Site-diluted three-dimensional Ising model with long-range correlated disorder

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We present numerical simulations of the site-diluted Ising model in three dimensions in the presence of two different forms of quenched disorder with long-range correlations. We use finite-size scaling techniques to compute the critical exponents of these two systems, taking into account the strong corrections to scaling. We find a value of the critical exponent ν that is compatible with the analytical predictions.
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I. INTRODUCTION

Neutron and x-ray critical scattering experiments in some magnetic systems[1] revealed an unexpected feature: the coexistence of two length scales. In the theory of critical phenomena it is expected that the momentum dependence of the scattering intensity is a Lorentzian function with a width proportional to the inverse of the correlation length. Nevertheless, it was found[1] that the experimental results could be better interpreted if we suppose that the scattering intensity is the superposition of a broad Lorentzian and a sharper function similar to a simple or a squared Lorentzian function. Only the width of the first peak behaves according to theoretical expectations. In Ref. 2 it is proposed that the existence of this new component of the scattering intensity is due to defects, in particular to the presence of dislocations near the surface of the sample. Therefore there is a crossover between the bulk critical behavior (corresponding to the broader component) and the *disorder*-induced critical behavior (corresponding to the sharper component).

Dislocations constitute lines, instead of points, oriented at random, and therefore the quenched disorder presents long-range correlations.

A Gaussian disorder with correlations decaying with a power law was studied in Ref. 3 for the vector spin models. Analytical techniques were used to compute the critical exponents of the system. The authors calculated the expansion of the exponents in powers of $\epsilon=4-d$ and $\delta=4-a$, up to first order, where a is the power with which the correlation function of the disorder decays and d is the ordinary spatial dimension. The quantity a crucially depends on the origin of the quenched disorder. For example, straight dislocation lines with random orientation correspond to $a=d-1$. In this calculation a new fixed point was found in the case of quenched disorder with long-range correlations. Rather surprisingly, it was found that the renormalization group transformation near this new point has a pair of complex eigenvalues, and consequently we expect oscillating corrections to scaling. At this order, the critical exponents found for the fixed point corresponding to the long-range correlated disorder are

$$\nu=\frac{2}{a},$$

$$\eta=\mathcal{O}(\epsilon^2), \tag{1}$$

where, following the usual definitions, ν is the thermal critical exponent, associated with the correlation length, and η is the anomalous dimension of the order parameter. Using the usual scaling relations it is possible to obtain the other critical exponents of the system.

It could be argued that the first of the relations in Eqs. (1) should be valid for all orders in perturbation theory and that, therefore, it should be an exact relation. The more interesting case of non-Gaussian disorder has never been studied in detail, although it is possible that the results for non-Gaussian disorder are the same as for the Gaussian case. It should be noted that the previous relation is valid only if the disorder decays in a sufficiently slow manner: for large values of a, i.e., if Eq. (1) predicts a value of the ν exponent smaller than the value for the system with short-range disorder (i.e., ν_{SR}), one should obtain $\nu=\nu_{\mathrm{SR}}$ (this phenomenon has been studied also in the case of self-avoiding walks[4]).

We should also notice that a useful criterion due to Harris[5] indicates when the disorder is an irrelevant perturbation in the pure system, in terms of the pure critical behavior. This criterion states that the short-range disorder is not relevant when $d\nu_{\mathrm{pure}}-2=-\alpha_{\mathrm{pure}}>0$. In Ref. 3 the criterion is extended to the long-range correlated case. This kind of disorder will become an irrelevant perturbation if the condition $2/a>\nu_{\mathrm{pure}}$ holds, i.e., when the ν exponent given by relation (1) is larger than the ν_{pure} exponent of the model without disorder.

For three dimensions, a line of defects corresponds to a correlation function between defects decaying as x^{-a} with $a=2$ (x being the distance between defects), but with a non-Gaussian distribution of the disorder. In this case, using the data from the pure Ising case [$1>\nu=0.6294(5)(5)$ (Ref.6)] the previous criterion indicates that the disorder is a relevant perturbation. Neglecting the non-Gaussian effects and applying the results of Eq. (1) to this case, a ν exponent is found in Ref. 2 that is close to the experimental data for different materials.[1] Furthermore, the local fluctuations in the critical

temperature due to the presence of line defects were computed and an acceptable agreement with experiments was found.

