Electronic correlations, electron-phonon interaction, and isotope effect in high-T_c cuprates

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Using a large-N expansion we present and solve the linearized equation for the superconducting gap for a generalized t-J model which also contains phonons within a Holstein model. Keeping all terms up to $O(1/N)$ the kernel of the gap equation consists of an electron-phonon part with self-energies and vertex functions renormalized by the interactions of the t-J model, and a t-J part unaffected by phonons. Considering first the electron-phonon part we find that the leading superconducting instability always occurs in the s-wave channel with a T_c which is lower compared to that of noninteracting electrons, especially at large dopings. The corresponding isotope coefficient β is larger than 1/2 and increases with decreasing doping. Including also the t-J part in the gap equation the leading T_c has always d-wave symmetry with phonons giving a positive contribution to T_c. β is very small at optimal doping and increases towards the classical value 1/2 with increasing doping similar as in several cuprates. Considered as a function of the phonon frequency ω_0 at a fixed coupling strength β increases monotonously from zero to about 1/2 with increasing ω_0.
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I. INTRODUCTION

High-T_c cuprates show strong electronic correlations but also a non-negligible electron-phonon interaction. The latter causes, for instance, the observed isotope effect in the transition temperature T_c, especially, away from optimal doping.[1] It is thus of interest to calculate superconducting instabilities taking both the electron-electron and the electron-phonon interaction into account. There are several calculations dealing with the case of weak electronic correlations.[2–7] In these calculations the phonon-mediated part of the effective interaction is unaffected by electronic correlations. The electronic part to the effective interaction is calculated using a Hubbard model and assuming U to be small compared to the bandwidth. It is repulsive and strongly peaked near the M point in the Brillouin zone. As an alternative, Refs. 3 and 6 use the spin-fluctuation term of Ref. 8. Since the phonon part is attractive throughout the Brillouin zone it is plausible that the d-wave pairing due to the electronic part will be weakened if the electrons couple strongly to phonons near the M point and only weakly affected if only phonons with small momentum are involved.[2,4–6,9] References 4 and 7, for instance, found that the T_c for d-wave superconductivity is always lowered by phonons and that the isotope coefficient β is negative for a constant electron-phonon coupling function. If only phonons with small momenta play a role, β becomes positive.[4] Reference 6 even finds a change of sign in β as a function of the phonon frequency.

The case of strong electronic correlations has quite different features compared to the weak correlated case. The purely electronic part of the effective interaction is rather a smooth function of momentum without a sharp peak at the M point due to spin fluctuations.[10] Furthermore, electronic correlations modify substantially the phonon-mediated part of the effective interaction.[11–13] It is the purpose of this communication to present results for this case of strong electronic correlations. A suitable model for such calculations is a generalized t-J model which also includes phonons within a Holstein model. Extending the spin degrees from 2 to N systematic approximations for the effective interaction in terms of powers in $1/N$ can be carried out.[11–14] In a first step we consider only the phonon-mediated effective interaction which, unlike in the case of weak correlations, is strongly modified by vertex corrections transforming the original constant electron-phonon coupling of the Holstein model into a strongly momentum- and frequency-dependent function. Explicit results for T_c and β will be given for the leading symmetry channels of the superconducting order parameter as a function of the doping. In a second step these results are extended to the case where also the purely electronic contribution to the effective interaction is taken into account.

II. LINEARIZED EQUATION FOR THE SUPERCONDUCTIVITY GAP

Our Hamiltonian for the t-J model plus phonons can be written as

$$H=\sum_{\substack{ij\\p=1\ldots N}}\frac{t_{ij}}{N}X_i^{p0}X_j^{0p}+\sum_{\substack{ij\\p,q=1\ldots N}}\frac{J_{ij}}{4N}X_i^{pq}X_j^{qp}-\sum_{\substack{ij\\p,q=1\ldots N}}\frac{J_{ij}}{4N}X_i^{pp}X_j^{qq}+\sum_i\omega_0\left(a_i^\dagger a_i+\frac{1}{2}\right)+\sum_{\substack{i\\p=1\ldots N}}\frac{g}{\sqrt{N}}[a_i^\dagger+a_i](X_i^{pp}-\langle X_i^{pp}\rangle). \quad (1)$$

