l,l_0) = \sum_{l_1=0}^{l_0-1} H_l(l_1,l_1-l_0). \quad (40)$$

In the calculation of K_+ , the summation over l_1 must be done before the $N \rightarrow \infty$ limit is taken. This is because the expressions (9) are derived by assuming that l is fixed, and therefore finite, so that $l/N \rightarrow 0$ as $N \rightarrow \infty$. They are, therefore, not correct when l is of order N , and such values of l_1 are indeed included in the sum in K_+ .

The calculations of K_+ and K_- are rather lengthy, so we only give the final results ($\bar{\delta}_{l,l'} \equiv 1 - \delta_{l,l'}$):

$$K_+(l,l_0) = \delta_{l,0} \delta_{l_0,0} \frac{1}{12} + \delta_{l,0} \bar{\delta}_{l_0,0} \frac{1}{2\pi^2 l_0^2} + \delta_{l_0,0} \bar{\delta}_{l,0} \frac{1}{2\pi^2 l^2} + \bar{\delta}_{l,0} \delta_{l,l_0} \frac{(-1)^{l+1}}{4\pi^2 l^2}, \quad (41)$$

$$\begin{aligned}
K_-(l, l_0) = & \left[\frac{\sin(\pi l_0/2)}{2\pi l_0} \right]^2 \delta_{l,0} + \frac{\cos^2(\pi l/2) \cos^2(\pi l_0/2)}{\pi^4} \left\{ \bar{\delta}_{l,0} \bar{\delta}_{l_0} \left[\frac{\psi\left(\frac{1+l}{2}\right) - \psi\left(\frac{1+l_0}{2}\right) + \psi\left(\frac{1}{2}\right)}{l_0(l+l_0)(l-l_0)} \right. \right. \\
& - \frac{1}{2ll_0} \left(\frac{\psi\left(\frac{1+l+l_0}{2}\right)}{l+l_0} + \frac{\psi\left(\frac{1+l-l_0}{2}\right)}{l-l_0} \right) \left. \right] + \delta_{l,0} \left[\frac{2\left(\psi\left(\frac{1+l_0}{2}\right) - \psi\left(\frac{1}{2}\right)\right)}{l_0^3} \right. \\
& \left. \left. + \frac{\psi'\left(\frac{1-l_0}{2}\right) - \psi'\left(\frac{1+l_0}{2}\right)}{4l_0^2} \right] + \delta_{l,l_0} \left[\frac{\frac{\pi^2 l}{2} + \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1-2l}{2}\right) - l\psi'\left(\frac{1-l}{2}\right)}{4l^3} \right] \right\}. \quad (42)
\end{aligned}$$

Here, $\psi(z)$ is the digamma function, defined as $\psi(z) = \Gamma'(z)/\Gamma(z)$, where $\Gamma(z)$ is the gamma function.²² For large arguments, $\psi(z) \sim \ln(z)$, which in this context must be regarded as a constant, since it does not contribute a power-law behavior with nonzero exponent. The asymptotic behavior of $K(l, l_0)$ is therefore to leading order given by $K(l, l_0) \propto 1/l^2$.

B. The x and t dependence of the correlation functions

The explicit calculation of F is performed by bosonization of the chain field operators,^{23,3} writing

$$\psi_{\alpha ls}(x, t) = \frac{1}{\sqrt{2\pi\epsilon}} U_{\alpha ls}^\dagger e^{-\alpha\varphi_{\alpha ls}(x, t) + i\alpha k_F x}, \quad (43)$$

where ϵ is a short-distance cutoff, which should be sent to zero at the end of all calculations. In principle, the phase field φ contains both finite modes ($n_x \neq 0$) and zero modes ($n_x = 0$, corresponding to number operators), but the zero modes may be neglected in the thermodynamic limit.⁴ Furthermore, it is easily seen that the ladder operators U do not give rise to any minus signs in the expectation values we consider, because there is no need to reorder the U 's before eliminating them using $U_{\alpha ls} U_{\alpha ls}^\dagger = U_{\alpha ls}^\dagger U_{\alpha ls} = 1$, and they can therefore be neglected. The phase field is $\varphi_{\alpha ls}(x, t) = \sum_\nu \varphi_{\alpha ls}^\nu(x, t)$, where we have defined

