Dynamic scaling for two-dimensional superconductors, Josephson-junction arrays, and superfluids

Stephen W. Pierson*
Department of Physics, Worcester Polytechnic Institute (WPI), Worcester, Massachusetts 01609-2280

Mark Friesen†
Department of Physics, Purdue University, West Lafayette, Indiana 47907-1396

S. M. Ammirata
Department of Physics, Ohio State University, Columbus, Ohio 43210

Jeffrey C. Hunnicutt and LeRoy A. Gorham
Department of Physics, WPI, Worcester, Massachusetts 01609-2280

(Received 28 September 1998)

The value of the dynamic critical exponent z is studied for two-dimensional superconducting, superfluid, and Josephson junction array systems in zero magnetic field via the Fisher-Fisher-Huse dynamic scaling. We find $z \simeq 5.6 \pm 0.3$, a relatively large value indicative of nondiffusive dynamics. Universality of the scaling function is tested and confirmed for the thinnest samples. We discuss the validity of the dynamic scaling analysis as well as the previous studies of the Kosterlitz-Thouless-Berezinskii transition in these systems, the results of which seem to be consistent with simple diffusion ($z=2$). Further studies are discussed and encouraged. [S0163-1829(99)08325-3]

I. INTRODUCTION

The dynamics of two-dimensional (2D) zero-field systems has been studied continually over the last few decades,[1–6] usually in the context of the Kosterlitz-Thouless-Berezinskii[7–9] (KTB) transition. The dynamic critical exponent z characterizes the critical behavior of the dynamics of a transition. Despite ongoing studies of KTB dynamics, the value of z is usually not questioned to be anything but the value that describes simple diffusion: $z=2$. This paper, through an analysis of various transport data sets from systems including superconductors (SC's), superfluids (SF's), and Josephson Junction arrays (JJA's) using the dynamic scaling of Fisher, Fisher, and Huse,[10] presents ample evidence that the value of z in these systems is much higher: $z \simeq 5.6$. The purpose of this paper is to convince the reader that despite the many previous reports consistent with $z=2$, the question of the value of z is still an open one. Perhaps the single element that distinguishes this analysis from past analyses is that z is not taken *a priori* to have a value 2. In the dynamic scaling analysis, varying z to optimize the description of the data is standard.

This paper is organized as follows. We begin in this section with a general description of the KTB transition, previous scaling attempts of I-V data, and previous findings of anomalous vortex diffusion. In Sec. II, we will discuss the various length scales whose competition results in the interesting critical behavior of this transition. In Sec. III, we present scaling results on SC's, JJA's, and SF's and also check for universality of the scaled data. We discuss the validity of the dynamic scaling approach as well as that of the conventional approach in Sec. IV. We summarize the paper in Sec. V.

The KTB transition is driven by the unbinding of vortex pairs. Below the transition temperature T_{KTB} vortices are thermally induced and can only be excited in pairs that have a finite energy and not as ''free'' vortices, which have an infinite energy in an infinite system. As the temperature is increased, the number and size of the vortex pairs increase and these pairs screen one another's interactions. At the transition temperature, the vortex pairs start to unbind and free vortices are formed. It is the largest pairs that unbind first, leaving a finite density of smaller pairs above the transition temperature. The size of the largest pairs decrease as one goes further above T_{KTB}. As we review below, for $T > T_{KTB}$, the free vortices result in an Ohmic I-V curve at low currents while the smaller pairs result in a nonlinear I-V relationship at larger currents.

Scaling techniques applied to zero-field I-V data from SC's or JJA's have been reported in the past but without finding the results reported here. In Ref. 11, Wolf *et al.* scaled their I-V data from granular 30-Å thick films. There, however, the dynamic universality class was not explicitly studied and no value of z was determined. Harris *et al.*[12] reported $z=2$ in a dynamic scaling analysis of Josephson junction array I-V data, although it appears that they did not allow z to vary. We address that data in Sec. III B. Miu *et al.*[13] mention peripherally a dynamic scaling analysis on $Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$ in which it is found that $z \simeq 4.5$ (assuming a 2D system). However, those results were not pursued further by those authors. In the case of superfluids, Brada *et al.*[14] have performed a finite-size scaling analysis (in contrast to a dynamic scaling analysis) of their frequency-dependent superfluid density and dissipation data from helium films. They however did not include a study of the value of z in their work.

There has been experimental work that points toward anomalous vortex diffusion. Théron *et al.*[15] used impedance measurements on weakly frustrated JJA's to provide evidence of nonconventional vortex dynamics. Unfortunately, this work did not probe the value of z. There has also been theoretical[16] and simulational[6,17] work that indicates anomalous vortex diffusion, but none conclude that $z>2$ at $T=T_{KTB}$. In this work, in order to limit the scope of this already long paper, we will not discuss the theoretical and simulational work, but concentrate only on experimental data.

In comparing the ''conventional'' approach with the dynamic scaling approach, we will make use of many of the formulas derived from the former. However, because the dynamic scaling approach indicates that the dynamics is nondiffusive, the assumptions used to derive the formulas for the dynamics in the conventional approach may be incorrect. For that reason, we will use those formulas only to address the validity of the conventional approach and not the validity of the dynamic scaling.

It is useful for us to clarify our use of the phrase ''conventional approach.'' Because the dynamic scaling has only been used sparingly in analyses of the type of data that we look at here, we view this approach as nonconventional. We will therefore refer to any other approach to analyzing this data as ''conventional'' or traditional.

The value of z that is found here runs contrary to some of the conclusions of previous studies. However, we stress that our results are not inconsistent with all of the results in the literature. In particular, the dynamic scaling results presented here do not contradict (or even pertain to) the conventional findings for static behavior of the KTB transition. Further, our results differ from only some of the experimental studies of the dynamics. This will be discussed in Sec. IV B but we mention some of those now. If the measurement does not involve z or if the measurement does involve z but can measure only the product bz (where b is a material-dependent constant that enters through the correlation length [see Eq. (1)]), then our findings do not contradict those measurements. An example of the first type of measurement is ''static'' kinetic inductance measurements.[18] Examples of the second class of measurement are the resistance, noise spectrum measurements,[19] and helium torsion measurements.[20,21]

II. KTB TRANSITION IN 2D SUPERCONDUCTING SYSTEMS: BACKGROUND

In this paper, we propose an interpretation of transport data on superconductors, superfluids, and Josephson junction arrays that is very different than that which has been accepted for the last 20 years. In order to judge the two approaches, a thorough understanding of the KTB transition and critical behavior as well as the approaches used to study them is needed. In this section, we will review this background, directing our discussion primarily at superconducting systems. (For a discussion of JJA's or superfluids, see the reviews listed below.) We will discuss the various relevant length scales, the criteria for a phase transition, and the approaches one can take to study the KTB dynamic critical behavior. We do not intend our review to be comprehensive and refer the reader to any of the many excellent reviews.[22–24]

A. Length scales

The competition of the length scales[25] in the system determines the critical behavior of the KTB transition. For this reason it is important to review each of them. One can subdivide the length scales of the system into two categories: intrinsic and extrinsic. By extrinsic, we mean those length scales that are determined by an applied current or magnetic field.

1. Intrinsic

The intrinsic length scales include the vortex correlation length $\xi(T)$, the 2D penetration depth $\lambda_{2D}=2\lambda^2/d$ (where λ is the London penetration depth and d is the sample thickness), and sample size.

There are three important aspects of the correlation length that we discuss here. The first is its distinctive temperature dependence for temperatures above the transition temperature T_{KTB}:[8]

$$\xi_+(T)\propto\exp[\sqrt{b/(T/T_{KTB}-1)}], \tag{1}$$

where b is a nonuniversal constant. This unique temperature dependence is in contrast to the common power-law dependence one finds, for example, in Ginzburg-Landau theory.

The second aspect is the behavior of $\xi_-(T)$ below T_{KTB}. Because the susceptibility below the transition temperature is infinite, Kosterlitz originally defined $\xi_-(T)$ to be infinite. Based on the critical behavior of the dielectric constant, Ambegaokar, Halperin, Nelson, and Siggia[1] (AHNS) defined a finite diverging correlation length for $T<T_{KTB}$. The two results do not contradict one another since they have different meaning. The AHNS correlation length for $T<T_{KTB}$ can be thought of as the size of the largest vortex pairs.[26] Ambegaokar *et al.*[2] estimate that $\xi_-(T)$ has a smaller magnitude than $\xi_+(T)$:

$$\xi_-(T)\propto\exp[\sqrt{b/2\pi(1-T/T_{KTB})}]. \tag{2}$$

In this paper, we will take $\xi_-(T)$ to represent the size of the largest pairs.

The third aspect of the correlation length that is important in this paper is its behavior in an applied current I. The effect of an applied current is to unbind vortex pairs down to zero temperature and therefore to destroy the phase transition. As a result, the correlation length no longer diverges for finite I at T_{KTB} and has the following behavior:

$$\xi_\pm(T,I)\propto\frac{T}{I}f[I\xi_\pm(T,I=0)/T], \tag{3}$$

where f is a nonsingular function.

The next two intrinsic length scales are associated with finite-size effects and give a single vortex a finite ''bare'' energy. The first, which does not apply to superfluid helium, is the 2D penetration depth $\lambda_{2D}=2\lambda^2/d$. At distances less than this length from the vortex core, the superfluid velocity goes as $1/r$. Beyond this length, the superfluid velocity decreases as $1/r^2$.[27] In a ''perfect'' ($d=0$) superconductor,

$\lambda_{2D}=\infty$ and the superconductor behaves as a superfluid would. The second finite size length is sample size, which is the smaller of the sample width W and length L (typically $W \ll L$). The energy of a free vortex is

$$E_{FV}=[q^2/2]\ln(L_{fs}/\xi_0)+E_c, \tag{4}$$

where ξ_0 is the size of the vortex core and the finite size

$$L_{fs}=\min[\lambda_{2D},W]. \tag{5}$$

The vortex interaction strength q^2 and the core energy E_c depend on the system (SC, SF, or JJA). Finite energy for a single vortex means that there will be free vortices below the transition temperature, which in turn precludes a true phase transition as we will discuss in the next section.

2. Extrinsic

Three extrinsic length scales characterize the application of dc and ac electric and magnetic fields. The first is the length scale r_c,[2,28] which is the length scale probed by a dc applied current. For a superconducting film, the energy of a vortex pair with separation R is[24,29]

$$E(R)=[\pi n_s^{2D}\hbar^2/2m]\ln(R/\xi_0)-\pi\hbar IRd/eA+2E_c, \tag{6}$$

where the vortex interaction strength has been expressed in terms of the superconducting parameters $q^2=\pi n_s^{2D}\hbar^2/2m$ ($n_s^{2D}=n_s d$ is the areal superfluid density, n_s is the superfluid density, and m is the mass of a free electron,) and $A(=Wd)$ is the cross-sectional area through which the current I flows. For small separations ($R<r_c$), the 2D, logarithmic term dominates and the interaction is attractive. For $R>r_c$, the linear, current-induced term dominates and so the interaction is repulsive. As a result, the interaction energy peaks at

$$r_c=4k_BT_{KTB}We/\pi\hbar I, \tag{7}$$

where we have written the interaction strength in terms of the transition temperature,[29] $\pi n_s^{2D}\hbar^2/2m=4k_BT_{KTB}$. The nonlinear I-V relationship originates in thermally activated hopping over this barrier at a rate Γ which depends on the value of the vortex pair energy $E(R)$ at this separation: $\Gamma \propto \exp[E(r_c)/k_BT]$. Therefore, the dc I-V curves probe length scales of $O(r_c)$.[2,28]

In ac measurements (e.g., kinetic inductance) with circular frequency ω, the probing length is the diffusion length, $r_\omega=(14D/\omega)^{1/2}$, where D is the vortex diffusion constant.[1,2,30] This result is derived by analyzing the linear response of the dielectric constant. It should be pointed out that the two quantities r_c and r_ω do not compete *per se* with the other length scale but rather indicate the length scale being probed.

The final extrinsic length scale that we mention is due to an applied magnetic field and characterized by the average distance between field-induced vortices $l_B \simeq (\Phi_0/B)^{1/2}$ where Φ_0 is the superconducting flux quantum and B is the magnetic induction. The field-induced vortices are ''free'' and present at all temperatures, which precludes a true phase transition.