The first three terms correspond for $N=2$ to the usual t-J model. For $N=2$ X is identical with Hubbard's projection operator $X_i^{pq}=|_i^p\rangle\langle_i^q|$, where $|_i^p\rangle$ denotes for $p=0$ an empty and for $p=1,2$ a singly occupied state with spin up and down, respectively, at the site i. t_{ij} and J_{ij} are hopping and Heisenberg interaction matrix elements between the sites i,j. The fourth term in Eq. (1) represents one branch of dispersionless, harmonic phonons with frequency ω_0. The fifth term in Eq. (1) describes a local coupling between the phonon and the change in the electronic density at site i with the coupling constant g. $\langle X\rangle$ denotes the expectation value of X. The extension from $N=2$ in Eq. (1) to a general N has been discussed in detail in Ref. 12. The label p runs then not only over the two spin directions but also over $N/2$ identical copies of the orbital. The symmetry group of H is the symplectic group $Sp(N/2)$ which allows us to perform $1/N$ expansions for physical observables. In Eq. (1) the electron-phonon coupling is scaled as $1/\sqrt{N}$ whereas the free phonon part is independent of N. As a result the leading contributions to superconductivity from the t-J model alone and from the phonons are both of order $O(1/N)$ which allows us to treat them on an equal footing.

Instabilities towards superconductivity in the t-J model have been studied in the above framework in Refs. 10,15, and 16. The contributions to the anomalous self-energy from the phonon-mediated effective interaction have been derived in Refs. 11 and 12 for the case $J_{ij}=0$. Generalizing the latter treatment to a finite J similar as in Ref. 13 the linearized equation for the superconducting gap $\Sigma_{\rm an}$ for the entire Hamiltonian Eq. (1) can be written as

$$\Sigma_{\rm an}(k)=-\frac{T}{NN_c}\sum_{k'}\Theta(k,k')\frac{1}{\omega_{n'}^2+\epsilon^2(\mathbf{k}')}\Sigma_{\rm an}(k'). \quad (2)$$

N_c is the number of cells and k the supervector $k=(\omega_n,\mathbf{k})$, where ω_n denotes a fermionic Matsubara frequency and $\mathbf{k}$ the wave vector. $\epsilon(\mathbf{k})$ is the one-particle energy which is unchanged by the phonons in $O(1)$ and thus given by Eq. (41) of Ref. 10. The kernel Θ in Eq. (2) consists of two parts

$$\Theta(k,k')=\Theta^{t\text{-}J}(k,k')+\Theta^{e\text{-}p}(k,k'). \quad (3)$$

The first contribution $\Theta^{t\text{-}J}$ in Eq. (3) comes from the t-J model. Explicit expressions for it have been given in Eqs. (42)–(52) in Ref. 10. The second term $\Theta^{e\text{-}p}$ in Eq. (3) is due to phonon-mediated interactions and is given by

$$\Theta^{e\text{-}p}(k,k')=-\frac{2g^2\omega_0}{(\omega_n-\omega_{n'})^2+\omega_0^2}\times\gamma_c(k',k-k')\gamma_c(k,k'-k). \quad (4)$$

γ_c is the charge vertex for which an explicit expression has been derived in Ref. 13. For uncorrelated electrons there are no vertex corrections, i.e., $\gamma_c=-1$. Since g and ω_0 are assumed to be independent of $\mathbf{k}$, $\Theta^{e\text{-}p}$ is also independent of $\mathbf{k}$ and only s-wave superconductivity is possible. When correlations are present the kernel $\Theta^{e\text{-}p}$ depends on $\mathbf{k}$ and ω_n through the charge vertex γ_c and general symmetries for the order parameter may become possible.

III. CALCULATION OF T_c AND β FROM THE PHONON-MEDIATED PART

Taking a square lattice with point group C_{4v} $\Theta(k,k')$ and $\epsilon(\mathbf{k})$ in Eq. (2) are invariant under C_{4v} which means that $\Sigma_{\rm an}(k)$ can be classified according to the five irreducible representations Γ_i of C_{4v}. s-wave symmetry corresponds to Γ_1, d-wave symmetry to Γ_3, etc. In the weak-coupling case $\Theta(k,k')$ can be approximated by its static limit $\Theta(\mathbf{k},\mathbf{k}')$. Putting all momenta right onto the Fermi line the sum over $\mathbf{k}'$ in Eq. (2) can be transformed into a line integral along the Fermi line. Assuming a certain irreducible representation Γ_i for the order parameter, the line integral can be restricted to the irreducible Brillouin zone (IBZ) introducing a symmetry-projected kernel $\Theta_i(\mathbf{k},\mathbf{k}')$ with $\mathbf{k},\mathbf{k}'\in$ IBZ. Finally, the line integral was discretized by a set of points $[\mathbf{k}_\alpha^F]$ along the Fermi line in the IBZ with line elements $[s(\mathbf{k}_\alpha^F)]$. Denoting the smallest eigenvalue of the symmetric matrix

