Anisotropic impurities in anisotropic superconductors

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The physical properties of anisotropic superconductors such as the critical temperature and other important properties depend sensitively on the electron mean free path. The sensitivity to impurity scattering and the resulting anomalies are considered a characteristic feature of strongly anisotropic pairing. These anomalies are usually analyzed in terms of *s*-wave impurity scattering which leads to universal pair-breaking effects depending on only two scattering parameters, the mean free path and the impurity cross section. We investigate here corrections coming from anisotropies in the scattering cross section, and find not only quantitative but also qualitative deviations from universal *s*-wave isotropic pair breaking. The properties we study are the transition temperature, quasiparticle bound states at impurities, and pinning of single- and double-flux lines by impurities. [S0163-1829(99)12341-5]

I. INTRODUCTION

Unconventional anisotropic pairing is evidently realized in high-temperature superconductors (HTS's) (see reviews in Ref. 1), and probably in some heavy fermion superconductors (HFS's) (see review in Ref. 2). There is good evidence for *d*-wave pairing in optimally doped HTS oxides[1] but the type of pairing in HFS's is still unclear.[2] The effect of impurities on unconventional pairing is an important tool in analyzing the symmetry of the pairing amplitude, and is the subject of a number of experimental[3,4] and theoretical works.[5,9] Most theoretical considerations and calculations were done assuming an *s*-wave impurity scattering potential $u_{imp}(\mathbf{p},\mathbf{p}')=\text{const}$, and taking either the Born limit $[N(0)u_{imp}\ll 1]$ or unitarity limit $[N(0)u_{imp}\gg 1]$.

Surprisingly, a number of experiments on the optimally HTS oxides[3,4] have shown that *d*-wave pairing is quite robust, i.e., not very sensitive to various kinds of impurities and defects. For instance, the decrease of the critical temperature $T_c(\rho_{imp})$ with increasing residual resistivity ρ_{imp} is much smaller,[3,4] than the theory with the *s*-wave impurity scattering predicts.[5,9] A way out of this experimental and theoretical discrepancy of pair-breaking effects by impurities in HTS oxides was proposed by the authors of Refs. 6 and 8, who invoked a momentum-dependent impurity scattering potential with an appreciable contribution in the *d* channel. The microscopic theory in Ref. 8 accounts for the renormalization of the impurity potential by strong correlations, which gives rise to a pronounced forward scattering peak, while backward scattering is suppressed, as first proposed in Ref. 10. Application of this theory to impurity scattering shows that in addition to the contribution in the *s* channel there is a significant contribution to the Born amplitude from the *d* channel of the same magnitude,[10] in particular for low (hole) doping concentration $\delta<0.2$. As a consequence, the decrease of $T_c(\rho_{imp})$ with increasing ρ_{imp} is much slower than the theory with exclusively *s*-wave impurity scattering predicts.[9] This renormalization effect explains the robustness of *d*-wave pairing in HTS oxides.

One may raise the question whether this robustness also holds far away from T_c and for a very strong scattering potential, for instance in the unitarity limit. To answer this question we shall analyze in Sec. II a class of models by calculating the scattering T matrix, with an impurity potential that depends on the scattering angles, and its effect on the $T_c(\rho_{imp})$ dependence.

Many-band models belong to the special class of anisotropic superconductors. Recently, several models were suggested for the pairing mechanism in HTS oxides based on two-band and multiband models,[11–15] where impurity effects are studied in the Born approximation.[11–13] Magnetic and nonmagnetic interband scattering can lead in this model to a lowering of the critical temperature and also to a relative sign change of the order parameters in different bands.[13] In Sec. III we analyze the changes in the two-band model when going beyond the Born limit. It is shown that in some range of the parameters and in the unitarity limit the thermodynamic properties are unaffected by impurities; i.e., the Anderson theorem holds.

While in Secs. II and III a homogeneous superconductor with homogeneously distributed impurities is studied, in Sec. IV selected inhomogeneous problems are studied, such as bound states at an impurity and the pinning energy at an impurity (defect) of single- and double-quantized vortices.

II. ANISOTROPIC SCATTERING IN ANISOTROPIC AND HOMOGENEOUS SUPERCONDUCTORS

In the following we analyze the superconducting properties of anisotropic superconductors in the presence of momentum-dependent nonmagnetic impurity scattering by the quasiclassical equations of Eilenberger[16] and Larkin and Ovchinnikov[17] (ELO equations). For a homogeneous distribution the quasiclassical Green's function matrix $\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)$ is independent of $\mathbf{R}$, and the quasiclassical equations read

$$[i\omega_n\hat{\tau}_3-\hat{\Delta}(\mathbf{p}_F,\omega_n)-\hat{\sigma}_{imp}(\mathbf{p}_F,\omega_n),\hat{g}(\mathbf{p}_F,\omega_n)]=0, \quad (1)$$

$$\hat{g}^2(\mathbf{p}_F,\omega_n)=-\hat{1}. \quad (2)$$

The brackets $[,]$ mean the commutator. We assume weak-coupling superconductivity with $\hat{\Delta}(\mathbf{p}_F)[=i\Delta(\mathbf{p}_F)\hat{\tau}_2]$, where $\Delta(\mathbf{p}_F)$ is real. The 2×2 matrices $\hat{\tau}_0\equiv\hat{1}$ and $\hat{\tau}_{1,2,3}$ are Nambu-Gor'kov matrices in the electron-hole space. The effect of nonmagnetic impurities is described by the self-energy $\hat{\sigma}_{imp}$, given in terms of the forward scattering part of the t matrix $\hat{t}(\mathbf{p}_F,\mathbf{p}'_F,\omega_n)$:[16–18]

$$\hat{\sigma}_{imp}(\mathbf{p}_F,\omega_n)=c\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n), \quad (3)$$

where $c(\ll 1)$ is the impurity concentration.

For simplicity we assume an isotropic Fermi surface but pairing and impurity scattering are angle dependent, i.e., $\Delta(\mathbf{p}_F)\equiv\Delta(\mathbf{s})$, $\hat{g}(\mathbf{p}_F,\omega_n)\equiv\hat{g}(\mathbf{s},n)$ and $\hat{t}(\mathbf{p}_F,\mathbf{p}'_F,\omega_n)\equiv\hat{t}(\mathbf{s},\mathbf{s}',n)$ where $\mathbf{s}=\mathbf{p}_F/p_F$. The generalization to the anisotropic Fermi surface is straightforward. The t matrix is the solution of the equation

$$\hat{t}(\mathbf{s},\mathbf{s}',n)=u(\mathbf{s},\mathbf{s}')\hat{1}+N(0)\times\int d\mathbf{s}''u(\mathbf{s},\mathbf{s}'')\hat{g}(\mathbf{s}'',n)\hat{t}(\mathbf{s}'',\mathbf{s}',n), \quad (4)$$

where $\int d\mathbf{s}\{\cdots\}\equiv\int\{\cdots\}d\Omega/4\pi$. Since $\Delta(\mathbf{s})$ is real, one has $\hat{g}=g_2\hat{\tau}_2+g_3\hat{\tau}_3$ and $\hat{t}$ is given by $\hat{t}=t_0\hat{\tau}_0+t_1\hat{\tau}_1+t_2\hat{\tau}_2+t_3\hat{\tau}_3$.

