Angular-dependent torque magnetometry on single-crystal $HgBa_2CuO_{4+y}$ near the critical temperature

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Angular-dependent torque magnetization measurements on a $HgBa_2CuO_{4+y}$ microcrystal performed close to the critical temperature T_c are analyzed within the framework of three-dimensional (3D) XY critical behavior. In a finite magnetic field the magnetic torque is given by the derivative of a universal scaling function $dG(z)/dz$, where the scaling argument z depends on temperature, field, and the angle between the field and the c axis of the sample. We are able to determine dG/dz quantitatively by scaling the angular-dependent torque data. Universal constants and critical amplitudes for $HgBa_2CuO_{4+y}$ are calculated. [S0163-1829(99)03425-6]

I. INTRODUCTION

Reversible magnetic torque measurements on high-temperature superconductors performed at temperatures below T_c have been interpreted in terms of the three-dimensional (3D) anisotropic London model, where the magnetization M depends logarithmically on the applied field B_a (Refs. 1,2). On this basis angular-dependent measurements yield information on crucial superconducting parameters, such as the effective mass anisotropy $\gamma=\sqrt{m_c^*/m_{ab}^*}$ (Refs. 2,3). However, the London model cannot explain magnetic measurements on cuprates in the temperature regime where a 2D ''crossing point'' phenomenon (M^*,T^*) occurs,[4,5] and at temperatures near T_c. In order to explain the existence of the ''crossing point,'' where $M=M^*$ is independent of B_a, (2D) vortex fluctuations have been included in the London model.[6] The ratio M^*/T^* is predicted to be inversely proportional to the interlayer distance s. This is in contradiction to experiments, which show that M^*/T^* does not depend on s (Ref. 7). In torque measurements an unusual angular dependence has been observed in $La_{2-x}Sr_xCuO_4$ (Refs. 8,9) and in $HgBa_2CuO_{4+y}$ (Ref. 10) in the vicinity of T_c. Similar angular-dependent curves have been obtained by taking Gaussian fluctuations into account,[11] though in a more narrow temperature regime than observed by experiment.

In this work we will use a different approach to describe the observed deviations from the 3D anisotropic London model. Since cuprates have a high critical temperature T_c, a short coherence length $\xi_{ab}(0)$, and a large effective mass anisotropy γ, the temperature regime around T_c where critical fluctuations dominate is large compared to the fluctuation regime in conventional superconductors.[12,13] In optimally doped $YBa_2Cu_3O_{7-y}$ single crystals the temperature dependence of the in-plane penetration depth follows the 3D XY behavior down to temperatures 10 K below T_c (Ref. 14). Thermal expansivity experiments on the same compound showed the presence of critical fluctuations within the same temperature region.[15] For more anisotropic materials this temperature region should be even larger. Therefore, the deviations from the London model mentioned above should lie in the temperature regime, where critical fluctuation theory can be applied. In the framework of this theory the existence of the ''crossing point'' follows from 2D scaling behavior, which applies when the coherence length perpendicular to the CuO_2 planes $\xi_c(T)=\xi_{ab}(T)/\gamma$ is getting smaller than the interlayer distance s. The system can then be regarded as a stack of 2D superconducting slabs with effective thickness $d_s<s$ (Ref. 16). Within fluctuation theory M^* does not depend on s, but on d_s. The value of d_s is not known a priori but seems to be rather material independent.[7,16]

Although cuprates have a pronounced anisotropy, they still have a finite coupling between adjacent layers. Therefore, 2D effects can only be observed in fields that are larger than a crossover field. Below this field the stack of the 2D slabs behaves as a 3D system due to correlations along the c axis between the slabs. The crossover field is of the order of 1 T for field orientations along the c axis, but it rises up to about 100 T for orientations in the ab plane.[16] Therefore, 2D behavior is not observable for field orientations close to the ab plane, unless in very high fields. In fact, no ''crossing point'' could be observed in the most anisotropic cuprate BSCCO with the field applied in the ab plane.[5] In the same material a crossover from 2D to 3D behavior upon rotating the magnetic field from the c axis towards the ab plane was observed at low temperatures.[17] Since torque measurements are dominated by orientations close to the ab plane where the torque signal is large, angular-dependent torque measurements will mainly reveal 3D behavior, whereas 2D features showing up for field orientations close to the c axis are strongly suppressed. Therefore, we will analyze the observed torque curves near T_c in the framework of 3D fluctuation

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theory. On the other hand, at lower temperatures where 3D fluctuations freeze out, the 3D London model[1,2] applies for angular-dependent torque measurements in moderate fields for a rather large angular regime around the ab plane.

When critical fluctuations dominate, in an applied magnetic field $\vec{B}_a$ the magnetic torque along the a axis acting on a sample with volume V is given by the scaling expression[18]

$$\tau = \frac{V Q_1^{\pm} k_B T B_a \gamma}{2\Phi_0 \xi_{a,0}^{\pm} |T/T_c - 1|^{-\nu}} \left(1 - \frac{1}{\gamma^2}\right) \frac{\sin(2\delta)}{\epsilon(\delta)} \frac{dG^{\pm}(z)}{dz}, \tag{1}$$

if $\vec{B}_a$ is confined to the bc-plane of the sample. $\xi_{a,0}^{\pm}$ is the critical amplitude of the correlation length along the a direction, diverging as $\xi_a^{\pm}(T) = \xi_{a,0}^{\pm}|T/T_c - 1|^{-\nu}$, and $\pm = \mathrm{sgn}(T/T_c - 1)$. Several experiments give evidence that the zero-field transition of cuprates is a critical point belonging to the 3D XY universality class (see, e.g., Refs. 12,14,15,19–22). We therefore use critical exponents and universal constants belonging to this universality class, where $\nu \simeq 2/3$. We assume the system to be isotropic in the CuO_2 plane ($m_a^* = m_b^*$), which is reasonable in the mercury compounds due to their tetragonal structure. $Q_1^{\pm}$ is an universal constant, and the scaling function $G^{\pm}(z)$ depends on the dimensionless variable

$$z = \frac{[\xi_a^{\pm}(T)]^2 B_a}{\Phi_0} \epsilon(\delta), \tag{2}$$

where δ is the angle between $\vec{B}_a$ and the c axis of the sample, and $\epsilon(\delta) = (1/\gamma^2 \sin^2\delta + \cos^2\delta)^{1/2}$.