B. Existence of the phase transition

Because of λ_{2D}, Kosterlitz and Thouless originally wrote that this critical behavior would not apply to superconductors.[7] It was later realized[29,31] that, in practice, λ_{2D} can be larger than the system size and so superconducting films should not behave much differently than superfluid films. Whether λ_{2D} is larger than the system size or vice versa, there will be a finite density of free vortices below the transition temperature. The density of free vortices will be[29,32]

$$n_F \propto L_{fs}^{q^2/2k_BT}, \tag{8}$$

where q^2 here is the renormalized vortex interaction strength and, again, $L_{fs}=\min[\lambda_{2D},W]$. Note that system size and λ_{2D} enter Eq. (8) in the same manner, implying that one cannot tune the ratio of λ_{2D} to W to observe the transition. This is contrary to a perception in the literature[32–34] that the transition can be observed if $\lambda_{2D}>W$. This begs the question of why one should see critical behavior at all if there is no true phase transition in finite-size systems. The answer of course is that critical behavior can be seen if there is a diverging length. This occurs in the KTB system, provided that $\xi_\pm(T)<L_{fs}$.

C. Approaches to studying KTB dynamics in superconductors

A variety of approaches can be used to study KTB dynamic behavior. Here we review the two main approaches used to study the dynamics of superconducting and JJA films and that therefore determine a value for the dynamic critical exponent z. Brief mention of the methods used for superfluid helium systems will be made in Secs. III C and IV B 2. There are other approaches for investigating KTB behavior that do not uniquely determine a value for z; we will not review here but we will discuss them in Sec. IV B in the context of previous evidence that $z=2$.

In this section, we will first review the conventional results and its derivation. We will then generalize these formula to a general value of z. The dynamic scaling analysis will then be introduced and, finally, the connections between the two approaches will be discussed.

1. Conventional approach

In the first, more ''conventional'' approach, the current-voltage (I-V) isotherms are measured and analyzed in terms of their $I \to 0$ limit:[24,29]

$$V/I \propto I^{\alpha(T)-1}. \tag{9}$$

The signature of a KTB transition is a jump from nonlinear behavior below the transition temperature to Ohmic above the T_{KTB}:

$$V/I \equiv R(T) \propto \exp[-2\sqrt{b/(T/T_{KTB}-1)}], \tag{10}$$

where b is a nonuniversal constant. [Minnhagen[24] has generalized this to take into account the underlying superfluid. See Eq. (22).] In particular, the exponent $\alpha(T)$ will decrease linearly with increasing temperature until the transition temperature is reached at which point $\alpha(T)$ will, in the $I \to 0$ limit, jump from 3 to 1 (Ref. 29) because of the ''universal

jump'' in the superfluid density.[35] Indeed, the condition $\alpha(T=T_{KTB})=3$ is commonly used to determine the transition temperature.

Equation (9) is derived by determining $R(T,I)$ above and below T_{KTB} and using $V=IR(T,I)$. This derivation is well documented[28,29,36] but we will highlight the key points. To find $R(T,I)$, the density of free vortices n_f must be determined.

Below the transition temperature, free vortices in the limit of a weak current are produced by thermal activation over the barrier in $E(r_c)$ as mentioned above [below Eq. (6)]. This is done using the kinetic equation for the rate of change in the number of free vortices:

$$dn_f/dt=\Gamma(T,I)-n_f^2. \tag{11}$$

As mentioned above, $\Gamma(T,I)$ is the rate at which vortex pairs are unbound [$\Gamma(T,I)\propto\exp\{-E(r_c)/k_BT\}$.] The second term in Eq. (11) takes into account free vortices combining to form pairs. In a steady state,

$$n_F=\Gamma(T,I)^{1/2}\propto I^{q^2/2k_BT}. \tag{12}$$

Above the transition temperature,

$$n_f\propto\xi_+^{-2}, \tag{13}$$

since ξ_+ is the average distance between free vortices.

The final step in determining Eqs. (9) and (10) is to relate $R(T,I)$ to n_f:[28,29,37]

$$R\propto n_f. \tag{14}$$

Substituting Eq. (12) and Eq. (13) into Eq. (14) and using $V=IR(T,I)$ one arrives at Eqs. (9) and (10), respectively. Kadin *et al.*[28] have made extensions of this work to finite current. Many workers in the field however take Eq. (9) to be valid over wide ranges of I.

Equation (14) is based on the Bardeen-Stephen flux-flow formula[38] (as stated by subsequent authors[18,24]). For subsequent discussion, it will be useful to outline the derivation of this equation,[29] whose starting point is the electrodynamic Josephson relation

$$V=[\hbar/2e]d\Delta\phi/dt, \tag{15}$$

where $\Delta\phi$ is the change in the phase of the superconducting order parameter across the width of the sample. $d\Delta\phi/dt$ is proportional to the number of vortices that cross the width of the sample per unit time,

$$|d\Delta\phi/dt|=2\pi L n_F|v_D|, \tag{16}$$

where v_D is the vortex drift velocity. Finally, one assumes

$$v_D=\mu\pi\hbar I/eA, \tag{17}$$

where μ is the vortex mobility and e is the electron charge. Because the vortex mobility is taken to be local and therefore independent of n_F, Eqs. (15)–(17) result in Eq. (14): $R\equiv V/I\propto n_F$.

The linear relationship between $R(T)$ and n_f in Eq. (14) presumes single-vortex diffusion. If one were to allow more complicated dynamic or critical behavior, then Eq. (14) must be modified. The general expression for $R(T)$ in the critical region depends upon the dynamic exponent z:

$$R\propto\xi^{-z}\propto n_f^{z/2}. \tag{18}$$

Of course the two expressions are identical for $z=2$. Using Eq. (18) in place of $R\propto n_f$ in the conventional derivation,[18,28,29] one would find that $\alpha(T)$ will jump from $z+1$ to 1 in the $I\to0$ limit and that

$$R(T)\propto\exp[-z\sqrt{b/(T/T_{KTB}-1)}]. \tag{19}$$

We will discuss the ramifications of using $z\neq2$ in Eq. (18) in Sec. II C 3.

2. Dynamic scaling approach

Dynamical scaling is motivated by the observation of critical slowing down near a continuous transition. This phenomenon is well established for the KTB transition, and is marked by the divergence of the relaxation time scale τ. The dynamic scaling hypothesis asserts that critical slowing down is related crucially to the divergence of the static correlation length:[39] $\tau\propto\xi^z$.

In general, many types of dynamics may be associated with a particular static universality class, and these should fall into distinct dynamical universality classes.[40] For two-dimensional systems, including SC's, SF's, and JJA's, the conventional KTB dynamical theory[1,2] is consistent with model A universal dynamics,[40] and $z=2$. For bulk superconductors, however, the correct dynamical universality class is presently unclear, and seems *not* to belong to model A.[41]

In light of this situation, it is important to test the conventional theory by performing a scaling analysis, in which the dynamical exponent, z, and the scaling functions are *a priori* unspecified. z is then determined by collapsing different data sets onto a single curve. Although conventional KTB dynamical theories do not support such a general approach, a scaling ansatz has recently been proposed by Fisher, Fisher, and Huse[10] (FFH) for superconducting critical phenomena. This successful ansatz has been applied to a wide variety of systems and transitions, including simulations of KTB dynamics.[42] We expect that dynamical scaling will be appropriate in the same regimes where the static KTB theory is applicable.

In the FFH theory, for 2D superconductors, the I-V curves should scale as

$$V=I\xi^{-z}\chi_\pm(I\xi/T), \tag{20}$$

where $\chi_{+(-)}(x)$ is the scaling function for temperatures above (below) T_{KTB}. The two important asymptotic behaviors of $\chi(x)$ are $\lim_{x\to0}\chi_+(x)=\text{const}$ (Ohmic limit) and $\lim_{x\to\infty}\chi_\pm(x)\propto x^z$ (critical isotherm). The universal jump appears as the difference in (log-log) slopes between the two asymptotic limits of $\chi_+(x)$, as we will discuss below.

It is convenient to rewrite the Eq. (20) as

$$\frac{I}{T}\left(\frac{I}{V}\right)^{1/z}=\varepsilon_\pm(I\xi/T), \tag{21}$$

where $\varepsilon_\pm(x)\equiv x/\chi_\pm^{1/z}(x)$. The advantage of Eq. (21) over Eq. (20) is that one can better judge the scaling, because only the x scale is stretched in Eq. (21). In Eq. (20), both the x scale

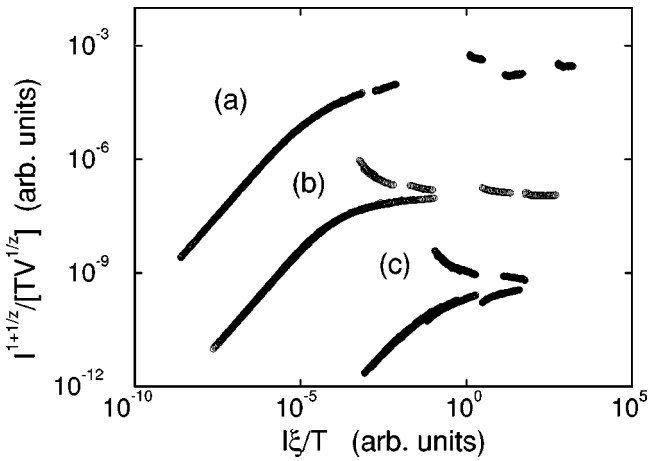

FIG. 1. The *I-V* curves from thin superconducting films scaled with Eq. (21): (a) 24-Å-thick YBCO layers in a multilayer structure from Vadlamannati *et al.*; (Ref. 44), (b) 24-Å-thick YBCO monolayer from Repaci *et al.* (Ref. 34), and (c) 100-Å-thick Hg/Xe film from Kadin *et al.* (Ref. 28). [Data sets (b) and (c) have been shifted arbitrarily.] The lower branch of these plots correspond to $T > T_{KTB}$. The limits of this branch are Ohmic in the weak-current limit to $V \propto I^{z+1}$ in the high-current limit. It is these limits that represent the jump in the exponent α.

and *y* scale are stretched, making it harder to judge a collapse of the scaled data. (Compare Fig. 4 of Ref. 12 with Figs. 1–4 here.)

3. Connections between the two approaches

While not apparent at first glance, the connections between the dynamic scaling approach and the conventional approach become clear when one considers the following. First, one must keep in mind that the conventional approach, as described in Sec. II C 1, is valid only in the limit $I \to 0$ while the dynamic scaling approach is valid for finite currents. Second, the relationship of one to the other should be

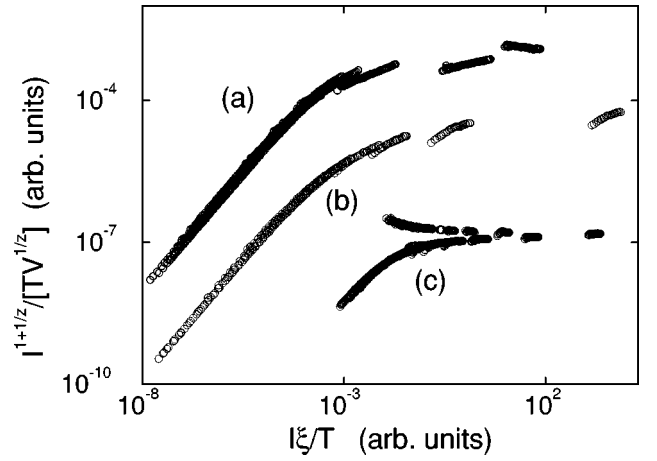

FIG. 2. The scaled *I-V* curves of (a) Ref. 45 on a YBCO monolayer, (b) Ref. 46 on a In/InO 150-Å-thick composite film, and (c) Ref. 43 from a 1000-Å-thick BSCCO crystal. [Data sets (b) and (c) have been shifted arbitrarily.] The collapse of curves (a) and (b) in this figure are not as complete as that of curve (c) or curves (a) and (b) of Fig. 1. Yet the collapse could not be improved using other values of *z*.

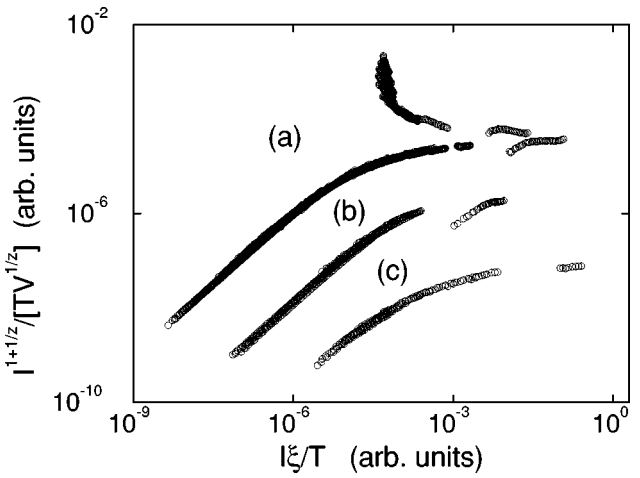

FIG. 3. The scaled *I-V* curves from Josephson junction arrays: (a) YBCO/Ag weak links from Ref. 49, (b) Nb-Ag-Nb proximity-coupled junctions from Ref. 12, and (c) Nb/Nb arrays of Ref. 48. [Data sets (b) and (c) have been shifted arbitrarily.]

considered for an arbitrary value of *z*. [For the generalization of the conventional theory to an arbitrary value of *z*, see the discussion around Eqs. (18) and (19).]

