Upper critical field H_{c_3} for a thin-film superconductor with a ferromagnetic dot

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We investigate the effect of a ferromagnetic dot on a thin-film superconductor. We use a real-space method to solve the linearized Ginzburg-Landau equation in order to find the upper critical field H_{c_3}. We show that H_{c_3} is crucially dependent on dot composition and geometry, and may be significantly greater than H_{c_2}. H_{c_3} is maximally enhanced when (i) the dot saturation magnetization is large, (ii) the ratio of dot thickness to dot diameter is of order one, and (iii) the dot thickness is large. [S0163-1829(99)04741-4]

I. INTRODUCTION

Recent experiments[1-4] involving thin-film superconductors have investigated the effects of nanosize artificial pinning centers in the form of ferromagnetic dots. In Ref. 1, magnetic dots with diameters on the order of 200 nm and thickness on the order of 40 nm were fabricated on a superconducting Nb film by electron-beam lithography. It was found that a regular array of magnetic dots can dramatically influence the transport properties in the presence of an applied magnetic field. In particular, the resistivity displayed minima at "matching fields" in which the number of flux quanta per dot was an integer. Such effects are known to occur in regular arrays of empty holes in thin-film superconductors.[1] These effects are believed to arise because empty holes can form effective pinning centers for multiple flux quanta vortices,[5] leading to particularly stable configurations at the matching fields. The strong pinning leading to such multiple vortices arises due to an enhancement of the order parameter near the edges of the hole, in a manner analogous to surface superconductivity.[6-8]

However, the physical situation for magnetic dots is significantly different. For empty holes, the superconducting order parameter must have a vanishing derivative[7,8] at the vacuum/superconductor interface. In a magnetic field, the states satisfying the linearized Ginzburg-Landau equation and this boundary condition turn out to have a maximum just inside the superconductor, leading to a magnetic field H at which the superconducting order parameter may be nonzero near the surface while it vanishes in the bulk of the sample (i.e., $H > H_{c_2}$). This is very similar to surface superconductivity,[8] and the maximum field at which superconducting order survives in the empty hole system is a direct analog of H_{c_3}.[5,8]

For magnetic dots, the strong field present inside the ferromagnet supresses the superconducting order parameter, and in such situations it is appropriate to adopt a boundary condition in which the order parameter itself vanishes. This spoils the effect that leads to surface superconductivity, and it is not at first obvious why magnetic dots should support the relatively large supercurrents associated with multiple vortices. However, the problem of a ferromagnetic dot in a superconducting thin film has a dimension not present in the empty hole analog: the magnetization and fringing magnetic field of the dot itself. (Throughout this work, we will consider only dots small enough to be treated as single domain ferromagnets.) For dots with diameter $2R$ larger than their height t (see Fig. 1),[9] shape anisotropy dictates that the magnetization of the dot will lie in the plane of the superconductor[10] in the absence of any external field. The application of a perpendicular magnetic field tilts the magnetization out of the plane of the dot, introducing a component to the dot fringing field that partially cancels the external applied flux passing through the superconducting film. This introduces a region just outside the dot in which the net field intensity is smaller than the applied field, allowing an enhancement of superconducting order.