The calculation of the scaling function was not possible up to now. Recently we derived its functional form in the limits $z \to 0^-$ ($T < T_c$, $B_a \to 0$), $z \to 0^+$($T > T_c$, $B_a \to 0$), and $z \to \infty$ ($B_a \neq 0$, $T \to T_c$) from expressions (6), (5), and (4) (describing the magnetic penetration depth, the magnetic suscebtibility, and the magnetization in the corresponding regimes) given in Ref. 18. The expression for the magnetic torque then reads as

$$\lim_{z \to 0^-} \tau = \frac{V Q_1^- C_{2,0}^- k_B T B_a \gamma}{2\Phi_0 \xi_{a,0}^- |T/T_c - 1|^{-\nu}} \left(1 - \frac{1}{\gamma^2}\right) \frac{\sin(2\delta)}{\epsilon(\delta)} \ln(z) \tag{3}$$

and

$$\lim_{z \to 0^+} \tau = \frac{V Q_1^+ C_0^+ k_B T \xi_{a,0}^+ |T/T_c - 1|^{-\nu} B_a^2 \gamma}{2\Phi_0^2} \left(1 - \frac{1}{\gamma^2}\right) \sin(2\delta), \tag{4}$$

below and above T_c, respectively. These limits should be observable in fields of about 1 T if one is not to close to T_c. At the critical temperature one finds

$$\lim_{z \to \infty} \tau = \frac{V Q_1^{\pm} C_\infty^{\pm} k_B T_c B_a^{3/2} \gamma}{2\Phi_0^{3/2}} \left(1 - \frac{1}{\gamma^2}\right) \frac{\sin(2\delta)}{[\epsilon(\delta)]^{1/2}}. \tag{5}$$

This limit is very hard to reach by experiment due to the finite width of the superconducting transition in high-T_c compounds.

Equation (3) shows the same angular dependence as the expression derived from the 3D anisotropic London model by Kogan *et al.*[1,2] For temperatures and fields where Eq. (3) is valid, the torque data is therefore equally well described by both expressions. The agreement is nicely seen if one writes the prefactor of Eq. (3) in terms of the in-plane penetration depth $\lambda_a(T)$ using the universal relation $k_B T_c = \Phi_0^2 \xi_a^-(T)/[4\pi^2 \mu_0 \lambda_a^2(T)\gamma]$. The agreement between Eq. (3) and the Kogan expression is comprehensible, since the free energy is calculated in Ref. 1 under the assumption that vortex cores do not overlap ($B_a \ll B_{c2}$), which is fulfilled in the low vortex density limit $B_a \to 0$ ($z \to 0$). However, the absolute values of $\lambda_a(T)$ and $\xi_a^-(T)$ that one can extract from the data using these expressions will be different.

With the three limits leading to Eqs. (3)–(5) one gets a rough overview on the z dependence of the derivative of the scaling function dG/dz (Ref. 18). In this work we will derive dG/dz in a quantitative way from the measured torque curves by scaling them according to Eq. (1), rewritten in the form

$$Q_1^{\pm} \frac{dG^{\pm}(z)}{dz} = \frac{\tau(z) 2\Phi_0 \xi_{a,0}^{\pm} |T/T_c - 1|^{-\nu}}{V k_B T B_a \gamma (1 - 1/\gamma^2)} \frac{\epsilon(\delta)}{\sin(2\delta)}, \tag{6}$$

using $\nu = 2/3$. Even for fixed values of temperature and magnetic field, the z dependence of dG/dz can be probed over a rather large interval of the argument, due to the angular dependence of the scaling variable z [Eq. (2)].

II. EXPERIMENTAL DETAILS AND RESULTS

The single crystal used in this work was grown using a high pressure growth technique[23] and annealed in 1.4 bar O_2 at 250 °C for 100 h. This procedure resulted in an almost optimally doped sample showing a superconducting onset at $T_c \simeq 95.6$ K with a transition width (10–90 % in $B_a = 1$ mT) of $\simeq 1.5$ K. The microcrystal (mass $\simeq 9$ μg) was mounted on a cantilever with piezoresistive readout between the poles of a conventional NMR electromagnet.[24] The field strength was kept constant and the magnetic torque τ was recorded as a function of the angle δ for increasing (τ_+) and decreasing angles (τ_-). The reversible torque was calculated using $\tau_{\mathrm{rev}}(\delta) = [\tau_+(\delta) + \tau_-(\delta)]/2$. Such rotation measurements were performed for different temperatures between 87 and 119 K. Below T_c, field-dependent measurements were performed at a fixed angle $\delta = 45°$ in order to obtain the irreversibility field $B_{\mathrm{irrev}}(T)$. The Meissner slope was determined at different temperatures in order to calibrate the torquemeter.[10] The background signal of the cantilever was minimized by compensation with a second, almost identical lever placed near the sample.[24] A small remaining cantilever background signal (amplitude $\simeq 8 \times 10^{-13}$ Nm) was subtracted from all measurements.