$$\frac{1}{4\pi^2}\sqrt{\frac{s(\mathbf{k}_\alpha^F)s(\mathbf{k}_\beta^F)}{|\nabla\epsilon(\mathbf{k}_\alpha^F)|\cdot|\nabla\epsilon(\mathbf{k}_\beta^F)|}}\Theta_i(\mathbf{k}_\alpha^F,\mathbf{k}_\beta^F) \quad (5)$$

by λ_i Eq. (2) yields for $\lambda_i<0$, $N=2$, and in the weak-coupling case the BCS formula

$$T_{ci}=1.13\omega_0 e^{1/\lambda_i}. \quad (6)$$

As usual we took ω_0 as a suitable cutoff. If $\lambda_i>0$ we have, of course, $T_{ci}=0$. According to Eq. (6) the absolute value of λ_i characterizes the strength of the effective interaction in the symmetry channel i. The overall strength of the electron-phonon coupling is conventionally expressed in terms of the dimensionless coupling λ defined by

$$\lambda=\frac{g^2}{8\omega_0}. \quad (7)$$

In Eq. (7) we have introduced a factor 1/2 to account for the prefactor 1/2 in Eq. (2) after setting there $N=2$. We also used the average density of states $1/(8t)$ for the density of states factor in λ and put t equal to 1. For the range of dopings we will be interested in, i.e., $0.15<\delta<0.8$, the density of states varies only little with doping so that the definition Eq. (7) of λ is appropriate for all dopings.

Figure 1 shows the eigenvalues λ_i for the two leading symmetries $i=1,3$ using the parameter value $J=0.3$ and t as the energy unit. δ is the doping measured from half-filling. For λ we have chosen the value 0.75 which is a typical value obtained in local-density approximation calculations.[17,18] The lowest eigenvalue occurs always in s-wave symmetry, i.e., for $i=1$. It depends only weakly on doping for $0.3<\delta<0.8$. In this region the order parameter also varies only slowly along the Fermi line, describing thus rather isotropic s-wave superconductivity. Below $\delta=0.3$, λ_1 rapidly decreases with decreasing doping which is caused by a soft mode which freezes into an incommensurate bond-order wave at $\delta_{\rm BO}\sim 0.14$.[10] As a result the s-wave order parameter becomes less and less isotropic with decreasing δ changing, for instance, for $\delta=0.2$ by about a factor 3 along the Fermi line but it does not pass through zero. The d-wave coupling constant λ_3 would be zero in the uncorrelated case. Decreasing δ from large values correlation effects increase and the

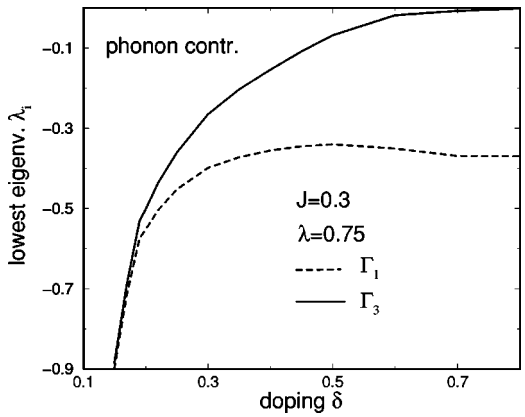

FIG. 1. Phonon contribution to the lowest eigenvalues λ_i of the static kernel Θ for the representations Γ_1 and Γ_3 of C_{4v} as a function of the doping δ.