Because the unperturbed solution has the form $[\omega_n=\pi T(2n+1)]$

$$\hat{g}^{(0)}(\mathbf{s},n)=-\frac{i\omega_n\hat{\tau}_3-i\Delta_0(\mathbf{s})\hat{\tau}_2}{\sqrt{\omega_n^2+\Delta_0^2(\mathbf{s})}}, \quad (5)$$

then $\hat{g}(\mathbf{s},n)$ is searched for in the form

$$\hat{g}(\mathbf{s},n)=-\frac{i\tilde{\omega}_n(\mathbf{s})\hat{\tau}_3-i\tilde{\Delta}(\mathbf{s},\omega_n)\hat{\tau}_2}{\sqrt{\tilde{\omega}_n^2(\mathbf{s})+\tilde{\Delta}^2(\mathbf{s},\omega_n)}}, \quad (6)$$

where

$$\tilde{\omega}_n(\mathbf{s})=\omega_n(\mathbf{s})+ic_it_3(\mathbf{s},\mathbf{s},n), \quad (7)$$

$$\tilde{\Delta}(\mathbf{s},\omega_n)=\Delta(\mathbf{s})-ic_it_2(\mathbf{s},\mathbf{s},n). \quad (8)$$

The self-consistency equation for $\Delta(\mathbf{s})$ is given by

$$\Delta(\mathbf{s})=N(0)T\sum_n\int d\mathbf{s}'V(s,s')g_2(\mathbf{s}',n), \quad (9)$$

where the pairing potential $V_p(s,s')$ is assumed in the factorized form, i.e. $V_p(s,s)'=V_pY(\mathbf{s})Y(\mathbf{s}')$, with $\langle Y^2(\mathbf{s})\rangle_{\mathbf{s}}=1$. The latter implies that the order parameter has the form $\Delta(\mathbf{s})=\Delta Y(\mathbf{s})$. For convenience we define the scattering amplitude (in the unitarity limit) $\Gamma_u(\equiv c\gamma_u)=c[\pi N(0)]^{-1}$ and $v(\mathbf{s},\mathbf{s}')\equiv\pi N(0)u(\mathbf{s},\mathbf{s}')$. In what follows we consider the effects of anisotropic impurity scattering on the anisotropic pairing where $\langle Y(\mathbf{s})\rangle_{\mathbf{s}}=0$.

A. Anisotropic impurity scattering and nodeless anisotropic pairing

First, we consider the nodeless d-wave-like pairing $\Delta(\mathbf{s})=\Delta\cdot Y(\mathbf{s})$ which is characterized by $\langle Y(\mathbf{s})\rangle_{\mathbf{s}}=0$ and $Y(\mathbf{s})=\pm 1$.[19] This means that there is a finite gap everywhere on the Fermi surface, i.e., $\Delta(\mathbf{s})\neq 0$. It is interesting to mention that besides the simplicity of this kind of pairing and its adequacy in some qualitative understanding of d-wave pairing it also appears to be a solution of the spin-bag model[20] for HTS oxides. In this model the nodeless d-wave-like pairing is due to residual (longitudinal and transverse) spin fluctuations on the antiferromagnetic background, where the antiferromagnetic (AF) order is distorted locally by hole doping and the spin bag is formed around doped holes. The impurity scattering potential is assumed to have the form

$$v(\mathbf{s},\mathbf{s}')=v_0+v_2Y(\mathbf{s})Y(\mathbf{s}'); \quad (10)$$

i.e., it contains an anisotropic contribution in the same channel as the unconventional pairing. The solution of Eq. (4) for t_3 and t_2 is searched in the form

$$t_3(\mathbf{s},\mathbf{s}')=[\tilde{t}_{30}(n)+\tilde{t}_{32}(n)Y(\mathbf{s})Y(\mathbf{s}')]g_3, \quad (11)$$

$$t_2(\mathbf{s},\mathbf{s}')=\tilde{t}_2(n)[g_2(\mathbf{s},n)+g_2(\mathbf{s}',n)]. \quad (12)$$

[Note that in this model one has $g_2(\mathbf{s},n)=\tilde{g}_2(n)Y(\mathbf{s})$, $g_2^2(\mathbf{s},n)=\tilde{g}_2^2(n)$, $g_3(\mathbf{s},n)=g_3(n)$, and due to Eq. (2), one has $g_3^2(\mathbf{s},n)+\tilde{g}_2^2(\mathbf{s},n)=-1$.] The solution is given by

$$\tilde{t}_{30}(n)=\gamma_uv_0^2\frac{1+v_2^2}{(1+v_0^2)(1+v_2^2)+(v_0-v_2)^2\tilde{g}_2^2(n)}, \quad (13)$$

$$\tilde{t}_2(n)=\gamma_uv_0v_2\frac{1+v_0v_2}{(1+v_0^2)(1+v_2^2)+(v_0-v_2)^2\tilde{g}_2^2(n)}, \quad (14)$$

while $\tilde{t}_{32}(n)=\tilde{t}_{30}(n,v_0\leftrightarrow v_2)$. Several interesting results comes out in this case.

