Existence of a Bose metal at $T=0$

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This paper aims to justify the existence of a two-dimensional Bose metal, i.e., a metallic phase made out of Cooper pairs at $T=0$. To this end, we consider the physics of quantum phase fluctuations in (granular) superconductors in the absence of disorder and emphasize the role of two order parameters in the problem, viz. phase order and charge order. We focus on the two-dimensional (2D) Bose Hubbard model in the limit of *very large* fillings, i.e., a 2D array of Josephson junctions. We find that the algebra of phase fluctuations is that of the Euclidean group E_2 in this limit, and show that the model is equivalent to *two* coupled XY models in $(2+1)$ dimensions, one corresponding to the phase degrees of freedom, and the other to the charge degrees of freedom. The Bose metal, then, is the phase in which both these degrees of freedom are disordered (as a result of quantum frustration). We analyze the model in terms of its topological excitations and suggest that there is a strong indication that this state represents a surface of critical points, akin to the gapless spin liquid states. We find a remarkable consistency of this scenario with certain low-T_c thin film experiments.
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I. INTRODUCTION

The superconductor-insulator (SI) transition in low-T_c thin film systems[1] has drawn a lot of attention over the past couple of decades. These systems undergo transition from superconductor (SC) to insulator as the disorder, thickness or magnetic field is tuned. The problem has received a strong impetus after the experiment by Goldman *et al.*[2] on homogeneous lead and bismuth films, which went from a superconducting phase to an insulating phase as a function of thickness, and which, right at the interface, was probably metallic. Since electrons usually do not form a metallic state in two dimensions, it was argued that [Ref. 3(b)] this $T=0$ transition is due to the localization of preformed Cooper pairs. It was also claimed that the resistivity at the transition is universal [Ref. 3(b)]. Similar SI transitions have been observed in granular superconductors and Josephson Junction arrays.[1] This scenario has been called into question after a recent magnetic field tuned experiment in the Mo-Ge sample,[4] where the metal is no more a point in the phase diagram, but exists as a separate phase. We would like to point out that this is not the first observation of a metallic phase in a two-dimensional, otherwise superconducting, system. We came across at least two separate instances of this phenomenon in granular superconductors — one in Ga films[5] and the other in granular Pb films,[6] where a metallic phase is found to be sandwiched between the superconducting and the insulating phases. A similar observation has been reported in Josephson Junction arrays.[7] Each of these observations has probably a detailed explanation within the scope of the specific experimental system being measured. However, there is something common in these systems which is quite hard to overlook, viz. the preformed Cooper pairs are very much alive when the metallic phase is seen; and continue to exist till the insulating transition and beyond. Motivated by this fact, we would like to ask the question — is it possible for the charged bosons, i.e., Cooper pairs, to form an incoherent metallic phase at $T=0$? The intriguing feature of this phase is that although the bosons are mobile, they do not bose condense at any temperature. Instead, they are dissipative and fail to drive a supercurrent even at $T=0$, unlike a superconductor. We regard this phase as a Bose metal (BM). In this paper, we give arguments justifying the existence of a Bose metal in a physically realizable system.

The natural question, then is, what has been missing in the current theoretical models where a metallic phase was not obtained. Our thought on this issue is the following: most of the earlier theories[3,8,9] tried to attack this problem from the superconducting side of the phase diagram and projected it onto a basis diagonal in the phase states. In all these theories, there was a single order parameter, viz. $\psi=\langle e^{i\phi}\rangle$, where ϕ refers to the phase of the charge boson. In the superconducting phase, this order parameter is well developed. At the SI transition, $\psi\rightarrow 0$ and the phases of the charge bosons get scrambled. The scrambling of the phases[3,10] had so far been taken as the indication of onset of the insulating phase. However, this is not enough to characterize the insulator. A Bose insulator (BI) phase is characterized by an *extra order parameter*, viz. the charge density. It is like a charge density wave but built out of Cooper pairs. This piece of physics has been missing in the existing theories.[3,8–10] The central point of this paper is that the phase fluctuation physics of superconductors should be viewed as a *two* order parameter problem, viz. there is a competition between phase order and charge order. It almost follows from this fact that the destruction of one order parameter does not necessitate the growth of the other order parameter. This implies a possible existence of a disordered phase where both the order parameters are zero at $T=0$. We consider this to be a Bose metal phase.

Secondly, we work in the limit when the average filling of the bosons (n_0) per site is very large. In this limit, the Bose Hubbard model [Ref. 3(b)], which plays a central role in the SI transition, becomes equivalent to a Josephson junction array (JJA) model. Most of the existing work on the JJA model does not treat this large n_0 limit consistently. In many cases, the constraint due to the average filling, Eq. (2) below is neglected;[46] in certain other cases, there is a tendency to replace the E_2 algebra, which is the appropriate algebra for this limit (see Secs. II and V for more details), by a qualitatively different algebra, viz. SU(2) algebra, which holds in the hard core limit, i.e., for small fillings. Hence, we shall give a quite detailed description of the basic formulation of this limit in this paper.

Thus, we consider a pure model of Josephson-coupled Cooper pairs with extremely large fillings, interacting via on-site and near neighbor repulsive forces in two dimensions. This model captures the basic physics of granular superconductors,[3] except that disorder is absent. It is applicable to a limited extent to the Josephson junction arrays as well. We are able to demonstrate that the physics of phase fluctuations, within such a model, can be described by two coupled anisotropic XY models in $(2+1)$ dimensions, one corresponding to the charge degrees of freedom and the other the phase degrees of freedom, where the coupling is ''XY-like.'' The two mechanisms which drive the two transitions are as follows — disordering of the vortices and the Bose condensation of vortices. Phase order is destroyed when the vortex-antivortex pairs unbind [or, vortex loops blow up in $(2+1)$ dimensions] as a result of the quantum fluctuations; the charge order grows when the vortices Bose condense.[11] In light of this, a search for a completely disordered phase in the charge picture translates into a search for a non superfluid (SF) liquid in the vortex picture. The presence of unbound, as yet uncondensed, dissipative vortices makes the disordered phase metallic. The two mechanisms mentioned above are separate processes owing to the presence of retardation, or equivalently, dissipative effects, coming from a gauge field mediated interaction in the vortex picture (for more details see Secs. II D and III B). In the charge picture, the existence of a disordered phase like that of a Bose metal results from *quantum frustration* effects, i.e., the zero point motion is not strong enough to set up superconducting correlations and the long range interactions are not sufficient to set up a charge ordered state. This state seems to be intrinsically related to the (gapless) spin liquid states. Although disorder is not explicitly present in our model, our results on the nature of the Bose metal phase can be readily generalized to the case where disorder is present. We find a remarkable agreement of our predictions with the Gallium film experiment.[5] We believe this provides evidence of the existence of such a phase and supports the scenario described above. All of our considerations in this paper are restricted to $T=0$ and zero magnetic field unless mentioned otherwise. This situation is relevant to the experiments in Refs. 5 and 6.

Before we go over to the main part of the paper, we would like to mention that the idea of two order parameters is not completely new and has appeared in the discussion of the hard-core limit of the Bose-Hubbard model[12] and also in the gauge theory description of the Josephson junction arrays.[13] The problem with the former is that superconductivity is never destroyed away from half-filling in their model (please see Sec. II B), while the latter invokes the idea of self-duality which implies logarithmic interaction among Cooper pairs and is not a good representation of the realistic thin film samples.

Apart from the existence of a disordered Bose liquid state, there is another important feature which emerges from our work. It is well known that[14] noninteracting electrons cannot support a metallic state at low temperatures in two dimensions (2D) in the absence of spin-orbit scattering. The situation is much less clear when both interactions and disorder are present. This fact has been brought into the limelight with the recent observation[15] of metal-like states in two dimensional electron and hole systems in semiconductor based materials. In the wake of these observations, our investigations suggest that a metallic state is a possibility if the electrons bind themselves into Cooper pairs and behave as if they are bosons.

The plan of the paper is as follows. In Sec. II, we introduce the model and straighten out some of the basic issues which relate to the model. Using a combination of qualitative and quantitative arguments, we explain why the BM phase is feasible. The mapping onto the coupled XY models is demonstrated here. We quantify the arguments in Sec. III and sketch the phase diagram of the model. Section IV contains an estimate of the resistivity of the Bose metal phase and a comparison with the Gallium film experiment. The thermodynamics of this strange metallic state and its connection with the spin liquids are discussed in Sec. V. We wind up with a discussion of certain relevant issues and conclusions in Sec. VI. Some of the calculational details may be found in the Appendices.

II. THE FOUNDATIONS

A. The model

We shall consider the following model Hamiltonian in this paper:

$$H=-J\sum_{\langle i\alpha\rangle}\cos(\phi_i-\phi_{i+\alpha})+V_0\sum_i(\delta\hat{n}_i)^2+V_1\sum_{\langle i\alpha\rangle}(\delta\hat{n}_i+\delta\hat{n}_{i+\alpha})^2-\bar{\mu}\sum_i\delta\hat{n}_i-\mu N n_0, \quad (1)$$

where $\delta\hat{n}_i=\hat{n}_i-n_0$, with $\hat{n}_i=$number density operator, n_0 =neutralizing background charge density (or, equivalently, the average density of Cooper pairs), $N=$number of lattice sites, and $\bar{\mu}=\mu+V_0$ the renormalized chemical potential in the problem. Also, $\alpha=\hat{x},\hat{y}$ refers to the spatial unit vectors. The first term in Eq. (1) represents the kinetic energy of the bosons (Cooper pairs for our case), the second term the on-site repulsion among them (which should be nonzero to prevent any collapse of the bosons onto a single site), the next term the repulsion among the nearest neighbors which acts to set up a charge order in the system, and the last two terms are the chemical potential terms. Equation (1) has to be supplemented with the constraint $(1/N)\Sigma_i\langle\hat{n}_i\rangle=n_0$, which in terms of the charge fluctuation operators convert into

$$\sum_i \langle \delta\hat{n}_i \rangle = 0. \tag{2}$$

This equation implies $\bar{\mu}=0$ and, in the rest of the paper, we shall forget about the chemical potential terms completely.