The amplitude $A = [\tau_{\max} - \tau_{\min}]/2$ of the angular-dependent torque measured in a field of 1.4 T is shown in

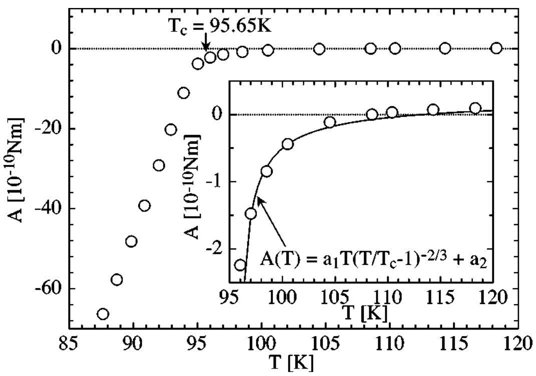

FIG. 1. Amplitude A of the torque signal as a function of temperature. $|A|$ is monotonically decreasing with increasing temperature up to $T \simeq 108.5$ K. For $T > 108.5$ K the paramagnetic background of the sample dominates. Inset: $A(T)$ for temperatures above T_c. The solid line corresponds to the temperature dependence predicted by Eq. (4) (see text).

Fig. 1 as a function of temperature. $|A|$ is monotonically decreasing with increasing temperature up to $T \simeq 108.5$ K. At this temperature a sign change of the amplitude is observed. This sign change, as well as the strong temperature dependence of A above T_c cannot be understood without taking superconducting fluctuations into account. For temperatures $T > 108.5$ K the paramagnetic background of the sample is larger than the diamagnetic signal arising from these fluctuations. Below 108.5 K the fluctuation signal dominates, increasing rapidly upon approaching T_c. The paramagnetic background can be estimated from the temperature dependence of the amplitude for $T > T_c$, where a $\sin(2\delta)$ signal is observed, which is slightly distorted for temperatures very near T_c (see also Ref. 9). The temperature dependence of A for $T > T_c$ is shown enlarged in the inset of Fig. 1. As predicted by Eq. (4), $|A|$ is increasing as the temperature approaches T_c from above. Assuming that the paramagnetic background is temperature independent in the range 97 K $< T <$ 105 K, according to Eq. (4), the temperature dependence of the $\sin(2\delta)$ amplitude is expected to be $A(T) = a_1 T(T/T_c - 1)^{-\nu} + a_2$ with $\nu = 2/3$. This dependence is shown in the inset of Fig. 1 as a solid line with a paramagnetic background $a_2 = 4 \times 10^{-11}$ Nm. Only data in the temperature regime 97 K $< T <$ 105 K, where Eq. (4) should be valid, are considered in order to determine a_1 and a_2. The paramagnetic background is less than 1% of the total signal amplitude at temperatures $T < T_c - 5$ K, but it is roughly of the same order of magnitude for temperatures above T_c. In what follows, this contribution was subtracted from the measured data.

The universal constant $Q_1^+ C_0^+$ entering Eq. (4) can be calculated from a_1, if the critical amplitude of the correlation length $\xi_{a,0}^+$ and the effective mass anisotropy γ are known. These quantities can be derived from the data taken at $T = 90.88$ K ($T = 0.95 T_c$) shown in Fig 2(a). Although this temperature is lower than $T^* \simeq 92.4$ K,[25] the applied field is too small for 2D features to show up, unless for angles $\delta \simeq 0°$ where the torque is very small. The observed angular dependence of the magnetic torque is well described by Eq. (3). A least square fit yields $\gamma = 29(1)$ and $\xi_{a,0}^- = 26(2)$ Å [from the scaling argument z, Eq. (2), using $T_c = 95.65$ K].

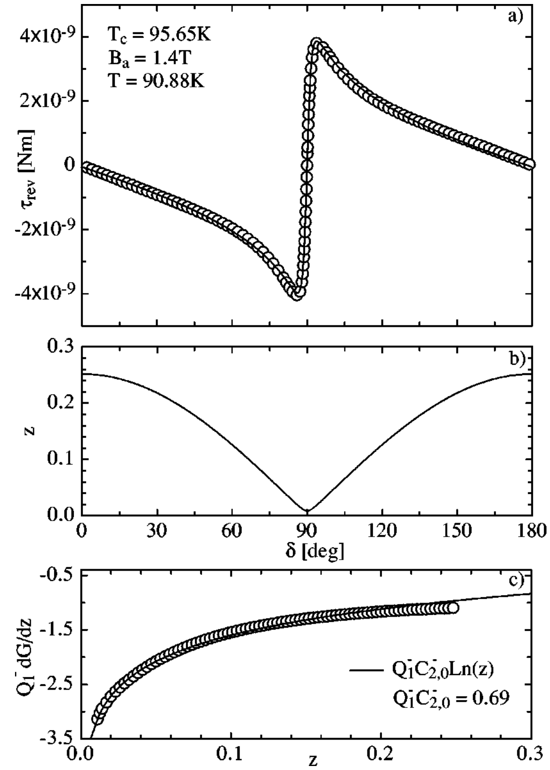

FIG. 2. (a) Angular-dependent torque measurement at $T = 90.88$ K in an applied field $B_a = 1.4$ T. The solid line is a fit to Eq. (3), yielding $\gamma = 29(1)$, $\xi_{a,0}^- = 26(2)$ Å, and $Q_1^- C_{2,0}^- \simeq 0.69$. (b) Scaling variable z as a function of the angle δ. The condition $z \ll 1$ for Eq. (3) to hold is reasonably well fulfilled. (c) $Q_1^- dG/dz$ as a function of z calculated using Eq. (6). The logarithmic dependence (solid line) is quite well fulfilled. For clarity not all measured data points are shown.