$$\varphi_{\alpha ls}^\nu(x, t) = \frac{\kappa_{\nu s}}{2\sqrt{2N}} \sum_n \frac{e^{-\epsilon|k_x|/2}}{n_x} \nu_\alpha(\vec{n}, t) e^{-ik_x x - ip_y t}, \quad (44)$$

where $\kappa_{\rho s} = 1$, $\kappa_{\sigma s} = s$ with $s = 1$ (-1) for spin up (down). The time dependence is given by $\nu_\alpha(\vec{n}, t) = e^{i(H_0 + H_B)t} \nu_\alpha(\vec{n}) e^{-i(H_0 + H_B)t}$. From $\nu_\alpha^\dagger(\vec{n}) = \nu_\alpha(-\vec{n})$, we obtain $\varphi_{\alpha ls}^\dagger(x, t) = -\varphi_{\alpha ls}(x, t)$. This gives

$$\begin{aligned}
F_{\beta, s_1 s_2}(l_1 - l_2, x, t) = & (2\pi\epsilon)^{-2} \exp[-ik_F x(1 + \beta)] \\
& \times \prod_{\nu=\rho, \sigma} \exp[E_{\beta, s_1 s_2}^\nu(l_1 - l_2, x, t)], \quad (45)
\end{aligned}$$

$$\begin{aligned}
& \exp[E_{\beta, s_1 s_2}^\nu(l_1 - l_2, x, t)] \\
& = \langle | e^{\varphi_{+l_1 s_1}^\nu(x, t)} e^{\beta\varphi_{-l_2 s_2}^\nu(x, t)} e^{-\beta\varphi_{-l_2 s_2}^\nu(0,0)} e^{-\varphi_{+l_1 s_1}^\nu(0,0)} \rangle. \quad (46)
\end{aligned}$$

Next, we perform the canonical transformation with the operator (21). Its effect on the spin and charge operators is given by

$$e^{S_\nu} \nu_\alpha(\vec{n}, t) e^{-S_\nu} = \sum_{\lambda=\pm 1} h_\nu^{\alpha\lambda}(\vec{n}) \nu_\lambda(\vec{n}) e^{i\lambda k_x \nu_\nu(\vec{n})t}, \quad (47)$$

where

$$h_\nu^{\alpha, \alpha}(\vec{n}) = \cosh \xi_\nu(\vec{n}), \quad h_\nu^{\alpha, -\alpha}(\vec{n}) = -\sinh \xi_\nu(\vec{n}). \quad (48)$$

The canonical transformation will mix the operators for the “+” and “-” faces in the phase fields:

$$e^{S_\nu} \varphi_{\alpha ls}^\nu(x, t) e^{-S_\nu} = \sum_{\lambda=\pm 1} \tilde{\varphi}_{\alpha ls}^{\nu\lambda}(x, t), \quad (49)$$

where

$$\begin{aligned}
\tilde{\varphi}_{\alpha ls}^{\nu\lambda}(x, t) = & \frac{\kappa_{\nu s}}{2\sqrt{2N}} \\
& \times \sum_n \frac{e^{-\epsilon|k_x|/2}}{n_x} h_\nu^{\alpha\lambda}(\vec{n}) \nu_\lambda(\vec{n}) e^{i\lambda k_x \nu_\nu(\vec{n})t - ik_x x - ip_y t}. \quad (50)
\end{aligned}$$

This gives

$$\begin{aligned}
& \exp[E_{\beta, s_1 s_2}^\nu(l_1 - l_2, x, t)] \\
& = \prod_{\lambda=\pm 1} \langle G | e^{\tilde{\varphi}_{+l_1 s_1}^{\nu\lambda}(x, t)} e^{\beta\tilde{\varphi}_{-l_2 s_2}^{\nu\lambda}(x, t)} e^{-\beta\tilde{\varphi}_{-l_2 s_2}^{\nu\lambda}(0,0)} \\
& \quad \times e^{-\tilde{\varphi}_{+l_1 s_1}^{\nu\lambda}(0,0)} | G \rangle, \quad (51)
\end{aligned}$$

where $|G\rangle \equiv \exp(\sum_\nu S_\nu)|\rangle$ is the ground state of H_D .