To demonstrate this effect, in this work we study the analog of H_{c_3} in the presence of a single ferromagnetic dot with a diameter greater than its height $(2R > t)$. To do this, we solve the linearized Ginzburg-Landau (GL) equation for a two-dimensional superconductor (i.e., thickness smaller than coherence length ξ) using a real-space method to be described below. The resulting equations specify a maximum magnetic field $H_{c_3}(l)$ at which a nonzero superconducting order parameter may be present for each value of the vorticity l. A typical example of our results is shown in Fig. 2. The form of this figure is easily understood if one keeps in mind the analogy between the linearized GL equation and the problem of electrons in a magnetic field.[11] In this analogy the vorticity l plays the role of angular momentum, and it is well known that, in the absence of a dot, the lowest-lying single-
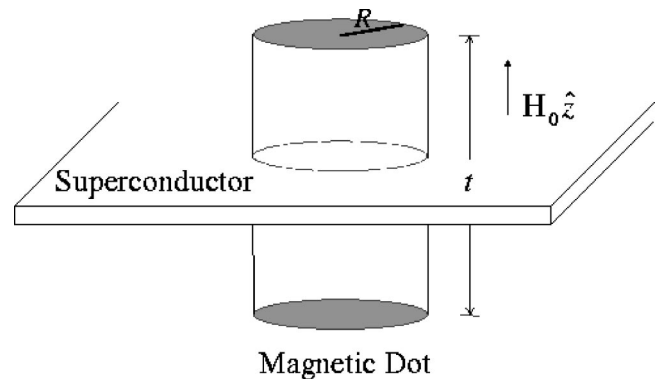

FIG. 1. Magnetic dot with a radius R and a thickness $t(<2R)$ at the center of a thin-film superconductor. An external field $H_0\hat{z}$ is applied through the sample, where $\hat{z}$ is the direction parallel to the normal of the sample plane.
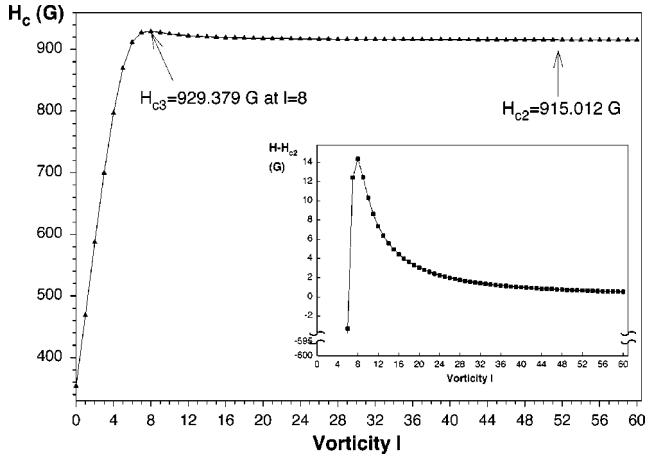
FIG. 2. Magnetic field versus vorticity curve for a Nb superconducting film with a Ni dot ($R=100$ nm and $t=40$ nm) at $T=8.2$ K. For large l, the field is unaffected by the presence of the dot, giving $H_{c_2}=915.012$ G. The maximal H_{c_3} (= 929.379 G) occurs at $l=8$. The inset shows that the difference between H_c and H_{c_2} is maximal at $l=8$.

particle states of a given l in a magnetic field are localized near a radius $R_l=a_m\sqrt{2l}$, where $a_m=\sqrt{\hbar c/e^*H_0}$ is the magnetic length, e^* the charge of the carriers, and H_0 is the applied field. While the presence of the dot and the appropriate boundary condition changes the precise relation between l and R_l, it nevertheless remains generally true that R_l increases with l. Thus, for small values of l, $H_{c_3}(l)$ is suppressed due to the boundary condition on the order parameter, whereas for large l, $H_{c_3}(l) \rightarrow H_{c_2}$, the value one expects in the absence of the dot. For intermediate values of l, one generically sees a peak in $H_{c_3}(l)$, due to the fringing field effect described above.

This peak value of $H_{c_3}(l)$ gives the maximum applied field in which the thin film may sustain superconducting order, and is the analog of H_{c_3}. That it occurs at a finite value of l indicates that it is indeed true that magnetic dots support and presumably pin multiple vortices. It is interesting to note that for appropriate parameters, this peak may become quite pronounced, and that it may exceed the value of $H_{c_3}=1.695H_{c_2}$ that occurs for a simple infinite surface and represents the maximum value possible in an empty hole.[5] Such large values of H_{c_3} occur when (i) the saturation magnetization of the dot is large, (ii) the height of the magnetic dot t is large, and (iii) the dot diameter $2R$ is close to the height t.