For the universal constant we find $Q_1^- C_{2,0}^- \simeq 0.69$. The angular dependence of the scaling argument z is given in Fig. 2(b). It is in the range $0.01 < z < 0.25$, and the condition $z \ll 1$ for Eq. (3) to hold is quite well fulfilled. Figure 2(c) shows the derivative of the scaling function $Q_1^- dG/dz$, obtained by scaling the torque data according to Eq. (6). The logarithmic dependence on z, which is expected in this limit,[18] is shown as a solid line, using $Q_1^- C_{2,0}^- = 0.69$. Slight deviations from this dependence around $z = 0.25$, where $\delta \simeq 0°$, may be due to small 2D effects showing up in this angular regime.

In order to analyze the data above T_c we estimate the critical amplitude of the correlation length to be $\xi_{a,0}^+ \approx 0.38 \times \xi_{a,0}^- = 9.8(8)$ Å.[18] Assuming $\gamma = 29$ also above T_c, with $a_1 = -1.2 \times 10^{-13}$ Nm determined from the above temperature dependence of A, we find for the universal constant $Q_1^+ C_0^+ = -1.01$. This universal constant can also be obtained within a different approach. Figure 3(a) shows the torque data at $T = 100.46$ K in an applied field $B_a = 1.4$ T. A $\sin(2\delta)$ signal is observed (solid line). As seen in Fig. 3(b), the limes $z \rightarrow 0$ for Eq. (4) to be valid is well fulfilled for all angles. $Q_1^+ dG/dz$ shows the expected linear behavior[18] for $z < 0.02$, revealing $Q_1^+ C_0^+ = -1.01$ [Fig. 3(c)]. This value is in excellent agreement with the value found from the temperature dependence of the $\sin(2\delta)$ amplitude A.

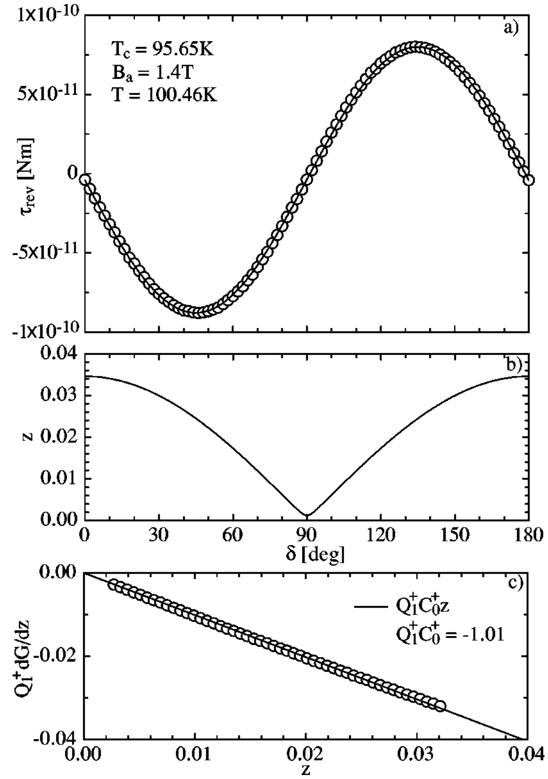

FIG. 3. (a) Angular-dependent torque measurement at $T = 100.46$ K in an applied field $B_a = 1.4$ T. As predicted by Eq. (4), a $\sin(2\delta)$ signal is observed. (b) Scaling variable z as a function of the angle δ ($\xi^+_{a,0} = 9.8$ Å). The condition $z \ll 1$ for Eq. (4) to hold is well established for all angles. (c) $Q_1^+ dG/dz$ as a function of z. A linear behavior is found revealing the universal constant $Q_1^+ C_0^+ = -1.01$. For clarity not all measured data points are shown.

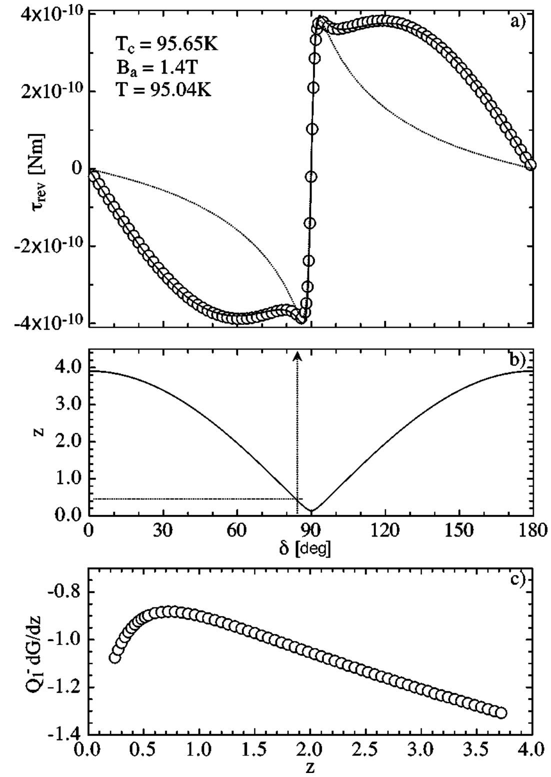

FIG. 4. (a) Angular-dependent torque measurement at $T = 95.04$ K in $B_a = 1.4$ T. Strong deviations from Eq. (3) (dashed line) are observed for angles $\delta < 84°$, $\delta > 96°$. The solid line is based on an analytical approximation for dG/dz given in Eq. (7) (see text below). (b) The scaling variable $z(\delta)$ shows a crossover from the limit $z \ll 1$ to the limit $z \gg 1$ upon turning the applied field from the ab plane of the sample ($\delta = 90°$) towards the c axis. (c) $Q_1^- dG/dz$ as a function of z. The crossover in the z dependence from the limit $z \ll 1$ to the limit $z \gg 1$ is clearly seen (see text). For clarity not all measured data points are shown.