1. Critical temperature T_c

In the limit $T\to T_c$, Eqs. (9), (13), and (14) give the equation for T_c,

$$\ln\frac{T_c}{T_{c0}}=\Psi\left(\frac{1}{2}\right)-\Psi\left(\frac{1}{2}+\frac{\Gamma_{pb}}{2\pi T_c}\right), \quad (15)$$

where the pair-breaking parameter Γ_{pb} is given by $(\Gamma_u=c\gamma_u)$

$$\Gamma_{pb}=\Gamma_u\frac{(v_0-v_2)^2}{(1+v_0^2)(1+v_2^2)}. \quad (16)$$

Note that T_c vanishes for $\Gamma_{pb}^c\approx 0.88T_{c0}$ and this pairing is in some respects similar to d-wave pairing. It is apparent from Eqs. (15) and (16) that the pair-breaking effect of impurities is weakened in the presence of momentum-dependent scattering and it is even zero for $v_0=v_2$ in this specific example. In the case $v_0v_2<0$ the pair-breaking parameter Γ_{pb} can be even increased in the presence of anisotropic scattering. This property is impossible to obtain in the approach of Refs. 6

and 7 because it is assumed that the impurity scattering potential in the Born approximation has the form $|w(k,k')|^2 = |w_0|^2 + |w_2|^2 Y(k)Y(k')$ [see Eq. (6) Ref. 6], where $w_{0,2} = \mathrm{const}$. As a result they get in the Born approximation $\Gamma_{pb}^{(w)} \sim (|w_0|^2 - |w_2|^2)$ instead of the correct result $\Gamma_{pb} \sim (v_0 - v_2)^2$. Note that $\Gamma_{pb}^{(w)}$ gives that it is unphysical that $\Gamma_{pb}^{(w)} < 0$ for $|w_2|^2 > |w_0|^2$ and accordingly an increase of T_c (see Ref. 7). The correct expression for Γ_{pb} gives $\Gamma_{pb} \geqslant 0$ and decreases T_c in the presence of impurities. In fact w_0 must be momentum dependent, i.e., $|w_0|^2 = |w_0(k,k')|^2$, except in the case with $Y(\mathbf{s}) = \pm 1$,[19] where $|w_0|^2 = v_0^2 + v_2^2$ and $|w_1|^2 = 2v_0 v_2$ for $v_0 v_2 > 0$. The latter result has been previously derived in the Born approximation.[8]

For $v_2 \approx v_0$ in the model with $Y(\mathbf{s}) = \pm 1$,[19] the slope $dT_c/d\rho_{imp}$ can be very small even for appreciable values of $\rho_{imp} \sim \Gamma_{tr} = \Gamma_u(\overline{\sigma}_0 + \overline{\sigma}_2)$, because in that case $\Gamma_{pb} \ll \Gamma_{tr}$ as indicated by the experimental results of Ref. 3. The parameters $\overline{\sigma}_i$ are given by

$$\overline{\sigma}_i = \frac{v_i^2}{1+v_i^2}, \quad i = 0,1,2 \ldots . \tag{17}$$

The resistivity ρ_{imp} and the reduction of T_c due to impurity scattering, $T_c(\rho_{imp})$ depend on the classical transition rate $W(\mathbf{s},\mathbf{s}') = \Gamma_u |t_N(\mathbf{s},\mathbf{s}',n)|^2$ in the normal state.[21] This transition rate comprises all the needed information on impurity scattering for either solving the normal state Boltzmann equation to determine ρ_{imp} or the linearized gap equation (9) to determine T_c. For the latter purpose one needs a linear (integral) equation for $g_2(\mathbf{s},n)$ which reads

$$|\omega_n| g_2(\mathbf{s},n) - \Delta(\mathbf{s}) + \int d\mathbf{s}' W(\mathbf{s},\mathbf{s}')[g_2(\mathbf{s},n) - g_2(\mathbf{s}',n)] = 0, \tag{18}$$

where the normal state t matrix $t_N(\mathbf{s},\mathbf{s}',n)$ is the solution of the equation

$$t_N(\mathbf{s},\mathbf{s}',n) = v(\mathbf{s},\mathbf{s}') - i\pi\, \mathrm{sgn}(\omega_n) \times \int d\mathbf{s}'' v(\mathbf{s},\mathbf{s}'') t_N(\mathbf{s}'',\mathbf{s}',n). \tag{19}$$

Hence, measurements of the $T_c(\rho_{imp})$ curve do not carry enough information on the microscopic scattering data, i.e. on the scattering t matrix. More such information is contained in spectroscopic data on anisotropic superconductors at temperatures $T \ll T_c$, such as tunneling data or optical data at about gap frequency.

2. Density of states

The density of states $N(\omega) = N(0)\mathrm{Im}\int d\mathbf{s} \; g_3(\mathbf{s}, i\omega_n \to \omega - i\eta)$ depends in the presence of pair-breaking impurities significantly on the values of v_0 and v_2. It is known[19] that in the case of s-wave scattering only ($v_2 = 0$) one has $N(\omega = 0) \neq 0$ for $\Gamma_u v_0^2 > \Delta$, and the highest value is $N(\omega = 0) = N(0)/\{0.5 + 0.5[1 + (2\Delta/\Gamma)^2]^{1/2}\}^{1/2}$, where $\Gamma \equiv \Gamma_u \overline{\sigma}_0$, is reached in the unitarity limit. On the other hand, in the limiting case $v_0 = v_2$ one obtains a restoration of the gap ω_g, i.e., $N(\omega < \omega_g) = 0$. Despite the strong scattering limit, $N(\omega)$ is BCS-like.

B. Isotropic impurity scattering and *d*-wave pairing

Let us study a two-dimensional superconductor with the pairing function $\Delta(\mathbf{s}) \equiv \Delta(\varphi)[= \Delta Y_2(\varphi)] \sim \cos 2\varphi$ d-wave pairing. Note that this case is more realistic for HTS oxides than the previous one, because the "$\cos 2\varphi$" pairing has nodes at the simply connected Fermi surface. We assume that the isotropic impurity potential depends on the transferred scattering angle

$$v(\varphi,\varphi') = v_0 + 2v_1 \cos(\varphi - \varphi') + 2v_2 \cos 2(\varphi - \varphi'), \tag{20}$$

where $v(\varphi,\varphi')$ contains the pairing channel $[\sim Y_2(\varphi)Y_2(\varphi')]$ too. (This problem but with $v_2 = 0$ is studied in Ref. 22 but there is an inappropriate sign in the t_2 matrix, which in fact corresponds to a magnetic impurity scattering.) From Eqs. (13) and (14) one obtains the pair-breaking parameter Γ_{pb}

$$\Gamma_{pb} = \Gamma_u \left[\overline{\sigma}_0 \frac{(1-\alpha)^2 + \alpha^2(1+v_0^2)}{1+\alpha^2 v_0^2} + \overline{\sigma}_1 \right], \tag{21}$$

where $\alpha = v_2/v_0$ and $\overline{\sigma}_i$ are given by Eq. (17).