On the experimental side, the three parameters of the Hamiltonian (1) can be determined in the following way:[16]

$$J=(R_Q/2R_n)\Delta_0. \tag{3}$$

Here $R_Q=(h/4e^2)=6.45\ K\Omega$, $R_n=$ normal resistance of the film, and $\Delta_0=$ pairing gap. The interaction constants V_0 and V_1 are related to the inverse of the capacitance matrix C_{ij} of the grains.

B. The commutation relations

The model discussed in Eq. (1) needs to be supplemented with the phase fluctuation algebra, which constitutes the appropriate commutation relations for this problem:

$$[\delta\hat{n}_i, \hat{\phi}_j] = i\delta_{ij}. \tag{4}$$

Equation (4) implies an angular momentum representation [Ref. 3(a)]

$$\delta\hat{n}_i = i\frac{\partial}{\partial\phi_i}.$$

Further, let us define the operators

$$L = i\frac{\partial}{\partial\phi}, \quad P = e^{i\phi}. \tag{5}$$

Thus, Eq. (4) can be recast as

$$[L,P] = -P, \tag{6a}$$

$$[L,P^\dagger] = P^\dagger, \tag{6b}$$

$$[P,P^\dagger] \simeq 0, \tag{6c}$$

with $PP^\dagger = I$. It is well known in quantum optics that the phase operators being ladder operators [as is seen from Eqs. (6a) and (6b)] usually do not commute.[17] However, in the large n_0 limit, they do (see Appendix F). Thus, Eq. (6c) is strictly valid in the large n_0 limit of the problem and, hence, our discussion holds good in this limit only. Now, Eqs. (6a)–(6c) constitute the *algebra of the Euclidean group* E_2,[18] the group of translations and rotations in 2D, with the square of linear momentum (P) restricted to unity for our case. It deserves to be mentioned here that the SU(2) algebra used in the context of the hard-core limit of the Bose problem[12] is distinctly different from this algebra and is obtained in the opposite limit, i.e., the small n_0 limit of the Bose Hubbard model. The effects of the change in the group structure are quite significant. In the hardcore model, away from half filling, increased interactions change the SF to a supersolid phase. Thus, superfluidity is never destroyed away from half filling. On the other hand, our model allows superfluidity to be quenched at arbitrary fillings. Further, the conserved quantities supported by the two algebras are different: the invariant of E_2 algebra is (square of linear momentum) $P_x^2 + P_y^2 = p^2$ ($=1$ for our case), as compared to $S_x^2+S_y^2+S_z^2 = s(s+1)$ of the SU(2) algebra. Thus, unlike the SU(2) case, (a) here the constraint on the z component of the spin is much weaker and (b) the z component of the spin enters very anisotropically in the algebra compared to the x and y components of the spin. (Here operator L is referred to as the z component of spin for E_2 algebra; see Sec. V for more details). Hence, in the disordered/symmetric phase, although the x and y components of E_2 spin might acquire a gap like the SU(2) spin, the z component of the former might remain gapless unlike the latter. Put in simple words, there may be a length scale determining the local superfluidity in this phase and as yet no length scale associated with the charge ordering in the system. A calculation of the charge charge correlation function indeed justifies this, as discussed in Sec. V. Thus, given a disordered phase within a model built out of E_2 spin operators, it can support gapless excitations. We shall give detailed arguments in Secs. II D and III as to why the model (1) along with commutation rules (6a)–(6c) contain a completely disordered phase.

Now, if we rotate the charge fluctuations L_i at each site, viz.

$$L_i \rightarrow e^{i\vec{Q}\cdot\vec{r}_i}L_i,$$

where

$$\vec{Q} = (\pi,\pi),$$

we obtain the Hamiltonian as

$$H = \sum_k J_k P_k^\dagger P_k + \sum_k V_k L_k^\dagger L_k, \tag{7}$$

where

$$J_k = -J(\cos k_x + \cos k_y),$$

$$V_k = (V_0 + 4V_1) - 2V_1(\cos k_x + \cos k_y),$$

and P,L couple through the commutation relations (6a)–(6c). In this form, it assumes the shape of a two order parameter problem. (Macroscopic occupation of $k=0$ mode of P reflects superconductivity and that of L a charge density wave.) However, because of commutation rules (6a)–(6c), it is very hard to diagonalize this Hamiltonian in this form. So, we seek alternative means.

C. The charge picture

In this section, we demonstrate that the model Hamiltonian (1) is equivalent to two coupled XY models in $(2+1)$ dimension. To do this, we first write the model (1) in terms of a path integral representation[18,19] keeping the commutation relations (6a)–(6c) in mind. The partition function $Z = Tre^{-\beta H}$, where β is the inverse temperature ($\hbar$ and k_B are taken to be one, unless mentioned otherwise), can be written as

$$Z = \sum_{\{m_i(\tau)\}} \int_0^{2\pi} \mathcal{D}\phi_i(\tau) e^{-S},$$

where

$$S = i\int_0^\beta \sum_i m_i(\tau)\frac{\partial\phi_i}{\partial\tau} + \int_0^\beta d\tau\Bigg[-J\sum_{i\alpha}\cos(\phi_i(\tau) - \phi_{i+\alpha}(\tau)) + V_0\sum_i m_i^2(\tau) + V_1\sum_{i\alpha}(m_i(\tau) + m_{i+\alpha}(\tau))^2\Bigg], \tag{8}$$

with periodic boundary conditions in the imaginary time direction implied. Here $\alpha = \hat{x}, \hat{y}$ refers to the nearest neighbors in the space direction and $m_i(\tau)$ are integers meaning the change in the number of Cooper pairs from the average at the site i. We shall consider only the case $T=0$, so that the integral over imaginary time extends to infinity. Next, we discretize the time axis with an interval $\Delta\tau$ and rescale the lattice constant in the space directions to be unity. Further, we rotate the integers $m_i(\tau) \to e^{-iQ\cdot r_i} m_i(\tau)$, where $\vec{Q} = (\pi, \pi)$. Thus, we obtain

$$S = i\sum_i e^{-iQ.r_i} m_i(\nabla_\tau\phi_i) + V_0\Delta\tau\sum_i m_i^2 + V_1\Delta\tau\sum_{i\alpha}(\nabla_\alpha m_i)^2 - J\Delta\tau\sum_{i\alpha}\cos(\nabla_\alpha\phi_i). \tag{9}$$

Here summation over i refers to the time axis as well, and so does the index i in m_i and ϕ_i. The derivatives ∇_μ in Eq. (9) and also in what follows are lattice derivatives.[20] To avoid any confusion, we shall reserve the notation r_i for the spatial coordinates of the ith point and x_i its space-time coordinates.

To show that the action (9) is equivalent to two coupled XY models, we follow the following sequence of steps. (1) We decouple the $(\nabla_\alpha m_i)^2$ term using a Hubbard-Stratanovich field $p_{i\alpha}$, viz.

$$\int \mathcal{D}p_{i\alpha} e^{-(1/4V_1\Delta\tau)\Sigma_{i\alpha}p_{i\alpha}^2 - i\Sigma_i m_i(\bar{\nabla}_\alpha p_{i\alpha})}.$$

(2) First, we notice that the coupling term with m_i is invariant under shifts of $p_{i\alpha}$ by 2π. So, we break up the integral over $p_{i\alpha}$ from $-\infty$ to $+\infty$ into that of periods of 2π. Further, we split up $p_{i\alpha}$ into a curl and a gradient part. Since the divergence of $p_{i\alpha}$ couples to m_i, only the gradient part enters the dynamics. Thus, we obtain for this part,

$$\sum_{\{l_{i\alpha}\}}\int_0^{2\pi}\mathcal{D}\theta_i e^{-i\Sigma_i m_i\nabla^2\theta_i - (1/4V_1\Delta\tau)\Sigma_{i\alpha}(\nabla_\alpha\theta_i - 2\pi l_{i\alpha})^2} \simeq \int_0^{2\pi}\mathcal{D}\theta_i e^{(1/2V_1\Delta\tau)\Sigma_{i\alpha}\cos(\nabla_\alpha\theta_i)} e^{-i\Sigma_i m_i\nabla^2\theta_i},$$

where we have used an (inverse) Villain transformation.[13] (3) Now, using an (inverse) Villain transformation again, one can execute the sum over integers m_i

$$\sum_{\{m_i\}}\exp\Bigg(-i\sum_i m_i(e^{-iQ.r_i}\nabla_\tau\phi_i + \nabla^2\theta_i) - V_0\Delta\tau\sum_i m_i^2\Bigg) \simeq e^{(1/2V_0\Delta\tau)\cos(e^{-iQ\cdot r_i}\nabla_\tau\phi_i + \nabla^2\theta_i)}.$$

Putting all these together, one obtains

$$Z = \int_0^{2\pi}\mathcal{D}\phi_i\mathcal{D}\theta_i e^S,$$

with

$$S = J\Delta\tau\sum_{i\alpha}\cos(\nabla_\alpha\phi_i) + \frac{1}{2V_0\Delta\tau}\times\sum_i\cos(e^{-iQ\cdot r_i}\nabla_\tau\phi_i + \nabla^2\theta_i) + \frac{1}{2V_1\Delta\tau}\sum_{i\alpha}\cos(\nabla_\alpha\theta_i), \tag{10}$$

where the phase ϕ is associated with superfluidity and the phase θ with charge density wave. Equation (10) explicitly shows that the phase fluctuation physics of (granular) superconductors, in the absence of disorder, is equivalent to two coupled XY models in $(2+1)$ D, where the coupling is XY-like. A few comments are in order. In the limit $V_1 = 0$, we have $\theta_i = 0$, the θ-terms in action (10) drop out, and we obtain a single XY model in $(2+1)$ D, as has been discussed previously.[3,10,19] Also, the coupling term (the second term) is highly anistropic in ϕ and θ, explicitly breaking self-duality in this system, *assumed* in Ref. 13. Equation (10) is one of the key results of this paper. Thus, we see that the destruction of superfluid state is driven by one XY model, whereas the other XY model characterizes the growth of the charge ordered state. And, hence, as the parameters are tuned one transition does not necessarily accompany the other.

