

Superconductivity versus tunneling in a doped antiferromagnetic ladder

J. P. Rodriguez

*Instituto de Ciencia de Materiales, Consejo Superior de Investigaciones Científicas, Cantoblanco, 28049 Madrid, Spain
and Department of Physics and Astronomy, California State University, Los Angeles, California 90032**

(Received 14 August 1998; revised manuscript received 19 January 1999)

The low-energy charge excitations of a doped antiferromagnetic ladder are modeled by a system of interacting spinless fermions that live on the same ladder. A relatively large spin gap is assumed to “freeze out” all spin fluctuations. We find that the formation of rung hole pairs coincides with the opening of a single-particle gap for charge excitations along chains and with the absence of coherent tunneling in between chains. We also find that such hole pairs condense into either a crystalline or superconducting state as a function of the binding energy. [S0163-1829(99)10625-8]

One of the strangest features of high-temperature superconductors is the semiconducting nature of electric transport perpendicular to the copper-oxygen planes common to these materials shown in the normal state.¹ Anderson and co-workers have proposed that this behavior is intrinsic, and that it results from the microscopic coincidence of Cooper pairing with incoherent tunneling in between adjacent metallic planes.² Analogous behavior has been observed recently in the doped “ladder” materials.³ In particular, a large anisotropy in the electronic conduction with respect to the ladder direction appears⁴ at compositions that exhibit superconductivity.^{5,6} The authors of Ref. 4 suggest that the chain version of the above interlayer-pair-tunneling (IPT) mechanism is responsible for the phenomenon.^{7,8}

Such “ladder” materials are made up of a parallel arrangement of magnetically isolated antiferromagnetic ladders that fill copper-oxygen planes similar to those of the cuprate superconductors. They notably show a spin gap on the order of $\Delta_{\text{spin}} \sim 500$ K when undoped, in accord with theoretical expectations based on the spin-1/2 antiferromagnetic Heisenberg ladder.³ The actual ladders in these materials can also be doped with a concentration x of mobile holes,⁹ in which case the spin gap persists. Again, theoretical studies of the t - J model for a lightly doped ladder, $tx < J$, find evidence for the formation of *hole pairs* along rungs that leave the remaining singlet bonds along the rungs intact.¹⁰ The spin gap persists, as a result, and charge excitations are left as the only low-energy excitations.

In this paper, we shall model the low-energy singlet charge excitations of a doped antiferromagnetic ladder by interacting spinless fermions that live on the same ladder in the vicinity of half filling.¹¹ We presume, therefore, that spin excitations are frozen out due to a relatively large spin gap of order $\Delta_{\text{spin}} \sim J/2$. A bosonization analysis based on the corresponding Luther-Emery model,¹²⁻¹⁵ and generalizations thereof,^{16,17} yields that the appearance of hole pairs along rungs coincides with the absence of coherent tunneling in between chains, as well as with the creation of a gap for single-particle excitations along chains. Also, while the hole pairs generally crystallize into a charge-density-wave (CDW) state in the weak-coupling limit,³ we predict that they Bose condense into an IPT-type superconductor at large enough binding energies (see Fig. 1). Last, the correlations in be-

tween pairs that form along the chains are found to be short range when hole pairing along rungs occurs.

To motivate the spinless fermion analysis that will follow, consider first the standard nearest-neighbor t - J model Hamiltonian

$$H = \sum_{\langle i,j \rangle} [-t(\tilde{c}_{i,s}^\dagger \tilde{c}_{j,s} + \text{H.c.}) + J \vec{S}_i \cdot \vec{S}_j] \quad (1)$$

for a doped antiferromagnetic ladder. Above, $\tilde{c}_{i,s}^\dagger$ creates a spin s electron on site i as long as this site is unoccupied, while \vec{S}_i measures the spin at site i . At half filling, the t - J model (1) reduces to the Heisenberg ladder, which is known to have a spin gap $\Delta_{\text{spin}} \cong J/2$.³ The latter persists in the presence of a dilute hole concentration, $tx < J$.¹⁰ In general, long wavelength electronic excitations within chains can be described via the standard Luther-Emery (LE) model.¹²⁻¹⁵ The

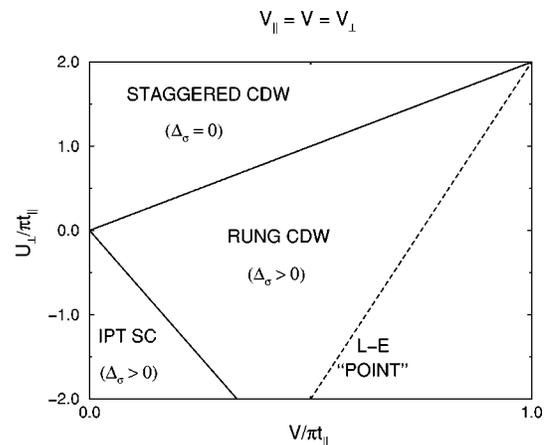


FIG. 1. The phase diagram of the SU(2)-invariant model (5) for the charge excitations of a doped antiferromagnetic ladder is displayed in the regime of effective repulsion within each chain, $V_{\parallel} = V_{\perp} > 0$, and in the absence of hopping in between chains. The rung-hole-pair regime ($\Delta_{\sigma} > 0$) is, nevertheless, expected to persist for small enough interchain hopping matrix elements $t_{\perp} \leq \Delta_{\sigma}/2$. In such a case, the line separating dominant CDW correlations from superconducting ones among the rung-hole pairs is determined by the condition $K_{\rho} = 1$.