Taking these considerations into account and looking at the asymptotic limit $I \to 0$ of the dynamic scaling function, one finds that the two approaches are indeed compatible: (i) both theories predict that the critical isotherm ($T = T_{KTB}$) should be a power law $V \propto I^{z+1}$ [i.e., $\alpha(T_{KTB}) = z+1$], (ii) for $T < T_{KTB}$, both theories agree that the voltage remains a power law of the current, and (iii) for $T > T_{KTB}$, both theories give $\alpha(T) = 1$, with $R(T)$ defined as in Eq. (19).

III. SCALING RESULTS

In this section we will apply the scaling theory [Eq. (21)] of Fisher, Fisher, and Huse[10] to transport data from superconductors, Josephson junction arrays, and superfluids. The universality of the scaled data will then be checked. Preliminary results on both high-temperature superconductors (HTSC's) and low-temperature (conventional) superconduct-

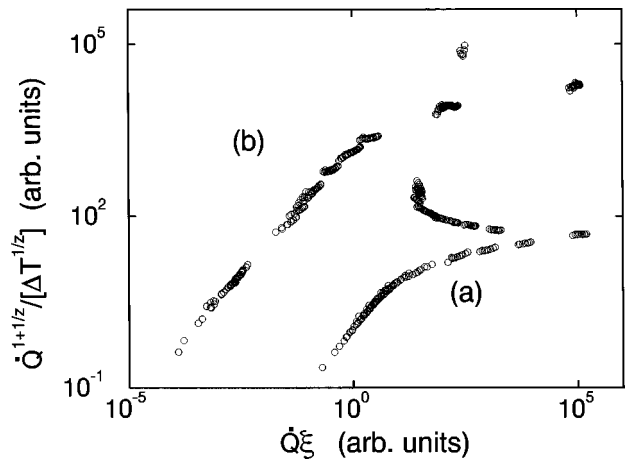

FIG. 4. The scaled $\dot{Q}$-ΔT curves from superfluid ^{4}He films of (a) Ref. 53 and (b) Ref. 54. [Data set (b) has been shifted arbitrarily.]

ors (LTSC's) were presented in Ref. 43 where it was concluded that $z \simeq 5.7 \pm 0.3$.

A preliminary estimate for the value of z indicates that $z \gg 2$ for these systems. The critical I-V isotherm ($T = T_{KTB}$) is easily identified on a log-log plot by the fact that it is straight [i.e., $\alpha(T_{KTB})$ is independent of current]. The slope of this isotherm [$\alpha(T_{KTB}) = z + 1$] gives an estimate for z. A visual check for this condition on the I-V data of Repaci *et al.*,[34] Vadlamannati *et al.*,[44] Matsuda *et al.*,[45] or any of the others clearly shows that, indeed, $z \gg 2$. With this initial evidence, we move on to a more rigorous scaling analysis of the data.

To perform the scaling analysis, $\xi_{\pm}(T)$ in Eq. (21) must be specified. This can be done for superconductors and Josephson junction array by exploiting the Ohmic limit of Eq. (21): $R(T) \propto \xi_{+}^{-z}$. (For superfluids, the thermal conductance K is used in place of the resistance: $K \propto \xi_{+}^{z}$.) For ξ_{-}, we will assume that the vortex correlation length is symmetric about the transition (modulo some prefactor) in this section. We will explore the validity of this assumption in Sec. IV A 1.

Note that in the dynamic scaling theory, there are no requirements for the temperature dependence of $\xi_{\pm}(T)$. In this work, we will assume that the KTB form $\xi(T) \propto \exp[\sqrt{b/(T/T_{KTB}-1)}]$ provides the most efficient parametrization of the correlation length. It is through this assumption that the explicit connection with KTB theory is made. Any other temperature dependence for $R(T)$ could be used to check the scaling collapse, which would leave the type of transition more ambiguous.

We determine $R(T)$ in two ways where possible. The first method is to extract it from the Ohmic part of the I-V curves and the second method is to digitize the $R(T)$ data. The two results are then compared to one another. In the case of discrepancies, the $R(T)$ determined from the I-V is used since thermal equilibrium is more likely in that case. Another advantage of using $R(T)$ determined from the I-V is that one is assured that the $R(T)$ is Ohmic. The disadvantage of course is that fewer data points are available for this $R(T)$. It should also be noted that we fit $R(T)$ only over the temperature range over which we have I-V isotherms.

The fitting parameters for Eq. (21) are z (universal) and T_{KTB} and b (nonuniversal). Three requirements must be fulfilled self-consistently in our scaling procedure: (i) $V \propto I^{z+1}$, along the critical isotherm $T = T_{KTB}$; (ii) $R(T) \propto \xi^{-z}$, in the high-temperature range; and (iii) scaling collapse of the I-V isotherms, according to Eq. (21). Condition (i), which says that the I-V curves are straight on a log-log scale at T_{KTB}, is used first to estimate a value of T_{KTB} and z. That value of T_{KTB} is then used in (ii) to fit the Ohmic resistance data to obtain an expression for $\xi_{\pm}(T)$. Finally, condition (iii) is checked. Because there may be a couple isotherms that appear to be straight on the log-log scale, this process is repeated, in the manner of Shaw *et al.*,[19] for the acceptable range of T_{KTB}'s to satisfy all three conditions.

In some of the data sets that we examined, the I-V curves crossed over to an Ohmic region at large I.[32] This behavior is not due to the critical behavior of the vortices but rather to a breakdown of the underlying superfluid. For that reason, we have omitted such data from our analysis.

Note that in the following, we have displayed all the scaling results (except for a few noted exceptions) without first making a judgment of the quality of the data. As a result, the quality of the scaling also varies. Nonetheless, we stress that each scaling result displayed here has been optimized for the best collapse and not for a value of z in agreement with the other samples. This makes the result that all of the collapses occur for roughly the same value of z all the more striking. Furthermore, we believe that there is a strong correlation between the quality of the raw I-V data and the scaled data. But we leave this for the reader to judge.

A. Superconducting films

Figure 1 shows the scaling of three separate data sets to Eq. (21) (previously reported in Ref. 43 but plotted on a different scale here.) The first data set[44] [marked *a* in Fig. 1] covers a temperature range 30.06–46.09 K and is from a $YBa_2Cu_3O_{7-\delta}$ (YBCO)/$PrBa_2Cu_3O_{7-\delta}$ multilayer in which the YBCO layers have a thickness of 24 Å and are reported to be nearly electrically isolated from one another by $PrBa_2Cu_3O_{7-\delta}$ (PBCO) barrier layers. The scaling procedure leads to the results $T_{KTB} = 32.0$ K, $b = 14.0$, and $z = 5.6 \pm 0.3$ where the resistance was fit over the range 43.5–47.0 K. (See Fig. 8 below.) Curve (b) in Fig. 1 is the scaled I-V data from Ref. 34 taken on a 12-Å-thick YBCO monolayer and includes isotherms ranging in temperatures from 10 to 40 K. The resistance was fit over the range 25–34.5 K, yielding, along with the other two criteria, the following parameter values: $T_{KTB} = 17.6$ K, $b = 7.79$, and $z = 5.9 \pm 0.3$. The scaled data set denoted by (c) in Fig. 1 and covering the temperature range 2.6–3.4 K corresponds to a conventional, 100-Å-thick, superconducting sample[28] (Hg-Xe alloy). The parameters which led to this collapse are $T_{KTB} = 3.04$ K, $b = 3.44$, and $z = 5.6 \pm 0.3$. The resistance was fit over the temperature range 3.3–3.4 K. While this collapse is not as complete as those of the others in this figure, we emphasize that the best collapse was obtained for the reported value of z.

A few features of the scaled data in Fig. 1 should be pointed out. First, the upper branch corresponds to temperatures below the transition temperature and the lower branch to temperatures above the transition temperature. Second, an Ohmic I-V relationship here is represented by a slope 1 on the log-log scale. One can see in each of the three curves in this figure (and the scaled data in the following figures) that the lower ($T > T_{KTB}$) branch is Ohmic at low values of the scaling variable x (typically low currents) and curves over and approaches a horizontal line as x is increased. The horizontal line corresponds to the I-V relation, $V \propto I^{z+1}$.

For this paper, we have extended this analysis to two more 2D superconducting films. The collapse is shown in Fig. 2. The curve marked (a) in that figure is from a monolayer of YBCO sandwiched between two PBCO layers of different thicknesses[45] and covers isotherms ranging from 16.32 to 41.31 K. For the parameters, we find $T_{KTB} = 18.3$ K, $b = 31.04$, and $z = 5.3 \pm 0.5$. The value of T_{KTB} is similar to that of the YBCO monolayer of Ref. 34 but the value of (b) is nearly 4 times larger. Curve *b* in Fig. 2 is from I-V data on 150-Å-thick In/InO composite film from Ref. 46. The I-V isotherms cover a temperature range 3.010–3.182 K. Here, the resistance data determined from the I-V curves covered only a limited range and did not match well

the $R(T)$ data from Fig. 3 of that paper. For these reasons, the scaling criterion (ii) was not fully met. Nevertheless, the scaling collapse occurred for a value of z near that of the other samples: $T_{KTB}=2.97$ K, $b=10.21$, and $z=5.2\pm0.5$.

We also applied this scaling analysis[43] to a thicker (1500 ± 500 Å) $Bi_2Sr_2CaCu_2O_{8+\delta}$ crystal. For such a thick crystal of a layered material, one would expect a crossover to 3D behavior near the critical temperature and a failure of the 2D scaling and perhaps a breakdown in the scaling. We however did not see any breakdown in the scaling for this sample. Indeed, as shown in curve (c) of Fig. 2, we found a good collapse of the data with the 2D scaling form with $z=5.6\pm0.3$, $T_{KTB}=78.87$ K, and $b=0.20$. (See also Fig. 2 of Ref. 43.) A difference between this scaled data and the others was found when we looked at universality. We examine this issue in Sec. III D and discuss the reason why a thicker crystal may scale in the same way as the thinner samples in Sec. IV A 3.

A scaling analysis of the Hebard-Fiory I-V data on a In/InO film[18,47] was not possible since the temperatures of their isotherms were not published.

B. Josephson junction arrays

I-V characteristics of Josephson junction arrays are expected to be similar to that of superconducting films.[33] In this section, we apply Eq. (21) to data from two JJA systems.[48,49] A primary difference for these systems is that their resistance is not described by Eq. (19) because the temperature is renormalized and depends upon the temperature-dependent critical current $i_c(T)$. In the data that we analyze below, we did not have access to $i_c(T)$ and so we could not determine if $R(T)$ followed the KTB behavior for JJA's. Nonetheless, following the discussion at the beginning of this section (Sec. III), we will use Eq. (19) to check for a collapse of the data according to Eq. (21). As a result, the value of b will not have the significance it had in Sec. III A.

Curve (a) in Fig. 3 is scaled I-V data from a high-temperature superconducting Josephson junction array (YBCO/Ag),[49] where the parameters used were $T_{KTB}=74.3$ K, $z=5.8\pm0.4$, and $b=0.72$. The resistance was fit over the temperature range 78–90 K. One can see that the collapse is very good except for the isotherms furthest below T_{KTB}. This breakdown could be because those isotherms are out of the critical region. An attempt to optimize the $T\ll T_{KTB}$ collapse by letting the correlation length be asymmetric in accordance with Eq. (2) was unsuccessful.

The scaled data denoted by curve (b) in Fig. 3 is from a Nb-Ag-Nb proximity-coupled junction array.[12,50] (We used the ''100%'' data from Ref. 50.) The parameters which produced the best fit are $T_{KTB}=6.84$ K, $z=5.8\pm0.3$, and $b=0.32$ and the resistance fit was over the temperature range 7.3–7.8 K. The scaled I-V's covered the range 6.9–7.8 K. One can see that the 6.9 K data set does not scale well for low currents. We believe that this is not a real effect since all of those noncollapsing data have a voltage $V<10^{-9}$ V and do not parallel the behavior of other data in that range. The data do scale well for $V>10^{-9}$ V . Harris *et al.*[12] have previously checked the dynamic scaling of these data and concluded that $z=2$. We believe that they would have reached the same result as found here had they allowed z to vary to optimize the scaling and had they used Eq. (21) instead of Eq. (20).