D. The vortex picture

Although the action (10) shows that the superfluid order and charge order are driven by different XY models, it is not clear whether the coupling between them, which is quite complicated, guarantees a completely disordered phase. In order to answer such a question, we now consider the model (1) in the dual picture, i.e., of the vortices. To do this, we invoke a duality transformation.[3,13,11,19] Starting from action (9) (Ref. 21) one can show that

$$S = \frac{\pi^2}{\ln\left(\frac{2}{J\Delta\tau}\right)}\sum_{q,\omega} j^0_{q,\omega}\frac{1}{\hat{q}^2}j^0_{-q,-\omega} + \frac{\pi^2}{\ln\left(\frac{2}{J\Delta\tau}\right)}\sum_{q,\omega} j^\alpha_{q,\omega}G(\omega,q)j^\alpha_{-q,-\omega}, \tag{11}$$

where $j^\mu_{q,\omega}$ are Fourier transforms of the integer vortex variables j^μ_i (j^0_i = vortex density, j^α_i = vortex current), and

$$G^{-1}(\omega,q) = \Bigg[\hat{\omega}^2 + \frac{V_0\Delta\tau}{\ln\left(\frac{2}{J\Delta\tau}\right)}\hat{q}^2 + \frac{V_1\Delta\tau}{\ln\left(\frac{2}{J\Delta\tau}\right)}\hat{q}^2(\hat{q}-Q)^2\Bigg]. \tag{12}$$

The intervening steps are quite standard[22] and are discussed in Appendix A. $\hat{\omega}, \hat{q}$ refer to lattice frequency and momentum respectively (Appendix A). One may like to note here

that there is no Magnus force term on the vortices, corresponding to the disappearance of n_0 from the problem in the large n_0 limit.

In the previous subsection, we observed that when $V_1 \to 0$, the model is equivalent to a single XY model rather than two coupled XY models. The question obviously is, how this change is reflected in the dual picture. The basic answer lies in the appearance or disappearance of the zero point motion term for the vortices (more popularly known as the vortex mass term). To show this, we look at the long wavelength low frequency modes. In the limit $\omega, q \to 0$ and $\omega \ll c_s q$ (where c_s is the plasmon velocity), the Green's function $G(\omega,q)$ splits up into a singular part $G_s(\omega,q)$ and a constant part G_0, viz.

$$G(\omega,q) \simeq G_s(\omega,q) + G_0, \tag{13}$$

where

$$G_s(\omega,q) = 1/(\omega^2 + c_s^2 q^2), \quad G_0 = b^2/8(b^2+c^2)^2,$$

with

$$c_s^2 = b^2 + c^2, \quad b^2 = 8\frac{V_1 \Delta\tau}{\ln\left(\frac{2}{J\Delta\tau}\right)}, \quad c^2 = \frac{V_0 \Delta\tau}{\ln\left(\frac{2}{J\Delta\tau}\right)}.$$

Thus the action is

$$S = \frac{\pi^2}{\ln\left(\frac{2}{J\Delta\tau}\right)} \sum_{q,\omega} j^0_{q,\omega} \frac{1}{q^2} j^0_{-q,-\omega} + \frac{\pi^2}{\ln\left(\frac{2}{J\Delta\tau}\right)} \sum_{q,\omega} j^\alpha_{q,\omega} [G_s(\omega,q) + G_0] j^\alpha_{-q,-\omega}. \tag{14}$$

Since G_0 is a constant, it allows us to identify this term as the vortex kinetic energy, i.e., mass term. From Eqs. (14) and (13), we notice that this term does not exist when $V_1 = 0$. Thus, we have shown that the change of the nature of the XY models is tied to the existence or nonexistence of a vortex mass term.

We now look at the other terms in the action (14). The first term in the action is the usual logarithmic interaction term among the vortices, and the second term, because of the retardation effects, leads to a dissipative term for the vortices.[16] In the limit $|r_i - r_j| \ll c_s|\tau - \tau'|$ (which is the same quasistatic limit discussed so far), this part of the action takes the form

$$\frac{\pi^2}{c_s^2 \ln(2/J\Delta\tau)} \sum_{i,j} q_i q_j \sum_{\omega} \omega^2 \ln\left(\frac{1}{|\omega|}\right) r_{i,\omega} r_{j,-\omega},$$

where $q_i = \pm 1$ refers to the charge on the ith vortex. Physically, the aforesaid limit corresponds to the slow motion of vortices. The source of this heat bath (or, dissipation) is the gauge field (discussed in Appendix A), or more precisely, the transverse modes arising from the quantum fluctuations in the system. (There will be additional retardation effects in a real system from external heat bath mechanisms, e.g., electrons in the vortex core, etc.) These features of vortices in granular superconductors have been discussed previously by Eckern and Schmid.[16]

One may also undo some of the steps in Appendix A, and recast Eq. (14) in terms of a gauge field with appropriate action as

$$S = \ln\left(\frac{2}{J\Delta\tau}\right) \sum_{q,\omega} [q^2 A^0_{q,\omega} A^0_{-q,-\omega} + (\omega^2 + c_s^2 q^2) A^\alpha_{q,\omega} A^\alpha_{-q,-\omega}] + 2\pi i \sum_i j_i^\mu A_i^\mu + \pi^2 \frac{G_0}{\ln\left(\frac{2}{J\Delta\tau}\right)} \sum_{q,\omega} j^\alpha_{q,\omega} j^\alpha_{-q,-\omega} \tag{15}$$

(with the gauge condition $\nabla_\alpha A_i^\alpha = 0$ imposed). A few comments are in order. Equation (15) shows that, considered in the vortex picture, the quantum phase fluctuations in a 2D superconductor, as described by model (1), are equivalent to a *two*-component *quantum* plasma (bosons of two flavors, viz. vortices and antivortices) moving in a fluctuating gauge field A^μ. Secondly, the above scenario is a simple quantum mechanical extension of classical phase fluctuations in a 2D superconductor, which is described by a two component *classical* plasma undergoing screening by a static electric field ($\vec{E} = -\vec{\nabla} A_0$), as described by Kosterlitz and Thouless (KT).[25] The effect of including quantum mechanics in the problem, apart from bringing up the importance of the quantum statistics of vortices, is to make the electric field dynamical, viz. $\vec{E} = -\vec{\nabla} A_0 - (1/c_s)\partial\vec{A}/\partial\tau$ with a magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$. Whereas the importance of the statistics is to allow for the superfluidity of the vortices, an important consequence of the dynamical nature of the electromagnetic field is that there are retardation effects, viz.

$$\sum_{i,j} \int d\tau d\tau' \frac{\dot{\vec{r}}_i(\tau) \cdot \dot{\vec{r}}_j(\tau')}{\sqrt{(\vec{r}_i(\tau) - \vec{r}_j(\tau'))^2 + c_s^2(\tau - \tau')^2}},$$

which break Galilean invariance. (This feature is intensified in a real system by external heat bath mechanisms mentioned before.)[16] This is not surprising because the action (15) has the structure of Maxwell's action, which is reputed to have Lorentz invariance but lacks Galilean invariance. Now the absence of Galilean invariance will have a strong effect on our system, because it is bosonic. It is well known that *all* the delocalized bosons will condense into the superfluid state only if the system is Galilean invariant.[26] If this invariance is absent in a Bose system, then as the relevant parameter is tuned, the (vortex) condensate is gradually depleted and at one point the superfluidity will be completely lost. At this stage, it is important to recollect what important processes are going on in this system. There are two of them: destruction of vortex (and antivortex) superfluidity (owing to retardation effects) and the binding of vortex-antivortex pairs, corresponding respectively to the destruction of charge order and the growth of phase order. Now, these two processes are controlled effectively by two separate parameters, viz. $c^2/g^2 \sim (V_0 + 8V_1)\Delta\tau$ (corresponding to the strength of the retardation effects) and $g^2 \sim J\Delta\tau$ (corresponding to the strength of the logarithmic interaction), respectively, where $\Delta\tau \sim 1/\Delta_0$[34] (please refer to Appendix D and Sec. III for the

notation and appropriate details). As a result, the vortices (and antivortices) do not necessarily condense into the superfluid state as soon as they unbind. This leads to the possibility of a non-SF vortex liquid, or equivalently, a BM phase. In fact, that is what we find when we quantitatively evaluate these processes in Sec. III. There is a simpler way to see what is happening here. As we noted earlier, following Eq. (14), the kinetic energy of vortices originates from two sources — (a) quantum zero point motion (the G_0 term) and (b) action of an *effective* heat bath (the G_s term). In the delocalized state, if the source (a) dominates, the vortices move coherently, and since they are bosons, they form a superfluid. On the other hand, when the source (b) dominates, because of the random nature of the effective heat bath, the motion of the vortices is necessarily incoherent, and one is in a metallic phase. In this phase, no charge order is set up and one obtains a Bose metal phase.

These features of dual vortices are not special to a lattice model, but observable in the continuum formulation as well, as discussed in Appendix B.

III. THE PHASE DIAGRAM

In the previous section, we argued why the Hamiltonian described by Eq. (1) may contain an incoherent metallic phase. In this section, we quantify these arguments by calculating the phase diagram of model (1): we shall locate the phase boundary where superconductivity is destroyed and the one where charge order is established.