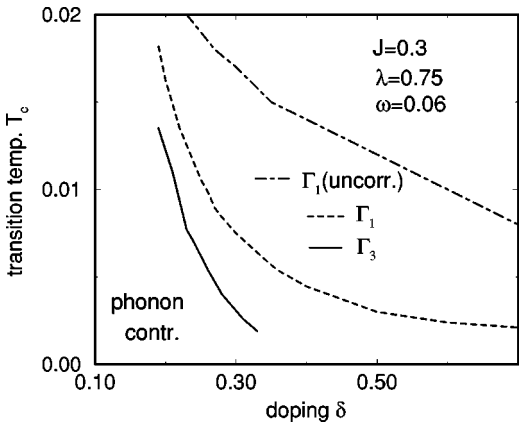

FIG. 2. Transition temperature T_c in units of t as a function of the doping δ for the uncorrelated (dash-dotted line, Γ_1 symmetry) and correlated (dashed and solid lines for Γ_1 and Γ_3 symmetries, respectively) cases, using only the phonon contribution.

static vertex function $\gamma(\mathbf{k},\mathbf{k}-\mathbf{k}')$ develops more and more a forward scattering peak in the transferred momentum $\mathbf{k}-\mathbf{k}'$.[11,12] In the extreme case where γ is proportional to $\delta(\mathbf{k}-\mathbf{k}')$ λ_1 and λ_3 would become degenerate. Figure 1 shows that this degeneracy is nearly reached at low dopings. The strong localization of the effective interaction is caused by the forward scattering peak in γ and, in addition, by the incipient instability at δ_{BO} which is also very localized in $\mathbf{k}$ space.

The neglect of retardation effects which leads to the BCS formula Eq. (6) is doubtful for several reasons. First, $\lambda = 0.75$ no longer corresponds really to a weak-coupling case. Secondly, T_c is no longer small compared to the frequency of the soft mode near δ_{BO} which means that the frequency dependence of Θ cannot be neglected for these dopings. Thirdly, and most severely, the formation of the forward scattering peak in the static vertex function γ is accompanied by a strong frequency dependence which should be taken into account on the same footing as its momentum dependence. We thus have solved the gap equation Eq. (2) assuming only that the momenta can be put right onto the Fermi line but keeping the full frequency and momentum dependence along the Fermi line.

Figure 2 shows the obtained results for T_c for an s-wave (dashed line) and a d-wave (solid line) order parameter, together with the T_c curve for the uncorrelated case (dash-dotted line). The first observation is that correlation effects always suppress s-wave superconductivity. The suppression is large for $\delta > 0.25$ and becomes less pronounced at smaller dopings in agreement with previous results based on the static approximation.[12] The order parameter associated with the dashed line in Fig. 2 varies only slowly along the Fermi line, i.e., we have a usual s-wave order parameter without nodes. In a Gutzwiller description this order parameter would be exactly zero if retardation effects can be neglected and $N=2$ is taken: Integrating out the phonons the effective interaction becomes in the static limit proportional to the double occupancy operator. Any matrix element formed with Gutzwiller wave functions thus would be zero. In contrast to that the enforcement of our constraint at large N's, namely, that only $N/2$ out of the total N states at a given site can be occupied at the same time, gives rise to another effect, which is absent in the Gutzwiller treatment: the effective interaction becomes more and more long-ranged with decreasing doping. This makes isotropic s-wave superconductivity possible even in the presence of a $N=2$ constraint. Figure 2 nevertheless shows that the reduction of T_c due to the constraint is substantial except at very small dopings where the effective interaction becomes extremely long-ranged and the suppression by the constraint is small.

The kernel $\Theta^{e\text{-}p}$ is in the absence of correlations and in the static limit negative for all arguments $\mathbf{k}$ and $\mathbf{k}'$. This implies that the s-wave symmetry has always the highest T_c. According to our numerical studies the same is also true in the correlated case which explains why the solid line in Fig. 2, describing d-wave superconductivity, is always below the dashed line. As indicated previously the very existence of finite values for T_c with d-wave symmetry is in our model due to electronic correlation effects. Figure 1 showed that the momentum dependence of γ_c causes in the static limit a finite coupling strength in the d-wave channel. The solid line in Fig. 2 proves that a finite T_c in the d-wave channel results from this even if the strong frequency dependence of γ_c is also taken into account.