Curve (c) in Fig. 3 is the scaled I-V data from a Nb/Nb Josephson junction array.[48] The parameters used to optimize this were $\tau_{KTB}=0.51$ K, $z=5.5\pm0.5$, and $b=5.7$, where $\tau=k_BT/[(\hbar/2e)i_c(T)]$. A collapse of the scaled data could be obtained over a relatively large range of z and T_{KTB}. We attribute this to the fact that the I-V's isotherm only covered two to three decades of voltage and that there were no isotherms for temperatures below T_{KTB}. The resistance was fit over the range $\tau=0.8$–1.3.

In a conventional analysis, Abraham *et al.*[51] reported a jump in the I-V exponent α of 3 to 1 for PbBi/Cu arrays, a result which implies $z=2$. As far as we could tell, those data were not published and so were not available for our analysis. We tried to apply Eq. (21) to sample 6-18-81 of Ref. 52 but no definitive conclusions were reached due to the fact that the I-V isotherms only covered two to three decades of voltage and a limited temperature range.

C. Superfluid ^{4}He films

For superfluid ^{4}He films, the analogs of electrical conductance and I-V curves are thermal conductance and $\dot{Q}$-ΔT curves, where $\dot{Q}$ is power through the film and ΔT is the temperature gradient across the film. These measurements are done by injecting heat at one end of a thin superfluid film adsorbed on a surface (e.g., Mylar). Because of dissipation from vortex pairs, there is a temperature gradient across the film that is measured and is the analog of the voltage in the superconducting measurements. In reality, $\dot{Q}$ is not the heat through the film but rather represents ^{4}He mass flow from the cool end of the film to the warm end, which occurs to replenish the ^{4}He which evaporates from the warmer end at a quicker rate than from the other end. This is thoroughly discussed in Ref. 53.

As we shall shortly see, these results do show that $z\simeq5.6$. However, we are careful to point out that the results we are about to present can only be said to be consistent with such a value of z but cannot be taken to be evidence that $z\simeq5.6$ for two reasons. First, there are no reliable thermal conductance K data in the ''Ohmic'' limit (i.e., $T>T_{KTB}$ zero-power $\dot{Q}$ limit). Furthermore, one has to account for the conductance of the gas K_g surrounding the film which can only be approximated to within a factor of 2. These two points allow us more freedom to obtain the optimal scaling. Nonetheless, the best scaling does yield a z which is similar to that of superconductors and Josephson junction arrays.

The collapsed data in Fig. 4 marked curve (a) are from Ref. 53. Instead of varying the temperature, those authors varied the film thickness d which in turn varies the transition temperature. The independent variable then is d and not T and so the correlation length depends upon $|d-d_c|$ instead of $|T-T_{KTB}|$ where d_c is the thickness whose T_{KTB} corresponds to the temperature of the experiment. [See Eq. (17) of Ref. 53.] By adjusting d, they were able to obtain $\dot{Q}$-ΔT data for both above and below the transition temperature. The parameters that we obtained were $d_c=5.4$, $z=5.4\pm0.4$, and $K_g=8.0\times10^{-4}$ W/K. We assumed that K_g was a constant

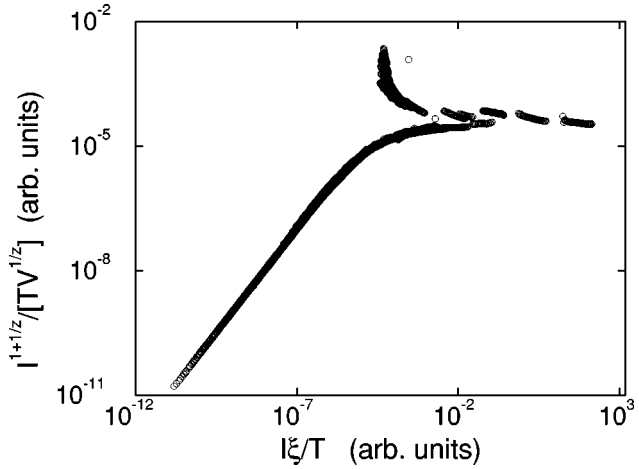

FIG. 5. The JJA data of Fig. 3(a) (Ref. 49), the YBCO SC data of Fig. 1(b) (Ref. 34), and the Hg/Xe SC data of Fig. 1(c) (Ref. 28) plotted together. The scaled data sets have been shifted to show universality of the scaling function.

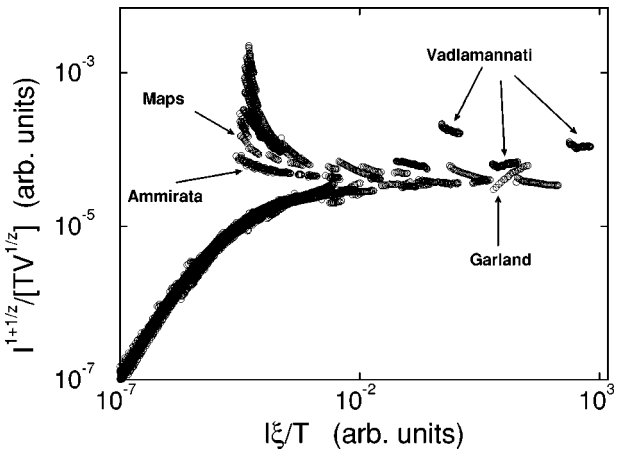

FIG. 6. The data from all of the data sets plotted together. The $T>T_{KTB}$ data collapse very well, suggesting universality of the scaling function for that temperature regime. The $T<T_{KTB}$ data also scale well if three data sets, belonging to layered materials [labeled Ammirata (Ref. 43) and Vadlamannati (Ref. 44)] and superfluid helium [labeled Maps (Ref. 53)], are neglected. This is discussed in the text. The Garland-Lee data (Ref. 46) scaled well except for the anomalous isotherm just above T_{KTB}, labeled Garland (Ref. 46).

over the parameter space that the $\dot{Q}$-ΔT curves covered and estimated it based on the small-d behavior of K. This placed some limits on this quantity but we were still able to vary it by a factor of 2. We found that the scaling collapse was relatively insensitive to the value of K_g because most of the power was flowing through the film. As one can see, the scaling starts to break down for thicknesses ($d\sim 9$ layers) much larger than d_c.

Curve (b) in Fig. 4 is from Ref. 54. It contains more data corresponding to $T>T_{KTB}$ but does not cover as wide a range of thicknesses overall as the data of Maps and Hellock[53] The collapse was found to be consistent with the results we have presented in this paper: $d_c=5.2$, $z=5.6\pm 0.5$, and $K_g=5.0\times 10^{-4}$ W/K. Like the JJA data of Ref. 48, the error bars here are large because the data extend only over a couple orders of magnitude.

D. Universality

One of the fundamental properties of critical behavior and scaling is universality: the idea that the same function $\varepsilon_{\pm}(x)$ in Eq. (21) and the same value of z describe all of the data independent of the system or material. For the $T>T_{KTB}$ branch, we found universality for nearly all of our scaled data. For the $T<T_{KTB}$ branch, on the other hand, the same function $\varepsilon_{\pm}(x)$ described the data sets of the thinnest 2D samples but not that of the layered materials[43,44] or superfluid helium.[53] We explore these issues in this section.

In Fig. 5, the HTSC data from the YBCO monolayer,[34] the LTSC data from the Hg/Xe thin film,[28] and the YBCO JJA data[49] are plotted together. Because the scaling functions are dimensionless, we multiply the x and y axes by nonuniversal constants which enter the scaling theory to account for the sample dependence. In order to test universality, all the data sets must be scaled with the same value of z. So we have adjusted each data set within the error bars of the parameters so that each has $z=5.6$. As one can see, the collapse is very good, and bolsters the evidence that these data scale and that $z\simeq 5.6\pm 0.3$. The agreement of the data of Repaci *et al.*[34] and Herbert *et al.*[49] is particularly impressive since both data sets have extensive $T<T_{KTB}$ branches and because they come from two different systems: JJA's and SC's.

In Fig. 6, we plot all of the SC, JJA, and SF data that we have presented here in a single plot to test universality. As in the previous figure, we adjusted each data set within the error bars so that $z=5.6$. We also removed some of the individual scaled isotherms that did not scale well with the other scaled isotherms from the same sample. This includes two of the isotherms from the data of Matsuda *et al.*[45] [the second and third from the right in data set (a) of Fig. 2], parts of two of the data sets from data set (b) of Fig. 2 (second and third from the right), and part of one data set from Harris *et al.*[12] [the rightmost set labeled (b) in Fig. 3]. One can see clearly that the $T>T_{KTB}$ data scale well and strongly suggest universality for this temperature regime. The Garland-Lee[46] data are the weakest of these since their lowest-temperature isotherm does not lie on the other scaled data as we have pointed out in this figure.

Universality was not found for all of the $T<T_{KTB}$ data. Besides the data plotted in Fig. 5, there are only a few data sets that have significant $T<T_{KTB}$ branches: the HTSC BSCCO thin crystal of Ref. 43, the ^{4}He data of Ref. 53, and the YBCO multilayer data of Ref. 44. (We have labeled each of these data sets in Fig. 6.) The low-temperature branches of these data sets fail to collapse with the other data. It is likely that the BSCCO and YBCO multilayer data fail to collapse in this temperature range due to their layered structure. We will discuss this point in Sec. IV A 3. As to why the $T<T_{KTB}$ ^{4}He scaled data do not collapse with the other data, a possible explanation is that the approximate determination of $\xi(T)$ was inaccurate. (Recall the lack of ''Ohmic'' conductivity data for the superfluid measurements.) We recommend further studies of such data to more accurately determine $\xi(T)$ and to check the universality. In short, we cannot conclude whether the lack of universality of the scaling function for $T<T_{KTB}$ indicates a breakdown of the dynamic scaling results or if it is due to the data or systems considered. We believe it is the latter.

IV. DISCUSSION

The main result of Sec. III is that $z \simeq 5.6$ for superconductors, superfluids, and Josephson junction arrays. The large value of z is nearly 3 times the expected value for these systems. Indeed there are many reports for superconductors and Josephson junction arrays that the I-V exponent α jumps from 3 to 1 at the transition temperature, a result that is consistent with $z=2$. In this section, we explore the discrepancy between the approaches by discussing the validity of each.

A. Dynamic scaling

Dynamic scaling is a powerful technique which has proved particularly useful in investigating the nature of the H-T phase diagram in high-temperature superconductors. Yet one must be careful with the results obtained with a scaling analysis because this technique is not without its weaknesses. For example, in Sec. III D, universality of the scaled data was examined, and while universality was found for the $T>T_{KTB}$ data and some of the $T<T_{KTB}$ data, it did not hold for all of the latter. In this section, we will examine other aspects of the dynamic scaling analysis presented here and its validity.

1. Vortex correlation length

There are various aspects of assumptions that we made regarding the vortex correlation length that may detract from the dynamic scaling analysis. Here we make considerations on its symmetry, the value of the nonuniversal constant b, and the expected range of the validity of the temperature dependence.

Throughout Sec. III it was assumed that the vortex correlation length was symmetric about the transition temperature even though Ambegaokar *et al.*[2] predict that it should be asymmetric. [See Eq. (2).] Surprisingly, where there were data for both above and below the transition temperature, the scaling did not seem to suffer from this assumption. This is particularly so for the data on the superconductors. One notices that for the data of Refs. 28, 34, and 43, the scaling works well for both branches. (See Figs. 1 and 2.)

When we allowed the correlation length to be asymmetric by allowing for different values of b below T_{KTB} (b_-) and above T_{KTB} (b_+), significantly better scaling could not be achieved, possibly indicating that ξ is symmetric, in accordance with the scaling of the numerical results of Lee and Teitel.[42] There were of course a few exceptions,[49,53] but in neither case was prediction $b_+ = 2\pi b_-$ (Ref. 2) verified. In fact, in these two cases, b_- tended to be larger than b_+. For the superfluid ^{4}He data of Ref. 53, we found $b_- \simeq 3b_+$. We do not view this result as conclusive because of the aforementioned problems with determining $\xi(T)$ for helium and we suggest further study of this topic.