A. Destruction of charge superfluidity

This is done nonperturbatively by estimating where the vortex loops blow up in $(2+1)$ D.[27] This happens when the entropy of the loops overcomes their interaction energy.[28] A good estimate of the interaction energy is the self energy of the loops, simply because dipole-dipole interactions fall off as $1/r^3$ and the mutual interaction energy of the links in a loop is much smaller than the self-energy of the loops when the loops are fairly large. Thus, the effective free energy of the loops is given by

$$\mathcal{F}=\left[\frac{\pi^2}{\ln\left(\frac{2}{J\Delta\tau}\right)}G(0)-\mu_l\right]N, \tag{16}$$

where N is the number of the links in a vortex loop, $G(0)$ = diagonal part of the Green's function $=\int_{-\pi}^{\pi}(d\omega/2\pi)[d^2q/(2\pi)^2]G(\omega,q)$, obtained from Eq. (12), and μ_l = entropy of the loops $=\ln 3$ for our case.[29] An estimate of $G(0)$ is given in the Appendix C. The loops blow up when $\mathcal{F}<0$. Thus, superconductivity is destroyed when[30]

$$(V_0/J)+8(V_1/J)>\tilde{b}_0, \tag{17}$$

where $\widetilde{b_0}=2(b_0/\mu_l)^2$ and $b_0=$ a number of order unity $\simeq 3.1725$, defined by Eq. (C6) in Appendix C. This phase boundary has the character of $(2+1)$ D XY model, at least when $V_1 \ll V_0$ [as seen from Eq. (10)], and hence, the superfluid density changes continuously across this transition line.

B. Destruction of vortex superfluidity

It is well established that the existence of charge order implies superfluidity of vortices and vice versa.[11] We shall follow this notion here and estimate the growth of charge order in terms of the superfluidity of vortices. We mentioned in Sec. II D that the vortices and antivortices move in the presence of a *dynamical* gauge field and argued that because of the latter there are retardation effects which deplete the (vortex) Bose condensate. As a result, the vortices and antivortices do not necessarily condense into a superfluid state as soon as they unbind. This physics of suppression of Bose condensation as a result of gauge field fluctuations is not new but has been explored substantially in the context of spin charge separation theories in high T_c superconductors.[31,23] The discussion here is very similar in spirit to that piece of work.

To estimate the strength of the parameters where vortex superfluidity is destroyed, we follow the self-consistent functional approach of Ioffe *et al.*[23] They did the calculation for a very similar piece of physics, i.e., how the coupling to a gauge field can kill superfluidity in a bosonic system (with logarithmic interaction) as a result of broken Galilean invariance stemming from retardation effects. We refer the reader to that paper for a full description of this technique. A short discussion of this is given in Appendix D. We can follow their approach here, simply because the lattice action and the continuum action have identical structure in the low frequency long wavelength limit, as discussed in Appendix B. To do this, we first replace the two component plasma by a one component plasma, i.e., charges (vortices) of one flavor moving in the background of fixed neutralizing charges of the other flavor. This approximation is very standard and captures the salient features of the problem, until and unless the plasma is extremely dense.[32] Then, the results of Sec. V in Ref. 23 can be directly carried over here with the identification of the Coulomb interaction parameter α_c and transverse gauge field coupling constant α_g as[30] (Appendix D)

$$\alpha_c=(\pi^2/2)v_1/(v_0+8v_1)^2,$$

$$\alpha_g=(1/\pi n_v)[1+(v_0/8v_1)], \tag{18}$$

where $n_v=$ average vortex density and $v_i=(V_i/J),(i=0,1)$. Since what counts in the destruction of vortex superfluidity are the free vortices (and antivortices), we take $n_v \sim n_f$, which goes inversely as the square of the correlation length ξ_+ which diverges at the SC-BM boundary from the BM side. Thus, from Eq. (17), $n_f \sim [\overline{b_0}(V_0+8V_1)/J-1]^{2\nu}$, $(\overline{b_0}=1/\tilde{b}_0)$ with $\nu \sim 2/3$, since this phase boundary has the character of $(2+1)$ D XY model. Also, if we are above the phase boundary of Eq. (17) (see Fig. 1), we have $\alpha \ll 1$ as in Ref. 23, where $\alpha=\sqrt{\alpha_c}$. Since as we shall note below that the transition from SF to non-SF state takes place at a small value of α, this assumption of small α is self-consistent.

Let us first consider the simple case when we are slightly above the phase boundary (16) and along the V_1 axis (see Fig. 1), so that $V_0=0, V_1 \sim 2.1J$. Then, we have, $\alpha \simeq 0.2$ and $\alpha_g \gg 1$; the calculation of Ref. 23, in that case, suggests we are in the disordered BM phase. Thus, there is at least a small region close to the phase boundary (17), where the system is metallic.

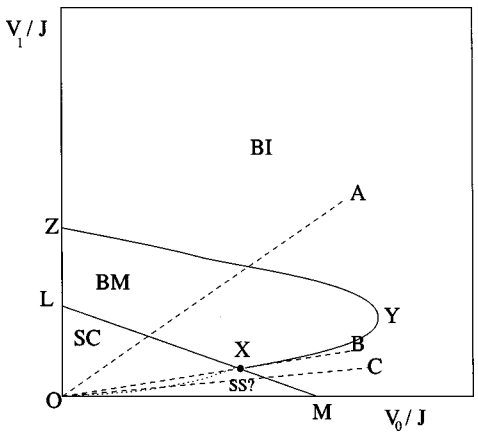

FIG. 1. A schematic phase diagram for model (1). The various phases are demarcated by solid boundaries. X denotes the multicritical point. The dashed lines (OA,OB,OC) denote typical trajectories executed as the normal resistance R_n of a superconducting film is tuned. Please see the text for details.

We now complete the calculation of phase boundary of BM-BI transition, following Ref. 23. Since $0<n_v<1$, from Eq. (18), $\alpha_g>0.32$. From Fig. 7 of Ref. 23, one can see that in this regime, the phase boundary tends to saturate at $\alpha=\alpha_{cr}\simeq 0.08$. Actually, for larger values of α_g, α_{cr} is a little less; but, the point is that α_{cr} is always finite, however small it be, for large values of α_g. That is, the vortices always form a superfluid for small enough α. A physical way of seeing why there is always superfluidity for small enough Coulomb repulsion is as follows. The strength of the Coulomb repulsion is controlled by the effective charge g (see Appendix D), which also controls the strength of the coupling between the particles (vortices) and the transverse gauge field. So, when the magnitude of Coulomb repulsion is small, g is small, and hence, the coupling with the transverse gauge field is small as well. As a result, the particles do not feel the effect of retardation strongly enough in this limit and condense into a superfluid phase. Thus, the vortices are in a superfluid state when $\alpha<\alpha_{cr}$, which means

$$\frac{1}{(8V_1/J)}[(V_0/J)+(8V_1/J)]^2>c_0, \tag{19}$$

where $c_0=4(\pi/8\alpha_{cr})^2\simeq 96$ for $\alpha_{cr}\simeq 0.08$. This calculation implies a jump in the (vortex) superfluid density at this phase boundary.[23] This means that the phase transition is either of first order or has a KT character. More calculations are necessary to resolve this point.

We display a schematic phase diagram determined by Eqs. (17) and (19) in Fig. 1. Equation (17) is represented by the curve LXM and Eq. (19) by ZYX. They seem to meet at a *tricritical* point X. Since the retardation effects are very strong in this model (meaning that the value of α_{cr} is small), the constant c_0 in Eq. (19) is roughly an order of magnitude larger than the constant $\widetilde{b_0}$ appearing in Eq. (17); and hence we expect that this crossing between the two curves will always occur, implying the existence of the Bose metal (see Sec. VI as well). The various phases determined by these equations are as marked in Fig. 1 when the zero point motion of the Cooper pairs is large, the system is superconducting (region $LXMO$); when the interactions dominate, the system is charge ordered and insulating as a result we call it a Bose insulator (BI); and in the intermediate region, the system is disordered and, hence, metallic (please see Sec. IV below) the BM phase (region $ZYXL$).

What one sees on the phase diagram of Fig. 1 is that the metallic phase is more prominent towards the V_1 axis rather than the V_0 axis. Further, from Eq. (1) we note that the V_1 term contains both on-site and nearest neighbor repulsion energies. This shows that the Bose metallic phase is to be expected in cases where these energy scales are of *comparable* order of magnitude, a situation well represented by the granular superconductors.[7]

We can provide a physical explanation in the charge picture as to why the metallic phase opens up along the V_1 axis: let us focus on Eq. (1) and say that superconductivity is already quenched. First consider the case V_0 large and V_1 small, so that we are along (or, close to) the V_0 axis. Naively, one would expect from the V_1 term that $\delta n_i+\delta n_{i+\alpha}\sim$ large. But, this costs a large energy from the V_0 term. As a result, what is favored is $\delta n_i\simeq 0$, i.e., $n_i\simeq n_0$, a (rather trivial) charge ordered state. Now, consider the opposite case: V_1 fairly large and V_0 small, so that we are close to (or, along) the V_1 axis. Since V_1 is appreciable, Eq. (1) suggests $\delta n_i+\delta n_{i+\alpha}\simeq 0$. A nontrivial configuration may be $\delta n_i=-\delta n_{i+\alpha}=1$, for some i's and zero otherwise. This state represents an RVB-like state of fluctuating charge (particle-hole) dipoles, the equivalent of spin singlets here (please see Sec. V for more on this point). V_0 being small, the $V_0\Sigma_i(\delta n_i)^2$ term does not cost much energy for this kind of state. This is the disordered BM phase. When V_1 is increased further, the dipoles freeze into a charge ordered solid. Thus, very crudely speaking, the smallness of V_0 is a source of frustration in this model for finite V_1.