This article is organized as follows. In Sec. II we describe our model of a ferromagnetic dot in a thin superconducting film in detail and present the corresponding GL equation for the system. Section III outlines our method for solving the equation, and in Sec. IV we present our results. We conclude in Sec. V with a summary.

II. MODEL OF A FERROMAGNETIC DOT IN A THIN-FILM SUPERCONDUCTOR

Consider a thin-film superconductor with a small magnetic dot at its center (Fig. 1). The radius of the dot is R, and the thickness is t. We assume that the dot is small enough so
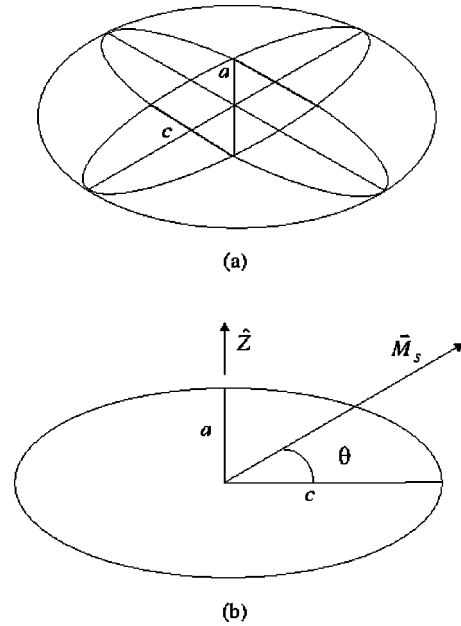

FIG. 3. (a) Oblate spheroid with semimajor axis c and semiminor axis a. (b) $\hat{z}\|a$. The angle between M_s and the sample plane is θ.

that its magnetization density is uniform throughout the dot; i.e., there are no domains. We wish to compute the largest magnetic field H_{c_3} for which the order parameter is nonvanishing inside the superconductor.

The magnetization of the ferromagnet is maximal, but its direction may vary. The orientational energy in any given direction can be conveniently estimated if we approximate its cylindrical shape as an ellipsoid whose semimajor axis length is $c(=R)$ and semiminor axis length is $a(=t/2)$, as shown in Fig. 3.

The energy for a given orientation of $\vec{M}$ (the dipole moment of the ferromagnet) in an ellipsoid is[10]

$$F_M^0=\frac{1}{2}\Omega[(M_s\cos\theta)^2 N_c+(M_s\sin\theta)^2 N_a], \quad (1)$$

where Ω is the volume of ellipsoid, M_s is the saturation magnetization of the ferromagnet, and N_a and N_c are the demagnetizing factors along a and c. θ is the angle between $\vec{M}$ and the sample plane. Since $N_a>N_c$, $\sin\theta=0$ in the absence of an applied magnetic field and the magnetization is perpendicular to the a axis, i.e., anywhere in the sample plane.

When there is an external field $H_0\hat{z}$ ($\hat{z}\|a$), we must add $-\Omega\vec{M}\cdot H_0\hat{z}$ to the dot energy. Therefore the total magnetization energy of the dot is

$$F_M=-\Omega M_s H_0\sin\theta+F_M^0(\theta). \quad (2)$$

For a thin-film superconductor, the free energy may be written as

$$F_S = F_N + \int_{r>R} d^2r \left[\alpha |\psi|^2 + \frac{1}{2}\beta |\psi|^4 \right.$$

$$\left. + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*\vec{A}}{c}\right)\psi\right|^2 \right]. \tag{3}$$