In our fitting of the resistance $R(T)$ to determine $\xi_\pm(T)$, we were sometimes able to fit the resistance to the Kosterlitz form over an extended temperature range. A notable example is the YBCO monolayer data[34] where we fit the resistance from 25 to 35 K. The upper limit of this fit is twice the T_{KTB} which is remarkable since true KTB critical behavior is expected to be valid over a very narrow temperature range.[24,56] This result could suggest that the critical region is larger than expected or, more likely, that KTB-like behavior remains valid outside the critical region.[24] Indeed, Minnhagen and Olsson[57] indicate that Eq. (1) is a useful phenomenological form over wider temperature regimes as long as b is taken to be a phenomenological parameter.[42] We adopt this interpretation.

In the literature, it is commonly stated that b is material dependent but that $b=O(1)$.[8] In our results for superconductors, we found b to roughly $O(10)$ but as small as 3.44 and 0.2, thereby varying by an order of magnitude or two. We also found that its value could vary within materials. For example, the value of b for the two-YBCO-monolayer systems examined in Sec. III A varied by a factor of 4,[34,45] which is concerning. However, we do not think it is troubling that the b varied from material to material by an order of magnitude. The systems considered here are diverse from one another. For example, the electron density can vary significantly from the conventional superconductors to the HTSC's. Another interpretation is that the value of b is a phenomenological one and not equivalent to its true asymptotic critical value.[42,57]

2. Universal jump: 6.6 to 1

If one is to believe that $z \simeq 5.6$ in these materials, then one expects to see a jump in the I-V exponent α of 6.6 to 1 in the $I \to 0$ limit. Yet evidence of this is not observed in any of the samples. We believe this is because previous measurements have not gone to weak enough currents to observe this behavior. For example, based on the scaling curve of the YBCO monolayer data[34] in Fig. 1(b) where $T_{KTB}=17.6$ K, one can see that the scaling curve is Ohmic for $x \lesssim 10^{-5}$ (x is the scaling variable). This means that the 18 K isotherm would not become Ohmic until the current $I \lesssim 10^{-11}$ A. Clearly, voltage sensitivity is far from detecting that crossover. [In fact, that isotherm would become first Ohmic due to finite-size effects at a much higher current (Sec. IV C 1).] It is a similar situation for the data of Herbert *et al.*[49] where the scaling curve is Ohmic for $x \lesssim 10^{-6}$. The 74.722 K isotherm, the first isotherm above $T_{KTB}=74.3$ K, would not become Ohmic until the current $I \lesssim 10^{-8}$ A. This is several orders of magnitude smaller than the minimum current for that isotherm.

The ''jump'' is evident in the scaled data only in the following way. For the $T>T_{KTB}$ data, the scaled data go from Ohmic behavior ($\alpha=1$) for small values of the scaling variable to $\alpha=6.6$ at large value of the scaling variable. In this way, the ''jump'' is only manifest as a crossover from the small-x behavior to the large x behavior of the $T>T_{KTB}$ branch of the scaled data.

In Sec. IV C 2, we will use a conventional approach to show that the behavior of α for the YBCO monolayer data[34] is consistent with the value of z that we found for that sample: $z \sim 5.9$.

3. Three-dimensional effects

We have examined two samples which could be viewed as layered. (The first layered sample that we examined was the YBCO/PBCO multilayer system of Ref. 44 where the two unit-cell-thick YBCO layers were believed to be electri-

cally isolated. The second sample was the 1000-Å-thick BSCCO crystal.[43]) In layered superconductors, three-dimensional behavior is expected in a small region of the current-temperature phase diagram near T_{KTB}. One could then ask why such samples would scale in the same way as the much thinner samples and why 3D effects are not manifest.

To discuss the 3D effects, two new lengths are introduced. The first is related to the energy of the vortex pair. This length is the Josephson length $\lambda_J = \gamma s$, which incorporates the effects due to Josephson coupling between the layers. (γ is the anisotropy ratio for layered superconductors and is equivalent to the ratio of the coherence length in the ab planes to the coherence length in the c direction: $\gamma \equiv \xi_{ab}/\xi_c$. s is the distance between layers.) For separations less than λ_J, the vortices interact with the 2D (logarithmic) potential. For larger separations however, the potential becomes linear due to the Josephson coupling between the layers.[58] (The Josephson coupling also introduces a term to the interaction energy for lengths less than λ_J, but that term is very small compared to the 2D logarithmic one.)

The second length is the c-axis vortex correlation length ξ_c^v (to be distinguished from the c-axis superconducting coherence length) and characterizes how far along the z direction the vortices are correlated. This length can also be viewed in terms of the length-scale-dependent layer decoupling length $l_{3D/2D} = \gamma \xi_c^v$ (defined for $T > T_c$) since it determines the extent over which the 3D effects are important in the in-plane direction.[59] Because of vortex screening, the Josephson interaction is screened out beyond lengths $l_{3D/2D}$, making the interaction 2D at large separations for $T > T_c$. $l_{3D/2D}$ becomes small quickly above the transition temperature.[59]

Because of these two competing lengths, 3D behavior is expected only over a small range of the I-T phase space. On the temperature axis, this region is limited by $s < \xi_c^v(T) < D$, where D is the thickness of the sample. On the I axis, the 3D region is limited to intermediate currents: $\lambda_J < r_c < l_{3D/2D}$. Above T_{KTB}, the renormalized γ (and hence λ_J) grows quickly[60] while $l_{3D/2D}$, because of the temperature dependence of ξ_c^v, decreases rapidly, further limiting the 3D behavior. This leaves only a small window in which 3D near T_{KTB} effects can be observed.

Returning our attention to the layered samples that we examined here, it is plausible that 3D effects are being seen. After all, it is primarily the layered superconductors of Refs. 44 and 43 that do not obey universality. For the multilayer sample,[44] the $T > T_{KTB}$ scaled data fail to collapse with the other curves (see Fig. 6) for the isotherms nearest T_{KTB}. That this sample could have 3D behavior is not in contradiction with the reports of those authors that their layers are electrically isolated[44] since magnetic coupling extends over larger distances. For the BSCCO sample of Ref. 43 whose $T < T_{KTB}$ data do not collapse with the others in Fig. 6, we have done the following calculation based on our above discussion to try to estimate where 3D effects should be seen. Since $\gamma \simeq 160$ for BSCCO 2212, 3D effects should start to occur for currents $\lesssim 1.2$ mA (where $r_c \gtrsim \lambda_J$) and persist up to the current associated with the minimum of L_{fs} (~ 2.4 mA) or the decoupling length, very near T_{KTB}. Based on these rough estimates, which do not incorporate renormalization effects, it seems unlikely that 3D effects could be observed in these samples for $T > T_{KTB}$. For the $T < T_{KTB}$ branch, it is more likely that the deviation from the universal curve is due to the thickness of that crystal since it is well known that the 3D region is much larger below the transition temperature than above.[55,61]

B. Validity of ''conventional'' results

In the previous section (Sec. IV A), we have addressed the validity of the dynamic scaling, whose results indicate that $z \simeq 5.6$. Since this contradicts the evidence from the conventional approach that $z = 2$, we now address those results. We do not claim that each paper is incorrect in their claims of $z = 2$ but we do hope to convince the reader that the question of the value of z is still an open one.

The conventional results fall into roughly two classes, dc and ac. The dc measurements are the most common and include the determination of $\alpha(T)$ from the I-V measurements. dc magnetoresistance measurements have also been used but less frequently. The ac measurements include the torsion oscillator ^{4}He measurements of Bishop and Reppy[20,21] and the ac inductance measurements of Fiory, Hebard, and Glaberson.[18] Clearly, we cannot address each paper that reports evidence for $z = 2$ and so we will discuss them in broad terms using particular examples where appropriate.

1. I-V and R(T) dc measurements

Most of the papers that report evidence for a KTB transition or KTB behavior make their determinations based only on $\alpha(T)$ and $R(T)$ measurements. (There are a few notable exceptions to this that we discuss below.[18,46]) We suggest here that such an approach cannot be taken as evidence of KTB behavior and $z = 2$ but only as being consistent with $z = 2$ within the conventional approach.

We begin with a brief description of this method. From the I-V data, a value of T_{KTB} is determined from the condition $\alpha(T = T_{KTB}) = 3$ (which, of course, assumes $z = 2$). It is then that this value of the transition temperature is consistent with the $R(T)$ data and the Minnhagen[24] form of the resistance,

$$R(T) = A \exp[-2\sqrt{b(T_{c0} - T_{KTB})/(T - T_{KTB})}], \quad (22)$$

or the traditional form for $R(T)$ [Eq. (14).] The mean-field temperature T_{c0} and constant A, in addition to b, amount to three fitting parameters. [If Eq. (10) is used, then there are only two fitting parameters.] A further check that is sometimes used is to verify that $\alpha(T)$ decreases quickly above T_{KTB}.

There are several reasons why this approach can be a misleading check of $z = 2$. The first and most important is that determining where $\alpha(T) = 3$ is difficult. It is well known that the predicted jump in the I-V exponent $\alpha(T)$ exists only in the $I \to 0$ limit[28,32,34,62] and that, for temperatures above the transition temperature, the value of $\alpha(I,T)$ can vary quickly from a value of 1 to a value of $z+1$ as a function of current. Hence, one must be sure that the value of reported

$\alpha(T)$ will not dip to a lower value at currents whose voltages are below the voltage sensitivity.

(Beyond the problem of probing the weak current limit, actually detecting the jump can be very difficult because, in the presence of finite-size effects, small magnetic fields, and disorder, the jump gets rounded considerably, due to the many competing length scales. Note that most papers[44,47,63,64] do not report a jump in the I-V exponent.)

The second problem with this approach is that, most of the time, a misleading criterion is used to determine $\alpha(T)$. Because it is seldom (if ever) the situation that one knows that the value of $\alpha(T)$ determined for a particular isotherm represents the weak current limit, one should examine $\alpha(T)$ for a given length scale (or common current), as originally suggested by Kadin *et al.*[28] Here, one looks for a change in the behavior of this quantity near the transition temperature. This approach is usually not followed rigorously, however. Instead of using a common current, investigators will determine $\alpha(T)$ for a common voltage range. This has the effect of biasing the results because, as one looks at higher-temperature isotherms at common voltages, one is looking at lower currents. This means that $\alpha(T)$ will have decreased not only due to temperature but also due to longer length scales. Therefore, any report rapid decreases of $\alpha(T)$ may be an artifact of examining the isotherms at a constant voltage. [There are also cases where not even a common voltage range is used; rather, the parts of the isotherms that deliver the desired rapid change in $\alpha(T)$ are studied.]

Third, once a value of T_{KTB} is obtained where $\alpha=3$, the number of fitting parameters in Eq. (22) used to verify the self consistency is large (3) and the temperature range over which one fits the $R(T)$ data is limited. So we contend that the conditions for checking $z=2$ in this approach are not stringent enough to be considered as evidence.

Finally, we note the inherent problem with this approach mentioned above, that z is not allowed to vary in order to optimize the analysis.

As an example, in Ref. 49 a jump in the I-V characteristics is reported for the YBCO/Ag weak-link JJA. [See Fig. 2, inset, in Ref. 49(a) or Figs. 5-7 and 5-8 in Ref. 49(b).] The sharp jump in Ref. 49 was obtained by fitting the Ohmic part of the curves at a temperature just above where the I-V has $\alpha=3$. In this case, no fit of $R(T)$ was done to check the value of T_{KTB} for consistency. Also, one cannot rule out that, if those authors had another decade or two of voltage sensitivity, isotherms at temperatures lower than their T_{KTB} would also become Ohmic at lower currents. Further, as is clear from the scaling analysis shown in Figs. 3(a) and 5, these data are consistent with $z=5.6$. As a second example of this, we look at the YBCO monolayer data of Ref. 45. By inspecting Fig. 4 of the reference, it is clear that some of the isotherms below their reported T_{KTB} could manifest Ohmic behavior if more decades of voltage sensitivity were possible. An inspection of the I-V data of Ref. 44 (Fig. 2) yields a similar conclusion. It is unlikely that the reported value of $\alpha=3$ at $T=T_{KTB}=40.1$ is the asymptotic value of that quantity at low currents. [The voltage sensitivity for that data also occurs at a larger value ($\sim 10^{-7}$ V) than that of other measurements ($\sim 10^{-9}$ V).]

To summarize, I-V along with $R(T)$ measurements do provide self-consistent results for $\alpha(T=T_{KTB})=3$ (and thus $z=2$), but cannot be taken as proof that $z=2$. This is because the flexibility in determining T_{KTB} and the three fitting parameters used in Eq. (22) to fit the smooth, monotonic $R(T)$ data do not pose tight enough constraints.