In Ref. 7 the same authors studied the influence of the long-range correlated disorder on the line shape of the narrow component of the scattering intensity function, finding that it can be steeper than a Lorentzian one, in agreement with the experimental data. They were assuming that both the Gaussian disordered model and that with line defects, where the disorder is non-Gaussian even at a large scale, belong to the same universality class.

In this study we will use Monte Carlo (MC) simulations to compute the critical exponents of the site-diluted Ising model with long-range correlated disorder, in the $a=2$ case, finding that our results are compatible with analytical predictions. In order to do this we will use the finite-size scaling techniques that have been recently applied to the study of random site-diluted Ising systems.[8,9] We will study both Gaussian and non-Gaussian disorder and we will find similar results for the critical exponents, which supports the analysis of Refs. 2 and 7.

The layout of the paper is the following. In Sec. II we will define the models we have simulated in the lattice and the two different ways used to introduce the long-range correlated disorder in the system. In Sec. III we will describe the finite-size techniques we used. The technical details of the simulation will be reported in Sec. IV. The numerical results will be shown in Sec. V. Finally, in Sec. VI, the conclusions that can be drawn from this study will be presented.

II. MODELS AND THE OBSERVABLES

We have considered the following Hamiltonian defined in a cubic lattice of linear size L with periodic boundary conditions:

$$H=-\beta\sum_{\langle i,j\rangle}\epsilon_i\epsilon_j\sigma_i\sigma_j, \tag{2}$$

where the sum is extended over the nearest neighbors, the σ's are the usual Z_2 spin variables, and the ϵ's are quenched random variables, with long-range spatial correlation. A given set of the ϵ_i variables will be called a *sample* from now on. We have studied two different ways of introducing the correlation between the ϵ_i variables.

In the first method we start by obtaining a set of $V=L^3$ correlated Gaussian random variables $\eta(\mathbf{x})$, where $\mathbf{x}$ is the position vector in the lattice, with the properties

$$\langle\eta(\mathbf{x})\rangle=0,$$

$$\langle\eta(\mathbf{x})\,\eta(\mathbf{y})\rangle\propto\frac{1}{|\mathbf{x}-\mathbf{y}|^a}[\equiv C(|\mathbf{x}-\mathbf{y}|)], \tag{3}$$

where $d>a>0$. In order to do this we have used the Fourier filtering (FF) method.[10]

Let us denote by $\tilde{C}(\mathbf{p})$ the Fourier transform of the function $C(|\mathbf{x}-\mathbf{y}|)$. Let us define the set $\tilde{\eta}(\mathbf{p})$ as

$$\tilde{\eta}(\mathbf{p})=\sqrt{\tilde{C}(\mathbf{p})}\,u(\mathbf{p}), \tag{4}$$

where $u(\mathbf{p})$ is a Gaussian set of random numbers in the complex plane with the following properties:

$$\langle u(\mathbf{p})\rangle=0,$$

$$\langle u(\mathbf{p})u(\mathbf{p})\rangle=1, \tag{5}$$

$$\langle u(\mathbf{p})u(\mathbf{p}')\rangle=0,\ \text{if}\ p\neq p'.$$

At this point we construct $\eta(\mathbf{x})$ as the inverse Fourier transform of the $\tilde{\eta}(\mathbf{p})$ set. In order to make sure that the $\eta(\mathbf{x})$ set is real we have to introduce the condition

$$\tilde{\eta}(-\mathbf{p})=\tilde{\eta}^*(\mathbf{p}). \tag{6}$$

The zero-mode divergence which is present in a finite-lattice treatment is eliminated by using the condition $u(\mathbf{p}=0)=0$. This choice agrees with the property $\langle u(\mathbf{p})\rangle=0$.

With these definitions $\eta(\mathbf{x})$ is a Gaussian random variable, because it is a sum of a large number of random variables. It is easy to prove that the relations given by Eq. (3) are satisfied. Furthermore, it is also possible to calculate the variance of this Gaussian distribution, as the zero-momentum inverse Fourier transform of $\tilde{C}(\mathbf{p})$.

We now use the variables $\{\eta(\mathbf{x})\}$ to choose whether each site is occupied ($\epsilon_i=1$) or not ($\epsilon_i=0$) for a given value of the mean concentration of the spins of the system, p. Thus we compute the η value $\hat{\eta}$, for which the probability to get a value $\eta<\hat{\eta}$ is p. For each x point we compare the actual value $\eta(\mathbf{x})$ with $\hat{\eta}(p)$ and decide that the site is occupied if $\eta(\mathbf{x})<\hat{\eta}$.