Since the superconductor is thin, we ignore any fields produced by supercurrents in the film. The vector potential $\vec{A}$ then is the sum of the vector potential due to the magnetic dot ($\vec{A}_m$) and the vector potential due to the external field ($\vec{A}_0$): $\vec{A} = \vec{A}_m + \vec{A}_0$. In a uniform external field $H_0\hat{z}$, $\vec{A}_0$, in cylindrical coordinates, is given by

$$\vec{A}_0 = \frac{1}{2}H_0 r\hat{\varphi}. \tag{4}$$

Furthermore, for a thin-film superconductor, magnetic-field components parallel to the film have no effect on the order parameter.[8] Thus we may write

$$\vec{A}_M(r) = M_s t \, \sin\theta a_0\left(\frac{r}{R}\right)\hat{\varphi}, \tag{5}$$

where

$$a_0(x) \equiv \int_{-\pi}^{\pi} d\varphi \frac{\cos\varphi}{[x^2 - 2x\cos\varphi + 1]^{1/2}}, \tag{6}$$

which is an elliptical integral.[12] Therefore the total free energy of the system is $F = F_S + F_M$, or

$$F = F_N + \int_{r>R} d^2r \left[\alpha |\psi|^2 + \frac{1}{2}\beta |\psi|^4 \right.$$

$$+ \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*\vec{A}}{c}\right)\psi\right|^2 \right] - \Omega M_s H_0 \, \sin\theta$$

$$\left. + \frac{1}{2}\Omega[(M_s\cos\theta)^2 N_c + (M_s\sin\theta)^2 N_a]. \right. \tag{7}$$

We need to minimize the total free energy with respect to ψ and θ (or $\sin\theta$). After minimizing with respect to $\sin\theta$, we obtain

$$-\frac{e^*}{2mc}\int_{r>R} d^2r \left[\psi^*\left(\frac{\hbar}{i}\nabla - \frac{e^*\vec{A}}{c}\right)\psi \right.$$

$$\left. + \psi\left(-\frac{\hbar}{i}\nabla - \frac{e^*\vec{A}}{c}\right)\psi^* \right] \cdot M_s t a_0\left(\frac{r}{R}\right)\hat{\varphi}$$

$$= -\Omega M_s H_0 + M_s^2 \Omega(N_a - N_c)\sin\theta. \tag{8}$$

To find H_{c_3}, we consider $|\psi| \ll 1$ and so drop terms involving ψ. Then we have

$$\sin\theta = \frac{H_0}{M_s(N_a - N_c)}. \tag{9}$$

After minimizing the free energy with respect to ψ, we obtain

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e^*\vec{A}}{c}\right)^2 \psi = |\alpha|\psi, \quad \alpha < 0, \tag{10}$$

where $\alpha \propto T - T_c$. Equation (10) is the linearized Ginzburg-Landau equation. It should be kept in mind that although the equation is linear in ψ, it is *nonlinear* in $\sin\theta$. This means our solution of Eq. (10) requires a level of self-consistency usually absent in solving the linearized GL equation, which we describe below.

Equation (7) essentially defines our model of the film-dot system, and Eqs. (9) and (10) are what need to be solved to obtain the critical field of the system. In the next section we present some details describing how this is done.

III. NUMERICAL SOLUTION OF LINEARIZED GINZBURG-LANDAU EQUATION

We need to find the largest value of H_0 (i.e., H_{c_3}) for which the eigenvalue equation [Eq. (10)] has a nontrivial solution with the boundary condition

$$\psi(r = R) = 0. \tag{11}$$

Due to circular symmetry, we can write the solution to Eq. (10) in the form

$$\psi(\vec{r}) = f(r)e^{il\varphi}, \tag{12}$$

where l is an integer. The orbital number l corresponds to the vorticity of the solution for the superconducting order parameter. Next, we scale out length by defining

$$\rho \equiv \frac{r}{a_m}, \tag{13}$$

where the magnetic length a_m is

$$a_m = \left(\frac{\hbar c}{e^* H_0}\right)^{1/2}. \tag{14}$$

Applying Eqs. (12) and (13), Eq. (10) becomes

$$\left[-\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho} + \left(\frac{l}{\rho} - \tilde{A}(\rho)\right)^2\right]f(\rho) = \varepsilon f(\rho), \tag{15}$$

where

$$\tilde{A}(\rho) = \frac{1}{2}\rho + M_s t\frac{\sin\theta}{a_m H_0}a_0\left(\frac{a_m\rho}{R}\right) \tag{16}$$

and

$$\varepsilon = \frac{2m|\alpha|}{\hbar^2}\left(\frac{\hbar c}{e^* H_0}\right). \tag{17}$$