The calculation of the expectation value is done by writing the $\tilde{\varphi}_{\alpha ls}^{\nu\lambda}$ fields as a sum of two contributions, one over positive n_x and one over negative n_x . One of the contribu-

tions will contain creation operators, the other will contain annihilation operators. By using the relations

$$e^A e^B = e^{A+B} e^{[A,B]/2}, \quad (52)$$

$$e^A e^B = e^B e^A e^{[A,B]}, \quad (53)$$

we can move the exponentials containing creation operators to the left and the exponentials containing annihilation operators to the right. In doing so, we will generate a string of c-numbers due to the reordering of the exponentials which we can take outside the expectation value brackets. Since the annihilation operators acting to the right and the creation operators acting to the left annihilate the ground state, the expectation value of the operators which are left inside the expectation value brackets is just 1.

First, the phase fields are split into the two different contributions,

$$\tilde{\varphi}_{als}^{\nu\lambda}(x,t) = \tilde{\varphi}_{als}^{\nu\lambda,+}(x,t) + \tilde{\varphi}_{als}^{\nu\lambda,-}(x,t), \quad (54)$$

where the superscripts + and - denote creation and annihilation, respectively, and

$$\begin{aligned} \tilde{\varphi}_{als}^{\nu\lambda,\pm}(x,t) &= \pm \frac{\lambda \kappa_{\nu s}}{2\sqrt{2N}} \sum_n' \frac{e^{-\epsilon k_x/2}}{n_x} h_v^{\alpha\lambda}(\vec{n}) \nu_\lambda(\pm \lambda \vec{n}) \\ &\times e^{\pm [ik_x v_\nu(\vec{n})t - i\lambda k_x x - i\lambda p_y t]}, \end{aligned} \quad (55)$$

where we also used the evenness of $\xi_\nu(\vec{n})$, and where the prime after the summation sign indicates that only positive n_x should be summed over. Then, we use Eq. (52) to write each exponential in Eq. (51) as a product of exponentials of creation and annihilation parts of the phase field. For this, and further calculations, we will need the commutator

$$\begin{aligned} [\tilde{\varphi}_{als}^{\nu\lambda,+}(x,t), \tilde{\varphi}_{a'l's'}^{\nu\lambda,-}(x',t')] &= \frac{\kappa_{\nu s} \kappa_{\nu s'}}{4N} \sum_n' \frac{e^{-\epsilon k_x}}{n_x} h_v^{\alpha\lambda}(\vec{n}) h_v^{\alpha'\lambda}(\vec{n}) \\ &\times e^{ik_x v_\nu(\vec{n})(t-t') - i\lambda [k_x(x-x') + p_y(t-t')]}. \end{aligned} \quad (56)$$

Using Eq. (52) on all exponentials in Eq. (51) we generate the factor $\exp[C^\nu]$ where

$$C^\nu = -\frac{1}{2N} \sum_n' \frac{e^{-\epsilon k_x}}{n_x} \cosh 2\xi_\nu(\vec{n}), \quad (57)$$

where we also used $\kappa_{\nu s}^2 = 1$. Using Eq. (53) to move all annihilation operators to the right in Eq. (51), we generate the factor $\exp[D_{\beta, s_1 s_2}^{\nu\lambda}(l_1 - l_2, x, t)]$, where

$$\begin{aligned} D_{\beta, s_1 s_2}^{\nu\lambda}(l_1 - l_2, x, t) &= \frac{1}{4N} \sum_n' \frac{e^{-\epsilon k_x}}{n_x} [\beta \kappa_{\nu s_1} \kappa_{\nu s_2} \sinh 2\xi_\nu(\vec{n}) \\ &\times (1 - e^{-ik_x v_\nu(\vec{n})t + i\lambda k_x x}) \cos p_y(l_1 - l_2) \\ &+ \cosh 2\xi_\nu(\vec{n}) e^{-ik_x v_\nu(\vec{n})t + i\lambda k_x x}]. \end{aligned} \quad (58)$$