Very unusual angular dependences of the torque are observed at temperatures slightly below T_c. Figure 4(a) shows an angular-dependent torque measurement at $T = 95.04$ K in an applied field $B_a = 1.4$ T. Equation (3) is represented by the dashed line. It clearly fails to describe the data over a large angular regime. The observed angular dependence of the torque data can be explained by a crossover from the regime $z \ll 1$ to the regime $z \gg 1$. The scaling argument $z(\delta)$ is shown in Fig. 4(b). Equation (3) describes the torque data well in the angular regime $84° < \delta < 96°$, where $z \lesssim 0.4$. Upon rotating the field towards the c axis ($\delta = 0°, 180°$), z increases up to 3.9, thus leaving the regime where Eq. (3) is valid. Therefore strong deviations from the angular dependence given by Eq. (3) are seen for angles $\delta < 84°$ and $\delta > 96°$, respectively. The crossover from a logarithmic dependence of dG/dz on z for $z \ll 1$ to a square root dependence for $z \gg 1$ is clearly seen in Fig. 4(c).

Above T_c the regime $z \gg 1$ is difficult to reach by experiment, since $\xi^+_{a,0}$ is rather small. Therefore one has to measure very close to T_c, where effects due to the finite width of the superconducting transition are starting to play an important role. Torque data taken at $T = 96.04$ K ($T = 1.004 T_c$) in an applied field $B_a = 1.4$ T are shown in Fig. 5. Although the small step observed close to the ab plane of the sample is predicted by Eq. (5) (solid line), in this angular regime the scaling variable z, calculated with $T_c = 95.65$ K, is of the order of 0.1, and the condition $z \gg 1$ for Eq. (5) to be valid is

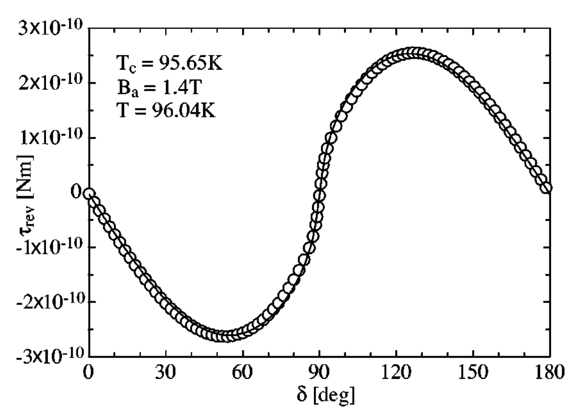

FIG. 5. Angular-dependent torque measurement taken in $B_a = 1.4$ T and $T = 96.04$ K, slightly above $T_c = 95.65$ K. Although the condition $z \gg 1$ for Eq. (5) (solid line) to be valid is not fulfilled for this measurement, the data are rather well described by Eq. (5). This seems to indicate that the observed characteristic signal is a precursor of the $T \rightarrow T_c$ curve. For clarity not all measured data points are shown.

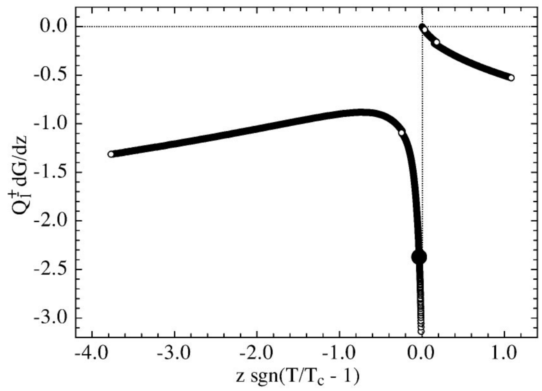

FIG. 6. Derivative of the scaling function $Q_1^{\pm} dG/dz$ as a function of z sgn(T/T_c-1), calculated from measurements at $T=90.88$, 95.04, 96.04, 97.03, and 100.46 K in $B_a=1.4$ T using Eq. (6). An excellent agreement with the qualitative behavior derived in Ref. 18 is found. The black dot marks z_{irrev}, where a crossover from irreversible behavior ($z<z_{\mathrm{irrev}}$) to reversible behavior ($z>z_{\mathrm{irrev}}$) is found (see text).

not fulfilled at all. However, due to the finite width of the superconducting transition Eq. (5) may already hold for a small part of the sample at slightly higher temperatures. Since the amplitude of the torque signal is dramatically increasing on approaching T_c from above, this small part may give a large contribution to the torque signal. The maximum of the angular dependent torque signal moves closer to the ab plane of the sample as the temperature approaches T_c from above.[9] Therefore effects of a finite transition width should be enhanced for angles around $\delta=90°$. We believe that the angular dependence of the torque data shown in Fig. 5 is a precursor of the curve expected for $T\rightarrow T_c$. Due to the finite width of the superconducting transition, it already shows up at temperatures where the condition $z\gg 1$ (calculated using $T_c=95.65$ K) is not yet fulfilled.

III. DISCUSSION

By use of Eq. (6) it is possible to determine the derivative of the scaling function dG/dz from the experiment. The result from our measurements on a $HgBa_2CuO_{4+y}$ microcrystal is shown in Fig. 6, where we plot $Q_1^{\pm} dG/dz$ as a function of z sgn(T/T_c-1) over the experimentally accessible z regime. The data shown are combined from measurements at $T=90.88$ K, 95.04 K, 96.04 K, 97.03 K, and 100.46 K in an applied field $B_a=1.4$ T. With $T_c=95.65$ K, the scaled data obtained from these measurements collapse onto a single curve. As discussed above, at $T=96.04$ K effects due to the finite width of the superconducting transition are observed. They become especially important at angles close to 90°, where $z\simeq 0.1$, leading to a value of $|dG/dz|$ that is too large for $z<0.15$. Therefore, the data obtained from the measurement at $T=96.04$ K in this z regime have been omitted in Fig. 6. For $z>0.15$ the finite transition width plays a less important role. In this z regime the crossover to a square root z dependence of dG/dz is clearly seen (rightmost part of the curve in Fig. 6).