spin gap present in lightly doped t - J ladders indicates then that the intrachain LE backscattering term has the form

$$H_{\text{back}} \sim -(\Delta_{\text{spin}}/a) \sum_{l=1}^2 \int dx \cos[\phi_{\uparrow}(x,l) - \phi_{\downarrow}(x,l)], \quad (2)$$

where $\phi_s(x,l)$ is the bosonic field that represents collective particle-hole excitations of spin s electrons at position $x = ia$ along chain l of the ladder.¹⁸ Yet long-wavelength charge excitations have a typical energy scale on the order of the kinetic energy $\Delta_{\text{charge}} \sim tx$ which is small in comparison to the spin gap Δ_{spin} in the low doping limit $tx \ll J$. We may therefore take $\Delta_{\text{spin}} \rightarrow \infty$ in this limit. The bosonic spin degrees of freedom are frozen in such a case: $\phi_{\uparrow} = \phi = \phi_{\downarrow}$. The remaining (bosonic) charge degree of freedom $\phi(x,l)$ must then correspond to an effective spinless fermion on each chain. This idea is developed below.

In general, the Hamiltonian for a system of N consecutively coupled chains of spinless fermions⁸ can be divided into parallel and perpendicular parts $H = H_{\parallel} + H_{\perp}$, where

$$H_{\parallel} = \sum_{l=1}^N \sum_i [-t_{\parallel}(f_{i,l}^{\dagger} f_{i+1,l} + \text{H.c.}) + V_{\parallel} n_{i,l} n_{i+1,l}] \quad (3)$$

and

$$H_{\perp} = \sum_{l=1}^N \sum_i [-t_{\perp}(e^{i\Phi/N} f_{i,l}^{\dagger} f_{i,l+1} + \text{H.c.}) + U_{\perp} n_{i,l} n_{i,l+1} + V_{\perp}(n_{i,l} n_{i+1,l+1} + n_{i,l+1} n_{i+1,l})] \quad (4)$$

describe, respectively, the quantum mechanics within and in between chains. Here, $f_{i,l}$ denotes the annihilation operator for the spinless fermion on the i th site of chain l , with occupation number $n_{i,l} = f_{i,l}^{\dagger} f_{i,l}$. Also, t_{\parallel} and t_{\perp} are the nearest-neighbor hopping matrix elements, while V_{\parallel} , U_{\perp} , and V_{\perp} are the model interaction energies. In the case of open perpendicular boundary conditions, we shall set $f_{i,N+1} = 0 = f_{i,N+1}^{\dagger}$. On the other hand, the identifications $f_{i,N+1} = f_{i,1}$ and $f_{i,N+1}^{\dagger} = f_{i,1}^{\dagger}$ produce periodic boundary conditions, in which case Φ denotes the magnetic flux along the parallel direction. Consider now the simple ladder, with $N=2$ chains and open perpendicular boundary conditions ($\Phi=0$). Since low-energy spin excitations are frozen out due to the formation of singlet bonds along the rungs,³ it is natural to identify the true electron field $c_{i,l,s}$ with the spinless fermion field following $f_{i,1} = c_{i,1,s(i)}$ and $f_{i,2} = c_{i,2,-s(i)}$, where $s(i)$ represents the antiferromagnetic spin configuration on a given chain. After some algebraic manipulations, it can be shown that the above spinless-fermion Hamiltonian takes the form of an extended Hubbard model¹⁴ in transverse magnetic field

$$H = \sum_i \left[-t_{\parallel} \sum_{l=1}^2 (f_{i,l}^{\dagger} f_{i+1,l} + \text{H.c.}) - t_{\perp} (f_{i,1}^{\dagger} f_{i,2} + \text{H.c.}) + (U_{\perp} + 2V_{\perp}) n_{i,1} n_{i,2} - \frac{1}{2} V (n_{i+1} - n_i)^2 - \frac{1}{4} V' (m_{i+1} - m_i)^2 \right], \quad (5)$$

where $n_i = n_{i,1} + n_{i,2}$ and $m_i = n_{i,2} - n_{i,1}$ and where $V = (V_{\parallel} + V_{\perp})/2$ and $V' = V_{\parallel} - V_{\perp}$. It is important to remark that the interaction terms in this ladder model are invariant with respect to SU(2) rotations of the chain labels if $V_{\parallel} = V_{\perp}$.