Since J is inversely proportional to the normal resistance R_n [Eq. (3)], as the normal resistance of a thin film is tuned in an experiment we gradually cross from superconducting to nonsuperconducting regions according to the phase diagram of Fig. 1. Some such typical traces are shown in the figure — the dashed lines OA, OXB, and OC. Traces OXB and OC represent a superconductor-insulator transition, a case which has been discussed very widely in the literature.[3] On the other hand, the trace OA represents a superconductor-metal-insulator transition. In this part of the phase diagram, the retardation effects are very strong and the system passes through an intermediate disordered phase. This is a new prediction made by our analysis. Model (1) which leads to this is good for superconductor-insulator-superconductor junctions.

IV. RESISTIVITY OF THE METALLIC PHASE

The discussion in the previous section suggests that the appropriate model for the Bose metal phase is that of uncondensed bosons (vortices and antivortices) in a transverse gauge field. In the frustrated BM phase, the vortices are unbound, but they fail to bose condense due to retardation, or equivalently dissipative, effects. As a result, the system exhibits metallic behavior. This section is devoted to making a simplest possible estimate of the resistivity of this metallic phase. We shall evaluate the resistivity of the charge bosons ρ_c in terms of the conductivity σ_v of the dual variables, viz.

the vortices. The relation between the two is given by[9] (we mention the factor of h explicitly in this formula)

$$\rho_c = (h/4e^2)\sigma_v . \tag{20}$$

Also, σ_v is given by the Drude formula $\sigma_v = n_{vf}\tau_{tr}/m_v$, where τ_{tr} refers to the transport time and n_{vf} is the free vortex density. There are three contributions to the conductivity of the vortices — (i) dissipation due to the action of a heat bath (coming from the trasverse modes), (ii) scattering from impurities, and (iii) Bardeen-Stephen processes. In this part, we shall focus on the contribution from process (i). We give here a short description of how this dissipation mechanism comes about in this BM phase. As one enters this phase from the SC phase, say (following curve A in Fig. 1, for example), the vortices and antivortices unbind in 2D. These unbound charges screen each other leading to a finite screening length, i.e., a gap in the longitudinal part of the gauge field. However, the transverse part of the gauge field is gapless, because there is no spontaneous symmetry breaking. These transverse modes have their source in quantum fluctuations and mediate the dissipation process. They represent the plasmons. (Physically, this makes sense, since the plasmons are gapless in 2D.) The physics is essentially that of damping which results when a charge (vortex) moves through a background of other charged particles. This lends a hydrodynamical character to this dissipation mechanism. In Sec. II D, we saw that the most important modes are $\omega \ll c_s k$. So, we shall consider contributions from these modes only. The leading order contribution, then, comes from the $\omega = 0, k =$ finite modes, and we make an estimate from this sector only. We think the higher order contributions will lead to simple renormalization of coefficients. Thus, we are led to evaluate the resistivity of a set of uncondensed bosons moving in a static random (but, annealed) gauge field, with a variance

$$\langle H(q)H(-q)\rangle = 1/2c_s^2 \ln(2/J\Delta\tau)$$

as seen from Eq. (15), where $H = \vec{\nabla}\times\vec{A}$. Now, also in this delocalized phase, the interactions between the vortices are screened (by antivortices) and, in a lowest order approximation, we shall treat them as noninteracting bosons. Thus, the transport time is the same as that of a noninteracting particle scattering from a random magnetic field, which is given by[33]

$$\tau_{\rm tr} = \frac{m_v}{\pi^2} 2(V_0 + 8V_1)\Delta\tau,$$

where we have used the above field distribution and the parameters as enumerated following Eq. (13). The lattice constant in the time direction, $\Delta\tau \sim 1/\Delta_0$.[34] Thus, we have

$$\rho_c \sim \frac{2}{\pi^2} R_Q (n_{vf}\xi_0^2)(V_0 + 8V_1)/\Delta_0, \tag{21}$$

where $R_Q = h/4e^2 = 6.45K\Omega$. The quantity $n_{vf}\xi_0^2$ is inversely proportional to the square of the superconducting coherence length which diverges at the SC-BM phase boundary, i.e., $n_{vf}\xi_0^2 \sim (\xi_0/\xi_+)^2 \sim [(V_0 + 8V_1)/\tilde{b}_0 J - 1]^{2\nu}$, obtained from Eq. (17). Using Eq. (3), we can rewrite this as

$$n_{vf}\xi_0^2 \sim \left(\frac{R_n}{R_c} - 1\right)^{2\nu}, \tag{22}$$

where $R_c = \tilde{b}_0 R_Q/[(V_0 + 8V_1)/\Delta_0]$ is the critical resistance of the film where the metallic phase sets in and ν = correlation length exponent. The exponent ν is dependent on the particular universality class of SC-BM phase transition. For our case, the SC-BM phase boundary has the character of a $(2+1)$ D XY model and, hence, $\nu \simeq 2/3$. However, a real material, like the low-T_c thin film systems, is heavily disordered and ν will certainly be very different from this, as we shall see below. So, from Eq. (21), we have

$$R_\Box/R_Q \sim \tilde{b}_0[(R_n/R_c) - 1]^{2\nu}(R_Q/R_c). \tag{23}$$

The case when disorder is present. The foregoing discussion can be readily generalized to the case when disorder is present. Owing to the Drude formula, the structure of Eq. (21) is not unique to model (1) but will come about when the bosons form a metallic phase as a result of phase fluctuations (in zero magnetic field), i.e.,

$$R_\Box \sim 2R_Q(n_{vf}\xi_0^2)\mu_D, \tag{24}$$

where μ_D represents the vortex mobility, or the friction factor, arising from the dissipation of vortices and is model dependent.[35] For the pure case discussed above, the damping is due to the transverse modes and $\mu_D \approx (V_0 + 8V_1)/\pi^2\Delta_0$. When the normal resistance is tuned through R_c, as we saw above for the pure case, the most important dependence of BM resistivity in the formula (24) comes from the free vortex density $n_{vf}\xi_0^2$, i.e.,

$$n_{vf}\xi_0^2 \sim \left(\frac{R_n}{R_c} - 1\right)^{2\nu},$$

where the critical exponent ν characterizing the SC-BM phase transition depends on the particular universality class being considered. As compared to this, the dissipation factor μ_D, in general being dependent on the morphological and normal properties of the film, is only weakly dependent on the normal resistivity about R_c.[35] Thus, in general,

$$R_\Box \sim R_Q[(R_n/R_c) - 1]^{2\nu}. \tag{25}$$

For the case of Ref. 5, $R_c \sim R_Q$. As R_n is tuned in such an experiment, one executes a typical trajectory A on the phase diagram of Fig. 1. The phase which exists in a real material like gallium and is not captured by a pure model like model (1) is the Bose glass (BG) phase. So, instead of SC-BM-BI scenario along A, one would probably have a SC-BM-BG(-BI) scenario. Either way, in the BM phase, because of formula (25) the resistivity of this metal increases continuously from zero through a wide spectrum of values as R_n is tuned through R_c, as is seen in the experiments.[5,6] We display the low temperature metallic resistivity from Ref. 5 [Fig. 2 of this reference] as a function of R_n in Fig. 2 and observe the remarkable fit to the formula (25) with $\nu \simeq 2$. This clearly points out that the low temperature metallic phase observed in Ref. 5 was, in all probability, due to an incoherent motion of the Cooper pairs. The aforementioned value of the correlation exponent is consistent with the no-

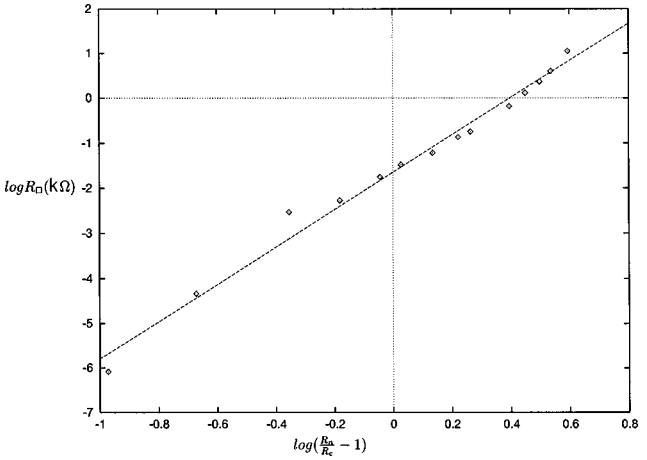

FIG. 2. The resistivity of the metallic phase as a function of R_n in Ga film experiment (Ref. 5) (the diamonds are the data points from Fig. 2 of this reference) and the best fit of Eq. (25) (the dashed curve). (We have not displayed those points of Fig. 2 in Ref. 5 which are in the insulating regime and, expectedly, deviate from the best fit shown here.)

tion that Ga films were highly disordered samples[36] and the Chayes theorem[37] along with it.

V. HOW NEW IS THIS STATE OF MATTER?

The question now arises whether the Bose metal discussed in the previous sections is actually an adiabatic continuation of some known state of matter or if it is completely new. In this section, we argue that this state is actually a mathematical variant of a quantum disordered spin liquid with *large* spin and that it is a liquid of nonfermionic variety.