The total exponent is

$$\begin{aligned} E_{\beta, s_1 s_2}^\nu(l_1 - l_2, x, t) &= C^\nu + \sum_{\lambda=\pm 1} D_{\beta, s_1 s_2}^{\nu\lambda}(l_1 - l_2, x, t) \\ &= -\frac{1}{\pi} \int_0^\pi dp_y \int_0^\infty dk_x [\cosh 2\xi_\nu(\vec{n}) \\ &\quad - \beta \kappa_{\nu s_1} \kappa_{\nu s_2} \sinh 2\xi_\nu(\vec{n}) \cos p_y(l_1 - l_2)] \\ &\quad \times \frac{e^{-\epsilon k_x}}{k_x} (1 - e^{-ik_x v_\nu(\vec{n})t} \cos k_x x). \end{aligned} \quad (59)$$

A momentum transfer cutoff must be imposed on the interactions. For fixed n_y , the treatment is as for the 1D case. In the equation above, we express the integral as a sum of a noninteracting and an interacting part, by writing $[\dots] = 1 + ([\dots] - 1)$. In both parts, we then multiply the integrand by $[e^{-\Lambda_\nu(n_y)k_x} + (1 - e^{-\Lambda_\nu(n_y)k_x})]$ to obtain two terms for each of the two parts. [For simplicity, we take the cutoff function $\Lambda_\nu(n_y)$ to be the same for both parts; since the noninteracting part is independent of β , $s_1 s_2$ and $l_1 - l_2$, this implies that the cutoff function be independent of these variables.] In the first term, the main contributions will come from $k_x \ll 1/\Lambda_\nu(n_y)$, where we can replace $\xi_\nu(\vec{n}) \rightarrow \xi_\nu(n_x = 0, n_y) \equiv \xi_\nu(n_y)$ and $v_\nu(\vec{n}) \rightarrow v_\nu(n_x = 0, n_y) \equiv v_\nu(n_y)$. In the second term, which can be neglected for the interacting part, the main contributions come from $k_x \gg 1/\Lambda_\nu(n_y)$, so we can replace $v_\nu(\vec{n}) \rightarrow v_0$ there. This gives

$$\begin{aligned} E_{\beta, s_1 s_2}^\nu(l_1 - l_2, x, t) &= -\frac{1}{2} \ln \left[\frac{(\epsilon + i v_0 t)^2 + x^2}{\epsilon^2} \right] \\ &\quad - \frac{1}{2\pi} \int_0^\pi dp_y \left\{ \ln \left[\frac{[\Lambda_\nu(n_y) + i v_\nu(n_y)t]^2 + x^2}{[\Lambda_\nu(n_y) + i v_0 t]^2 + x^2} \right] \right. \\ &\quad \left. + A_{\beta, s_1 s_2}^\nu(n_y, l_1 - l_2) \right. \\ &\quad \left. \times \ln \left[\frac{[\Lambda_\nu(n_y) + i v_\nu(n_y)t]^2 + x^2}{\Lambda_\nu^2(n_y)} \right] \right\}, \end{aligned} \quad (60)$$

where we have defined

$$\begin{aligned} A_{\beta, s_1 s_2}^\nu(n_y, l_1 - l_2) &= -1 + \cosh 2\xi_\nu(n_y) \\ &\quad - \beta \kappa_{\nu s_1} \kappa_{\nu s_2} \sinh 2\xi_\nu(n_y) \\ &\quad \times \cos p_y(l_1 - l_2). \end{aligned} \quad (61)$$