We now rotate to the bonding-antibonding basis, $f_{i,\pm} = 2^{-1/2}(f_{i,2} \pm f_{i,1})$, that diagonalizes the transverse kinetic energy (5). In the limit near (but not at) half filling, all umklapp processes are negligible. Taking the continuum limit of the ladder model (5) in the manner of Kogut and Susskind¹⁴ then yields the Luttinger model $H = H'_{\parallel} + H'_{\perp}$, where

$$H'_{\parallel} = \sum_n \int dx [2t_{\parallel} a (L_n^{\dagger} i \partial_x L_n - R_n^{\dagger} i \partial_x R_n) + 4Va L_n^{\dagger} R_n^{\dagger} R_n L_n - \mu_n (L_n^{\dagger} L_n + R_n^{\dagger} R_n)] \quad (6)$$

and

$$H'_{\perp} = H'_{\perp,1} + H'_{\perp,2} + H'_{\perp,4} + H'_{\perp,\text{pair}}$$

are rotated parallel and perpendicular pieces, with a backscattering term

$$H'_{\perp,1} = \sum_{n < n'} \int dx (U_{\perp} - 2V_{\parallel}) a [L_n^{\dagger} R_n^{\dagger} L_{n'} R_{n'} + \text{H.c.}], \quad (7)$$

with interband forward scattering terms

$$H'_{\perp,2} = \sum_{n < n'} \int dx 2(V_{\parallel} - V_{\perp}) a [L_n^{\dagger} R_n^{\dagger} R_{n'} L_{n'} + \text{H.c.}], \quad (8)$$

$$H'_{\perp,4} = \sum_{n < n'} \int dx (U_{\perp} + 2V_{\perp}) a : (L_n^{\dagger} L_n + R_n^{\dagger} R_n) :: (L_{n'}^{\dagger} L_{n'} + R_{n'}^{\dagger} R_{n'}) :, \quad (9)$$

and with an interband pseudotriplet pairing interaction

$$H'_{\perp,\text{pair}} = \sum_{n < n'} \int dx 2(V_{\parallel} - V_{\perp}) a [L_n^{\dagger} R_n^{\dagger} R_{n'} L_{n'} + \text{H.c.}]. \quad (10)$$

Here, $e^{ik_F x} R_n(x) = L^{-1/2} \sum_k e^{ikx} a_n(k)$ and $e^{-ik_F x} L_n(x) = L^{-1/2} \sum_k e^{ikx} b_n(k)$ denote field operators for right and left moving spinless fermions in the bonding or antibonding band $n = +, -$ for chains of length L , with a Fermi surface at $\pm k_F$. Above, the colon symbols represent normal ordering.¹⁴ Also, $\mu_{\pm} = \pm t_{\perp}$ are the chemical potentials for each band. Notice that Eqs. (6)–(10) describe a Luther-Emery model for pseudo-spin-1/2 fermions. Since such fermions experience pseudo-spin-charge separation, we have that the coupled chains factorize following $H = H_{\rho} + H_{\sigma}$, where

$$H_{\rho} = 2\pi \hbar v_{\rho} \sum_{q>0} \sum_{j=R,L} \rho_j(q) \rho_j(-q) + g_{\rho} \sum_q \rho_R(q) \rho_L(-q), \quad (11)$$

$$\begin{aligned}
H_\sigma = & 2\pi\hbar v_\sigma \sum_{q>0} \sum_{j=R,L} \sigma_j(q) \sigma_j(-q) \\
& + g_\sigma \sum_q \sigma_R(q) \sigma_L(-q) + H'_{\perp,1} + H'_{\perp,\text{pair}} \\
& - (2L)^{1/2} t_\perp [\sigma_L(0) + \sigma_R(0)] \quad (12)
\end{aligned}$$

are the respective commuting portions of the Hamiltonian. Here, $\rho_j(q) = 2^{-1/2}[\rho_j(q,+) + \rho_j(q,-)]$ and $\sigma_j(q) = 2^{-1/2}[\rho_j(q,+) - \rho_j(q,-)]$ are the standard particle-hole operators for total-charge and pseudo-spin excitations with respect to the bands $n = +, -$, with $\rho_R(q,n) = L^{-1/2} \sum_k a_n^\dagger(q+k) a_n(k)$ and $\rho_L(q,n) = L^{-1/2} \sum_k b_n^\dagger(q+k) b_n(k)$. The Fermi velocities and interaction strengths for each component are renormalized by the interband forward scattering processes [Eqs. (8) and (9)] to

$$v_{\rho,\sigma} = a[2t_\parallel \pm (U_\perp + 2V_\perp)/2\pi]/\hbar, \quad (13)$$

$$g_{\rho,\sigma} = a[4V \pm (U_\perp + 2V_\parallel)], \quad (14)$$

where the $+$ ($-$) signs above correspond to the $\rho(\sigma)$ labels. We remind the reader that it is assumed throughout that the system of spinless fermions (5) is near half filling.