The smallest eigenvalue of Eq. (15) corresponds to the largest H_0 for which there is a nonvanishing order parameter. Once we find ε_0, we can directly write

$$H_{c_3} = \frac{2mc|\alpha|}{\hbar e^*\varepsilon_0}. \tag{18}$$

To solve this eigenvalue problem, we used a real-space method as follows:

(1) Guess $H_0 = H_{c_3}$. Notice that H_0 enters explicitly in the vector potential and $\sin\theta$, and thus cannot be scaled out as would be the case for an empty hole.
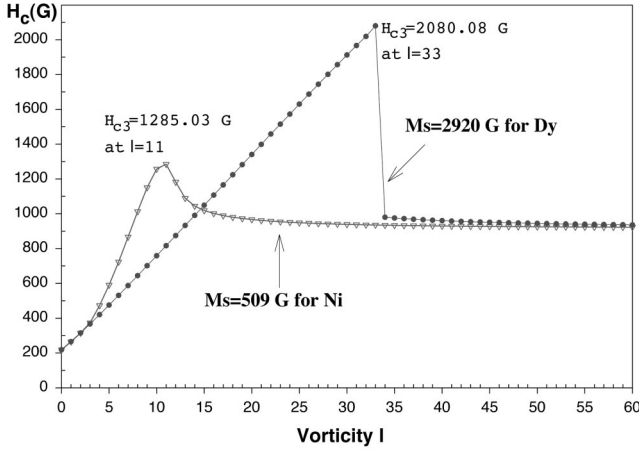
FIG. 4. Magnetic field versus vorticity curves for a Nb super-conducting film with a dot at $T = 8.2$ K. The dot size is fixed: $t = 160$ nm and $t/(2R) = 0.9$. If the dot material is Dy ($M_s = 2920$ G), $H_{c_3} = 2080.08$ G at $l = 33$. However, if the material is Ni ($M_s = 509$ G), $H_{c_3} = 1285.03$ G at $l = 11$.

(2) Define a set of N uniformly spaced points $\rho_1, \rho_2, \rho_3, \ldots, \rho_N$ and $\rho_0 \equiv R/a$.

(3) Turn the derivatives into differences, thereby transforming the differential equation into a *difference equation*.

(4) Set up the column vector

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_N \end{bmatrix},$$

and define $\Delta X_{n+1/2} = \rho_{n+1} - \rho_n, \Delta X_{n-1/2} = \rho_n - \rho_{n-1}$, turning the differential equation into a matrix equation:

$$\begin{bmatrix} \beta_1 & \alpha_1 & 0 & 0 & \cdots \\ \gamma_2 & \beta_2 & \alpha_2 & 0 & \cdots \\ 0 & \gamma_3 & \beta_3 & \alpha_3 & \cdots \\ 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_N \end{bmatrix} = \varepsilon_0 \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_N \end{bmatrix}, \quad (19)$$

where

$$\alpha_n = - \frac{8}{(\Delta X_{n+1/2} + \Delta X_{n-1/2})^3} \Delta X_{n-1/2}$$
$$- \frac{1}{\rho_n(\Delta X_{n+1/2} + \Delta X_{n-1/2})}, \quad (20)$$