In this work we will study the $a=2$ case corresponding to linear defects. With this value we have checked that the correlation obtained for the $\{\eta(\mathbf{x})\}$ set with the FF method, performed with double precision, is in good agreement with the expected correlation function. Although it is obvious that the ϵ_i are not Gaussian variables, their connected correlation functions for all points are equal to zero and therefore this model corresponds to a Gaussian model at a large scale (non-Gaussian effects are restricted to a short scale and are probably irrelevant). This model corresponds to what we called the Gaussian-distributed disorder in the Introduction.

A second way to obtain samples with long-range correlated disorder with decaying with the square of the distance is to remove some lines of spins in a random way. We start with a cubic lattice and remove lines until we get the fixed concentration p. The last line considered in this procedure is removed only with a given probability in order to get the right value (p) for the average concentration. We also want the probability of removal to be the same for all lattice points and lattice symmetries to be preserved. We have been able to reach this goal by only removing lines along the axes. It is clear that the connected correlation functions with this method are definitely different from zero even at long distances. The disorder is very far from being Gaussian and this model is what we called the non-Gaussian-distributed disorder in the Introduction.

In both cases, we will consider the case of quenched disorder. We first calculate the average of a given observable on the $\{\sigma_i\}$ variables with the Boltzmann weight given by the Hamiltonian of Eq. (2), the results on the different samples being *later* averaged. The quenched approximation is chosen

because the defect dynamics is much slower than that associated with the magnetic interaction. We will denote by brackets the thermal average and by overbars the sample average. The observables will be denoted with script letters, i.e., $\mathcal{O}$, and we will use italics for the double average $O = \overline{\langle \mathcal{O} \rangle}$.

Thus, we can define the nearest-neighbor energy as

$$\mathcal{E} = \sum_{\langle i,j \rangle} \epsilon_i \sigma_i \epsilon_j \sigma_j . \tag{7}$$

This quantity is extensively used for extrapolating the results for a given observable O, obtained at coupling β to a nearby coupling β',[11] as well as for calculating β derivatives through its connected correlation with the observable. For instance, one can define the specific heat as

$$C = \partial_\beta \overline{\langle \mathcal{E} \rangle} = \frac{1}{V} (\overline{\langle \mathcal{E}^2 \rangle - \langle \mathcal{E} \rangle^2}). \tag{8}$$

The order parameter of the phase transition is the usual normalized magnetization

$$\mathcal{M} = \frac{1}{V} \sum_i \epsilon_i \sigma_i . \tag{9}$$

In a finite lattice, its mean value M is zero and we consider only even powers of the magnetization. The second power is related to the susceptibility of the system:

$$\chi = V \overline{\langle \mathcal{M}^2 \rangle}. \tag{10}$$

With the fourth power we can construct another interesting quantity, the cumulant g_4, defined as

$$g_4 = \frac{3}{2} - \frac{1}{2} \frac{\overline{\langle \mathcal{M}^4 \rangle}}{\overline{\langle \mathcal{M}^2 \rangle^2}}. \tag{11}$$

In the finite-size scaling method we use it is very convenient to have a well-behaved estimate for the correlation length in a finite lattice. We have used the second-momentum definition, which reads[12]

$$\xi = \left(\frac{\chi / F - 1}{4 \sin^2 (\pi / L)} \right)^{1/2}, \tag{12}$$

where F is defined in terms of the Fourier transform of the spin distribution,

$$\mathcal{G}(\mathbf{p}) = \frac{1}{V} \sum_r e^{ip \cdot r} \epsilon_r \sigma_r , \tag{13}$$

as

$$F = \frac{V}{3} \overline{\langle |\mathcal{G}(2\pi/L, 0, 0)|^2 + \text{permutations} \rangle}. \tag{14}$$

III. FINITE-SIZE SCALING TECHNIQUES

In the scaling region, the mean value of a given observable O measured at (β, p) couplings can be written as

$$O(L, \beta, p) = L^{x_O/\nu} [F_O(\xi(L, \beta, p)/L) + \mathcal{O}(L^{-\omega})], \tag{15}$$

where x_O is the critical exponent of the operator O, F_O is a smooth scaling function which depends on the observable, and ω is the eigenvalue of the first irrelevant operator of the theory from the point of view of the renormalization group.