2. Kinetic inductance on SC's, magnetoresistance, and ^{4}He torsion experiments

We now turn to other measurements used to study dynamics in the SC, SF, or JJA system.

As mentioned above, there are a few notable papers that went beyond measuring only I-V and $R(T)$ curves. The most comprehensive study was done by Fiory, Hebard, and Glaberson[18] who looked at kinetic inductance and magnetoristance in addition to the usual I-V and $R(T)$ measurements on In/InO systems. The thoroughness of their approach is commendable. While their results seem to be consistent with $z=2$, they are not without their inconsistencies and cannot necessarily be taken as definitive evidence for $z=2$.

Fiory *et al.*[18] were able to determine the vortex interaction strength q^2 in two ways for a rectangular strip sample of In/InO. In their ac impedance measurements used to determine the kinetic inductance, the frequencies are small (160 Hz) and so they are probing only statics. In that case the kinetic inductance does not depend upon the density of free vortices n_f but only the superfluid density. In that case, q^2 is directly determined. In their I-V measurements, a dynamic value of q^2 is determined. They find that the values of q^2 from the two measurements agree over a limited temperature range (~ 30 mK), which would be consistent with a value of $z=2$. (See Fig. 9 of Ref. 18.) However, this result should be viewed cautiously since the two measurements disagree for most of the region $T<T_{KTB}$. Further, the four-probe contact method that they used to measure the data for the comparison of the values of q^2 is less accurate and less sensitive[18] than the two-coil contactless method that they used on circular sample from the same film. Moreover, their measurements of samples from the same film revealed variations in T_c that suggest that sample inhomogeneities may be significant.

To further test the value of T_{KTB} for the In/InO films, Fiory *et al.*[18] measured the magnetoresistance $R(H)$. According to the theory of Minnhagen,[65] $R(H)$ should be linear in H at T_{KTB} but sublinear above it and faster than linear below it. Their data at $T=1.782$ K do show a region which is nearly linear ($R\propto H^{1.07}$) over roughly two decades. This temperature for crossover is roughly consistent with their $T_{KTB}=1.782$ K determined from $\alpha(T=T_{KTB})=3$ criteria. As mentioned above, the roughly linear area is over only two decades and the samples do have a degree of inhomogeneity to them. Further, it is likely that surface barriers should be taken into account.[66]

Garland and Lee[46] have also used magnetoresistance data in addition to the I-V and $R(T)$ measurements on In/InO films. Based on their Minnhagen criteria for $R(H)$ they determine $T_{KTB}=3.123$ K. At this temperature, they find that $\alpha(T)$ drops from a value of roughly 4 to 1. One will also notice that their I-V at T_{KTB} is not a pure power law as required by the dynamic scaling and that their $T<T_{KTB}$ isotherms all have a positive curvature. They attribute this be-

havior to field-induced vortices. It is clear from Fig. 7 in their paper that the field plays a role for fields at least as low as 5 mG. Yet, in keeping with the discussion on finite-size effects in Sec. IV C 1, it is our view that the crossover to Ohmic *I-V* in their Fig. 5 should occur at a roughly common value of the current. This is because the magnetic length scale should not change significantly over the 100 mK that their *I-V* isotherms cover below their claimed T_{KTB}. This is not observed in those data. Choosing T_{KTB} at a lower temperature seems to be a better explanation, especially when one considers the good collapse of their data in the dynamic scaling analysis as seen in Fig. 2(b).

^{4}He torsion experiments[20,21] were among the first evidence for the Kosterlitz-Thouless-Berezinskii transition. Nevertheless, while we agree that these measurement are indicative of KTB behavior (i.e., vortex pair unbinding), we do not believe that they indicate $z=2$. In the case of ^{4}He torsion experiments,[21] Bishop and Reppy measured the period shift and *Q* value of an Andronikashvili cell. The former is proportional to the real part of the dielectric constant ϵ and the latter to the imaginary part of that quantity. It is the latter that has the predominant dependence on the free vortex density n_f. [See their Eq. (A2) or Eqs. (3.17) and (3.18) in Ref. 2.] They implicitly assume $z=2$ in writing $\mathrm{Im}(\epsilon)\propto n_f$. We point out that their method cannot distinguish a value of *z* but only the product bz. A value of *z* other than 2 would still make the values of their fitting parameters reasonable. The noise spectrum measurements of Shaw *et al.*[19] are another good example of a measurement that is not able to determine the value of *z* but only the product bz.[67] Hence these measurements cannot be taken as evidence for $z=2$.

C. Comparison of conventional approach with dynamic scaling

To further examine the validity of the finding $z\simeq 5.6\pm 0.3$ obtained from the Fisher-Fisher-Huse scaling, we will examine a data set using the conventional approach with an arbitrary value of *z*. We have chosen the YBCO monolayer data of Ref. 34 for several reasons. First, their data cover by far the largest current, voltage, and temperature range of any data set in the literature. Second, those authors have pointed out that their data do not satisfy the ''conventional'' criteria for KTB behavior: the *I-V* exponent does not vary rapidly from 3 to 1 near the transition temperature. Finally, they suggested finite-size effects to explain the lack of KTB behavior and performed a largely qualitative analysis.

In this section, we will use a quantitative analysis to show that finite size effects cannot account for the observed behavior in that data (Sec. IV C 1). We will then perform a conventional analysis of the *I-V* exponent to look for evidence of $z\simeq 5.9$ (Sec. IV C 2).

1. Finite-size effects

Repaci *et al.*[34] have been suggested that their YBCO monolayer data that have been shown here to scale so well with $z=5.9$ can be explained in a ''conventional'' manner by finite size effects, following similar suggestions regarding the low-current Ohmic behavior of others.[26,48,68] We investigate that possibility in this subsection, making an explicit comparison between the dynamic scaling analysis and the conventional approach. As the reader will see, we find no evidence of finite-size effects in any of the data that we examine.

This subsection is organized as follows. After a discussion of the principal differences between the two scenarios, we look for evidence of finite size effects in the *I-V* data from the YBCO monolayer data[34] first using the conventional picture and then in the dynamic scaling analysis.

a. Theory. The primary difference between the finite-size explanation and the dynamic scaling explanation of those data is the following: the former ascribes finite-size-induced, low-current Ohmic behavior to *I-V* isotherms with temperatures $T<T_{KTB}$; in the dynamic scaling explanation the low-current Ohmic behavior is associated with $T>T_{KTB}$ isotherms. The issue of course is the placement of the transition temperature. Because the scaling analysis indicates that $\alpha(T_{KTB})\simeq 6.8$ and the conventional analysis assumes $\alpha(T_{KTB})=3$, T_{KTB} will be at a lower temperature in the scaling analysis scenario.

We first examine the nature of the Ohmic to non-Ohmic crossover in the finite-size-effect scenario. Recall that with finite-size effects, there is always a density of free vortices. [See Eq. (8).] This means that at small currents where one is probing large length scales, the free vortices will dominate the resistance and the *I-V* characteristics will be Ohmic. As one probes smaller length scales by increasing the current ($r_c \ll L_{fs}$), current-induced vortex unbinding will start to dominate the resistance and the *I-V* curves will become non-Ohmic.[69] Because either the system size or the 2D penetration depth can be on the order of 1–10 μm, one would expect finite-size effects to have an influence.

We will now use the conventional picture to make the above discussion more quantitative for $T<T_{KTB}$. We will show that the *I-V* curves are Ohmic when $r_c \gg L_{fs}$ and non-Ohmic when $r_c \ll L_{fs}$. With finite-size effects, the energy of a vortex pair is generalized from Eq. (6) to a potential that is logarithmic for $R \ll L_{fs}$ and approaching a constant for $R \gg L_{fs}$.[27] To approximate this behavior, we use

$$E(R)=q^2[\ln(R/\xi_0)-\ln(R/L_{fs}+1)]-JR/\xi_0+2E_c, \tag{23}$$

where $q^2=\pi n_s^{2D}\hbar^2/2m$ and $J=\pi\hbar I\xi_0 d/eA$ is a current with dimensions of energy. It is the second term on the right hand side of the equation that causes the zero-current ($J=0$) pair energy to approach a constant as $1/R$ for $R>L_{fs}$. With this term, $E(R)$ no longer peaks at r_c. Rather, it peaks at

$$r_c^{fs}=-\frac{L_{fs}}{2}+\frac{\sqrt{L_{fs}^2/4+L_{fs}\xi_0 q^2/J}}{2}. \tag{24}$$

There are two limits to this equation. The large-current limit $J\gg q^2\xi_0/L_{fs}$ can be rewritten as $r_c \ll L_{fs}$ while the small-current limit can be expressed as $r_c \gg L_{fs}$. In the large-current limit where one is probing small lengths, r_c^{fs} approaches r_c while at small currents ($J\ll q^2\xi_0/L_{fs}$), $r_c^{fs}=\sqrt{L_{fs}\xi_0 q^2/J}$. Remember that it is the value of $E(R)$ at r_c^{fs} (i.e., the height of the barrier) that determines the density of free vortices. In one limit ($r_c \ll L_{fs}$),

$$E(r_c^{fs})=q^2[\ln(q^2/J)-1]. \tag{25}$$

On the other hand,

$$E(r_c^{fs}) = q^2 \ln(L_{fs}/\xi_0) \tag{26}$$

for $r_c \gg L_{fs}$. So, for $T < T_{KTB}$, the I-V's are Ohmic at small currents since $E(r_c^{fs})$ does not depend on current. At large currents, $E(r_c^{fs})$ depends upon the current in the traditional way, and one finds the usual I-V relationship: $V \propto I^{\alpha}$. The Ohmic to non-Ohmic crossover occurs when $I \sim 4k_B T_{KTB} We/\pi\hbar L_{fs}$ (i.e., $r_c \sim L_{fs}$.)

The $T > T_{KTB}$ effect of finite size on the transport behavior is a little more subtle than the low-temperature effect because another competing length scale is involved. Even in the absence of finite-size effects for this temperature range, the I-V curves cross over from thermally dominated Ohmic behavior at small I to current-induced non-Ohmic behavior at large I. The current at which this crossover occurs depends upon the size of the largest vortex pairs, $\xi_+(T)$,[26] a quantity which is strongly temperature dependent. With finite-size effects, since all currents that probe lengths greater than L_{fs} are Ohmic, L_{fs} competes with $\xi_+(T)$, yielding the following conditions: when $r_c \gg \min[L_{fs}, \xi_+]$ there is Ohmic behavior and non-Ohmic when $r_c \ll \min[L_{fs}, \xi_+]$. An important term in our discussion is ''premature-Ohmic'' behavior, which denotes Ohmic behavior at currents for which there would not be Ohmic behavior in the absence of finite-size effects. It occurs when $L_{fs} < \xi_+(T)$ (and when the temperature is sufficiently close to T_{KTB}.)

It is the temperature dependence of these conditions (as well as the magnitude of the crossover current) that mark the signature of finite-size effects for $T > T_{KTB}$. For $T \gtrsim T_{KTB}$, $\xi_+(T)$ is large and exceeds L_{fs} so that the value of the current at which the isotherms cross from one behavior to another will depend upon L_{fs} and not the size of the largest pairs. In this case, the crossover current will depend only weakly on temperature. As $\xi_+(T)$ becomes smaller than L_{fs} at temperatures further above the transition, it is $\xi_+(T)$ that sets sets the current scale for the Ohmic to non-Ohmic crossover. So the crossover current becomes strongly temperature dependent.

b. Conventional check of finite-size effects. As mentioned above a possible candidate for observing finite-size effects is the monolayer YBCO data of Ref. 34 because $\lambda_{2D} \simeq 40\ \mu$m. To determine the value of the current at which ''premature-Ohmic'' behavior should occur for this data, we use $r_c = L_{fs} = 40\ \mu$m to solve for the current. Using $W = 200\ \mu$m, we find $I_{crossover} \simeq 2.4\ \mu$A, a value much smaller than the observed crossover current: $\sim 100\ \mu$A for $T \simeq 28$ K. This would seem to indicate that λ_{2D} is not responsible for the Ohmic to non-Ohmic crossover in these materials. A renormalization group study[70] confirms that this remains the case after renormalization effects of L_{fs} are accounted for. This is because the condition for observing the finite-size effects is $r_c > L_{fs}$. Under renormalization, L_{fs} does shrink quickly but r_c shrinks even more quickly.[70]

The length scale that corresponds to the approximate crossover current for the data of Repaci *et al.*[34] for temperatures around 28 K is 1 μm. Because neither a field-induced vortex length nor a pinning (disorder) length could correspond to this value, it seems unlikely that the behavior observed in Ref. 34 is due to finite-size effects.