The overall z dependence of dG/dz is in excellent agreement with the qualitative behavior we derived from the three limits leading to Eqs. (3)–(5) (Ref. 18). Furthermore we are able to give the absolute value of dG/dz up to the universal constants $Q_1^{\pm}$. Table I lists the values for universal constants derived from the scaling analysis of our torque measurements. The values $Q_1^+C_\infty^+\approx -0.5$ and $Q_1^-C_\infty^-\approx -0.6$ are rough estimates obtained from the limiting behavior of dG/dz for $z\gg 1$ above and below T_c, respectively. Both values are equal within the uncertainties, as expected from Eq. (5). Using the universal relation $Q_1^+=-(R_\xi^+)^3/[\alpha(1-\alpha)(2-\alpha)]$ relating the free energy universal constant Q_1^+ to the specific heat universal constant and critical exponent $R_\xi^+\simeq 0.36$ (Ref. 26) and $\alpha\simeq -0.007$ (Ref. 27), we find $Q_1^+\approx 3.3$. With $Q_1^+C_\infty^+\approx -0.5$ this leads to $C_\infty^+\approx -0.15$. This value is in excellent agreement with a C_∞^+ value between -0.2 and -0.11 that was determined from magnetization measurements on $YBa_2Cu_2O_{7-y}$ by Hubbard *et al.*[21]

Table II lists the critical amplitudes for $HgBa_2CuO_{4+y}$. The value $\lambda_{a,0}\simeq 766$ Å seems to be rather small compared to typical values $\lambda_{ab}(0)\simeq 1500$ Å that are found in measurements at low temperatures. However, one has to consider that $\lambda_{a,0}$ is a parameter used to describe the temperature dependence of the magnetic penetration depth near T_c. A two-fluid temperature extrapolation $\lambda_{ab}(T)^{-2}=\lambda_{ab}(0)^{-2}[1-(T/T_c)^4]$ usually leads to low temperature values that are in agreement with values actually measured at low temperatures. Since $1-(T/T_c)^4$ shows a stronger curvature than $(1-T/T_c)^{2/3}$, the critical amplitude $\lambda_{a,0}$ is always smaller than the actual low temperature penetration depth $\lambda_{ab}(0)$. A similar consideration holds for the correlation lengths $\xi_{a,0}^-$ and $\xi_{ab}(0)$. From the measured $\xi_{a,0}^-$ we can calculate the critical amplitude A^- of the specific heat using the relation $(R_\xi^-)^3=A^-(\xi_{a,0}^-)^3/\gamma$. With $R_\xi^-\simeq 0.96$ (Ref. 26) we find for $HgBa_2CuO_{4+y}$ $A^-\approx 1.5\times 10^{-3}$ Å^{-3}. In order to extract A^- directly from specific heat measurements, one has to consider two points. First, it is quite difficult to subtract the enormous background arising from the crystal lattice in a proper way. Secondly, the asymptotic form $c/(Vk_B)=A^-/\alpha|t|^{-\alpha}$ normally used to extract A^- is only a first-

TABLE I. Universal constants determined from angular-dependent torque measurements using the scaling analysis.

Universal constant	Determined from
$Q_1^-C_{2,0}^-\simeq 0.69$	$\tau_{\mathrm{rev}}(\delta)$ at $T=90.88$ K, $B_a=1.4$ T
$Q_1^+C_0^+\simeq -1.01$	$\tau_{\mathrm{rev}}(\delta)$ at $T=100.46$ K, $B_a=1.4$ T
$Q_1^+C_0^+\simeq -1.01$	T dependence of A, $B_a=1.4$ T
$Q_1^+C_\infty^+\simeq -0.5$	$\tau_{\mathrm{rev}}(\delta)$ at $T=96.04$ K, $B_a=1.4$ T
$Q_1^-C_\infty^-\simeq -0.6$	$\tau_{\mathrm{rev}}(\delta)$ at $T=95.04$ K, $B_a=1.4$ T

TABLE II. Critical amplitudes of $HgBa_2CuO_{4+y}$ determined from angular-dependent torque measurements and calculated using universal relations.

Critical amplitude	Determined from
$\xi_{a,0}^-=26(2)$ Å	$\tau_{\mathrm{rev}}(\delta)$ at $T=90.88$ K, $B_a=1.4$ T
$\xi_{a,0}^+\simeq 9.8$ Å	calculated from $\xi_{a,0}^-$
$\lambda_{a,0}\simeq 766$ Å	calculated from $\xi_{a,0}^-$
$A^-\simeq 1.5\times 10^{-3}$ Å^{-3}	calculated from $\xi_{a,0}^-$

order approximation of the thermodynamic form given by $c/(Vk_B) = -T/k_B \partial^2 f_s / \partial T^2$. The temperature regime around T_c where the asymptotic form holds is very small. It is of the same order as the transition width of a high-T_c superconductor. Thus, we conclude that the determination of A^- from torque measurements is more accurate than any direct extraction from specific heat measurements.