In order to analyze the $T_c(\rho_{imp})$ dependence, where the residual impurity resistivity $\rho_{imp} = 4\pi\Gamma_{tr}/\omega_{pl}^2$ and ω_{pl} is the plasma frequency, we need to know the transport scattering rate Γ_{tr}. In the case of the scattering potential given by Eq. (20),

$$\Gamma_{tr} = \Gamma_u \left\{ \overline{\sigma}_0^2 + 2\overline{\sigma}_1^2 + 2\overline{\sigma}_2^2 - 2\overline{\sigma}_0 \left[\overline{\sigma}_1 \left(1 + \frac{1}{v_0 v_1} \right) + \overline{\sigma}_2 \left(1 + \frac{1}{v_0 v_2} \right) \right] \right\}. \tag{22}$$

If one wants to interpret the depairing effects of impurities and robustness of pairing in HTS oxides in terms of the above results, then the experiments[3] imply that the ratio, Γ_{pb}/Γ_{tr} must be small, i.e. $(\Gamma_{pb}/\Gamma_{tr}) \ll 1$. In the case when $v_2 \ll v_1$ one obtains $\Gamma_{pb}/\Gamma_{tr} = 2$ in both the Born and unitarity ($v_0, v_1 \to \infty$) limits. For $v_1 \ll v_2$, the pair-breaking parameter, Γ_{pb}, is minimized for $\alpha = 1/2$ which gives $\Gamma_{pb}/\Gamma_{tr} \approx 1/3$, in both limits. This means that the latter case ($v_1 \ll v_2$) is a more appropriate candidate than the case $v_2 \ll v_1$, for at least a qualitative explanation of the robustness of d-wave pairing in HTS oxides.[3,4,8]

III. TWO-BAND MODEL WITH NONMAGNETIC IMPURITIES

Interest in two-band (multiband) models and in the impurity effects was renewed after the discovery of HTS oxides,[11,14] where various kinds of intraband and interband pairing and impurity scattering are considered. Recently, a model was proposed for the anisotropic superconductors, like heavy fermions, which is based on the Fermi surface with multiple pockets.[15] In this model the pairing takes place on the pockets and the interpocket scattering of pairs is also taken into account, while impurity scattering is omitted. This model belongs to the class of multiband models which we analyze below by additionally taking into account nonmagnetic impurity scattering.

For simplicity we consider a two-band superconductor by assuming the intraband pairing Δ_α $(\alpha=1,2)$ only the interband pairing is highly improbable, and various types of intraband and interband nonmagnetic impurity scattering. The effect of the nonmagnetic impurities in the Born approximation is analyzed in Ref. 13 where the renormalized frequency $\tilde{\omega}_{\alpha n}$ and superconducting order parameters $\tilde{\Delta}_{\alpha n}$ are the solutions of the following equations (n enumerates Matsubara frequencies, and $\alpha,\beta,\gamma=1,2$):

$$\tilde{\omega}_{\alpha n}=\omega_n+\sum_\beta \frac{\tilde{\omega}_{\beta n}}{2Q_{\beta n}}\gamma_{\alpha\beta}, \tag{23}$$

$$\tilde{\Delta}_{\alpha n}=\Delta_\alpha+\sum_\beta \frac{\tilde{\Delta}_{\beta n}}{2Q_{\beta n}}\gamma_{\alpha\beta}, \tag{24}$$

$$\Delta_\alpha=\pi T\sum_{\beta,n}^{-\omega_D<\omega_n<\omega_D}\lambda_{\alpha\beta}\frac{\tilde{\Delta}_{\beta n}}{Q_{\beta n}}, \tag{25}$$

where $Q_{\alpha n}=\sqrt{\tilde{\omega}_{\alpha n}^2+\tilde{\Delta}_{\alpha n}^2}$, $\gamma_{\alpha\beta}=2\pi u_{\alpha\beta}^2N_\beta(0)$ for the nonmagnetic impurity scattering, and $\lambda_{\alpha\beta}=V_{\alpha\beta}^pN_\beta(0)$ are the corresponding coupling constants. In Ref. 13 are considered various possibilities for the suppression of the critical temperature, as well as the relative sign of Δ_1 and Δ_2, in the Born limit for nonzero values of $\lambda_{\alpha\beta}$ and $\gamma_{\alpha\beta}$. Before studying the strong scattering limit let us quote some interesting conclusions: (i) the diagonal scattering rate γ_{11} and γ_{22} disappear from the linearized equation (25) for T_c; (ii) in the case $\lambda_{11}\neq 0$, $\lambda_{22}=\lambda_{12}=\lambda_{21}=0$ the depression of T_c is given by $\delta T_c/T_c=-\pi\gamma_{12}/8T_c$; (iii) for $\lambda_{11}=\lambda_{22}\neq 0$ and $\lambda_{12}=\lambda_{21}=\lambda_\perp<0$ one has $\mathrm{sgn}(\Delta_1/\Delta_2)=-1$ and $\delta T_c/T_c=-\pi(\gamma_{12}+\gamma_{21})/8T_c$, while the sign of Δ_1 and Δ_2 is unchanged by the impurities.

The t-matrix equation in the two-band model with only s-wave scattering potential has the form (note that we consider the case of small impurity concentration and therefore neglect interband hybridization)

$$\hat{\mathbf{t}}(n)=\hat{\mathbf{u}}+\sum_\gamma \hat{\mathbf{u}}\mathbf{N}(0)\hat{\mathbf{g}}(n)\hat{\mathbf{t}}(n), \tag{26}$$

where $\hat{\mathbf{t}}(n)=\Sigma_{i=0}^3\mathbf{t}_i\otimes\hat{\tau}_i$, $\hat{\mathbf{g}}(n)=\mathbf{g}_3\otimes\hat{\tau}_3+\mathbf{g}_2\otimes\hat{\tau}_2$ and $\otimes$ is the direct product of matrices in the band space (bold) and in the Nambu space (caret). $\mathbf{g}_2$, $\mathbf{g}_3$, and $\mathbf{N}(0)$ are diagonal matrices in the band space.

In the case of nonmagnetic impurities one has $\hat{\mathbf{u}}^N=\mathbf{u}^N\otimes\hat{\tau}_0$, and since $\mathbf{g}_1=0$, one has

$$\mathbf{t}_0^N(n)=\mathbf{u}^N+\mathbf{u}^N\mathbf{N}(0)[\mathbf{g}_3(n)\mathbf{t}_3^N(n)+\mathbf{g}_3(n)\mathbf{t}_3^N(n)],$$

$$\mathbf{t}_1^N(n)=\mathbf{u}^N\mathbf{N}(0)[-i\mathbf{g}_3(n)\mathbf{t}_2^N(n)+i\mathbf{g}_2(n)\mathbf{t}_3^N(n)],$$

$$\mathbf{t}_2^N(n)=\mathbf{u}^N\mathbf{N}(0)[i\mathbf{g}_3(n)\mathbf{t}_1^N(n)+\mathbf{g}_2(n)\mathbf{t}_0^N(n)],$$

$$\mathbf{t}_3^N(n)=\mathbf{u}^N\mathbf{N}(0)[\mathbf{g}_3(n)\mathbf{t}_0^N(n)-i\mathbf{g}_2(n)\mathbf{t}_1^N(n)]. \tag{27}$$