To show this, we first note that the algebra which controls the quantum mechanics of our problem is E_2, as discussed in Sec. II. Now, E_2 is a group contraction of SO(3),[18] i.e., as the radius R of the sphere on which SO(3) operations are defined tends to infinity, SO(3) contracts to E_2, viz. $J_x/R \rightarrow -P_y, J_y/R \rightarrow P_x, J_z \rightarrow L$, where the operators L and P are as defined in Eq. (5). And, in the same limit, i.e., as the SO(3) angular momentum quantum number $j=pR \rightarrow \infty$, where p refers to the linear momentum quantum number of E_2, the irreducible representations of SO(3) map onto those of E_2. Thus, the model (1) is a variation of a quantum spin model and the Bose metal obtained here, which is a quantum disordered phase, may be thought of as analogous to the frustrated spin liquids obtained within such a model in the limit of spin becoming large.[38]

A more intriguing question is the issue of how consistent the existence of a disordered phase as that of a Bose metal at $T=0$ is with the third law of thermodynamics. To this end, we calculate the low temperature specific heat of the metal under the same approximation as in Sec. IV, i.e., noninteracting uncondensed bosons (vortices) moving in a transverse gauge field. The longitudinal part of the gauge field is gapped because of the screening effects and hence do not contribute significantly to the specific heat.[39] However, the transverse gauge field representing the quantum fluctuations associated with the transverse modes are gapless in this metallic phase[23] and, hence, make a dominant contribution to the specific heat. The calculation for uncondensed bosons interacting with a transverse gauge field[40,33] is analogous to the fermionic case for which an extensive literature exists.[39] Following Reizer,[39] we find at low temperatures (Appendix E)

$$C \sim A_p T^{2/3} \quad \text{(pure)}$$
$$\sim A_d T \ln(T_0/T) \quad \text{(with disorder)}. \tag{26}$$

This anomalous behavior originates sheerly from the fact that the transverse gauge field undergoes dynamical screening, which leads to the presence of diffusive modes $\omega \sim -iq^n$, where $n=2$ (disordered) or 3 (pure). The coefficient A_p scales as $A_p \sim n_f^{2/3}$, A_d as $A_d \sim n_f$, and T_0 as $T_0 \sim 1/n_f$, where n_f refers to the free vortex density (Appendix E). This displays the non-Fermi liquid behavior of the Bose metal phase and verifies that there is no consistency problem with the third law. Clearly, the BM phase has more entropy than a normal fermi liquid at low temperatures.

The foregoing temperature variation of the specific heat shows that there are weak singularities at $T=0$ in the metallic state. Thus, the BM state has the character of a *critical* point as $T \rightarrow 0$. This requires further study since the coefficients which enter the specific heat calculation would undergo strong renormalization and the actual temperature dependence may be somewhat different. These arguments also suggest that there is no good length scale in the quantum liquid phase at $T=0$. This can be seen by calculating the charge charge correlation function. Using Eqs. (A4) [or, equivalently Eqs. (B2)] and (E3), we obtained

$$\langle \delta n_{q,\omega} \delta n_{-q,-\omega} \rangle \approx \frac{q^2}{-i\tilde{a}\frac{\omega}{q} + \tilde{b}q^2}, \tag{27}$$

where the coefficients $\tilde{a}$ and $\tilde{b}$ may be read off from Eq. (E3). Thus, the excitation spectrum of the charge fluctuations is gapless and diffusive. As we pointed out in Sec. II B, because of the algebra of phase fluctuations, this does not contradict the fact that there is a length scale associated with superfluidity viz. the superfluid correlation length ξ_+ tied to the free vortex density n_f. Thus, the BM phase is analogous to a gapless quantum spin liquid, very much like an RVB state.

VI. DISCUSSION AND CONCLUSIONS

In summary, we have argued that the superconductor-insulator transition in low-T_c superconductors should be viewed as a *two* order parameter problem and observed that it is described by two coupled XY models in (2+1)D in the charge picture. This change from a single (2+1)D XY model, as conventionally thought, is tied to the inclusion of an appropriate vortex mass term in the vortex picture or nonlocal interactions in the charge picture. This leads to the interesting possibility of a novel disordered Bose metal phase distinct from the traditional superconductor and Bose insulator phases. On the basis of our analysis of the model, we expect a superconductor-insulator transition when $V_{\text{on-site}} \gg V_{\text{NN}}$ (NN=nearest neighbor), a case close to that of the Josephson junction arrays (with $B_{\text{ext}}=0$), and a superconductor-metal-insulator transition when $V_{\text{on-site}}$

$\sim V_{NN}$, a situation close to that of the granular superconductors [it may be helpful to note that $V_i \sim (2e)^2/C_i, C_i$ = capacitance].[7] The physics of the problem seems to be controlled by a multicritical point. We find the properties of the Bose metal to be critical. It is inherently connected to the quantum spin liquid states and is strongly reminiscient of the gapless RVB phase. The resistivity of this metallic phase predicted by our calculations finds an excellent match with the experiments on gallium films.[5] Dissipation in the bosonic system at low temperatures, within our model, is hydrodynamical: it comes from the fact that a moving charge (vortex) dissipates as it moves through a background of charged particles (vortices and antivortices). The heat is carried away by the gapless transverse modes representing the plasmons. This source of dissipation translates into quantum fluctuations in the charge picture.

To our knowledge, so far there has been no metallic phase proposed in the phase diagram of the Bose localization problem, within the scope of any physically realizable model. This paper is an attempt to propose this concept and establish some of the basic principles which underlie the existence of such a phase. As a result, many of the features which are tied to the current experiments have gone unaddressed. Some of these are existence of disorder, long-range $1/r$ interactions, nonzero magnetic field, finite temperature effects, etc. Also, in a realistic sample, an important source of heat bath in addition to the emission of transverse excitations is the electrons in the vortex core. This needs to be incorporated into the calculation.

An important concern is whether the BM phase will survive in improved calculations, given that our calculations have been mean field like. As we saw, the experiments argue in favor of the existence of this phase. Further, as we argued in Sec. II D, the retardation effects leading to the destruction of vortex superfluidity and the binding of vortex-antivortex pairs, the features corresponding to the loss of charge order and the growth of phase order respectively, are controlled effectively by two separate parameters. Plus, as we found in Sec. III, the constants which determine the curves LXM and ZYX in Fig. 1 differ by about an order of magnitude because of the strong retardation effects felt by the vortices in this model. In view of these facts and the self consistent nature of the calculation, it is reasonable to think that the curve ZYX will lie above the curve LXM and intersect it at a multicritical point even if the fluctuations are taken into account, implying a separate BM phase.[47] Either way, it will be very useful to check this via numerical simulations. An important issue which confronts us at the moment is what is the universality class of the critical phenomena associated with model (1)? Although we have commented on this above, one needs to perform renormalization group calculations or numerical simulations to answer this question completely. What is clear from our calculations, however, is that the critical behavior, both at $T=0$ and $T=$ small but finite, implies a universality class with a multicritical point which is very rich.

Given the considerable amount of work done on the Bose-Hubbard models, one might wonder why this phase was not observed in the other theoretical constructs, particularly in simulations. In view of this, we would like to mention a few points, in addition to the idea of two order parameters, which distinguishes our work from the previous ones. Firstly, we have worked in the large n_0 limit of the problem. Most of the current work on this model does not treat this limit consistently. Also, in this limit, the algebra which determines the quantum mechanics is qualitatively different from that of the hard core bosons, i.e., E_2, and the chemical potential μ does not play a significant role (as found in Sec. II B). In the small n_0 limit, μ plays a dominant role, especially in stabilizing the commensurate Bose insulator (CBI) phase (mostly called Mott insulator in the Bose literature). The weakening effect of μ as n_0 increases is clearly observable in the shrinking of CBI lobes with n_0, evidenced by the perturbative and quantum Monte Carlo calculations of Ref. 41. This provides additional support to the results obtained here in this approximation. Further, we also observed that the nearest neighbor interaction plays a crucial role in opening up the metallic phase in the system. There is no such phase with just on-site repulsion. This piece of physics has support from the RG flows constructed by Fisher and Grinstein.[10]

On the supersolid phase. It is somewhat tempting to use Eq. (19) to the fullest extent and extend the curve ZYX beyond the point X along the dotted line XO. Equation (19) is really not good beyond the point X; but if we take it at face value, it seems to suggest a region OXM where both phase order and charge order exist, i.e., a supersolid phase. This has some support, if we look at the action (10). Here, as we noted earlier, when $V_1 \ll V_0, \theta_i \simeq 0$, implying the presence of charge order in the system. This, however, corresponds to trivial charge ordering only, viz. $\delta n_i \simeq 0$, i.e., $n_i \simeq n_0$. This phase needs to be verified by better calculations. If this phase exists in improved calculations, the point X would be tetracritical.

Before we close, we would like to make a short comment on the Mo-Ge system,[4] a situation where magnetic field is present. As we argued in Sec II D, the physics of quantum phase fluctuations in 2D superconductors is a quantum mechanical generalization of the KT scenario. We expect this to happen when magnetic field is present as well. Let us recollect the classical case for completeness. Here, dislocation-antidislocation pairs are induced in the vortex lattice which unbind as a result of thermal fluctuations.[42] At low temperatures, quantum effects dominate and we expect the unbinding resulting from quantum fluctuations instead. But, as pointed out in this paper, their kinetic energy should receive two contributions — from the zero point motion and the action of a heat bath. The metallic state is realized when the dislocations in the vortex lattice (more precisely, vortex glass) phase move under the action of a heat bath. This is very much along the lines of what had been suggested in Ref. 4. Following our discussion in Sec. V, we suggest specific heat measurements be made to spot any nonfermionic behavior of this metal-like phase. We plan to take up some of the unresolved issues mentioned in this section in a future publication.