To proceed further, we first note that the pure Luttinger model (11) for the total-charge excitations along the ladder corresponds precisely to the (Efetov-Larkin) hardcore boson model¹⁹ for the rung hole pairs that is elaborated Ref. 20. The former is characterized entirely by the Luttinger liquid parameter $K_\rho = (2\pi\hbar v_\rho - g_\rho)^{1/2}/(2\pi\hbar v_\rho + g_\rho)^{1/2}$ that will reappear below. The pseudospin piece (12) of the present spinless fermion description for a doped antiferromagnetic ladder is less trivial, however. Along the SU(2)-invariant line $V_\parallel = V_\perp$, the pairing term $H'_{\perp,\text{pair}}$ in Eq. (12) is null. Application of the bosonic representation^{14,15} for the spinless fermions then reveals that a gap $\Delta_\sigma \neq 0$ opens in the spectrum of the pseudospin excitations (12) for $g_\sigma/a = 2V_\perp - U_\perp > 0$ in the absence of transverse hopping,¹³ $t_\perp = 0$. For general interchain hopping $t_\perp \neq 0$ and interactions $V_\parallel \neq V_\perp$ it is instructive to move along the Luther-Emery ‘‘line’’ $g_\sigma = 6\pi\hbar v_\sigma/5$, in which case the spinless fermions that correspond to the pseudospin system (12) are governed by the noninteracting Hamiltonian¹²

$$\begin{aligned}
H_\sigma = & \hbar v'_F \sum_k k (a_k^\dagger a_k - b_k^\dagger b_k) - 2^{1/2} t_\perp \sum_k (a_k^\dagger a_k + b_k^\dagger b_k) \\
& + \Delta_\sigma \sum_k (a_k^\dagger b_k + \text{H.c.}) + \Delta'_\sigma \sum_k (a_k^\dagger b_{-k}^\dagger + \text{H.c.}). \quad (15)
\end{aligned}$$

Here, the pseudospin gaps have values $\Delta_\sigma = (a/\alpha_0)[(U_\perp - 2V_\parallel)/2\pi]$ and $\Delta'_\sigma = (a/\alpha_0)(V'/\pi)$ where α_0^{-1} is the momentum cutoff of the Luttinger model, while $v'_F = (4/5)v_\sigma$. Consider first the SU(2) invariant line $V_\parallel = V_\perp$, in which case the gap Δ'_σ that originates from the pairing term (10) is null. The spinless fermions corresponding to the pseudospin sector therefore have energy eigenvalues $\varepsilon_p = \pm(v'_F p^2 + \Delta_\sigma^2)^{1/2}$. If N_n denotes the number of spinless fermions in band n , then it follows that the band occupations are equal,

TABLE I. Listed is the correlation exponent η obtained via the bosonization technique for various order parameters $O(x)$ in the doped spin ladder model (5); i.e., $\langle O(x)O^\dagger(O) \rangle \propto (\alpha/x)^\eta$. The SU(2)-invariant case is assumed (see Ref. 15). Below, the value $\eta = \infty$ indicates short-range order, while $K_\rho = (2\pi\hbar v_\rho - g_\rho)^{1/2}/(2\pi\hbar v_\rho + g_\rho)^{1/2}$.

Order	Order parameter	η (Staggered CDW)	η (Rung hole pairs)
CDW, \parallel	$R_1^\dagger L_1$	$K_\rho + 1$	K_ρ
Pair, \parallel	$R_1 L_1$	$K_\rho^{-1} + 1$	∞
CDW, \perp	$R_1^\dagger L_2$	$K_\rho + 1$	∞
Pair, \perp	$R_1 L_2$	$K_\rho^{-1} + 1$	K_ρ^{-1}
CDW ²	$R_1^\dagger R_2^\dagger L_2 L_1$	$4K_\rho$	$4K_\rho$
Pair ²	$R_1 R_2 L_2 L_1$	$4K_\rho^{-1}$	$4K_\rho^{-1}$

$N_+ = N_-$, for $t_\perp < \Delta_\sigma/2^{1/2}$ and that $N_+ - N_- = L\chi_0[t_\perp^2 - (1/2)\Delta_\sigma^2]^{1/2}$ for $t_\perp > 2^{-1/2}\Delta_\sigma$,²¹ where $\chi_0 = 2/\pi\hbar v'_F$ is the pseudospin susceptibility. Band splitting is therefore *absent* below a critical interchain hopping matrix element.⁷ The general case, $\Delta'_\sigma \neq 0$, off of the SU(2) invariant line can also be analyzed at the Luther-Emery ‘‘line’’ (15). One finds that the product of all of the energy eigenvalues is $\prod_{p>0} (v'_F p^2 + \Delta_\sigma^2 + \Delta'_\sigma)^2$, which never vanishes. We conclude that the net pseudospin gap $\Delta_{\text{charge}} = (\Delta_\sigma^2 + \Delta'_\sigma)^{1/2}$ is therefore robust with respect to SU(2) symmetry breaking in the spinless fermion model (5).