Let us consider for simplicity the case when $u_{11}^N,u_{22}^N=0$, but the interband scattering is finite, $u_{12}^N=u_{21}^N=u\neq 0$, and introduce three parameters

$$\sigma=\frac{\pi^2N_1(0)N_2(0)u^2}{1+\pi^2N_1(0)N_2(0)u^2} \tag{28}$$

and

$$\Gamma_i=\frac{c}{\pi N_i(0)},\quad i=1,2. \tag{29}$$

After some straightforward calculations one obtains the renormalized frequencies $\tilde{\omega}_{in}$ and order parameters $\tilde{\Delta}_{in}$:

$$\tilde{\omega}_{1n}=\omega_n+\Gamma_1\sigma\frac{(\sigma-1)(\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2)\tilde{\omega}_{2n}-\sigma\tilde{\omega}_{1n}\sqrt{\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2}\sqrt{\tilde{\omega}_{2n}^2+\tilde{\Delta}_{2n}^2}}{\det 1}, \tag{30}$$

$$\tilde{\Delta}_{1n}=\Delta_1+\Gamma_1\sigma\frac{(\sigma-1)(\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2)\tilde{\Delta}_{2n}-\sigma\tilde{\Delta}_{1n}\sqrt{\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2}\sqrt{\tilde{\omega}_{2n}^2+\tilde{\Delta}_{2n}^2}}{\det 1}, \tag{31}$$

where

$$\det 1=2(\sigma-1)\sigma\sqrt{\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2}(\tilde{\Delta}_{1n}\tilde{\Delta}_{2n}+\tilde{\omega}_{1n}\tilde{\omega}_{2n}) -[2(\sigma-1)\sigma+1](\tilde{\omega}_{1n}^2+\tilde{\Delta}_{1n}^2)\sqrt{\tilde{\omega}_{2n}^2+\tilde{\Delta}_{2n}^2}. \tag{32}$$

The solution for the second band is obtained from Eqs. (30)–(32) by replacing $1\Leftrightarrow 2$. In the Born limit one gets

$$\tilde{\omega}_{1n}=\omega_n+\pi cN_2(0)u^2\frac{\tilde{\omega}_{2n}}{\sqrt{\tilde{\omega}_{2n}^2+\tilde{\Delta}_{2n}^2}}, \tag{33}$$

$$\tilde{\Delta}_{1n}=\Delta_1+\pi cN_2(0)u^2\frac{\tilde{\Delta}_{2n}}{\sqrt{\tilde{\omega}_{2n}^2+\tilde{\Delta}_{2n}^2}}; \tag{34}$$

i.e., the interband scattering mixes both bands according to Eqs. (25) and (24). In the unitarity limit $\sigma\to 1$ $(u\to\infty)$ the bands are decoupled, i.e.,

$$\tilde{\omega}_{\alpha n}=\omega_n+\Gamma_\alpha \frac{\tilde{\omega}_{\alpha n}}{\sqrt{\tilde{\omega}_{\alpha n}^2+\tilde{\Delta}_{\alpha n}^2}}, \tag{35}$$

$$\tilde{\Delta}_{\alpha n}=\Delta_{\alpha n}+\Gamma_\alpha \frac{\tilde{\Delta}_{\alpha n}}{\sqrt{\tilde{\omega}_{\alpha n}^2+\tilde{\Delta}_{\alpha n}^2}}. \tag{36}$$

So in this unitarity for the interband scattering the Anderson theorem is restored; i.e., the thermodynamic properties are impurity independent.

The latter result can be generalized to the case when

$$\mathbf{u}^N=\begin{pmatrix} \alpha u & u \\ u & u_{22}\end{pmatrix}. \tag{37}$$

For $u\to\infty$ but α and u_{22} finite, α and u_{22} drop out from equations and the bands are decoupled with $\tilde{\omega}_{\alpha n}$ and $\tilde{\Delta}_{\alpha n}$ given by Eqs. (35) and (36). At T_c and for $\sigma\leqslant 1$ one has

$$\tilde{\omega}_{1n}=\omega_n+\sigma\Gamma_1\,\mathrm{sgn}(\omega_n),$$

$$\tilde{\Delta}_{1n}=\Delta_1+\sigma\Gamma_1\left(\frac{\sigma\tilde{\Delta}_{1n}}{|\tilde{\omega}_{1n}|}+\frac{(1-\sigma)\tilde{\Delta}_{2n}}{|\tilde{\omega}_{2n}|}\right). \tag{38}$$

From Eq. (38) it is seen that in the unitarity limit $\sigma\to 1$, the renormalized order parameters are decoupled and T_c is unrenormalized by such impurities. For $\sigma<1$ it can be easily shown that T_c is reduced with respect to the clean limit.

IV. SMALL ANISOTROPIC DEFECT IN ANISOTROPIC SUPERCONDUCTORS

In the following we consider the effect of a single impurity (small defect) with small scattering length a, which is supposed to be much smaller than the superconducting coherence length, $|a|\ll\xi_0$. The latter condition is with certainty fulfilled in many clean low-temperature superconductors for small defects due to large ξ_0. However, in HTS materials due to rather small $\xi_0\sim 20$ Å (in the ab plane) only some special defects fulfill this condition ($|a|\ll\xi_0$). For instance oxygen vacancies could play a role of small defect.

So if $|a|\ll\xi_0$, the impurity can be considered as a localized perturbation, but with negligible renormalization of $\hat{\Delta}(\mathbf{p}_F,\mathbf{R})$ which is of the order $(a/\xi_0)^2$ (see Refs. 23 and 24), giving rise to the quasiclassic equations[23,24]

$$\begin{aligned}&[(i\omega_n+e\mathbf{v}_F\cdot\mathbf{A}(\mathbf{R}))\hat{\tau}_3-\hat{\Delta}(\mathbf{p}_F,\mathbf{R}),\delta\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)]\\&\quad+i\mathbf{v}_F\nabla_{\mathbf{R}}\delta\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)\\&\quad=[\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n),\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)]\,\delta(\mathbf{R}-\mathbf{R}_{imp}).\end{aligned} \tag{39}$$