All the quantities in Eq. (15) are measurable in a finite lattice. In order to obtain the critical exponents we need to remove the unknown scaling function F_O. Let us define the quotient of a given observable O at two different lattice sizes and at the same coupling pair as

$$Q_O = O(sL, \beta, p) / O(L, \beta, p), \tag{16}$$

and let us compute this quotient at the coupling where the correlation length, in units of the lattice size, is the same for both lattices. Thus we get

$$Q_O|_{Q_\xi = s} = s^{x_O/\nu} + A_p^O L^{-\omega} + \cdots , \tag{17}$$

where A_p^O is a constant which depends on the observable and the spin concentration p and the ellipsis stands for higher-order scaling corrections. From this equation we can extract the critical exponent associated with a given observable.

The observables used to obtain the different critical exponents are the β derivative of the correlation length for ν ($x_{\partial_\beta \xi} = \nu + 1$) and the susceptibility χ for η [$x_\chi = \nu(2 - \eta)$].

In order to compute the critical coupling in the infinite-volume limit we will use the crossing points of the observables with $x_O = 0$, as g_4 or ξ/L, when measured at two different lattice sizes L and sL. The shift of these points from the critical coupling behaves as[13]

$$\Delta \beta_c^L \propto \frac{1 - s^{-\omega}}{s^{1/\nu} - 1} L^{-\omega - 1/\nu}. \tag{18}$$

IV. NUMERICAL METHODS

The best update method for an Ising model simulation is a cluster algorithm.[14] In particular, the most efficient one in the pure case is the single-cluster Wolff method.[15] Nevertheless, in a diluted system small groups of isolated spins appear, which are scarcely visited with this algorithm. Furthermore, in the non-Gaussian case isolated occupied lines also appear. In order to update all-sized spin clusters, after a fixed number of single-cluster updates we perform a Swendsen-Wang sweep. We call this procedure the MC step (MCS). We have discarded 100 MCS's for thermalization and then we have measured the different observables for each MCS. We have checked the correct thermalization of the system by starting from hot and cold configurations. We have chosen the single-cluster update number in such a way that the autocorrelation times for all the observables are nearly one MCS. The simulations are carried out in the RTNN machine at Zaragoza University.

Other interesting parameters are the number of measurements we perform for a given disorder realization, N_I, using the Ising Hamiltonian, Eq. (2), and the number of different disorder realizations, N_S. We refer to Refs. 8 and 9 for a discussion of the optimal choice of these parameters. In our case, we have performed $N_I = 100$ measurements in N_S

$=20\,000$ different samples for $L \leqslant 64$ and in $N_S = 10\,000$ samples for $L = 128$.

We have used in this study the usual β extrapolation.[11] Thus we are restricted to not too strong dilutions.

In order to work with large dilutions it is convenient to perform also a p extrapolation as a consequence of the phase diagram form of the systems. In the case of the random site-diluted Ising model this is possible because the distribution probability of the actual density of spins is known (it is a binomial one). In the Gaussian model, due to the correlation between the different sites, this distribution it is not known. Neither is it known in the non-Gaussian case, in the form presented here. Nevertheless, it is possible to construct a slight variation of this latter model allowing us to perform a dilution extrapolation. It is enough to choose whether a line is empty or filled with a given probability, but this possibility will not be considered in this study.

We recall[8,9] that a bias of order $2\tau/N_I$ is present in the β extrapolation, where τ is the correlation time between the energy and the observable under study. This fact is not relevant in the usual MC calculations, because the statistical errors are of order $1/\sqrt{N_I}$. But in diluted system investigations, when $\sqrt{N_S} \sim N_I$, this bias could be not negligible. We have performed a proper extrapolation procedure[8,9] in order to obtain unbiased estimates of the β derivatives and the values of the different observables in the neighborhood of the simulated couplings. The MCS is chosen in such a way that τ is nearly one measurement. For the largest lattice we have considered, $L = 128$, in the non-Gaussian case, the single-cluster update number for every Swendsen-Wang sweep is 1200 and for the Gaussian case it is 400. For the statistical error computation we have used the jackknife method with 50 blocks, which allows us to obtain a 10% of accuracy in the error bars.