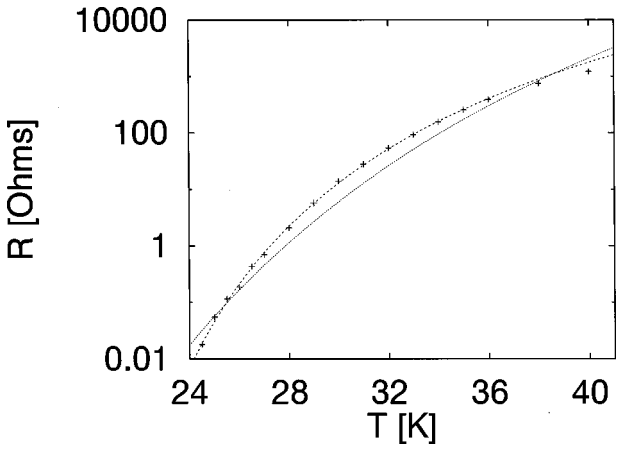

FIG. 7. The resistance data of Ref. 34 and fits to Eq. (19) (dashed line) and Eq. (8) (dotted line). The latter equation, based on assumptions of finite-size effects (finite $\lambda_\perp$), does not adequately describe the data.

Not only is the magnitude of the crossover current inconsistent with finite-size effects but so also is the temperature dependence of this quantity. If it were L_{fs} and not ξ_+ which determined the Ohmic to non-Ohmic crossover, the data would not collapse so well in the dynamic analysis since the temperature dependences of these two quantities are so different.

Another signature for finite-size effects in the conventional picture is that the resistance will have an Arrhenious temperature dependence: $R(T) = A_N \exp[B/K_B T]$ where B is related to the energy of a free vortex and A_N is a constant. [See Eq. (8).] We have fit this formula to the data, as shown in Fig. 7 (dotted line), but do not get a satisfactory result.

Finally, a finite-size analysis of the data of Repaci *et al.*[34] would indicate a T_{KTB} in excess of 30 K following the subsequent work of Herbert *et al.*[32] This is contrary to the mutual inductance data of Gasparov *et al.*[71] on YBCO monolayer films which probes the temperature at which the largest pairs unbind. [Remember, the size of the largest pairs is expected to decrease as $\xi_+(T)$ above T_{KTB}.] They find that vortex pairs of size $\sim 0.018\ \mu$m unbind at a temperature of roughly 47.0 K, vortex pairs of size $\sim 0.78\ \mu$m unbind at a temperature of roughly 27.8 K, and vortex pairs of size $\sim 1.58\ \mu$m unbind at a temperature of roughly 25.5 K. This trend is consistent with the value of T_{KTB} that we find (17.6 K) for the YBCO monolayer data of Ref. 34 using a dynamic scaling analysis.

c. Dynamic scaling check of finite-size effects. The signature of finite-size effects in the dynamic scaling depends upon the temperature. For $T < T_{KTB}$, the scaled data should peal off the scaling curve to go Ohmic (slope 1 in Figs. 1–4) for currents less than $4k_B T_{KTB} We/\pi\hbar L_{fs}$. For temperatures above the transition temperature, one would observe finite-size effects only if L_{fs} were shorter than the size of the largest vortex pairs. And in that case, the scaled data would break from the scaling curve to become ''prematurely Ohmic'' at $I = 4k_B T_{KTB} We/\pi\hbar L_{fs}$. This is not observed in any of the scaled data in this paper. Such behavior was observed however for the BSCCO data of Ref. 43. (See Fig. 2 there.) In that case, however, it seems more likely that this behavior is not due to finite-size effects but to voltage sen-

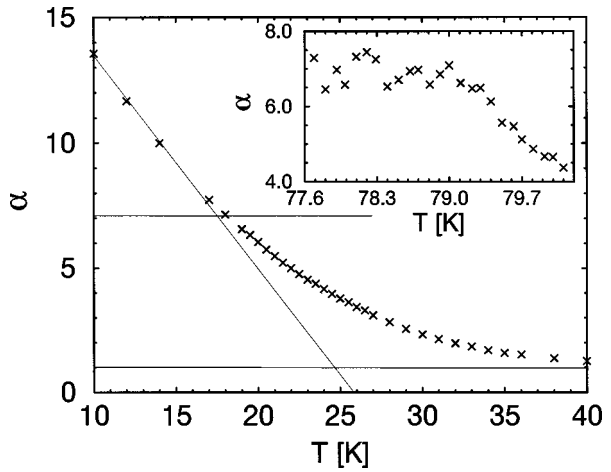

FIG. 8. $\alpha(T)$ data at a fixed current ($I=0.7$ mA) for an YBCO monolayer taken from the $d[\log V]/d[\log I]$ data in Fig. 3 of Ref. 34. Also plotted are a linear fit to the data less than the transition temperature and the lines $\alpha(T)=7.1$ and $\alpha(T)=1.0$ used to determine the parameters T_{c0}, z, and ϵ_c. [Inset: $\alpha(T)$ data at a fixed current ($I\sim 4.0$ mA) for the BSCCO film of Ref. 43.]

sitivity. The crossover to the premature-Ohmic behavior occurs more rapidly than one would expect for finite-size effects and also occurs at roughly the same voltage, which is near the voltage sensitivity limit.

d. *Finite-size-effect discussion.* As mentioned above, there is no ''true'' thermodynamic phase transition in superconductors because of the finite penetration depth. One could then ask why there is a critical isotherm at all. For example, in the data of Repaci *et al.*,[34] the 17 K isotherm is straight over nearly nine decades of voltage. The answer to this is that the correlation length at finite current does not become infinite (and is not longer than L_{fs}) even though it is very close to T_c. This is apparent from Eq. (3) where it is seen that the correlation length decreases as the reciprocal of current. It is only when $\xi(I,T)\simeq L_{fs}$ that one would begin to see deviations in the critical isotherm.

To summarize this section, we find no evidence of finite-size effects in any of the data that we examine. In principle, however, finite-size effects are inherent to superconductors and will manifest themselves if the probing current is small enough. It is our opinion that none of the data sets that we examined went to currents small enough to detect finite-size effects.

2. Conventional approach

We now examine the data of Repaci *et al.* with a conventional approach. In Fig. 8, we plot the *I-V* exponent ($V\propto I^{\alpha(T)}$) at a fixed current ($I=0.7$ mA) as determined from the $d[\log V]/d[\log I]$ data in Fig. 3 of Ref. 34. [The error bars were determined from that figure and from fits to the *I-V* curves, and are only shown for the near-linear region. The $\alpha(T<17$ K) data also came from power-law fits to the digitized *I-V* data from Fig. 2 of that reference.] As one can see, there are no features at $\alpha\sim 3$ that would suggest a phase transition, as originally pointed out by those authors. An interpretation of these data is difficult. A possible feature is a crossover from near-linear behavior of $\alpha(T)$ to nonlinear behavior near the value of T_{KTB} (17.6 K), obtained from the dynamic scaling behavior. Further, the value of α observed at this temperature produces an estimate for $z(\simeq 7)$ that is similar to that obtained from the scaling procedure. One can see that the $T>T_{KTB}$ behavior is concave up, contrary to the analytical work of Ref. 28, but more consistent with the simulational wor of Ref. 72.

The values of T_{KTB} and T_{c0} determined from the standard analysis of $\alpha(T)$ with arbitrary z are less definitive because one is not assuming a value of $\alpha(T_{KTB})$. To be consistent with the value of T_{KTB} determined from the scaling analysis, we have chosen $T_{KTB}=17.6$ K. The subsequent value of T_{c0} determined from a linear fit to the $\alpha(T<17.6$ K) is $T_{c0}=24.8$ K. (See Fig. 8.) z was found to be 6.1 ± 0.2, consistent with the dynamic scaling value. The renormalized dielectric constant also has a reasonable value: $\epsilon_c=1.59$.

One possible explanation for the absence of a clear jump in the exponent $\alpha(T)$ in Fig. 8 is the relatively short length scale it represents. In order to obtain well-defined and stable $\alpha(T)$ values (at the same reference current for all of the isotherms), it was necessary to choose a relatively large value for the reference current. This current value corresponds to the length scale 1400 Å ($\ll L_{fs}$), which is distant from the desired $I\rightarrow 0$ limit.

The final step in the conventional approach is to examine $R(T)$. Equation (19) was used in place of Eq. (22) to fit the data since many of the isotherms are obtained in the regime $T>T_{c0}=24.8$ K, where Eq. (22) is not valid. The dashed line in that figure is the fit using the parameters that optimized the scaling for Fig. 1(b): $T_{KTB}=17.6$ K, $b=7.79$, and $z=5.9\pm 0.3$. Clearly, the data are more consistent with KTB critical behavior than the finite-size effects discussed in the previous section.

We repeat this analysis for the BSCCO sample (1000 Å thick) of Ref. 43. The $\alpha(T)$ determined for a constant current is shown in the inset of Fig. 8. As one can see, these data are more noisy and cover a much smaller temperature range than that in Fig. 8, thereby precluding a complete conventional analysis. So while the value of α at $T_{KTB}=78.87$ K is consistent with that determined from the dynamic scaling analysis (6.6) and α seems to change behavior at that temperature, one could not claim these observations as evidence for $\alpha(T_{KTB})=6.6$. However, an important observation can be made by comparing this inset to the inset of Fig. 1 of Ref. 43, which shows $\alpha(T)$ determined from the same data, but for a constant voltage. As one can see, $\alpha(T)$ decreases much more rapidly for a constant voltage than for a constant current, reinforcing our claim in Sec. IV B 1 that a rapid decrease in $\alpha(T)$ could be an artifact of using the constant voltage $\alpha(T)$ data.

D. Theoretical considerations

The primary degrees of freedom associated with the KTB phase transition are vortices. The dynamic behavior should therefore be dissipative. Specifically, it has been argued[10] that superconducting dynamics in zero field may be purely relaxational [(Ref. 40) model A] for any dimension, with a diffusive exponent, $z\leqslant 2$. This interpretation is consistent with the conventional treatment of KTB dynamics. However, the present scaling analysis of *I-V* and $\dot{Q}$-ΔT data from SC's, JJA's, and SF's indicates that $z\simeq 5.6$, a result consis-

tent with subdiffusive dynamics. Here we mention some possible explanations for this large value of the dynamic exponent.

Pinning is known to play a crucial role in the large values of $z(\gg 2)$ observed in vortex glass phenomena in experiments[73–75] and Monte Carlo simulations[41,76,77] in high-temperature superconductors. However, we do not believe that pinning can explain the surprising values of z obtained in the present 2D analysis at zero field. The reason is that our result $z \simeq 5.6$ is obtained from very distinct systems: superconductors, JJA's, and superfluids. For superfluids, in particular, pinning effects should be negligible. A pinning explanation therefore appears inconsistent with the universal nature of z.

Collective excitations, such as vortex density waves,[78] may mediate the observed dynamic behavior. This is more likely to be true if the vortices cannot exit the sample easily, because of surface barriers. We can determine the dynamical critical exponent for this behavior as follows. Based upon the Coulomb gas analogy, the vortex plasma frequency is given by $\omega \propto \sqrt{n}$, where $n \sim \xi^{-d}$ is the vortex density. Using $\tau \propto \xi^z$, we find $z = 1.5$ in three dimensions and $z = 1.0$ in two dimensions. Vortex plasmons therefore cannot explain the large values of z.

Another possibility is that the suppositions leading to Eq. (17) are incorrect. To arrive at that equation, only two forces are included, a viscous force and the Lorentz force. Perhaps, with the inclusion of other forces (e.g., surface barrier forces[66]) an explanation for $z = 5.6$ can be found.

We believe the most likely explanation for large values of z lies in correlated vortex motion, described as ''partner transfer''[16] or ''collaborative dissociation.''[43] These suggest mechanisms whereby bound vortex pairs do not simply dissociate into free vortices. Instead, the process is mediated by neighboring vortex pairs, in terms of consecutive recombination-dissociation events. Further work is required to confirm this model.

V. SUMMARY

As stated in the introductory paragraph, the dynamic scaling approach presented here is different than most previous studies of dynamics in 2D SC's, JJA's, and SF's in that this approach allows one to vary z to optimize the analysis. In the ''conventional'' approach, the value of z is implicitly taken to be 2. By using the dynamic scaling analysis and allowing the value of z to vary, we have presented evidence which suggests nondiffusive behavior.