Figure. 3 shows results for the isotope coefficient β in the case of s-wave (dashed line) and d-wave (solid line) superconductivty. We assumed hereby that ω_0 is inversely and g

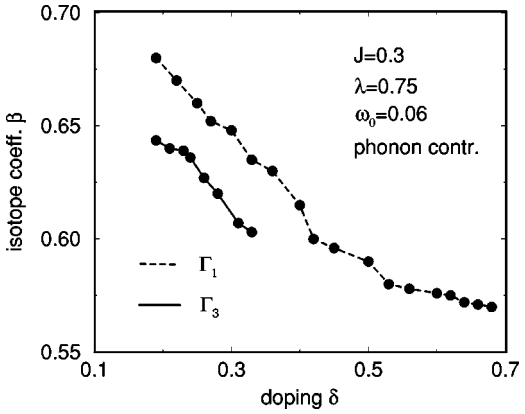

FIG. 3. Isotope coefficient β as a function of the doping δ considering only the phonon contribution for Γ_1 (dashed line) and Γ_3 (solid line) symmetries. The filled circles are calculated values.

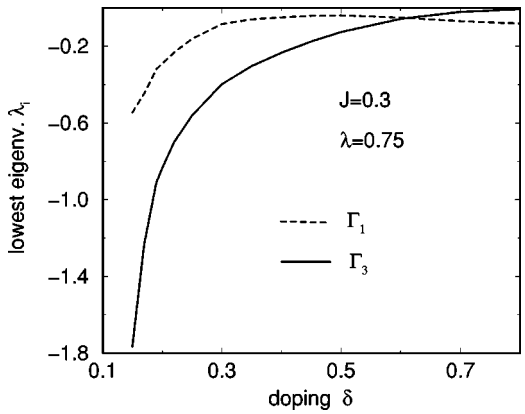

FIG. 4. Lowest eigenvalues λ_i of the total static kernel Θ for the representations Γ_1 and Γ_3 of C_{4v} as a function of the doping δ.

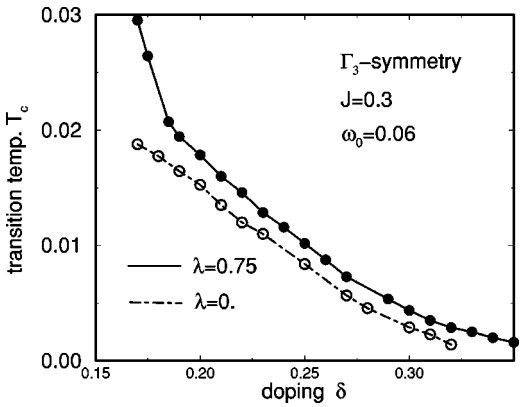

FIG. 5. Transition temperature T_c in units of t as a function of the doping δ for Γ_3 symmetry for $\lambda=0$ (dash-dotted line) and $\lambda=0.75$ (solid line).

directly proportional to the the square root of the mass rendering λ independent of the mass. Neglecting the frequency dependence of γ_c there is only one energy scale involved in the gap equation which yields $\beta=1/2$ independent of doping, λ, etc. This is true because at large N's phonon renormalizations and vertex corrections due to the electron-phonon interaction are absent. In contrast to that the curves in Fig. 3 show that β increases monotonically with decreasing doping and is always larger than 1/2. The deviation of β from the value 1/2 is due to the frequency dependence of γ_c which increases with decreasing doping[10] and acts as a second energy scale. The nonadiabatic effects produced by electronic correlations thus always increase β for our range of parameter values. This should be contrasted to the case where vertex corrections due to the electron-phonon interaction have been studied,[19,20] and where β may be larger or smaller than 1/2. We can conclude from our results that models based on phonon-induced effective interactions in the presence of correlations seem not to be very attractive models for high-T_c superconductivity because (a) the largest transition temperatures are found for s-wave symmetry for all dopings and (b) the isotope coefficient is larger than 0.5, especially at low dopings. Experimentally, high-T_c cuprates exhibit d-wave superconductivity and, at least near optimal dopings, small isotope effects.

IV. CALCULATION OF T_c AND β FROM THE FULL EFFECTIVE INTERACTION

Superconducting instabilites of the pure t-J model have been studied in Refs. 10,15, and 16. The smallest eigenvalue of $\Theta^{t\text{-}J}$ and, correspondingly, the largest T_c always occur in the d-wave symmetry channel. On the other hand, the phonon-induced effective interaction described by $\Theta^{e\text{-}p}$ favors in the t-J model mainly s-wave superconductivity. In this section, we will consider the case where both effective interactions are present and study the symmetry of the resulting superconducting state, its transition temperature and isotope coefficient.