$$\beta_n = \frac{8}{(\Delta X_{n+1/2} + \Delta X_{n-1/2})^2} + \left(\frac{l}{\rho_n} - \tilde{A}(\rho_n) \right)^2, \quad (21)$$
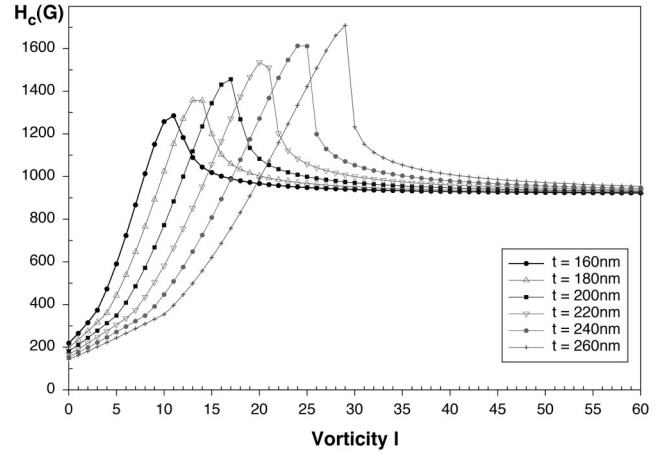
and

FIG. 5. Magnetic field versus vorticity curves for a Nb super-conducting film with a Ni dot at $T = 8.2$ K. The dot thickness varies from 160–260 nm, while keeping the value of $t/(2R)$ fixed. H_{c_3} increases with t.

$$\gamma_n = - \frac{8}{(\Delta X_{n+1/2} + \Delta X_{n-1/2})^3} \Delta X_{n+1/2}$$
$$+ \frac{1}{\rho_n(\Delta X_{n+1/2} + \Delta X_{n-1/2})}. \quad (22)$$

(5) Diagonalize the matrix to find the lowest eigenvalue, ε_0, which gives the highest magnetic field $H^* = 2mc|\alpha|/\hbar e^* \varepsilon_0$.

(6) Finally, one must check if the solution is self-consistent. If $H^*[\varepsilon_0]$ equals the guessed H_0 in step (1), then $H_0 = H_{c_3}$. However, if $H^*[\varepsilon_0]$ is not equal to the initial guess, then set $H^* \to H_0$ and return to step (1). The element of self-consistency arises because one must determine the orientation of the dot magnetization θ in the field H_{c_3}. Note that if the field H_{c_3} is large enough, then $\sin\theta = 1$ and the magnetization is fully parallel to the applied field; in this situation Eq. (9) is not appropriate [except precisely at $H_0/(M_s(N_a - N_c)) = 1$], and we do not need to iterate the equations.

IV. RESULTS

One of the interesting results that was found in this study is that H_{c_3} may be very large for this system. In order to examine when this happens, we studied how the variables M_s, t, and a/c (or $N_a - N_c$) affect H_{c_3}. First, we fixed t and a/c to compute H_{c_3} for different values of M_s. Clearly, we expect a large M_s to create large fringing field, capable of canceling large applied fields near the dot. Presumably this will lead to large values of H_{c_3}. Figure 4 illustrates this for a 160-nm-thick dot with $a/c[=t/(2R)] = 0.9$ (or $N_a = 4\pi \times 0.361\,472$ and $N_c = 4\pi \times 0.305\,689$) in a Nb film at $T = 8.2$ K ($\xi \approx 60$ nm) with Ni ferromagnetic dots ($M_s = 509$ G). (These parameters correspond to those of the magnetic dots studied in Ref. 1.) For comparison, we also show H_{c_3} for a Dy dot ($M_s = 2920$ G) with otherwise the same parameters of the system. The enhancement of H_{c_3} for the larger value of M_s is quite apparent.

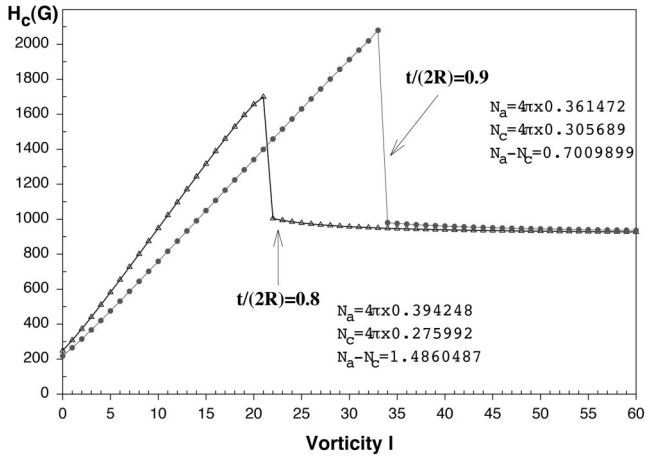We also note in this figure and several that follow that