Yet what is the physical character of the present ladder model at zero temperature? To answer this question, it is convenient to look again along the line, $V_\parallel = V_\perp$, in which case the SU(2) noninvariant interaction term in the model Hamiltonian (5) is absent. Let us start by assuming no interchain hopping, $t_\perp = 0$. Then the boundary at $U_\perp = 2V$ that marks the appearance of the pseudospin gap Δ_σ can be identified with the phase boundary that exists between the staggered CDW state and the rung (hole-pair) CDW state in the strong-coupling limit (see Fig. 1 and Ref. 22). A self-consistent calculation in terms of the CDW mean field $\langle R_n^\dagger L_n \rangle$ yields the approximate formula $\Delta_\sigma = \hbar\omega_0 / \sinh(\pi\hbar v_\sigma/g_\sigma)$ for the pseudospin gap in the hole-pair regime,¹⁷ $g_\sigma > 0$, with prefactor $\omega_0 = v_\sigma/\alpha_0$. It agrees reasonably well with the previous exact result along the Luther-Emery point. The same exact analysis indicates that band splitting generally remains absent at small enough interchain hopping amplitudes $t_\perp < \Delta_\sigma/2^{1/2}$. We now address the initial question posed by computing the correlation functions at long distances and at long times within this hole-pair regime (see Table I). Then SU(2) invariance yields the identity $\langle f_{0,l}(0) f_{i,l}^\dagger(t) \rangle = \langle f_{0,n}(0) f_{i,n}^\dagger(t) \rangle$ for the intrachain one-particle propagator, where $\langle f_{0,n}(0) f_{i,n}^\dagger(t) \rangle = G_R(x,t) + G_L(x,t)$ is the propagator in the bonding-antibonding basis ($x = ia$), with right and left moving components G_R and G_L , respectively. Note that the latter independence of the spinless fermion propagation with the band index n is a result of the equal band occupation $N_+ = N_-$ present in the hole pair regime $t_\perp < \Delta_\sigma/2^{1/2}$. The application of the bosonization method plus pseudo-spin-charge separation yields the forms $G_{R,L} = G_{R,L}^{(\rho)} \cdot G_{R,L}^{(\sigma)}$ for the right and left propagators, with a Luttinger liquid factor

$$G_{R,L}^{(\rho)} \sim (x \mp v'_\rho t)^{-1/2} [2\pi\alpha_0 / (x^2 - v'^2_\rho t^2)^{1/2}]^{\alpha_\rho} \quad (16)$$

due to excitations of the total charge,¹⁴ and with pseudospin factor²³

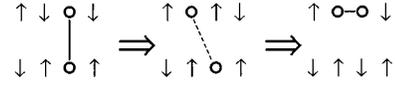
$$G_{R,L}^{(\sigma)} \sim (x \mp v_\sigma t)^{-1/2} e^{-(\Delta_\sigma / \hbar v_\sigma)(x^2 - v_\sigma^2 t^2)^{1/2}}. \quad (17)$$

The exponent and velocity that appear in expression (16) have the forms $\alpha_\rho = \sinh^2 \psi_\rho$ and $v'_\rho = v_\rho \operatorname{sech} 2\psi_\rho$, respectively, with the hyperbolic angle ψ_ρ set by the relation $\tanh 2\psi_\rho = -g_\rho / 2\pi\hbar v_\rho$. A gap $\Delta_{\text{charge}} = \Delta_\sigma$ therefore exists for one-particle charge excitations along chains. The interchain single-particle propagator, on the other hand, is by definition $\langle f_{0,1}(0) f_{i,2}^\dagger(t) \rangle = (1/2) [\langle f_{0,+}(0) f_{i,+}^\dagger(t) \rangle - \langle f_{0,-}(0) f_{i,-}^\dagger(t) \rangle]$. Again, the band occupations N_+ and N_- are equal for $t_\perp < \Delta_\sigma / 2^{1/2}$, which implies that $\langle f_{0,1}(0) f_{i,2}^\dagger(t) \rangle$ vanishes at long times and at long distances in such case. In other words, we find that *coherent* single-particle tunneling of charge in between chains is entirely suppressed in the hole-pair regime. Notice that this is consistent with renormalization group calculations that find interchain hopping to be an irrelevant perturbation for Luttinger liquids with a pseudo gap in the density of states:²⁴ $N(\omega) \propto |\omega|^\alpha$, with exponent $\alpha > 1$.