V. NUMERICAL RESULTS

We have studied the Gaussian case at two different dilutions $p = 0.8$ and $p = 0.65$, performing simulations on lattices of sizes $L = 8$, 16, 32, 64, and 128. In the non-Gaussian case we have only considered $p = 0.8$ with the same lattice sizes.

In Fig. 1 we show the phase diagram for the Gaussian model. The percolation critical point $p_c \simeq 0.25$ was obtained by studying the behavior of the g_4 function in a $L = 128$ lattice. In the thermodynamical limit, we know that $g_4 = 0$ in the disordered phase and $g_4 = 1$ in a ferromagnetic ordered one. The corresponding phase diagram for the non-Gaussian case is qualitatively the same, with a ferromagnetic ordered phase for large β, provided that p is larger than its percolation threshold.

A. Thermal exponent

In Table I we present the results for the ν exponent in the two cases considered, the Gaussian and non-Gaussian disorder. It was computed by applying Eq. (17) to $\partial_\beta \xi$ using $s = 2$.

As we can see, there are visible scaling corrections in all the cases. Nevertheless, the values we have obtained for the ν exponent are very different from those of the pure Ising model, $\nu = 0.6294(5)(5)$,[6] and from those of the three-dimensional random site-diluted case, $\nu = 0.6837(24)(29)$.[8] In order to obtain the critical exponent, we have to perform an infinite-volume extrapolation procedure. It is possible that the scaling corrections we observe could be complicated by the presence of oscillatory terms, as is suggested by the first order of the ϵ expansion, but we are unable to confirm or discard this possibility. Strong corrections to simple scaling are present, as we will see later, so it is rather difficult (although we study lattices ranging from 8^3 to 128^3) to get conclusive statements on the nature of finite-volume corrections.

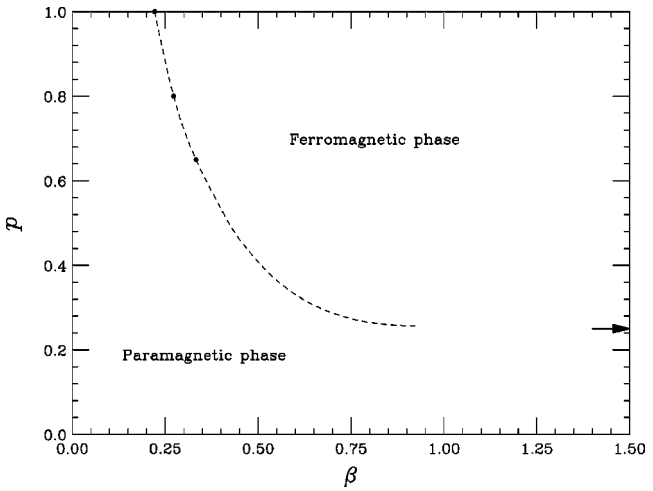

FIG. 1. Phase diagram of the site-diluted Ising model with long-range correlated Gaussian disorder, in the inverse temperature–dilution plane. The dots correspond to the simulated points, while the arrow points to the percolation limit ($\beta = \infty$).

We can try to parametrize the scaling corrections as in Eq. (17), keeping only the first term. We have used the data from the two different dilutions of the Gaussian case to perform a joint fit where a single value for the ω and ν exponents is assumed, according to the picture of a single universality class along the critical line. Using $L \geqslant 8$ data and the full covariance matrix to compute the statistical function χ^2, we find a very large value of $\chi^2/\mathrm{DOF} = 13.9/4$. Nevertheless, if we discard the data from the $L = 8,16$ pair, we find $\chi^2/\mathrm{DOF} = 1.20/2$. The value obtained for the thermal exponent, $\nu = 1.012(16)$, is compatible with the analytical predictions. We also find in this analysis $\omega = 1.01(13)$.

We can control the presence of the higher-order corrections in a simple and naive way. We could perform a quadratic fit for each dilution with $L \geqslant 8$ data, assuming $\omega = 1$, which is compatible with our results, and using only the

TABLE I. Critical exponent ν computed using $\partial_\beta \xi$ when measured in $(L,2L)$ lattice pairs at the couplings where $Q_\xi = 2$ for both models at the different concentrations considered.

	Gaussian model		Non-Gaussian
L	$p = 0.8$	$p = 0.65$	$p = 0.8$
8	0.7626(19)	0.871(3)	0.8335(24)
16	0.833(3)	0.942(6)	0.934(4)
32	0.907(4)	0.969(7)	1.009(9)
64	0.964(9)	0.996(11)	1.009(13)

TABLE II. Magnetic exponent η computed from χ, using lattice $(L,2L)$ pairs at the couplings where $Q_\xi=2$ for both Gaussian and non-Gaussian cases at the different concentrations simulated.