Introducing n_y -averaged velocities \bar{v}_ν and cutoffs $\bar{\Lambda}_\nu$, and adding and subtracting terms in the integrand, we may write

$$\begin{aligned}
\prod_{\nu=\rho,\sigma} \exp[E_{\beta,s_1s_2}^\nu(l_1-l_2,x,t)] &= \frac{\epsilon^2}{(\epsilon+i\nu_0t)^2+x^2} \prod_{\nu=\rho,\sigma} \left[\frac{(\bar{\Lambda}_\nu+i\nu_0t)^2+x^2}{(\bar{\Lambda}_\nu+i\bar{\nu}_\nu t)^2+x^2} \right]^{1/2} \left[\frac{\bar{\Lambda}_\nu^2}{(\bar{\Lambda}_\nu+i\bar{\nu}_\nu t)^2+x^2} \right]^{d_{\beta,s_1s_2}^\nu(l_1-l_2)} \\
&\times \exp \left[-\frac{1}{2\pi} \int_0^\pi dp_y \left\{ \ln \left(\frac{[\Lambda_\nu(n_y)+i\nu_\nu(n_y)t]^2+x^2}{[\Lambda_\nu(n_y)+i\nu_0t]^2+x^2} \right) / \left[\frac{(\bar{\Lambda}_\nu+i\bar{\nu}_\nu t)^2+x^2}{(\bar{\Lambda}_\nu+i\nu_0t)^2+x^2} \right] \right\} \right. \\
&\left. + A_{\beta,s_1s_2}^\nu(n_y,l_1-l_2) \ln \left(\frac{[\Lambda_\nu(n_y)+i\nu_\nu(n_y)t]^2+x^2}{\Lambda_\nu^2(n_y)} / \left[\frac{(\bar{\Lambda}_\nu+i\bar{\nu}_\nu t)^2+x^2}{\bar{\Lambda}_\nu^2} \right] \right) \right], \quad (62)
\end{aligned}$$

where we have defined the charge and spin exponents

$$d_{\beta,s_1s_2}^\nu(l_1-l_2) = \frac{1}{2\pi} \int_0^\pi dp_y A_{\beta,s_1s_2}^\nu(n_y, l_1-l_2). \quad (63)$$

Summing up the results so far, the correlation functions (32)–(36) are given by Eqs. (37)–(42), (45), and (61)–(63).

C. Correlation function exponents

From Eqs. (33) and (34) we see that the correlation functions for the x and z components of the SDW operator differ only in the sign of s_1s_2 . Assuming spin-rotation invariance of the underlying model, these correlation functions must be identical. From Eq. (61) it is seen that this is obtained if $\xi_\sigma(n_y)=0$, which gives $A_{\beta,s_1s_2}^\sigma(n_y, l_0) = d_{\beta,s_1s_2}^\sigma(l_0) = 0$. Consequently, the correlation functions become independent of s_1s_2 , and the s_1s_2 subscript may therefore be omitted from now on. The spin-rotation invariance also has the effect of making the SS and TS correlation functions identical. As for the 1D case,^{24,25} a more sophisticated (renormalization-group) treatment is needed to lift the degeneracy between CDW and SDW fluctuations, and between SS and TS fluctuations.

For $t=0$ it is seen that the factor $\exp[\dots]$ on the rhs in Eq. (62) is unity for $\nu=\sigma$, and for $\nu=\rho$ its leading behavior for $x \rightarrow \infty$ is independent of x . Thus, the exponent of the leading x dependence of Eq. (62) is $-2[1+d_\beta^\rho(l_0)]$. For asymptotic values of t the analysis is not so straightforward, since both the square roots and the factors $\exp[\dots]$ will depend on x and t to leading order. However, as long as all velocities involved are nonzero (which we assume to be the case), it seems reasonable to neglect this additional dependence, so that the following leading-order asymptotic approximation may be used:

$$F_\beta(l_0, x, t) \propto \frac{e^{-ik_F x(1+\beta)}}{(\epsilon+i\nu_0t)^2+x^2} \left[\frac{\bar{\Lambda}_\rho^2}{(\bar{\Lambda}_\rho+i\bar{\nu}_\rho t)^2+x^2} \right]^{d_\beta^\rho(l_0)}. \quad (64)$$