A similar analysis can be employed to obtain the static auto-correlators for various CDW and pair order parameters at long distance. These results are compiled in Table I. The intrachain CDW correlator, for example, has the form¹²⁻¹⁵ $\langle L_1^\dagger(0) R_l(0) R_l^\dagger(x) L_l(x) \rangle \sim \cos(2k_F x) (\alpha_0/x)^{K_\rho}$. It coincides with the form obtained for the density-density correlator of the hardcore boson model²⁰ for the rung-hole pairs,¹⁹ $b_i^\dagger = f_{i,1} f_{i,2}$. On the other hand, both interchain CDW order and chain-hole-pair autocorrelations are short range in the rung-hole-pair regimes, with a unique correlation length $\xi_\sigma = \hbar v_\sigma / 2\Delta_\sigma$. Finally, the static autocorrelator for rung-hole pairs has the asymptotic form $\langle L_1(0) R_2(0) R_2^\dagger(x) L_1^\dagger(x) \rangle \sim (\alpha_0/x)^{1/K_\rho}$. This form also coincides with that obtained from the previously cited hardcore boson model for the propagation, $\langle b_i b_j^\dagger \rangle$, of rung-hole pairs. In conclusion, the phase-boundary separating dominant rung-CDW correlations from dominant rung-pair autocorrelations is evidently determined by the condition $K_\rho = 1$ (see Table I). Given the absence of coherent tunneling that characterizes the rung-hole-pair regime in general, we interpret the latter phase ($K_\rho > 1$) as an IPT-type superconductor (see Fig. 1).

To address the question of the transverse conductivity of the ladder model (5), we shall now compute the transverse charge stiffness of the triangular three-leg ladder.⁸ In particular, consider the corresponding spinless fermion model [Eqs. (3) and (4), $N=3$] with a magnetic flux Φ threading each triangle formed by the rungs. Then along the line $V_\parallel = V_\perp$, the interaction terms of this model Hamiltonian are invariant with respect to SU(3) rotations of the chain labels. After making algebraic manipulations similar to those employed to achieve the form (5) for the simple ladder, we obtain the form

$$(S=0) \quad (S=1) \quad (S=0)$$



$$(\mathcal{E}_{\text{pair}}) \quad (\mathcal{E}_{\text{pair}} + \Delta_{\text{spin}}) \quad (\mathcal{E}_{\text{pair}})$$

FIG. 2. A diagrammatic representation is given for the second-order matrix element (t_\times) that connects rung and chain hole pairs. The spin gap (Δ_{spin}) is presumed to be large in comparison to any difference in the binding energy ($\mathcal{E}_{\text{pair}}$) between rung and chain hole pairs.

$$H = \sum_i \left[-t_\parallel \sum_{l=1}^3 (f_{i,l}^\dagger f_{i+1,l} + \text{H.c.}) - t_\perp \sum_{l=1}^3 (e^{i\Phi/3} f_{i,l}^\dagger f_{i,l+1} + \text{H.c.}) + (U_\perp + 2V) \sum_{l=1}^3 n_{i,l} n_{i,l+1} - \frac{1}{2} V (n_{i+1} - n_i)^2 \right] \quad (18)$$

for the triangular ladder model Hamiltonian, where $n_i = \sum_{l=1}^3 n_{i,l}$ is the manifestly SU(3) invariant number operator. After rotating to the basis $f_{i,0} = 3^{-1/2} \sum_{l=1}^3 f_{i,l}$ and $f_{i,\pm} = 3^{-1/2} \sum_{l=1}^3 e^{\pm i2\pi l/3} f_{i,l}$ that diagonalizes the transverse kinetic energy, we obtain the previous Luttinger model [Eqs. (6), (7), and (9)], but with the band index summed over the new basis $n=0, +, -$. The new chemical potentials for each band are $\mu_0 = 2t_\perp \cos(\Phi/3)$ and $\mu_\pm = 2t_\perp \cos[(\Phi \pm 2\pi)/3]$. Now consider the special line $U_\perp = -2V$, along which the interband forward scattering interaction (9) vanishes. What remains is a generalized backscattering model with three internal quantum numbers.¹⁶ A mean-field analysis of this model¹⁷ finds that long-range CDW order of the type $\langle R_n^\dagger L_n \rangle$ is stable for effective attraction between rungs, $U_\perp < 0$, with a single-particle gap $\Delta_\sigma \cong 2\hbar\omega_0 e^{-2\pi t_\perp/3|U_\perp|}$, and prefactor $\hbar\omega_0 \sim (a/\alpha_0)t_\parallel$. Hence for small enough interchain hopping, $t_\perp < \Delta_\sigma/2$, the chemical potential of each band lies within the gap, which means that the transverse charge stiffness, $\partial^2 E_0 / \partial \Phi^2|_0$, is *null*. In addition, since the band occupations, N_0 and N_\pm , are all equal for $t_\perp < \Delta_\sigma/2$, the single-particle intraband amplitudes $\langle f_{0,n}(0) f_{i,n}^\dagger(t) \rangle$ are then all equal. The interchain single-particle amplitude $\langle f_{0,l}(0) f_{i,l+1}^\dagger(t) \rangle = (1/3) \sum_n e^{i2\pi n/3} \langle f_{0,n}(0) f_{i,n}^\dagger(t) \rangle$ must therefore vanish as well, in agreement with the previous case of the simple ladder. On the basis of this mean-field analysis,¹⁷ we conclude that the present ladder model allows *no* coherent transport whatsoever in between chains in the (rung) hole-pair regime.