Once the scaling function is known, it is possible to approximate it by an analytic expression in the intermediate regime of z. This is especially useful for temperatures below T_c. In this temperature regime, for $z \lesssim 1$ the scaling function can be approximated by $dG/dz = b_1 \ln(z) + b_2 z$. Therefore, for temperatures not to close to T_c one can describe the angular dependence of the torque well with a combination of Eqs. (3) and (4), as demonstrated in Ref. 10. Over the whole z regime investigated, $0.01 < z < 3.77$, the scaling function is well approximated by the expression

$$dG/dz \simeq c_1 \ln(z) + c_2 z^{1/2} + c_3 z^{1/4} + c_4. \quad (7)$$

The first and second term are the leading terms for $z \to 0$ and $z \to \infty$, respectively. The other two terms provide a good approximation of dG/dz in the intermediate z regime. We are now able to calculate the angular-dependent torque at different fields and temperatures $T < T_c$, approximating dG/dz in Eq. (1) by the analytic expression Eq. (7) in the intermediate z regime. As an example the torque curve calculated for $T = 95.04$ K and $B_a = 1.4$ T is shown as a solid line in Fig. 4(a). In order to discuss the crossover from the case $z \to 0$ [Eq. (3)] to the case $z \to \infty$ [Eq. (5)] upon approaching T_c from below, the normalized angular-dependent torque data, τ/τ_{max}, measured at temperatures around T_c in $B_a = 1.4$ T are summarized in Fig. 7(a). As shown by the solid lines, the data recorded below T_c (open symbols) are well described by Eq. (1), using Eq. (7) for dG/dz. The data obtained slightly above T_c (black dots) cannot be described as above, since Eq. (7) is only valid below T_c. However, as discussed in the context of Fig. 5, these data appear to be a precursor of the behavior in the limit $T \to T_c$ [Eq. (5)], represented by the dashed-dotted line. This limit is valid irrespective of whether T_c is reached from above or from below. The evolution of the angular dependence of τ/τ_{max}, going from a temperature sufficiently below T_c [$z \to 0$, Eq. (3)] to a temperature $T \lesssim T_c$ [$z \to \infty$, Eq. (5)] is evident in Fig. 7(b), where we show calculated curves at different reduced temperatures T/T_c at $B_a = 1.4$ T, using Eqs. (1) and (7). The change in the shape of τ/τ_{max} occurs in two steps while T_c is approached. First, for $T/T_c \gtrsim 0.99$ a hump develops at angles $\delta \approx 45°$ and $\delta \approx 135°$, respectively. Finally, for $T/T_c \gtrsim 0.995$ the pronounced ''peak'' of the angular-dependent torque near $\delta = 90°$, which is a characteristic feature for T sufficiently below T_c, completely vanishes. This ''peak'' arises from the logarithmic divergence of dG/dz for $z \to 0$ [see Eq. (7)] and is located near $\delta = 90°$, since z adopts its minimal value at this angle. However, for $T/T_c \gtrsim 0.995$ the scaling variable z is larger than 0.2 for all angles δ, and the divergency of dG/dz vanishes (see Fig. 6). According to Eq. (2), the same change in the angular dependence of the torque signal as shown in Fig. 7(b) is expected to occur by changing the applied field at a fixed temperature. Indeed, measurements

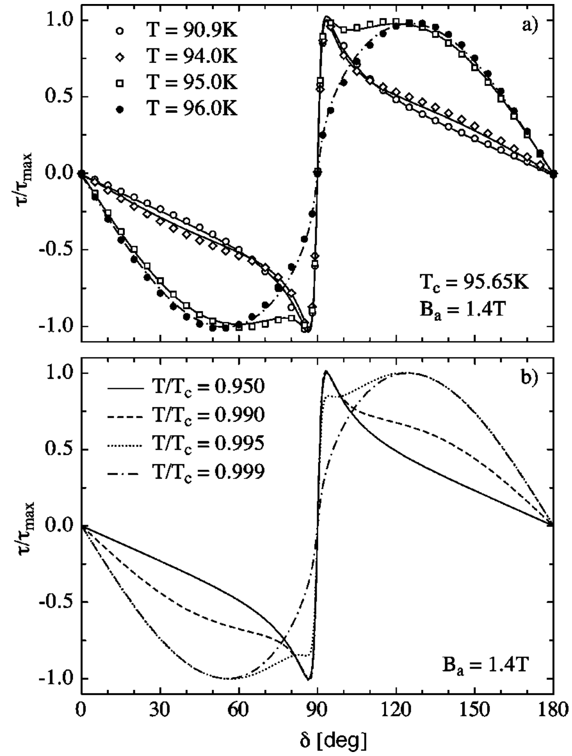

FIG. 7. (a) Normalized angular-dependent torque data taken in $B_a = 1.4$ T below (open symbols) and above T_c (black dots). The solid lines represent fits (with the signal amplitude as the only free parameter) to the data using Eq. (1) with the analytical approximation given in Eq. (7). The dashed-dotted line corresponds to Eq. (5) which is valid in the limit $T \to T_c$. For clarity not all measured data points are shown. (b) Normalized angular-dependent torque curves τ/τ_{max} for various reduced temperatures $T/T_c \lesssim 1$ and $B_a = 1.4$ T, calculated using Eqs. (1) and (7). Note the change in the shape of τ/τ_{max}, going from a temperature sufficiently below T_c [$z \to 0$, Eq. (3)] to a temperature very close to T_c [$z \to \infty$, Eq. (5)]. Details are given in the text.

taken at a fixed temperature slightly below T_c show a hump developing upon increasing the applied field.[28]