	Gaussian model		Non-Gaussian
L	$p=0.8$	$p=0.65$	$p=0.8$
8	0.0085(11)	0.0256(14)	−0.0513(14)
16	0.0082(14)	0.0274(16)	−0.0532(12)
32	0.0137(15)	0.0384(18)	−0.0259(18)
64	0.0259(19)	0.038(3)	0.0052(24)

diagonal part of the covariance matrix. If we do so, we obtain, for $p=0.8$, $\chi^2/\mathrm{DOF}=0.76/1$ with $\nu=1.012(10)$ and for $p=0.65$, $\chi^2/\mathrm{DOF}=0.73/1$ and $\nu=1.005(14)$. Therefore, the presence of second-order corrections for the thermal exponent data seems reasonable. Furthermore, we have found that the value of ν is not affected by the presence of these terms.

In the non-Gaussian case large finite-volume corrections are also present. Nevertheless, we find that the estimates from the two largest lattice pairs for the ν exponent are compatible with the analytical calculations.

B. Magnetic exponent

In Table II we present the estimates of the magnetic exponent η applying Eq. (17) to the susceptibility χ measured at the points where $Q_\xi=2$ for all the concentrations considered.

As we can see in the table, there are strong scaling effects in all cases, especially in the non-Gaussian case. An infinite-volume extrapolation procedure is therefore needed in order to get an η estimate.

If we only assume the presence of first-order corrections with our previously calculated ω value, we do not find reasonable fits to our data. Therefore, we should consider higher-order correction terms. As we have found $\omega\simeq 1$, the second-order terms and the analytical corrections are of the same order, so we can try a quadratic joint fit using the $\omega=1$ value. With the $L\geqslant 16$ data for all the concentrations studied, using only the diagonal part of the correlation matrix, we get $\chi^2/\mathrm{DOF}=1.63/2$ and $\eta=0.043(4)$. So we have found results compatible with the picture of a single η value, with scaling corrections parametrized by $\omega\simeq 1$, but with non-negligible higher-order correction effects. Nevertheless, this estimate has two different sources of systematic error: the first one is due to the possible dependence of the value of η on the minimum lattice size considered in the fits, and the second is due to the uncertainty on the fitted functional form.

We can compare this result for η with those from the random site-diluted Ising model, $\eta=0.0374(36)(9)$,[8] and with those from the Ising case, $\eta=0.0374(6)(6)$,[6] finding that they are similar to our estimate for η.

C. Critical couplings

In Table III we show the crossing points of g_4 and ξ/L from $(L,2L)$ lattice pairs for the Gaussian case. As we can see in the table, there is a nonmonotonic L behavior for g_4 crossing points in both concentrations, so the presence of high-order scaling corrections is expected. A way to extract the infinite-volume critical coupling is to perform a fit to the functional form of Eq. (18). In order to find a proper extrapolated value, we have to make sure that we are within the linear regime and that we can control the higher-order corrections. As the crossing points for g_4 show a minimum value around the $L=32,64$ pair, the former condition is not satisfied.

TABLE III. Crossing points from $(L,2L)$ pairs of g_4 and ξ/L for the Gaussian case at the different concentrations simulated.

	$p=0.8$		$p=0.65$	
L	ξ/L	g_4	ξ/L	g_4
8	0.274535(34)	0.273760(52)	0.335269(76)	0.33358(12)
16	0.273545(15)	0.272862(22)	0.333709(41)	0.332617(72)
32	0.2729883(96)	0.272604(14)	0.333099(19)	0.332682(28)
64	0.2727805(70)	0.272624(11)	0.332989(15)	0.332872(25)

We could fit the ξ/L crossing points to Eq. (18). Nevertheless, for both concentrations, using $L\geqslant 8$ data, we have found a large value of χ^2/DOF, being $\mathrm{DOF}=1$. We will then be forced to assume the presence of higher-order corrections to scaling.