Figure 4 shows the two lowest eigenvalues of the matrix Eq. (5) which occur in the s- and d-wave symmetry channel. For Θ both terms in Eq. (3) have been included. For $\delta<0.6$ the lowest eigenvalue has d-wave symmetry and decreases steadily and strongly with decreasing doping diverging finally at δ_{BO}. Comparing the solid lines in Figs. 1 and 4 of this work and Fig. 2 in Ref. 10, one sees that the lowest eigenvalue is roughly additive in the t-J and the phonon contributions though, of course, $\Theta^{t\text{-}J}$ and Θ^{e-p} do not commute with each other. Note also the considerable contribution of the phonon part to λ_3 though the overall dimensionless coupling constant has the rather moderate value $\lambda=0.75$. The eigenvalue λ_1 (dashed line in Fig. 4) is rather constant for $\delta>0.3$ and decreases much less towards lower dopings than λ_3. There is a crossover from d-wave to s-wave symmetry in the lowest eigenvalue at $\delta\sim0.6$ suggesting that the s-wave order parameter is more stable than the d-wave order parameter at high dopings. Such a crossover is expected for general reasons: For large dopings correlation effects play a minor role. As a result the d-wave part in the phonon-induced effective interaction as well as the whole t-J part becomes small whereas the s-wave part of the Holstein model will dominate. A comparison of the solid lines in Figs. 1 and 4 nevertheless shows that the latter is very effectively suppressed by the t-J part at practically all considered dopings. For instance, $|\lambda_1|$ in Fig. 1 is for $\delta>0.25$ about a factor of 4 larger than in Fig. 4.

The solution of the gap equation Eq. (2) is not straightforward because Θ contains both instantaneous and retarded contributions and because the various retarded contributions have different effective cutoff energies. We developed in Ref. 10 a method to solve Eq. (2) directly avoiding the use of pseudopotentials. The only simplification is that we put the momenta in retarded (but not in instantaneous) contributions to the Fermi line. This approximation has been checked numerically and found to be very well satisfied in our case. Using the fact that the instantaneous kernel consists only of a few separable contributions Eq. (2) can for a given Matsubara frequency reduced to a linear matrix problem of order 1 and 3 for d- and s-wave symmetry, respectively.

In Fig. 5 we have plotted the calculated values for T_c for d-wave symmetry Γ_3 as a function of the doping δ. The dashed-dotted line corresponds to $\lambda=0$, the solid line to $\lambda=0.75$. The figure shows that phonons always increase the T_c for d-wave superconductivity. The increase is especially large at low dopings. This is quite in contrast to calculations in the weak-coupling case[4,6] where the T_c of d-wave superconductivity is lowered by phonons. The physical picture in

the latter case is that the effective interaction of the electronic part is repulsive and strongly peaked near the M point whereas the phononic d-wave part is attractive and diminishes especially at the M point the repulsion causing a lowering of T_c. In our case the dependence of the static effective interaction of the pure t-J model on the transferred momentum can be inferred from Figs. 1(a) and 1(b) in Ref. 10. For small dopings the phonon part is restricted to small momenta which means that it would contribute only near the X point in Fig. 1(b) of Ref. 10. As a result the total effective interaction would become even more attractive around the X point and elsewhere not be changed. This clearly would enhance the d-wave part of the effective interaction which is also in agreement with Fig. 4. From Figs. 1, 4, and 5 it is evident that the large lowering of the eigenvalue λ_3 due to phonons corresponds only to a rather moderate increase in T_c. This means that the use of a BCS formula with a fixed effective cutoff would grossly overestimate the increase in T_c for d-wave superconductivity due to phonons. What has been overlooked in such an approach is that we deal with at least three energy scales, namely t, J, and ω_0. For instance, the instantaneous contribution of the t-J part is characterized by the energy scale t whereas the phononic one by ω_0. Since $t \gg \omega_0$ it is evident that the phononic part in λ_3 will contribute to T_c much less than the t-J part.

We were unable to find any finite transition temperatures T_{ci} for $i \neq 3$, i.e., for symmetries different from the d-wave symmetry. Taking the accuracy of our calculation into account this means that $T_{ci} < 0.002$ for $i \neq 3$. As shown in Fig. 2 the phonon-induced effective interactions lead to a considerable T_c for s-wave superconductivity. The t-J part to the effective interaction, however, is very repulsive in the s-wave channel prohibiting s-wave superconductivity. We cannot exclude that the crossover at $\delta \sim 0.6$ in Fig. 4 in the lowest eigenvalue from d wave to s wave could stabilize a s-wave order parameter at large dopings. Another possibility is that other symmetries than d or s wave become stable at large dopings in view of the approximate degeneracy of the eigenvalues of practically all symmetries in the pure t-J model.[10] In any case, the corresponding T_c's would be smaller than ~ 0.002 for all dopings and thus be rather irrelevant.