ACKNOWLEDGMENTS

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APPENDIX A: THE DUALITY TRANSFORMATION

We start from action (9), but with the transformation $m_i \rightarrow e^{-iQ\cdot r_i} m_i$ undone, so that

$$S = i\sum_i m_i(\nabla_\tau \phi_i) + V_0 \Delta\tau \sum_i m_i^2 + V_1 \Delta\tau \sum_{i\alpha} (m_i + m_{i+\alpha})^2 - J\Delta\tau \sum_{i\alpha} \cos(\nabla_\alpha \phi_i). \tag{A1}$$

We now use Villain transformation on the J term. For small values of $J\Delta\tau$, one has[34]

$$e^{J\Delta\tau\Sigma_{i\alpha}\cos(\nabla_\alpha\phi_i)} \simeq e^{-\ln(2/J\Delta\tau)\Sigma_{i\alpha} n_{i\alpha}^2 + i\Sigma_i \phi_i(\bar{\nabla}_\alpha n_{i\alpha})}.$$

Integrating out the phase degrees of freedom ϕ_i, one obtains

$$Z = \sum_{\{m_i\}} \sum_{\{n_{i\alpha}\}} e^{-S},$$

$$S = \ln\left(\frac{2}{J\Delta\tau}\right) \sum_{i\alpha} n_{i\alpha}^2 + V_0 \Delta\tau \sum_i m_i^2 + V_1 \Delta\tau \sum_{i\alpha} (m_i + m_{i+\alpha})^2, \tag{A2}$$

supplemented with the constraint

$$\bar{\nabla}_\tau m_i + \bar{\nabla}_\alpha n_{i\alpha} = 0 \tag{A3}$$

(with the summation over α implied). Defining $n_i^\mu = (m_i, n_i^\alpha)$, $(\mu = 0,1,2)$, one obtains the constraint (A3) as

$$\bar{\nabla}_\mu n_i^\mu = 0.$$

This allows us to define an integer gauge field A_i^μ, via

$$n_i^\mu = \varepsilon_{\mu\nu\rho} \bar{\nabla}_\nu A_i^\rho. \tag{A4}$$

A purist might object to our not using the shifted lattice operators $A^\rho_{i-\hat{\rho}}$ in Eq. (A4).[13] This does not matter for our case since the model involves a single gauge field. Now, transforming over to the Fourier space $[m_i = (1/N_0)\Sigma_{q,\omega} e^{i\tilde{q}\cdot x_i} m_{q,\omega}$, with $\tilde{q} = (\omega, \vec{q})$ and x_i referring to the ith site on the space-time lattice, with N_0 the total number of lattice sites], one obtains

$$Z = \sum_{\{A_i^\mu\}} e^{-S},$$

with

$$S = \ln\left(\frac{2}{J\Delta\tau}\right) \sum_{q,\omega} [\hat{q}^2 A^0_{q,\omega} A^0_{-q,-\omega} + G^{-1}(\omega, q) A^\alpha_{q,\omega} A^\alpha_{-q,-\omega}] \tag{A5}$$

with $G^{-1}(\omega,q)$ given by Eq. (12) and $A^\mu_{q,\omega}$ refer to the Fourier transform of A_i^μ. Here $\hat{q}$ and $\hat{\omega}$ refer to lattice momentum and frequency respectively, and $\hat{q}^2 = \Sigma_\alpha \bar{K}_\alpha(q) K_\alpha(q)$, with $\bar{K}_\alpha(q) = (1/i)(1 - e^{-iq_\alpha}), K_\alpha(q) = (1/i)(e^{iq_\alpha} - 1)$.[20] And, $(\hat{q} - Q)^2 = \Sigma_\alpha \bar{K}_\alpha(q-Q) K_\alpha(q-Q)$ in Eq. (12). Equation (14) is complete with the gauge condition,

$$\nabla_\alpha A_i^\alpha = 0. \tag{A6}$$

Now, we introduce the vortex variables j_i^μ via the Poisson summation formula[13]

$$Z = \sum_{\{j_i^\mu\}} \int \mathcal{D}A_i^\mu e^{-S - 2\pi i \sum_i j_i^\mu A_i^\mu}. \tag{A7}$$

Integrating out the gauge field degree of freedom, one obtains Eq. (11).

APPENDIX B: CONTINUUM VERSION

The features of the vortices discussed in Sec. II D are not special to the lattice case, but exist in the continuum case as well. This gives us a more general frame in which to study the model. The considerations in the continuum case start from the action[43,24]

$$\tilde{S} = \int d\tau d^2x \left[(m/2\bar{\rho})|\vec{J}|^2 + \frac{1}{2}\int d^2y\, V(x-y)\, \delta\rho(x)\, \delta\rho(y) + 2\pi i \tilde{j}_\mu A_\mu \right] \tag{B1}$$

where $\vec{J}$ refers to the charge current and $\delta\rho$ the charge fluctuations, with the gauge field A_μ given by

$$J^\mu(x) = (\delta\rho, \vec{J}) = \varepsilon_{\mu\nu\rho}\partial_\nu A_\rho. \tag{B2}$$

Also, $\tilde{j}_\mu = (\rho_v, \vec{J}_v)$ refers to the vortex three current density, m = mass of the Cooper pairs, and $\bar{\rho}$ = average Cooper pair density. We have not written down a Magnus force term on vortices due to a background charge condensate, because this is not contained in the lattice model (1) discussed in the paper. However, this may be important in a uniform system like Mo-Ge.[44] Working in the transverse gauge $\vec{\nabla}\cdot\vec{A} = 0$, we obtain after integrating out the gauge field

$$\tilde{S} = 2\pi^2 \frac{\bar{\rho}}{m} \sum_{k,\omega} \tilde{j}^0_{k,\omega} \frac{1}{k^2} \tilde{j}^0_{-k,-\omega} + 2\pi^2 \frac{\bar{\rho}}{m} \sum_{k,\omega} \tilde{j}^\alpha_{k,\omega} \tilde{G}(\omega,k) \tilde{j}^\alpha_{-k,-\omega}, \tag{B3}$$

where the Green's function $\tilde{G}(\omega,k)$ is given by

$$\tilde{G}^{-1}(\omega,k) = \omega^2 + (\bar{\rho}/m) k^2 V_k, \tag{B4}$$

where V_k is the Fourier transform of the potential $V(r)$. From here onwards we consider the screened Coulomb potential with

$$V_k = V_0 \alpha / \sqrt{k^2 + \alpha^2}.$$

In the limit $\omega, k \rightarrow 0$ and $\omega \ll \tilde{c}_s k$ (with $\tilde{c}_s$ being the plasmon velocity), the Green's function splits up into a singular part $\tilde{G}_s(\omega,k)$ and $\tilde{G}_0$ just like in the lattice model

$$\tilde{G}(\omega,k) \simeq \tilde{G}_s(\omega,k) + \tilde{G}_0, \tag{B5}$$

where

$$\tilde{G}_s(\omega,k)=1/(\omega^2+\tilde{c}_s^2k^2),\quad \tilde{G}_0=1/(2\overline{\rho}V_0\alpha^2/m),$$

with

$$\tilde{c}_s^2=\overline{\rho}V_0/m.$$

Thus, from Eqs. (B3) and (B5), we obtain

$$\tilde{S}=2\pi^2\frac{\overline{\rho}}{m}\sum_{k,\omega}\tilde{j}^0_{k,\omega}\frac{1}{k^2}\tilde{j}^0_{-k,-\omega}+2\pi^2\frac{\overline{\rho}}{m}$$

$$\times\sum_{k,\omega}\tilde{j}^\alpha_{k,\omega}[\tilde{G}_s(\omega,k)+\tilde{G}_0]\tilde{j}^\alpha_{-k,-\omega}. \tag{B6}$$

Thus, comparing Eqs. (B6) and (14), we see that the lattice and continuum actions of phase fluctuations, within the scope of our model, have identical low frequency and long wavelength limits.

Correspondence between the lattice version and the continuum version is established via the following identifications:

$$m/\overline{\rho}\leftrightarrow 2\ln(2/J\Delta\tau), \tag{B7}$$

$$\tilde{c}_s^2=\overline{\rho}V_0/m\leftrightarrow c_s^2=(V_0+8V_1)\Delta\tau/ln(2/J\Delta\tau), \tag{B8}$$

$$\tilde{G}_0=1/(2\overline{\rho}V_0\alpha^2/m)\leftrightarrow G_0=b^2/8(b^2+c^2)^2. \tag{B9}$$

Following Ref. 43, one can easily show that Eq. (B3) is equivalent to the following Hamiltonian for the vortices:

$$\tilde{H}_v=\frac{1}{2m_v}\sum_i(\vec{p}_i+2\pi q_i\vec{A}(x_i))^2+2\pi^2\frac{\overline{\rho}}{m}\sum_{i\neq j}q_iq_j\ln|r_i-r_j|, \tag{B10}$$

where the summation is over all vortices and antivortices and the gauge field $\vec{A}$ has the spectrum determined by $\tilde{G}_s^{-1}(\omega,k)$ as in Eq. (15). Here $q_i=\pm1$ refers to the charges on the vortices. Here m_v refers to the vortex mass.

APPENDIX C: AN ESTIMATE OF $G(0)$

From Sec. III A, we have

$$G(0)=\frac{1}{2}\int_{-\pi}^{\pi}\frac{d\omega}{2\pi}\int_{-\pi}^{\pi}\frac{d^2q}{(2\pi)^2}\frac{1}{(1-\cos\omega)+A^2}, \tag{C1}$$

where

$$A^2=[2-(\cos q_x+\cos q_y)][\kappa_0^2+2\kappa_1^2\{2+\cos q_x+\cos q_y\}], \tag{C2}$$

with

$$\kappa_0^2=c^2\quad\text{and}\quad\kappa_1^2=b^2/8,$$

where b^2 and c^2 have been introduced in Sec. II D.