In order to control the finite-volume effects, assuming our estimate for $\omega+1/\nu=2.00(13)$, we can discard the $L=8$ data and perform a linear fit for the ξ/L data. In the $p=0.65$ case we find a reasonable $\chi^2/\mathrm{DOF}=1.28/1$ in the central value, and we get $\beta_c(\infty)=0.332\,929(13)(12)$, where the second error bar is due to the uncertainty in $\omega+1/\nu$. Nevertheless, for $p=0.8$ we do not find a reasonable fit, which shows that the higher-order corrections are important even in the $L=16$ lattice. We can check this latter picture performing a fit with $1/L^2$ and $1/L^3$ terms, only considering the diagonal part of the covariance matrix for $L\geqslant 8$ data. Then we obtain $\chi^2/\mathrm{DOF}=1.69/1$ and $\beta_c(\infty)=0.272\,715(10)$. So the picture of second-order scaling corrections is compatible with our data in this case.

A similar analysis can be done by studying the g_4 and ξ/L crossing points measured with a (L_1,L_2) pair but with a fixed L_1 value.

In the $p=0.65$ case, by performing a linear fit for the crossing points of ξ/L with $L\geqslant 16$, and $\omega+1/\nu=2.00(13)$ we get $\chi^2/\mathrm{DOF}=1.43/1$ for the central value of this interval and $\beta_c(\infty)=0.332\,927(13)(15)$, where the second error bar is due to the uncertainty in the critical exponents.

In the $p=0.8$ case a diagonal fit for ξ/L with $1/L^2$ and $1/L^3$ terms using $L\geqslant 8$ data yields $\chi^2/\mathrm{DOF}=0.52/1$ and $\beta_c(\infty)=0.272\,722(10)$. The behavior we find is similar to our previous analysis, finding reasonable fits and compatible estimates for the infinite-volume critical couplings.

In Table IV we show the crossing points of g_4 and ξ/L measured at L and $2L$ lattice sizes for the non-Gaussian case with $p=0.8$ mean concentration.

Also in this case we see that the g_4 crossing point is not a monotonic function of L. In the ξ/L case we find that with our previous ω estimate, a linear fit for $L\geqslant 16$, is not reasonable, so we have to conclude that also in this case the higher-order terms are present. In order to check this assumption in a simple way, we perform a fit with $1/L^2$ and $1/L^3$ terms,

TABLE IV. Crossing points of g_4 and ξ/L from $(L,2L)$ pairs for $p=0.8$ in the non-Gaussian disorder.

L	ξ/L	g_4
8	0.25926(4)	0.25803(5)
16	0.257935(22)	0.25706(3)
32	0.257375(13)	0.25708(21)
64	0.257188(9)	0.257110(13)

using $L \geqslant 8$ and using only the diagonal part of the covariance matrix. We obtain $\chi^2/\mathrm{DOF}=0.34/1$ and $\beta_c(\infty) = 0.257\,126(14)$, so this picture is compatible with our data.

In order to compute the value of the scaling functions g_4 and ξ/L at the critical coupling in the thermodynamical limit, we have measured the values of these quantities at the crossing points of g_4 and ξ/L, respectively. In the g_4 case, the finite-volume corrections are large, and we find values for this observable in the range 0.58–0.64. In the ξ/L case, we have also found that an infinite-volume extrapolation procedure is needed. Performing a $1/L$ extrapolation we quote for this quantity the value 0.36(2).

VI. CONCLUSIONS

We have studied the three-dimensional site-diluted Ising model, with long-range spatially correlated disorder by Monte Carlo simulations. We have considered Gaussian and non-Gaussian disorder in order to study the influence of the long-range correlations in the disorder on the critical behavior of the system. We have used finite-size scaling techniques for the computation of the critical exponents.

We have found strong scaling corrections for the ν exponent. In the Gaussian case, we succeed to parametrize them with the first corrections-to-scaling term, and find an infinite-volume ν value that is compatible with the analytical prediction in this model. In the non-Gaussian case, the value we obtain for the two largest lattice-size pairs is also compatible with this calculation.

For the η exponent, large finite-volume effects are also present. Our data for $L \geqslant 16$ are compatible with the picture of a single value of η independent from the type of disorder and from the concentration considered, but with non-negligible second-order correction terms.

Therefore, we have obtained a consistent picture of the existence of a single fixed point (single η, ν, and ω values) using Gaussian and non-Gaussian correlated disorder, but with non-negligible second-order corrections to scaling. This fact introduces systematic errors in our analysis that are very difficult to measure, and it is not easy to obtain solid estimates of the final errors for the critical exponents.

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