The behavior of the isotope coefficient β as a function of doping is another interesting test for any theory of high-T_c superconductivity. Figure 6 shows the calculated β for $\lambda = 0.75$, $J = 0.3$, $\omega_0 = 0.06$ in the case of Γ_3 symmetry. It is always positive, starting from small values near the bond-order wave instability at δ_{BO}, which can be identified with optimal doping.[21] With increasing doping it increases monotonically approaching values near 1/2 at large dopings. Our calculated curve shows the typical features of the available experimental data in the optimally doped and the overdoped regime.[1] Experimentally, β increases away from optimal doping towards the overdoped but also towards the underdoped regime and most presently available data deal with the latter case. We cannot apply the above developed theory to the underdoped regime because of the occurrence of the bond-order wave instability at δ_{BO}. The curve in Fig. 6 is the result of several competing effects. We saw in Fig. 3 that correlation effects in the phonon-induced part cause a doping dependence of β which is opposite to that in Fig. 6, namely,

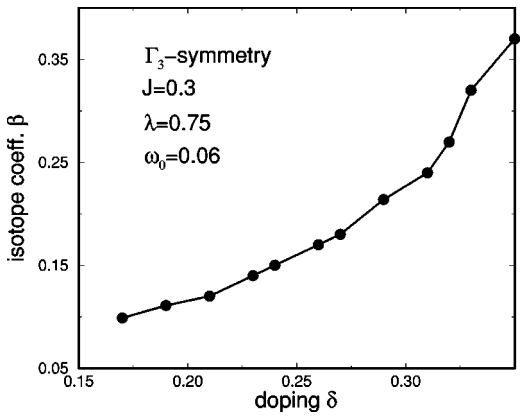

FIG. 6. Isotope coefficient β as a function of the doping δ for d-wave-like Γ_3 symmetry. The filled circles are calculated values.

a monotonically increasing β with decreasing doping starting at the classical value of 1/2 at large dopings. Including also the t-J part the large values of β at low dopings are nearly completely and at larger dopings partially quenched. Part of this effect may be understood from Fig. 5. It shows that the relative importance of the phonon contribution increases steadily with increasing δ except at very small dopings. In a simple picture one thus may argue that at small dopings T_c is mainly due to the t-J part. This part decays faster than the phononic part with doping so that T_c at larger dopings is mainly due to the phonons. A more realistic interpretation of Fig. 6 should, however, also take into account the complex competition between the three energy scales t, J, and ω_0, pair-breaking effects, etc.

In order to get more insight into the behavior of β we have calculated T_c and β in the Γ_3 symmetry channel for the fixed doping value $\delta = 0.20$ as a function of the phonon frequency ω_0. Figure 7 shows the result for $J = 0.3$ and $\lambda = 0.75$. The solid line in Fig. 7 represents the value for T_c, multiplied with 10. Since we keep λ fixed the coupling constant g approaches zero in the adiabatic limit $\omega_0 \to 0$ which means that T_c in this limit is solely due to the t-J part. With increasing phonon frequency T_c increases monotonically, passing over from an initial sublinear to a linear behavior. The solid line shows that phonons always increase T_c for

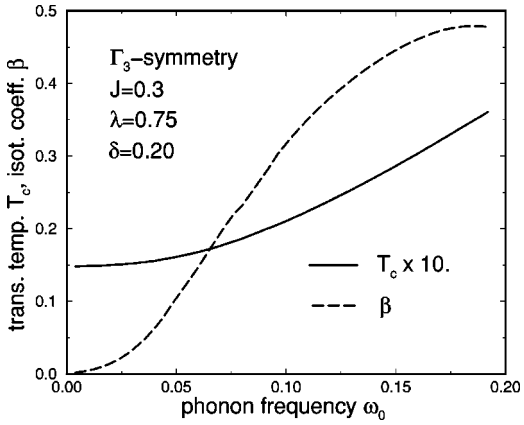

FIG. 7. Transition temperature T_c, multiplied by 10, and isotope coefficient β as a function of the phonon frequency ω_0 for d-wave-like Γ_3 symmetry.