Via a dynamic scaling analysis of transport data from SC's, JJA's, and SF's, we find $z \simeq 5.6 \pm 0.3$, contrary to the value assumed but not tested in previous reports. This analysis seems convincing in that the collapse is excellent in many data sets and the value of z is robust from system to system and material to material.

The results of the dynamic scaling analysis also go against the many studies consistent with $z = 2$. We have included in this work a discussion of those ''conventional'' approaches to studying the dynamics of the KTB transition. Like the dynamic scaling analysis, we find that this approach also has its drawbacks. Perhaps the most important is that the experiments do not yet have the sensitivity to actually observe the predicted jump in the I-V's in the $I \to 0$ limit which we estimated to be at 10^{-11} A for a particular sample.[34] Another drawback is that these approaches do not vary z to optimize the fits. Furthermore, the most common method of ''verifying'' KTB behavior [which is to obtain T_{KTB} from the condition $\alpha(T_{KTB}) = 3$ and then to do a three parameter fit to Eq. (22)] does not pose constraints tight enough to prove $z = 2$. Our analysis of the evidence for $z = 2$ raises the following questions

(i) Why has no scaling of zero-field I-V data with $z = 2$ been realized?

(ii) Why do critical isotherms have a much larger value of α than the value consistent with $z = 2$?

(iii) If finite size effects are present, why does the Ohmic to non-Ohmic crossover not coincide with $r_c = L_{fs}$?

We also compared directly the conventional approach and the dynamic scaling approach for data from a particular sample: the YBCO monolayer data of Ref. 34. In that reference, those authors found that the conventional approach cannot explain their data. In Sec. IV C 1, we found that an incorporation of the finite-size effects into the conventional approach is also not consistent with their data. Further, in Sec. IV C 2, we saw that a conventional analysis of the I-V exponent was consistent with $z \sim 5.6$. A dynamic scaling analysis of their data, however, resulted in a beautiful collapse, as shown in Fig. 1(b).

The primary purpose of this paper is to convey that the question of the value of z in these systems is still an open one, despite the conventional wisdom that $z = 2$. We believe that more study is needed. In particular, more data on all systems, especially JJA and SF, are needed and over wider temperatures and current regions (or $\dot{Q}$ regions for SF's). The impressive data of Repaci *et al.*[34] set a good standard. Not only should dynamic scaling analysis be tried on these data but so too should comprehensive ''conventional'' studies like those of Fiory, Hebard, and Glaberson.[18] [By ''comprehensive,'' we mean going beyond just the usual $\alpha(T_{KTB}) = 3$ and $R(T)$ measurements.] Of special importance would be a measurement of q^2 using static kinetic inductance data (in the appropriate frequency range) and the dynamic I-V exponent α. If the conventional theories are valid after generalization to a general z, then $\alpha - 1 = [z/2]q^2$. (Fiory *et al.* did do such a measurement but found agreement only over 30 mK.) Allowing the value of z in the conventional theory would also be a useful exercise.

ACKNOWLEDGMENTS

The authors gratefully acknowledge conversations with S. M. Girvin, L. I. Glazman, B. I. Halperin, A. Hebard, J. M. Kosterlitz, C. J. Lobb, P. Minnhagen, P. Muzikar, and S. Teitel. This work was supported by the Midwest Superconductivity Consortium through DOE Contract No. DE-FG02-90ER45427 and by NSF Grant No. DMR 95-01272. Acknowlegement (S.W.P.) is made to the The Petroleum Research Fund, administered by the ACS, for support of this research.

*Electronic address: pierson@wpi.edu
†Present address: Applied Superconductivity Center, University of Wisconsin-Madison, 1500 Engineering Drive, Madison, WI 53706. Electronic address: mgfriesen@facstaff.wisc.edu

[1] V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. Lett. **40**, 783 (1978).
[2] V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. B **21**, 1806 (1980).
[3] See, e.g., M. Takahashi, J. Phys. Soc. Jpn. **48**, 746 (1980).
[4] See e.g., F. G. Mertens, A. R. Bishop, G. M. Wysin, and C. Kawabata, Phys. Rev. Lett. **59**, 117 (1987).
[5] See, e.g., A. Jonsson and P. Minnhagen, Phys. Rev. B **55**, 9035 (1997), and references therein.
[6] B. J. Kim, P. Minnhagen, and P. Olsson, Phys. Rev. B **59**, 11 506 (1999).
[7] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[8] J. M. Kosterlitz, J. Phys. C **7**, 1046 (1974).
[9] V. L. Berezinskii, Sov. Phys. JETP **32**, 493 (1971).
[10] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
[11] S. A. Wolf, D. U. Gubser, and Y. Imry, Phys. Rev. Lett. **42**, 324 (1979).
[12] D. C. Harris, S. T. Herbert, D. Stroud, and J. C. Garland, Phys. Rev. Lett. **67**, 3606 (1991).
[13] L. Miu, P. Wagner, U. Frey, A. Hadish, D. Miu, and H. Adrian, Phys. Rev. B **52**, 4553 (1995).
[14] R. Brada, H. Chayet, and W. I. Glaberson, Phys. Rev. B **48**, 12 874 (1993).
[15] R. Théron, J.-B. Simond, Ch. Leemann, H. Beck, and P. Marinoli, Phys. Rev. Lett. **71**, 1246 (1993).
[16] D. Bormann, Phys. Rev. Lett. **78**, 4324 (1997).
[17] P. Minnhagen *et al.*, Phys. Rev. Lett. **74**, 3672 (1995); see also Ref. 16, and references therein.
[18] A. T. Fiory, A. F. Hebard, and W. I. Glaberson, Phys. Rev. B **28**, 5075 (1983).
[19] T. K. Shaw, M. J. Ferrari, L. L. Sohn, D.-H. Lee, M. Tinkham, and J. Clarke, Phys. Rev. Lett. **76**, 2551 (1996).
[20] D. J. Bishop and J. D. Reppy, Phys. Rev. Lett. **40**, 1727 (1978).
[21] D. J. Bishop and J. D. Reppy, Phys. Rev. B **22**, 5171 (1980).
[22] B. I. Halperin, in *Physics of Low-Dimensional Systems*, Proceedings of the Kyoto Summer Institute, 1979, edited by Y. Nagaoka and S. Hikami (Publication Office, Prog. Theor. Phys., Kyoto, 1979), p. 53.
[23] J. E. Mooij, in *Percolation, Localization and Superconductivity*, Vol. 109 of *NATO Advanced Study Institute, Series B: Physics*, edited by A. M. Goldman and S. A. Wolf (Plenum, New York, 1984), pp. 325–370.
[24] P. M. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
[25] A. Hebard, in *Strongly Correlated Electronic Materials: The Los Alamos Symposium, 1993*, edited by K. S. Bedell (Addison Wesley, New York, 1994), p. 251.
[26] M. V. Simkin and J. M. Kosterlitz, Phys. Rev. B **55**, 11 646 (1997).
[27] J. Pearl, Appl. Phys. Lett. **5**, 65 (1964).
[28] A. M. Kadin, K. Epstein, and A. M. Goldman, Phys. Rev. B **27**, 6691 (1983).
[29] B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 599 (1979).
[30] V. Ambegaokar and S. Teitel, Phys. Rev. B **19**, 1667 (1979).
[31] M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. **42**, 1165 (1979).
[32] S. T. Herbert, Y. Jun, R. S. Newrock, C. J. Lobb, K. Ravindran, H.-K. Shin, D. B. Mast, and S. Elhamri, Phys. Rev. B **57**, 1154 (1998).
[33] See C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983), and references therein.
[34] J. M. Repaci, C. Kwon, Q. Li, X. Jiang, T. Venkatessan, R. E. Glover, C. J. Lobb, and R. S. Newrock, Phys. Rev. B **54**, R9674 (1996).
[35] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
[36] S. W. Pierson, Philos. Mag. B **76**, 715 (1997).
[37] B. A. Huberman, R. J. Myerson, and S. Doniach, Phys. Rev. Lett. **40**, 780 (1978).
[38] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).
[39] See N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group* (Addison-Wesley, Reading, MA, 1992), p. 213.
[40] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).
[41] J. Lidmar, M. Wallin, C. Wengel, S. M. Girvin, and A. P. Young, Phys. Rev. B **58**, 2827 (1998).
[42] J.-R. Lee and S. Teitel, Phys. Rev. B **50**, 3149 (1994).
[43] S. M. Ammirata, M. Friesen, S. W. Pierson, L. A. Gorham, J. C. Hunnicutt, M. L. Trawick, and C. D. Keener, Physica C **313**, 225 (1999).
[44] S. Vadlamannati, Q. Li, T. Venkatesan, W. L. McLean, and P. Lindenfeld, Phys. Rev. B **44**, 7094 (1991).
[45] Y. Matsuda, S. Komiyama, T. Terashima, K. Shimura, and Y. Bando, Phys. Rev. Lett. **69**, 3228 (1992).
[46] J. C. Garland and H. J. Lee, Phys. Rev. B **36**, 3638 (1987).
[47] A. F. Hebard and A. T. Fiory, Phys. Rev. Lett. **50**, 1603 (1983).
[48] H. S. J. van der Zant, H. A. Zant, H. A. Rijken, and J. E. Mooij, J. Low Temp. Phys. **79**, 289 (1990).
[49] (a) S. T. Herbert, J. Markus, S. E. Hebboul, and J. C. Garland, J. Low Temp. Phys. **106**, 607 (1997); (b) S. T. Herbert, Ph.D. thesis, Ohio State University, 1993.
[50] D. C. Harris, Ph.D. thesis, Ohio State University, 1989.
[51] D. W. Abraham, C. J. Lobb, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **26**, 5268 (1982).
[52] D. W. Abraham, Ph.D. thesis, Harvard University, 1983.
[53] J. Maps and R. B. Hallock, Phys. Rev. B **27**, 5491 (1983).
[54] R. J. Dionne, S. B. Crary, and R. B. Hallock, Phys. Rev. B **33**, 1997 (1987).
[55] S. W. Pierson, Phys. Rev. B **51**, 6663 (1995).
[56] M. Friesen, Phys. Rev. B **53**, R514 (1996).
[57] P. Minnhagen and P. Olsson, Phys. Rev. B **45**, 10 557 (1992).
[58] V. Cataudella and P. M. Minnhagen, Physica C **160**, 442 (1990).
[59] S. W. Pierson, Phys. Rev. Lett. **75**, 4674 (1995).
[60] M. Friesen, Phys. Rev. B **51**, 632 (1995).
[61] M. Friesen, Phys. Rev. B **51**, 12 786 (1995).
[62] S. Sujani, B. Chattopadhyay, and S. R. Shenoy, Phys. Rev. B **50**, 16 668 (1994).
[63] K. Epstein, A. M. Goldman, and A. M. Kadin, Phys. Rev. Lett. **47**, 534 (1981).
[64] D. P. Norton and D. H. Lowndes, Phys. Rev. B **48**, 6460 (1993).
[65] P. M. Minnhagen, Phys. Rev. B **23**, 5745 (1981).
[66] D. T. Fuchs, E. Zeldov, M. Rappaport, T. Tamegai, S. Ooi, and H. Shtrikman, Nature (London) **391**, 373 (1998).
[67] Shaw *et al.* (Ref. 19) do extract a value of z by assuming that the resistance $R(T) \propto \xi^{-2}$. [See paragraph around Eq. (8) in that reference.] This is tantamount to assuming $z=2$. See Eq. (18).

[68] S. Kim and M. Y. Choi, Phys. Rev. B **48**, 322 (1993).
[69] See, e.g., Fig. 29 of G. Blatter, M. V.Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
[70] S. W. Pierson and O. T. Valls (unpublished).
[71] V. A. Gasparov, I. E. Batov, C. Kwon, and Q. Li (unpublished).
[72] K. Holmlund and P. Minnhagen, Phys. Rev. B **54**, 523 (1996).
[73] R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1511 (1989).
[74] H. K. Olsson, R. H. Koch, W. Eidelloth, and R. P. Robertazzi, Phys. Rev. Lett. **66**, 2661 (1991).
[75] K. Moloni, M. Friesen, S. Li, V. Souw, P. Metcalf, L. Hou, and M. McElfresh, Phys. Rev. B **56**, 14 784 (1997).
[76] K. H. Lee and D. Stroud, Phys. Rev. B **46**, 5699 (1992).
[77] H. Weber and H. J. Jensen, Phys. Rev. Lett. **78**, 2620 (1997).
[78] S. E. Hebboul, D. P. Johnson, and M. Rokhlin, Phys. Rev. Lett. **82**, 831 (1999).