From Eq. (C1), we obtain

$$G(0)=\frac{1}{2}\int_{-\pi}^{\pi}\frac{d^2q}{(2\pi)^2}F(q), \tag{C3}$$

with

$$F(q)=\frac{1}{\sqrt{A^2(2+A^2)}}. \tag{C4}$$

Important contributions to Eq. (C3) come from the low momentum region. Thus,

$$F(q)\simeq 1/\sqrt{2(\kappa_0^2+8\kappa_1^2)}\sqrt{2-(\cos q_x+\cos q_y)}. \tag{C5}$$

Combining Eqs. (C3) and (C5), we are led to the condition (17), with

$$b_0=\int_0^{\pi/2}\frac{d^2x}{\sqrt{\sin^2x+\sin^2y}}. \tag{C6}$$

APPENDIX D: SELF-CONSISTENT FUNCTIONAL APPROACH

In this appendix, we give a brief review of the self-consistent functional approach[45] discussed by Ioffe *et al.*[23] This is essentially a hydrodynamical kind of approach. We start with the following electromagnetic Lagrangian of bosons in imaginary time (in $2-d$):

$$L_B=L_B^0+\tilde{L}_B+L_B^a. \tag{D1}$$

Here

$$L_B^0=\psi^*\frac{\partial\psi}{\partial\tau}+\frac{1}{2m}|\vec{\nabla}\psi|^2+\frac{g^2}{2}\rho(r)\ln|r-r'|\rho(r')-\mu|\psi|^2, \tag{D2a}$$

$$\tilde{L}_B=-\frac{i}{2m}\vec{a}\cdot[\psi\vec{\nabla}\psi^*-\psi^*\vec{\nabla}\psi]+\frac{1}{2m}\vec{a}^2|\psi|^2, \tag{D2b}$$

$$L_B^a=\frac{1}{2g^2}[(\partial_\tau\vec{a})^2+c^2(\vec{\nabla}\times\vec{a})^2], \tag{D2c}$$

where ψ,ψ^* refer to the Bose fields and $\rho=\psi^*\psi$. We work in the transverse gauge $\vec{\nabla}\cdot\vec{a}=0$. The dimensionless coupling constants determining the strength of the Coulomb repulsion and transverse gauge field, viz. α_c and α_g, respectively, are defined as

$$\alpha_c=\frac{g^2m}{16\pi^2n},\quad \alpha_g=\frac{g^2}{8\pi mc^2}, \tag{D3}$$

where n is the average density of bosons. Comparison of Eq. (D2) with Eq. (15) allows us to identify

$$g^2\leftrightarrow 2\pi^2/\ln(2/J\Delta\tau),\quad c^2\leftrightarrow c_s^2,$$

and

$$m/2n\leftrightarrow\pi^2G_0/\ln(2/J\Delta\tau). \tag{D4}$$

The last identification also follows from the fact that the weight of the zero point motion term $|\vec{j}|^2$ is $m/2n$. Using Eqs. (D3) and (D4), we are led to Eqs. (18).[30]

The calculation of the depletion of the Bose condensate from the action (D2) proceeds in two steps. The first involves determining the effective functional of the superfluid Bose system in the absence of a transverse gauge field. And, the

second involves determining the effect of the gauge field on this system, i.e., calculating the change in the superfluid density.

The effective functional of the superfluid Bose system (with $\vec{a}=0$) is obtained by expanding the Bose field in terms of the slow and fast modes, and integrating out the fast modes.[45] Galilean invariance of this system dictates $n_s=n$.[26] The leading terms in the effective action are (for small values of α)

$$S_0=\int d^2x\,d\tau\left[\frac{n}{2m}|\vec{\nabla}\phi|^2-i\pi\partial_\tau\phi+\frac{1}{8mn}|\vec{\nabla}\pi|^2\right]+\frac{g^2}{2}\sum_{q,\omega}\pi_{q,\omega}\frac{1}{q^2}\pi_{-q,-\omega}, \tag{D5}$$

where ϕ and $\rho(x,\tau)=n+\pi(x,\tau)$ are the phase and the amplitude of the slow mode $\psi_0=\sqrt{\rho(x,\tau)}e^{i\phi(x,\tau)}$. We now work in the real time and integrate out the phase variables. This leads to an action solely in terms of the density fluctuation variables π

$$\exp\left((i/2)\int d^3x\,d^3x'\,\pi(x)C^{-1}(x-x')\,\pi(x')\right),$$

where

$$C(\omega,q)=\frac{nq^2/m}{\omega^2-(q^2/2m)^2-ng^2/m+i\delta} \tag{D6}$$

and $d^3x=d^2x\,dt$.

To calculate the change of the superfluid density $n_s=n$ due to the action of the transverse gauge field, we focus on the diamagnetic term in Eq. (D2b)

$$-\frac{1}{2m}\vec{a}^2\psi_0^*\psi_0=-\frac{1}{2m}\vec{a}^2(n+\pi). \tag{D7}$$

The weight of the $\vec{a}^2$ term determines the superfluid density. One now integrates out the π-degrees of freedom. The coupling term in Eq. (D7) leads to an $\vec{a}^2-\vec{a}^2$ interaction term. One then (a) decouples this term in a mean field approximation and (b) rewrites the gauge field in terms of fast and slow modes, a_1 and a_0, and integrates out the fast modes. The weight of the a_0^2 term then gives the self-consistent equation for the superfluid density n_s,

$$n_s=n+\frac{i}{m}\int d^3x\,C(x)D(x), \tag{D8}$$

where $D(x)$ is the gauge field correlator

$$D(\omega,q)=\frac{g^2}{\omega^2-(c^2q^2+n_sg^2/m)+i\delta}. \tag{D9}$$

Equation (D8) leads to the phase diagram in $\alpha-\alpha_g$ plane, mentioned in Sec. III B.

APPENDIX E: SPECIFIC HEAT CALCULATION

To calculate the specific heat, we consider the Lagrangian (D2) without the log interaction, which is screened in the metallic phase. Connection with the original model is established via the mapping (D4).[30] We first rescale the transverse gauge field $\vec{a}\to g\vec{a}$ so that the effective charge g appears in the Eq. (D2b) rather than Eq. (D2c). Thus, the unperturbed gauge propagator is

$$D^0_{\alpha\beta}(q,\omega_n)=K_{\alpha\beta}(q)\frac{1}{\omega_n^2+c^2q^2}, \tag{E1}$$

where $\omega_n=2\pi nT(n=\text{integer})$ are Matsubara frequencies and

$$K_{\alpha\beta}(q)=\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}.$$

To obtain the contribution of specific heat from the transverse modes, one integrates out the particle degrees of freedom in a one-loop approximation.[39] This leads to the retarded polarization function

$$\Pi^{(R)}_{\alpha\beta}(q,\Omega)=-K_{\alpha\beta}(q)g^2\left[-i\frac{2n}{m}\frac{\Omega}{v_Bq}+c^2\chi_Dq^2\right], \tag{E2}$$

where $\chi_D=n_B(-\mu)/24\pi mc^2$ and $mv_B^2/2=|\mu_B|$, with $n_B(\xi)=1/(e^{\beta\xi}-1)$. The structure of Eq. (E2) is quite independent of the statistics because the integrals which enter the calculation are of the form $\int_{-\mu}^{\infty}d\xi n(\xi)$ and $\int_{-\mu}^{\infty}d\xi(\partial n/\partial\xi)$. The renormalized gauge propagator is then given by Dyson's equation

$$D=D_0+D_0\Pi D.$$

In the quasistatic approximation $\Omega\ll cq$, we have the retarded gauge propagator as

$$D^{(R)}_{\alpha\beta}(q,\Omega)=K_{\alpha\beta}(q)\left[1\Big/\left(-i\frac{2ng^2}{m}\frac{\Omega}{v_Bq}+\overline{c^2}q^2\right)\right], \tag{E3}$$

where $\overline{c^2}=c^2(1+g^2\chi_D)$. Integrating out the gauge fields leads to the free energy

$$\beta F=-\frac{1}{2}\mathrm{Tr}\ln D(q,\omega_n). \tag{E4}$$

The specific heat is obtained from $C=T\partial S/\partial T$. The details of the calculation may be found in Ref. 39. For the disordered case, the Green's function is written as $G(p,\omega_n)=1/[i\omega_n-\xi_p+i/2\tau\,\mathrm{sgn}(\omega_n)]$, with τ the scattering time for the vortices from the impurities (which appear as a static gauge field in the vortex picture).[24] In this case, the factor $1/v_Bq$ appearing in the frequency term in Eqs. (E2) and (E3) gets replaced by 2τ. The coefficients A_p, A_d and T_0 in Eq. (26) are approximately given by

$$A_p\approx(2ng^2/mv_B\overline{c^2})^{2/3}/\pi,$$

$$A_d\approx ng^2\tau/\pi^2m\overline{c^2},$$

$$T_0\approx\overline{q_c}^2m\overline{c^2}/4ng^2\tau, \tag{E5}$$

where $\overline{q_c}$ denotes an upper momentum cutoff $\sim 1/\xi_0$, with ξ_0 the Cooper pair size.

APPENDIX F: COMMUTATION RULE (6c)

In this appendix, we discuss how the commutation rule (6c) comes about. From commutators (6a) and (6b), we observe that P and $P^\dagger$ are ladder operators. Hence, in the angular momentum representation ($L|\Omega_m\rangle = m|\Omega_m\rangle$),[17,18]

$$P = i \sum_{-\{n_0\}+1}^{\infty} |\Omega_{m-1}\rangle\langle\Omega_m| \approx i \sum_{-\infty}^{\infty} |\Omega_{m-1}\rangle\langle\Omega_m|, \quad \text{(F1)}$$

$$P^\dagger = -i \sum_{-\{n_0\}}^{\infty} |\Omega_{m+1}\rangle\langle\Omega_m| \approx -i \sum_{-\infty}^{\infty} |\Omega_{m+1}\rangle\langle\Omega_m|, \quad \text{(F2)}$$

where $\{n_0\} = n_0$ for integer n_0, and the floor or ceiling of n_0 (appropriately taken) otherwise. The approximate expressions written on the right hold only when $n_0 \gg 1$.

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