We see that $F_\beta(l_0, x, t)$ has a form that resembles that of the correlation functions of the 1D Luttinger model. However, the exponents $d_\beta^\rho(l_0)$ depend on the chain difference l_0 , and in general have a rather different form than the Luttinger-model exponents. The exceptions are the exponents for the equal-chain terms $l_0=0$. We have, in our notation,²⁶

$$\text{CDW/SDW: } d_{\beta=+1}^\rho = -\frac{1}{2}(1-e^{-2\xi_\rho}) \quad (1D),$$

$$d_{\beta=+1}^\rho(l_0=0) = -\frac{1}{2\pi} \int_0^\pi dp_y (1-e^{-2\xi_\rho(n_y)}) \quad (2D)$$

$$\text{SS/TS: } d_{\beta=-1}^\rho = -\frac{1}{2}(1-e^{+2\xi_\rho}) \quad (1D),$$

$$d_{\beta=-1}^\rho(l_0=0) = -\frac{1}{2\pi} \int_0^\pi dp_y (1-e^{+2\xi_\rho(n_y)}) \quad (2D) \quad (65)$$

It is seen that the form of these exponents is very similar in the 1D and 2D case, the only difference being the averaging over p_y in the 2D expressions. If this had been the set of exponents that determined the leading behavior of our correlation functions, then the analogy to the Luttinger model would have been very strong indeed. However, things are not quite that simple. Each correlation function is given as a sum over contributions from different l_0 . Since for all l_0 , $K(l, l_0) \propto 1/l^2$ for asymptotically large l , the asymptotic behavior of the various contributions differ only in their $x-t$ dependence. Thus, for a given correlation function (i.e., a given value of β) the leading asymptotic behavior comes from the value of l_0 that gives the smallest $d_\beta^\rho(l_0)$, i.e., we must minimize

$$-\frac{\beta}{2\pi} \int_0^\pi dp_y \sinh 2\xi_\rho(n_y) \cos p_y l_0 \quad (66)$$

with respect to the integer l_0 . Assuming repulsive effective interactions [i.e., $\xi_\rho(n_y) > 0$], it is indeed true that $d_1^\rho(l_0)$ is minimal for the term with $l_0=0$, which therefore gives the slowest decay in the CDW/SDW correlation functions. However, for $d_{-1}^\rho(l_0)$ the term $l_0=0$ actually gives the *largest* exponent; the minimum exponent for the SS/TS correlation functions comes from some nonzero l_0 , and is therefore not of the form in Eq. (65). Consequently, the relation between the leading exponents for the CDW/SDW and SS/TS correlation functions is not as for the 1D Luttinger model, where these exponents are related through simple scaling relations. Since $d_1^\rho(0) < d_{-1}^\rho(l_0)$, regardless of l_0 , we conclude that the CDW/SDW fluctuations are dominant. This is the same conclusion as for the Luttinger model for repulsive interactions, although the detailed nature of the leading exponents is different.

Throughout this paper we have assumed that we are in the region of parameter space where the square shape of the Fermi surface is stable with respect to doping away from half filling. This requires, when spin-rotation invariance is invoked, that¹²

$$\frac{1}{\pi} \int_0^\pi \sinh^2 \xi_\rho(n_y) (1 - \cos p_y l_0) > 2, \quad \text{for all } l_0 \neq 0 \quad (67)$$

[since $l_0 = 1$ then minimizes the expression on the lhs of Eq. (67), an equivalent requirement is that this expression be > 2 for $l_0 = 1$]. Let us define $U = U_{++}^{(\rho)}(0) a / 4\pi v_0$ and $\gamma = U_{+-}^{(\rho)}(0) / U_{++}^{(\rho)}(0)$. Having repulsive interactions requires $U \geq 0$, $\gamma \geq 0$. Furthermore, Eqs. (23) and (24) are well defined and real only when $\gamma < 1 + 1/U$. In order to have Eq. (67) satisfied, there is a lower bound on γ , which is easily found by assuming $\gamma < 1$, and noting that for fixed γ , $\sinh^2 \xi_\rho(n_y)$ is maximal when $U \rightarrow \infty$, giving

$$\sinh^2 \xi_\rho(n_y) \approx \frac{1}{2} \left[\frac{1}{\sqrt{1 - \gamma^2}} - 1 \right]. \quad (68)$$