Here, $\delta\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)=\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)-\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)$. The extra term, which is proportional to $\delta(\mathbf{R}-\mathbf{R}_{imp})$, describes a jump in $\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)$ at the site $\mathbf{R}_{imp}$ of the impurity (defect), while the intermediate Green's function $\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)$ describes the quasiclassic motion in the absence of impurity (defect) and it is the solution of Eq. (39) with the right-hand side equal to zero. The Green's functions $\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)$ and $\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)$ are normalized according to Eq. (2). The t matrix entering Eq. (39) is the solution of Eq. (4) where $\hat{g}(\mathbf{p}_F,\omega_n)$ is replaced by $\hat{g}_0(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$. The change of the superconducting free energy in the presence of a single impurity (defect) is given by[23,24]

$$\begin{aligned}\delta F(\mathbf{R}_{imp})=N(0)T\sum_n\int_0^1 d\lambda\int\frac{d^2\hat{k}_F}{4\pi}\int d^3R\ \mathrm{Tr}\\ \times[\delta\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)\hat{\Delta}_b(\mathbf{p}_F,\mathbf{R})],\end{aligned} \tag{40}$$

where $\hat{\Delta}_b(\mathbf{p}_F,\mathbf{R})$ and the vector potential $\mathbf{A}_b(\mathbf{R})$ are calculated in the absence of the impurity. The Green's function $\delta\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)$ must be evaluated for an order parameter $\hat{\Delta}(\mathbf{p}_F,\mathbf{R})=\lambda\hat{\Delta}_b(\mathbf{p}_F,\mathbf{R})$.

In the following we study the consequences of anisotropic impurity scattering for three selected examples of inhomogeneous anisotropic superconductors.

A. Bound states due to the anisotropic impurity

Let us consider the local change of superconductivity in the presence of a single anisotropic impurity with the potential $v(\mathbf{s},\mathbf{s}')$ given by Eq. (10) and analyze the impurity-induced quasiparticle bound state and the change in the free energy $\delta F(\mathbf{R}_{imp})$. By assuming that $2\pi\bar{\sigma}_i\ll E_F/\Delta_0$, where $i=0,2$ and $\bar{\sigma}_i=v_i^2/(1+v_i^2)$, the t matrix is given by the same expression as Eqs. (13) and (14), but with $\tilde{g}_2(n)$ replaced by $g_2^{(0)}(n)$. The bound state energy $\omega_{B,anis}<\Delta_0$, which is due to the pair-breaking impurity effects, is obtained as a pole of the t matrix which gives

$$\omega_{B,anis}=\Delta_0\sqrt{1-\bar{\sigma}_{pb}}, \tag{41}$$

where

$$\bar{\sigma}_{pb}=\bar{\sigma}_0\bar{\sigma}_2\frac{(v_0-v_2)^2}{v_0^2v_2^2}. \tag{42}$$

In the unitarity limit for both channels, i.e., $v_0\gg 1$, $v_2\gg 1$, but v_2/v_0 finite, one has $\omega_{B,anis}\to\Delta_0$ contrary to the unitarity limit for the s-wave scattering ($v_0\gg 1$, $v_2=0$) where $\omega_{B,iso}\to 0$. This example tells us that the bound state can disappear (i.e., moves to the continuum) in anisotropic systems even in the case of strong quasiparticle scattering on impurities. However, the zero-energy bound state $\omega_{B,anis}\to 0$ appears when $v_0v_2=-1$; i.e., if one channel is in the unitarity limit, the other one must be in the Born limit.

Due to the bound state, there is a change (increase) of the free energy $\delta F(\mathbf{R}_{imp})\equiv\delta F_{imp}$. By solving Eq. (39) with $\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)$ given by Eq. (5) and $\hat{t}$ given by Eqs. (11)–(14) one gets δF_{imp} from Eq. (40):

$$\begin{aligned}\delta F_{imp}&=T\sum_n\int_0^1 d\lambda\,\bar{\sigma}_{pb}\frac{\lambda\Delta_0^2\omega_n^2}{[\omega_n^2+\lambda^2\Delta_0^2][\omega_n^2+\omega_{B,anis}^2]}\\&=2T\ln\frac{\cosh(\Delta_0/2T)}{\cosh[(1-\bar{\sigma}_{pb})^{1/2}\Delta_0/2T]},\end{aligned} \tag{43}$$

where $\overline{\sigma_{pb}}$ is given in Eq. (42). It is seen that there is a loss in the condensation energy, $\delta F_{imp}>0$, which is related to the pair-breaking effect of the impurity. For $v_0=v_2$ such an impurity does not affect superconductivity and $\delta F_{imp}=0$.

The obtained results tell us that for angle-dependent impurity scattering even a strong impurity potential may have a very weak effect on T_c, the bound state, and the free energy of anisotropic and unconventional pairing. In that case the anisotropic pairing is robust in the presence of impurities.

B. Pinning of a single vortex by a small anisotropic defect

Because in HTS oxides strong correlations give rise to strong momentum-dependent charge scattering processes, such as, for instance, impurity scattering, it is interesting to analyze the elementary flux-pinning potential of a small defect by using the approach of Thuneberg *et al.*,[23,24] They showed that in *s*-wave superconductors the pinning energy of a small (impurity) defect ($a\ll\xi_0$) is dominated by scattering processes at the defect. It is proportional to the product of the scattering cross section and coherence length ($\propto a^2\xi_0$), instead of (naively believed) a^3, thus giving rise to a larger pinning force by a factor $(\xi_0/a)\gg 1$. The case of anisotropic pairing but with *s*-wave impurities and near T_c was recently studied in Ref. 29.

In the following we study the effect of anisotropic scattering on the pinning energy of a small defect in an *anisotropic superconductor* at any temperature below T_c. We use the model potential given in Eq. (10) and assume that the vortex is placed at the defect. In order to calculate the elementary flux-pinning energy one has to solve the quasiclassical equations for various ballistic trajectories which go through the vortex core and with **R**-dependent vector potential $\mathbf{A}(\mathbf{R})$ and order parameter $\Delta_b(\mathbf{p}_F,\mathbf{R})$. For trajectories going through the vortex core it can be parametrized in the following way:

$$\Delta_b(\mathbf{p}_F,\mathbf{R})=|\Delta(\mathbf{p}_F,\mathbf{R})|e^{i\theta}Y(\theta). \tag{44}$$