Let us now apply the spinless fermion model (5) to doped antiferromagnetic ladders by first drawing a comparison with the corresponding t - J model.¹⁰ In the limit $\Delta_{\text{spin}} \rightarrow \infty$ that is assumed throughout due to the relatively small energy scale tx for charge excitations, triplet excitations are forbidden. This means that the only possible charge carriers are hole pairs aligned parallel to either the rungs or to the chains of the ladder (see Fig. 2). The spinless fermion system (5) describes the motion of these objects. Since rung hole pairs are responsible for coherent charge transport along the ladder,

we have $t_{\parallel} = t$. In other words, the hopping of the spinless fermion along chains accounts for the effective tunneling of rung hole pairs between rungs. Yet what is the value of the interchain model parameter t_{\perp} ? First, observe that the rotation of a rung hole pair into a chain hole pair is a two-stage process. As depicted in Fig. 2, the initial and final singlet pair states ($S=0$) pass through an intermediate triplet pair state ($S=1$) that is the lowest-energy spin-excitation of the system.^{3,20} Second order perturbation theory then yields the matrix element $t_{\times} = t_{\parallel} t_{\perp} / \Delta_{\text{spin}}$ for the rotation of a rung hole pair into a chain hole pair, and vice versa. Yet since such 90° rotations represent the low-energy single-particle charge excitations of the ladder, we have the identity $t_{\times} = \Delta_{\text{charge}}$. This yields the expression $t_{\perp} = (\Delta_{\text{spin}} / t_{\parallel}) \Delta_{\text{charge}}$ for the effective interchain hopping matrix element. Yet since $t_{\parallel} = t$ and $\Delta_{\text{spin}} < J/2$, we obtain the desired inequality

$$t_{\perp} < \Delta_{\text{charge}}/2 \quad (19)$$

for $J < t$. This indicates that the simple t - J ladder is indeed consistently within the rung-hole-pair regime per the spinless fermion description (5). Note also that Eq. (19) implies that coherent motion (11) of rung-hole pairs represents the only gapless charge excitation of the t - J ladder. This is consistent with exact diagonalization results²⁵ that find a single-particle gap Δ_{charge} of order $\frac{3}{2}tx$ (the energy to *add* an electron).²⁰ The latter energy scale is small in comparison to the spin gap at low doping, $tx \ll J$, which justifies use of the spinless fermion model (5).

The remaining effective interaction parameters of the spinless fermion model (5) will be considered to be phenomenological. For simplicity, let us move along the SU(2) invariant line $V_{\parallel} = V_{\perp} > 0$. We then notably predict a phase transition in between a rung-CDW state and an IPT-type superconductor at $K_{\rho} = 1$ (see Fig. 1). Comparison of the corresponding correlation exponents shown in Table I with those obtained from a density-matrix renormalization group analysis of the t - J ladder²⁶ indicate that such a doped antiferromagnet is in the vicinity of this superconductor-insulator transition; i.e., $K_{\rho} \sim 1$. On this basis, we conclude that coherent single particle tunneling in between chains is absent in a lightly doped antiferromagnetic ladder, since $\Delta_{\sigma} \neq 0$ by Eq. (19). This does not exclude the possibility of (coherent) Josephson tunneling of hole pairs in between adjacent ladders, however. To address this issue, consider two

neighboring doped ladders. Let us also suppose that adjacent ladders are shifted with respect to each other by half a lattice constant, which is in fact the case for real ladder systems that exhibit superconductivity.^{3,5,6} The dynamics of the rung-hole pairs $b_i^{\dagger} = f_{i,1} f_{i,2}$ is then equivalent to that of coupled spin- $\frac{1}{2}$ XXZ chains in magnetic field.^{19,20} The frustrating nature of the ‘‘zig-zag’’ (Josephson) coupling in between chains³ effectively reduces this system to isolated XXZ chains, each with a renormalized intrachain (Josephson) coupling.²⁷ We thus recover the previous superconductor-CDW transition, but with K_{ρ} now dependent on the interchain Josephson coupling as well.

Concerning the experimental situation, the incoherent tunneling that is characteristic of the rung-CDW phase could explain the large conduction anisotropy seen in the normal state of antiferromagnetic ladder materials.^{4,9,28} In addition, if such a rung-CDW state were to be pinned, then all components of the resistivity tensor would exhibit insulating behavior in the low-temperature limit. This is indeed observed experimentally.⁴ It must be pointed out, however, that whether or not the low-temperature conductivity in doped antiferromagnetic ladder materials is intrinsic remains to be determined (see Refs. 4, 9, and 28). If, on the other hand, the rung-CDW state would depin at some elevated temperature, then the generic Drude response characteristic of the present ladder model (5) would yield metallic behavior in the longitudinal resistivity. The latter is also observed in antiferromagnetic ladder materials at relatively high temperatures.^{4,28} Last, such materials are observed to go superconducting under extreme pressure.⁵ A transition under pressure from a rung-CDW state to an IPT superconductor due to a strong dependence in the binding energy¹⁰ $-U_{\perp} \sim J$ of the rung-hole pairs with the lattice constants, for example, could account for this phenomenon. Very recently, however, the observation of high-temperature superconductivity in doped antiferromagnetic ladder materials at ambient pressure has been reported.⁶ The above discussion suggests that this system could transit into a rung-CDW ground state, on the contrary, by appropriate variations in the doping levels, or by varying other parameters such as the pressure.