FIG. 6. Magnetic field versus vorticity curves for a Nb superconducting film with a Dy dot at $T = 8.2$ K. The dot's thickness is fixed at $t = 160$ nm, but the radius varies: $t/(2R) = 0.9$ (or $N_a - N_c = 0.700\,989\,9$) and $t/(2R) = 0.8$ (or $N_a - N_c = 1.486\,048\,7$). The curve for $t/(2R) = 0.9$ shows a greater H_{c_3} than that for $t/(2R) = 0.8$.

there is an apparent jump in $H_{c_3}(l)$ vs l. If l were a continuous variable this would not be a discontinuous jump but rather a continuous (albeit sharp) drop in $H_{c_3}(l)$. Nevertheless, the sharp behavior is a direct result of the nonlinearity of the equations in $\sin\theta$. The behavior represents a sharp crossover as a function of l in which the superconducting order parameters are localized relatively close to the dot and ones in which they are further away; in the latter case the dot potential is a relatively weak perturbation on the result in the absence of a dot.

Next, we fixed M_s and a/c to study the influence of dot thickness t on H_{c_3}. For large values of t one again expects the field generated by the dot can cancel a relatively large external field, leading to an enhancement of H_{c_3}. One can imagine a situation in which both the film and the dot thickness t are the same and are varied together. The results re-

ported here will apply provided the superconductor is effectively two dimensional, i.e., the coherence length must be larger than t. For many interesting materials that may not be possible; however, it is quite possible and often appropriate to consider systems in which the dot thickness is *different* than that of the film. This is the situation depicted in Fig. 1. Figure 5 illustrates H_{c_3} for a Ni dot with $a/c = 0.9$ in a Nb film at $T = 8.2$ K. As expected, thicker dots (bigger t) indeed yield higher values of H_{c_3}.

Finally, we fixed M_s and t and considered the effect of a/c on H_{c_3}. The aspect ratio of the dot is relevant because it enters into the demagnetizing factors N_a and N_c. Physically, if a/c is close to one, the dot should approach a limit in which it is relatively easy to tip the magnetization out of the plane.[13] This again maximizes the fringing field available to cancel the applied field. Figure 6 illustrates H_{c_3} for $a/c = 0.8$ and $a/c = 0.9$ (or $N_a - N_c = 1.486\,048\,7$ and $N_a - N_c = 0.700\,988\,9$, respectively), and demonstrates that greater values of a/c (or smaller values of $N_a - N_c$) give higher values of H_{c_3}.

V. SUMMARY

In this work, we studied H_{c_3} for magnetic dots in a thin-film superconductor. To find H_{c_3}, we used a real-space method to solve the linearized Ginzburg-Landau equation. We showed that the enhancement of the order parameter crucially involves the shape anisotropy of the magnetic dot and is qualitatively different from that for empty holes. We found the enhancement to be maximal when (i) the dot saturation magnetization M_s is large, (ii) the dot thickness is large, and (iii) the value of $t/(2R)$ of the dot is close to 1, for which the demagnetizing parameters N_a and N_c are of comparable magnitude.

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[11] See, for example, R. E. Prange and S. M. Girvin, *The Quantum Hall Effect* (Springer-Verlag, New York, 1987).

[12] This form for the dot vector potential is easily derived when one notes that the magnetic field produced by (the z component of) a uniformly magnetized cylinder is the same as that of a current sheet around the side of the cylinder.

[13] Note that if a/c is greater than one, then the equilibrium direction of the magnetization will be along the cylinder axis rather than in the plane. In this case fringing fields will be present and disrupt the superconducting order even in the absence of an applied field. This situation was considered in Ref. 9 above, and we do not address it in our present work.