In the gauge where θ is the angle with respect to the X axis, $\mathbf{A}(\mathbf{R})$ has no radial component. The solution of Eq. (39) requires for a realistic vortex numerical calculations with **R**-dependent $|\Delta(\mathbf{p}_F,\mathbf{R})|$. For a qualitative discussion we will adopt a simplified vortex model[23,24] which neglects the suppression of the order parameter in the vortex core and sets $|\Delta_b(\mathbf{p}_F,\mathbf{R})|=\Delta_0(\mathbf{p}_F)$; i.e., $|\Delta_b(\mathbf{p}_F,\mathbf{R})|$ is **R** independent, but keeps its phase dependence. Hence, the order parameter along a trajectory passing through the vortex center has constant magnitude but its phase changes abruptly by π when going through the vortex core. We stress that this zero-core model gives the right order of magnitude of the pinning energy $\delta F_{pin}(\mathbf{R}_{imp})$ and its temperature dependence in *s*-wave superconductors with *s*-wave impurity scattering, when compared with the numerical calculations.[24] In order to calculate $\delta F_{pin}(\mathbf{R}_{imp})$ two quantities are needed. First, the intermediate solution $\hat{g}_0(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ at the impurity position $\mathbf{R}_{imp}$ is the vortex solution $\hat{g}_v$ in the absence of the impurity, i.e., $\hat{g}_0(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)\equiv\hat{g}_v(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$. In the case of the zero-core vortex model the solution for $\hat{g}_v(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ is straightforward[24] and has the form ($\Delta=\Delta_1+i\Delta_2$)

$$\hat{g}_v(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)=\frac{1}{\omega_n}[(-\Delta_2\hat{\tau}_1+\Delta_1\hat{\tau}_2)Y(\theta)+(-i\alpha_n)\hat{\tau}_3]. \tag{45}$$

Second, the impurity t matrix is the solution of Eq. (4) where $\hat{g}(\mathbf{p}_F,\omega_n)$ is replaced by $\hat{g}_v(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ given by Eq. (45), which gives

$$\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n)=t_3\hat{\tau}_3=-i\gamma_u\alpha_n\omega_n\left[\frac{\overline{\sigma_0}}{\omega_n^2+\overline{\sigma_0}\Delta_0^2}+\frac{\overline{\sigma_2}}{\omega_n^2+\overline{\sigma_2}\Delta_0^2}\right]\hat{\tau}_3. \tag{46}$$

Here, $\alpha_n=\sqrt{\omega_n^2+\Delta_0^2}$. Note that $\hat{t}$ does not contain t_1 and t_2 because Δ_1 and Δ_2 are averaged to zero by integrating over $\mathbf{k}'$ in Eq. (4). Equation (39) can be solved by the Fourier (or Laplace) transform which gives the expression for the pinning free energy:

$$\delta F_{pin}=\delta F_{pin}^{(stiff)}(\overline{\sigma_0},\overline{\sigma_2})+\delta F_{pin}^{(pb)}(\sigma_{pb}), \tag{47}$$

$$\delta F_{pin}^{(stiff)}=-2T\ln\left\{\cosh\frac{\sqrt{\overline{\sigma_0}}\Delta_0}{2T}\cdot\cosh\frac{\sqrt{\overline{\sigma_2}}\Delta_0}{2T}\right\}, \tag{48}$$

$$\delta F_{pin}^{(pb)}=-\delta F_{imp}=-2T\ln\frac{\cosh(\Delta_0/2T)}{\cosh[(1-\overline{\sigma_{pb}})^{1/2}\Delta_0/2T]}. \tag{49}$$

Equations (47)–(49) imply that $\delta F_{pin}<0$ (because $\delta F_{pin}^{(stiff)}<0$ and $\delta F_{pin}^{(pb)}<0$), and the vortex is attracted (pinned) by the defect. A comparison of Eq. (47) with the corresponding results for *s*-wave superconductors with an *s*-wave scattering potential shows that in the former case two additional terms are present. The first one, depending on $\overline{\sigma_2}$, appears also in *s*-wave superconductors with anisotropic scattering accounted for. In fact $\delta F_{pin}^{(stiff)}$ describes the reduction of the superconducting stiffness in the presence of an impurity. For instance, near T_c, Eq. (48) is simplified:

$$\delta F_{pin}^{(stiff)}=-(\overline{\sigma_0}+\overline{\sigma_2})\frac{\Delta_0^2(T)}{4T_c}\approx-2.72\frac{\overline{\sigma_0}+\overline{\sigma_2}}{\hbar v_F N(0)}\xi_0 E_{cond}(T), \tag{50}$$

where $E_{cond}(T)$ is the condensation energy, i.e., $E_{cond}(T)=N(0)\Delta_0^2(T)/2$. It is seen from Eq. (50) that $\delta F_{pin}^{(stiff)}$ is proportional to the total scattering amplitude $\overline{\sigma_0}+\overline{\sigma_2}$ and to ξ_0. For a vortex far away from the impurity there is loss in the condensation energy δF_{imp} due to the pair-breaking effect of the impurity. For a vortex sitting on the defect the latter part of the energy is gained, i.e., $\delta F_{pin}^{(pb)}=-\delta F_{imp}<0$. Therefore this part enters into Eq. (47) with a negative sign, thus increasing the pinning energy when vortex is sitting on the defect and stabilizing it additionally. Near T_c one has

$$\delta F_{pin}^{(pb)}(\sigma_{eff}) = \frac{\overline{\sigma_{pb}}}{(\overline{\sigma_0}+\overline{\sigma_2})}\,\delta F_{pin}^{(stiff)}. \tag{51}$$

Note that $0>\delta F_{pin}^{(pb)}(\sigma_{eff})>\delta F_{pin}^{(stiff)}$ for $v_0 v_2>0$ and for $v_0 v_2<-1$, while for $-1<v_0 v_2<0$ one has $0>\delta F_{pin}^{(stiff)} >\delta F_{pin}^{(pb)}(\sigma_{eff})$. For $v_2=v_0$ the pair breaking of the impurity is absent, $\overline{\sigma_{pb}}=0$ and $\delta F_{pin}^{(pb)}=0$; i.e., in this case the pinning by the small defect is similar to that in s-wave superconductors. We stress that $\delta F_{pin}\sim a^2\xi_0$, contrary to the naive expectation where $\delta F_{pin}\sim a^3$.