This work was supported in part by National Science Foundation Grant No. DMR-9322427. The author thanks G. Gomez-Santos, A. Leggett, D. Poilblanc, P. Sacramento, and V. Vieira for discussions.

*Permanent address.

¹T. Ito, H. Takagi, S. Ishibashi, T. Ido, and S. Uchida, *Nature* (London) **350**, 596 (1991).

²J. M. Wheatley, T. C. Hsu, and P. W. Anderson, *Phys. Rev. B* **37**, 5897 (1988); S. Chakravarty, A. Sudbø, P. W. Anderson, and S. P. Strong, *Science* **261**, 337 (1993).

³T. M. Rice, *Z. Phys. B* **103**, 165 (1997).

⁴N. Motoyama, T. Osafune, T. Kakeshita, H. Eisaki, and S. Uchida, *Phys. Rev. B* **55**, R3386 (1997).

⁵M. Uehara, T. Nagata, J. Akimitsu, H. Takahashi, N. Mōri, and K. Kinoshita, *J. Phys. Soc. Jpn.* **65**, 2764 (1996).

⁶L. Leonyuk, G.-J. Babonas, R. Szymczak, H. Szymczak, M. Baran, A. Reza, V. Maltsev, L. Shvanskaya, and V. Rybakov, *Europhys. Lett.* **45**, 387 (1999).

⁷D. G. Clarke, S. P. Strong, and P. W. Anderson, *Phys. Rev. Lett.* **72**, 3218 (1994).

⁸S. Capponi, D. Poilblanc, and F. Mila, *Phys. Rev. B* **54**, 17 547 (1996).

⁹T. Osafune, N. Motoyama, H. Eisaki, and S. Uchida, *Phys. Rev. Lett.* **78**, 1980 (1997).

¹⁰E. Dagotto, J. Riera, and D. J. Scalapino, *Phys. Rev. B* **45**, 5744 (1992); H. Tsunetsugu, M. Troyer, and T. M. Rice, *ibid.* **49**, 16 078 (1994).

¹¹J. P. Rodriguez, *Bull. Am. Phys. Soc.* **43**, 790 (1998).

¹²A. Luther and V. J. Emery, *Phys. Rev. Lett.* **33**, 589 (1974).

¹³S. T. Chui and P. A. Lee, *Phys. Rev. Lett.* **35**, 315 (1975).

¹⁴V. J. Emery, in *Highly Conducting One-dimensional Solids*, edited by J. T. Devreese, R. P. Evrard, and V. E. van Doren (Plenum Press, New York, 1979).

- ¹⁵J. Voit, Rep. Prog. Phys. **58**, 977 (1995).
- ¹⁶J. P. Rodriguez, Europhys. Lett. **39**, 195 (1997).
- ¹⁷J. P. Rodriguez, Phys. Rev. B **58**, 944 (1998).
- ¹⁸The intrachain LE backscattering term (2) appears naturally in the weak-coupling analysis of the Hubbard ladder [see L. Balents and M. P. A. Fisher, Phys. Rev. B **53**, 12 133 (1996)].
- ¹⁹K. B. Efetov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **69**, 704 (1975) [Sov. Phys. JETP **42**, 390 (1976)].
- ²⁰M. Troyer, H. Tsunetsugu, and T. M. Rice, Phys. Rev. B **53**, 251 (1996).
- ²¹V. L. Pokrovskii and A. L. Talanov, Zh. Eksp. Teor. Fiz. **78**, 269 (1980) [Sov. Phys. JETP **51**, 134 (1980)].
- ²²J. E. Hirsch, Phys. Rev. Lett. **53**, 2327 (1984).
- ²³J. Voit, J. Phys.: Condens. Matter **8**, L779 (1996).
- ²⁴D. Boies, C. Bourbonnais, and A. M. S. Tremblay, Phys. Rev. Lett. **74**, 968 (1995).
- ²⁵C. A. Hayward, D. Poilblanc, Phys. Rev. B **53**, 11 721 (1996).
- ²⁶C. A. Hayward, D. Poilblanc, R. M. Noack, D. J. Scalapino, and W. Hanke, Phys. Rev. Lett. **75**, 926 (1995).
- ²⁷J. P. Rodriguez, P. D. Sacramento, and V. Vieira (unpublished).
- ²⁸T. Osafune, N. Motoyama, H. Eisaki, S. Uchida, and S. Tajima, Phys. Rev. Lett. **82**, 1313 (1999).