d-wave superconductivity. The absolute values in the figure are quite remarkable: Though the employed coupling constant for λ is rather moderate and though the phonon contribution to T_c in the *d*-wave channel is entirely due to correlation effects T_c increases by nearly a factor of 3 for $\omega_0 = 0.2$ corresponding to about the largest phonons in the cuprates. The dashed line in Fig. 7 represents the dependence of β on ω_0. For small ω_0 β is practically zero. β increases monotonously with increasing ω_0 and tends to the classical value 1/2 at large phonon frequencies. The dashed line in Fig. 7 may be understood in physical terms roughly in the following way. In our model the static effective coupling is independent of the phonon mass, i.e., of ω_0. β is thus determined by an effective energy cutoff. For small ω_0's this cutoff is mainly given by electronic parameters in the *t*-*J* part leading to a small value for β. At large ω_0's the phonon contribution to T_c, which is only due to electronic correlations, determines mainly the total effective energy cutoff causing a value of β near 1/2.

In conclusion, we have treated the electron-electron and the electron-phonon interactions in a generalized *t*-*J* model by means of a systematic $1/N$ expansion and have solved the resulting linearized equation for the superconducting gap by reliable numerical methods. We found that electronic correlations affect phonon-induced superconductivity in several ways: Instabilities towards *d* wave or other symmetries different from *s*-wave symmetry become possible. The corresponding transition temperatures are, however, always smaller than that of *s*-wave superconductivity. Moreover, the T_c for *s*-wave superconductivity is at larger dopings heavily and at small dopings somewhat suppressed by correlations. The isotope coefficient β is 1/2 at large dopings but increases with increasing correlations, i.e., decreasing dopings, both in the s- and *d*-wave channel. Including also the *t*-*J* part in the effective interaction we found that within our numerical accurracy only *d*-wave superconductivity is stable for dopings $0.15 < \delta < 0.8$ and that the phononic part always increases T_c for the above dopings. The *t*-*J* part in the effective interaction changes the dependence of β on doping: β assumes now small values at low dopings and increases monotonically with doping towards the classical value 1/2 at large dopings. Keeping λ fixed and varying the phonon mass, i.e., ω_0 we find that T_c and β increase monotonically with ω_0 and that β varies between zero at small and roughly 1/2 at large phonon frequencies within the interval $0 < \omega_0 < 0.2$.

Our findings differ in many aspects from the corresponding results based on weak-coupling calculations.[3–7] In these calculations T_c for *d*-wave superconductivity induced either by a U or a spin-fluctuation term is always suppressed by phonons. For instance, in the fluctuation-exchange approximation T_c drops to zero if the phonon-mediated on-site attraction U_p becomes comparable to the Hubbard term U.[7] We treated the opposite case where $U \gg U_p$ and found a different behavior, namely, that phonons enhance the T_c for *d*-wave superconductivity. Our different result is mainly caused by corrections to the bare electron-phonon vertex due to the strong electron-electron interaction. This vertex develops for not too large dopings a forward scattering peak so that only phonon with small momenta can couple to the electrons. As a result the electron-phonon and the *t*-*J* contributions to the gap decouple in **k** space. Our calculations show that the two contributions no longer cancel each other to a large extent but, on the contrary, enhance each other. For a momentum-independent bare electron-phonon coupling weak-coupling calculations yield a small, often negative value for the isotope coefficient β which is rather independent of doping. In our case the strong momentum-dependence of the effective electron-phonon coupling, induced by electronic correlations, causes a strong dependence of β on doping: being always positive, β is small at optimal doping and assumes values of roughly 1/2 at large dopings in agreement with the available experimental data in the cuprates in the optimal doped and overdoped region.

The presented results are accurate at large N's because we have taken into account only the leading terms of a $1/N$ expansion. Phonon renormalizations and vertex corrections due to the electron-phonon interaction are of $O(1/N)$ and thus have been omitted. On the other hand, it is known that in the physical case $N=2$ anharmonic effects in the atomic potentials and the formation of polarons occur if λ is about 1 or larger.[22] This suggests that keeping only the leading order of the $1/N$ expansion cannot describe adequately the case $N=2$ at large coupling strengths or if the Migdal ratio ω_0/t is no longer small. Correspondingly, we have restricted our numerical results to rather moderate values of λ and small Migdal ratios.

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