The condition (67) then requires $\gamma > \gamma_{\min} = 2\sqrt{6}/5 \approx 0.9798$. There is also a γ -dependent lower bound on U . The lowest such bound is found numerically to occur at the parameter space boundary, for $U \approx 376$, corresponding to $\gamma = 1 + 1/U \approx 1.00266$. As γ is decreased from this value towards γ_{\min} , the lower bound on U is found numerically to increase towards infinity. A lower bound $U \approx 376$ corresponds to $U_{++}^{(\rho)}(0) a / \pi v_0 \approx 1500$, an extremely high value. We do not know how large the bare couplings must be in order to renormalize to such high effective values. It may be that inclusion of scattering between orthogonal faces could reduce the effective values needed, but that is not clear. In the region of parameter space where Eq. (67) is satisfied, we find that $l_0 = 2$ minimizes $d_{-1}^\rho(l_0)$ if $\gamma < 1$ [the expression (66) is then negative], and $l_0 = \infty$ minimizes $d_{-1}^\rho(l_0)$ if $\gamma > 1$ [Eq. (66) is then 0]. We do not have any physical explanation for why these particular values of l_0 minimize the SS/TS exponent in each particular case.

IV. SUMMARY AND CONCLUDING DISCUSSION

We have evaluated various ground-state correlation functions for a 2D bosonic Hamiltonian with spin-charge separation, introduced in Ref. 12. The model was arrived at by mapping the problem of 2D electrons on a square lattice with nearest-neighbor hopping and Hubbard-like interactions on a square Fermi surface onto two orthogonal sets of 1D chains. In the presence of interactions, it was shown in Ref. 12 that

single-particle hopping between parallel chains was irrelevant; furthermore, for sufficiently strong interactions, the square shape of the Fermi surface was preserved even away from half filling. The bosonic Hamiltonian resulted from neglecting interactions between orthogonal chains and parallel-chain interactions of nonbosonic form.

In the mapping used, the physical 2D field operator is written as a sum of field operators on the 1D chains. It is the 2D field operators that enter in the definitions of the correlation functions. As a consequence, the resulting correlation function is a sum of many terms, each of which takes the form of a Luttinger-model correlation function in the time direction and the spatial direction parallel to the chains. The chain indices enter in the prefactors and in the exponents of the Luttinger-model correlation functions. In order to find the leading behavior of the correlation function for large times and distances, one must identify the term with the smallest decay exponent by minimizing the exponent with respect to the chain separation. The leading exponents were found to differ in form from the Luttinger model exponents due to the 2D nature of the problem. Specifically, the simple scaling relations valid for the Luttinger-model exponents were not valid for our model.

The fluctuations we considered were $2k_F$ charge-density wave and spin-density wave, s -wave singlet superconductivity, and triplet superconductivity. The condition of spin-rotation invariance fixed the spin part of the correlation functions, making only the charge part non-trivial. The CDW and SDW correlation functions turned out to be degenerate, and so did the SS and TS correlation functions. As for the Luttinger model, the CDW/SDW fluctuations were dominant for effective repulsive interactions.

One may ask whether our conclusion concerning the absence of divergent superconducting fluctuations for repulsive interactions holds for pairing in *any* symmetry channel. In view of the Kohn-Luttinger effect,²⁷ one would expect to see $T=0$ superconductivity in some, albeit possibly very high, angular momentum channel. It may be that possible superconductivity is either hiding in higher spin-triplet channels, or in higher singlet-channels, not considered in this paper. Since $O_{SS}(\vec{r})$ is a local object, it does not contain a $d_{x^2-y^2}$ component, and consequently our results do not rule out the possibility of a superconductive instability in this channel.

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