The black dot in Fig. 6 marks the value z_{irrev}, where a crossover from reversible to irreversible behavior is observed. We performed field-dependent measurements at a fixed angle $\delta = 45°$ in order to determine the irreversibility field. The field-dependent torque measured at $T = 90.88$ K is shown in Fig. 8. At the irreversibility field $B_{irrev}(T = 90.88\ \text{K}, \delta = 45°) = 0.25(2)$ T the data measured in decreasing field start to deviate significantly from the data measured in increasing field. This determination of the irreversibility field will result in a lower limit for B_{irrev}. All measurements were performed with a sweep rate of $dB_a/dt = 0.01$ T/s. The inset of Fig. 8 shows the temperature dependence of the irreversibility field in the range $86.7\ \text{K} < T < 93.9$ K. The data are well described by $B_{irrev}(T, 45°) = B_{irrev}(0, 45°)(1 - T/T_c)^{4/3}$ (solid line), in agreement with previous results reported for $YBa_2Cu_3O_7$ crystals.[22] Therefore, according to Eq. (2) for $z < z_{irrev} = (\xi_{a,0}^-)^2 B_{irrev}(0,45°) \epsilon(45°)/\Phi_0$ the system is in the irreversible regime. With $B_{irrev}(0,45°) = 14.3(8)$ T we find $z_{irrev} = 0.033(5)$. In the angular-dependent measurements a very small difference in τ_+ and τ_- is observed for orienta-

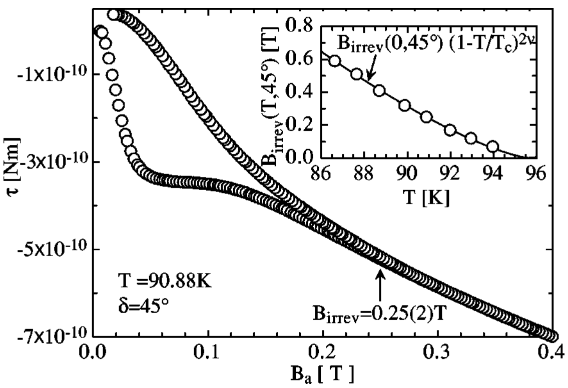

FIG. 8. Field-dependent torque measured at $T=90.88$ K, $\delta=45°$, with a sweep rate $dB_a/dt=\pm 0.01$ T/s. Irreversible behavior is observed for $B_a<B_{irrev}=0.25(2)$ T. Inset: The temperature dependence of the irreversibility field is well described by $B_{irrev}(T,45°)=B_{irrev}(0,45°)(1-T/T_c)^{2\nu}$ with $2\nu=1.35(2)$ and $B_{irrev}(0,45°)=14.3(8)$ T (solid line).

tions closer to the ab plane of the sample than δ_{irrev}, where $z(\delta_{irrev})\simeq 0.032$. This agreement between z_{irrev} resulting from field-dependent and angular-dependent measurements is expected, since the time constant of the measurement is almost the same in both cases.

There is considerable evidence from microscopic[29–31] as well as from macroscopic[32–35] experiments that flux-line lattice melting in high-T_c cuprates is a first-order phase transition. Thus, dG/dz, being proportional to the magnetization, should have a discontinuity at

$$z_m=[\xi_a^-(T)]^2 B_m(T,\delta)\epsilon(\delta)/\Phi_0, \tag{8}$$

where B_m denotes the melting field.[18] This discontinuity was observed in Ref. 34. Since z_m is constant for all temperatures, Eq. (8) predicts the temperature dependence of the melting field to be $B_m(T,\delta)=B_m(0,\delta)(1-T/T_c)^{4/3}$. This temperature dependence has been confirmed in $YBa_2Cu_3O_7$ (Refs. 31,34,36,35), yielding $B_m(0)\simeq 100$ T for $\delta=0°$. Together with $\xi_{a,0}^-=14.7$ Å for $YBa_2Cu_3O_7$ (Ref. 18) the value of the scaling variable, where a discontinuity in dG/dz is expected, is $z_m\simeq 0.088$. Due to universality there should be a discontinuity in dG/dz at the same z value in $HgBa_2CuO_{4+y}$. However, we do not expect to see this phenomenon in our torque measurements since the first-order transition might be smeared out due to the finite transition width.[35] The z_{irrev} for our sample is roughly three times smaller than the expected z_m. This discrepancy between the irreversibility field and the melting transition is not surprising for samples with very low pinning.[34] Furthermore, the irreversibility line that we determined is a lower bound, as discussed above.

From Eq. (8) follows that the critical amplitude of the correlation length and the melting transition are related as $\xi_{a,0}^-\propto[B_m(0)]^{-1/2}$. Since the melting line is strongly shifted to lower fields with increasing anisotropy,[37] $\xi_{a,0}^-$ should increase with increasing γ. This behavior is observed in $La_{2-x}Sr_xCuO_4$, where $\xi_{a,0}^-$ and γ are found to increase with decreasing Sr concentration x (Ref. 20).

The derivative of the scaling function dG/dz shows a logarithmic z dependence up to $z\simeq 0.3$, even for z values where the vortex lattice is melted. We conclude that macroscopic phase coherence, i.e., superconductivity also exists above the melting line, leading to a finite magnetic penetration depth and thus to the logarithmic z dependence. This is consistent with recent muon-spin-rotation measurements, where above the melting line the local field distribution $P(B)$ for a vortex lattice vanishes, but where a broadening of $P(B)$ due to macroscopic screening currents can still be seen.[29,38]

In summary, unusual angular-dependent magnetic torque curves observed in the vicinity of T_c may be interpreted in terms of critical fluctuation theory using the 3D XY scaling approach outlined in Ref. 18. Within this framework, the field and angular dependence of the magnetic torque follows the derivative of a scaling function $dG(z)/dz$. By scaling the torque data we are able to quantitatively determine dG/dz. This allows the calculation of several universal constants. The experimentally obtained dG/dz can be approximated by an analytical expression in the intermediate z regime. Using this analytical expression angular-dependent torque curves for different fields and temperatures within the critical regime can be calculated.

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