The (qualitative) physical picture of the vortex pinning by small defect given above is based on the microscopic derivation of the Ginzburg-Landau equations (near T_c) in the presence of impurities. For arbitrary temperatures the explanation of the impurity pinning is based on the quasiclassical approach which is given in Refs. 23 and 24. We briefly discuss it in order to develop an intuition for the case of a double-vortex pinning, which is studied below. Because in the presence of the vortex the order parameter changes its phase by π along the trajectories across the vortex core, it leads to the phase change of $\hat{g}_v(\mathbf{p}_F,\mathbf{R},\omega_n)$ $[\equiv \hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)]$ on the distance ξ_0, thus causing a cost in the condensation energy, i.e. the maximal increase of the free energy. Note that the function $\hat{g}_v(\mathbf{p}_F,\mathbf{R},\omega_n)$ describes the quasiclassical motion of particles (or pairs) along trajectories across the vortex core where the maximal phase change (π) occurs. In the presence of a defect (impurity) the motion of particles is described by the function $\hat{g}(\mathbf{p}_F,\mathbf{R},\omega_n)$ which contains scattering of particles to new directions where the phase change (mismatch) is less than π and it costs less condensation energy. Therefore the vortex is attracted to the defect because scattering helps superconductivity to sustain abrupt changes in the order parameter. The latter analysis explains the contribution $\delta F_{pin}^{(stiff)}$. However, in anisotropic superconductors due to the pair-breaking effect of the impurity, part (δF_{imp}) of the condensation energy is lost. On the other hand, if the vortex is sitting just on the impurity position, there is no pair-breaking effect and therefore there is a gain in the energy ($-\delta F_{imp}$) for vortex sitting on the defect.

C. Pinning of a double vortex by a small defect

We extend the method from Ref. 24 to the problem of pinning of a multiply quantized vortex on a small defect. The doubly quantized (two-quanta) vortex is theoretically proposed in Ref. 25, and the latter can be realized in some antiferromagnetic superconductors with metamagnetic or spin-flop transition, such as $DyMo_6S_8$ (see Ref. 26), the bct modification of $ErRh_4B_4$ (Ref. 27), and in HTS $GdBa_2Cu_3O_7$ (Ref. 28). If the lower critical field H_{c1} is of the order of the field for the metamagnetic transition H_m, then there is a redistribution of magnetic induction inside and outside the vortex core, leading to the magnetic doubly quantized vortex.[25]

In the following the s-wave superconductor is considered and we put the following question: is it possible to pin the double-flux-vortex ($\Phi=2\Phi_0$) by the small defect? The latter is for simplicity characterized by the parameter $\overline{\sigma_0}$ for the s-wave scattering only. We consider again the problem when the vortex is sitting on the defect and assume the zero-core model again. In that case the order parameter can be parametrized in the form

$$\Delta_b(\mathbf{p}_F,\mathbf{R}) = |\Delta(\mathbf{p}_F,\mathbf{R})|e^{2i\theta}. \tag{52}$$

For particle (pair) motion across the double vortex core the order parameter does not change its phase. The solution for the zero-core double-vortex $\hat{g}_0(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ $\equiv \hat{g}_{2v}(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ is given by

$$\hat{g}_{2v}(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n) = \frac{i}{\alpha_n}[\Delta_1\hat{\tau}_1+\Delta_2\hat{\tau}_2+(-\omega_n)\hat{\tau}_3]. \tag{53}$$

The solution for $\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n)$ in Eq. (4) with $\hat{g}(\mathbf{p}_F,\omega_n)$ replaced by $\hat{g}_{2v}(\mathbf{p}_F,\mathbf{R}=\mathbf{R}_{imp},\omega_n)$ has again simple form

$$\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n) = t_3\hat{\tau}_3 = -i\omega_n\alpha_n\gamma_u\frac{\overline{\sigma_0}}{\omega_n^2+\tilde{\sigma}_0\Delta_0^2}\hat{\tau}_3, \tag{54}$$

where $\tilde{\sigma}_0=\overline{\sigma_0}/v_0^2$. Note that the t_1 and t_2 terms are absent, like in the case of a single vortex, but the structure of t_3 is different from the single-vortex case. Then by solving Eq. (39) with $\hat{t}$ from Eq. (54) and by using Eq. (40) one obtains the pinning energy of the double vortex within the zero-core model:

$$\delta F_{2v,pin} = 2T\sum_n \int_0^1 d\lambda\,\frac{\lambda\Delta_0^2\omega_n^2\overline{\sigma_0}}{\alpha_n^2[\omega_n^2+\tilde{\sigma}_0\lambda^2\Delta_0^2]}>0. \tag{55}$$

From Eq. (55) follows a surprising result, that due to $\delta F_{pin}>0$, the double vortex in s-wave superconductors is repelled from the defect. This means that the zero-core double vortex can not be pinned contrary to the single-vortex case. While in the case of a single vortex the defect scatters particles to new directions where the phase change of the order parameter is smaller, thus lowering the energy, in the case of a double vortex the particles are scattered to directions where the phase change is larger, thus increasing the vortex energy. However, it might be that the above-obtained results are an artifact of the ''zero-core model,'' where there is no suppression of the superconducting order due to the vortex core, and numerical calculations are required for a realistic double-vortex structure.[30]

In the case of a double vortex in unconventional superconductors, such as that in Sec. II A, where the order parameter is given by $\Delta_b(\mathbf{p}_F,\mathbf{R})=|\Delta(\mathbf{p}_F,\mathbf{R})|\exp(2i\theta)Y(\theta)$, it may happen that the t matrix contains the terms t_1 and $t_2\neq 0$ which may reduce the jump $[\hat{t}(\mathbf{p}_F,\mathbf{p}_F,\omega_n),\hat{g}_0(\mathbf{p}_F,\mathbf{R},\omega_n)]$ in Eq. (39), thus making $\delta F_{2v,pin}$ less positive. In the case of unconventional pairing the pinning energy contains an additional term (gain in energy) due to the pair-breaking effect of the impurity, $\delta F_{pin}^{(pb)}=-\delta F_{imp}$, i.e., $\delta F_{pin}=\delta F_{2v,pin}-\delta F_{imp}$. Since $\delta F_{2v,pin}$ is expected to be less repulsive for unconventional pairing than for s-wave pairing and because of $-\delta F_{imp}<0$, it may happen that $\delta F_{pin}<0$ and even the zero-core double vortex can be pinned by the defect. A realistic calculation of δF_{pin} for anisotropic superconductors with anisotropic scattering of single and double vortexes will be discussed elsewhere.[30]

In conclusion anisotropic impurity scattering gives rise to new qualitative effects in unconventional and anisotropic superconductors, where, for instance, it ''screens'' the strength of the scattering in some quantities (such as T_c, bound states, pinning, etc.) even in the unitarity limit—robustness of pairing. It seems that this situation is partly realized in HTS oxides where *d*-wave pairing is robust in the presence of even very strong impurity scattering. In two-band models nonmagnetic impurities do not affect thermodynamic properties of *s*-wave superconductors in the unitarity limit for the interband scattering, contrary to the Born limit; i.e., in this case the Anderson theorem is restored in the unitarity limit. Anisotropic impurity scattering in unconventional superconductors gives additional pinning energy of single and double vortexes due to the reduction of the impurity pair-breaking effect for a vortex sitting on the impurity.

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