PHYSICAL REVIEW B VOLUME 60, NUMBER 15 15 OCTOBER 1999-I

Theory of phonon spectroscopy in the fractional quantum Hall regime

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(Received 24 March 1999; revised manuscript received 16 July 1999)

We describe a theoretical framework for the interpretation of time-resolved phonon absorption experiments carried out in the fractional quantum Hall regime of a magnetically quantized two-dimensional electron system. The only phonons which can be absorbed at low temperature are those whose energies exceed the magnetoroton gap predicted by Girvin, MacDonald and Platzman [Phys. Rev. B **33**, 2481 (1986)]. The rate of energy transfer from the phonons to the electron liquid is entirely controlled by the creation of these collective excitations. Using simple isotropic approximations for the phonon propagation and electron-phonon coupling, we obtain analytic results for the regime in which the electron temperature and the characteristic temperature of the phonons are much less than the gap, and identify the way in which the dispersion curve of the magnetorotons could be extracted from time- and angle-resolved experiments. [S0163-1829(99)09839-2]

I. INTRODUCTION

Most of what we know experimentally about two-dimensional electron systems has been gleaned from the analysis of dc transport measurements. Typically, a constant electric current is passed though a two-dimensional (2D) system, while the longitudinal and Hall voltages are monitored as the temperature, magnetic field, or density are varied. Recently there has been a move toward the development of complementary techniques for acquiring information about the states of electronic matter that arise when these 2D systems are subject to strong magnetic fields. One such technique is phonon absorption spectroscopy.[1–3] A pulse of nonequilibrium acoustic phonons is injected into the substrate of a semiconductor sample which has a 2D electron system formed at a heterojunction. The phonons travel ballistically across the substrate until they encounter a sheet of electrons where a small number of them are absorbed. Acoustic phonons are particularly well suited to the study of 2D electron systems (2DES's) since they have energies and wave vectors that are well matched to the low-energy collective excitations of these systems. Measurements of the rate at which energy is absorbed by the electrons therefore contain important spectroscopic information about the states of the 2D electron system.

The aim of this paper is to develop a framework for the interpretation of phonon absorption experiments carried out in the fractional quantum Hall regime. These experiments are now at a sufficiently sophisticated stage that both time[3] and, more recently, angle-resolved measurements[4] can be made. This paper will focus on our basic picture of what is happening in the experiments. The calculations presented will be entirely analytic, based on asymptotic analyses of general formulas making a wealth of simplifying approximations to expose the qualitative features to be expected. A more detailed numerical analysis which avoids some of the cruder approximations is currently in progress, and will be published as a sequel to this work.

The structure of this paper is as follows. In Sec. II an outline of the procedure employed in the phonon absorption experiments will be given. In Sec. III, the pertinent features of electrons and phonons in semiconductor materials will be briefly reviewed, while Sec. IV will review the standard picture of the fractional quantum Hall state. We hope that this paper will be of interest to both the community which studies the fractional quantum Hall effect and the community which studies phonons: Sec. III and IV are intended to introduce the relevant ideas and to fix our notation. Section V will discuss the phonon transport aspects of the experiments. In Sec. VI the basic formulas for the rate of energy transfer from phonons to the electron liquid in the fractional quantum Hall state will be introduced and the kinematics of the absorption discussed. In Sec. VII asymptotic expressions for the energy-transfer rate will be derived employing an isotropic approximation for the phonon properties. In Sec. VIII the time-dependent aspects of the experiments are discussed. In Sec. IX the processes leading to equilibrium within the 2DES are discussed qualitatively (a more quantitative discussion of some aspects of this matter will appear elsewhere). Section X contains a summary of our picture of what happens in the phonon absorption experiments, and what information could be extracted from them. Various technical details of the calculations are relegated to appendixes.

Throughout this paper we will be discussing phonons characterized by three-dimensional wave vectors and collective excitations of the electronic system characterized by two-dimensional wave vectors. In order to distinguish these we will use underlined characters $\underline{\mathbf{Q}}$ to represent 3D vectors, and bold characters $\mathbf{q}$ to represent 2D vectors. Where a 2D vector arises as the projection of a 3D vector onto the xy plane (see the inset to Fig. 1), this will be denoted by $\underline{\mathbf{Q}} = (\mathbf{q}, q_z)$.

II. EXPERIMENTAL DETAILS

An ideal phonon spectroscopy experiment would employ a monochromatic source of phonons on one side of a device, and a tunable phonon detector on the opposite side with the 2D electron system lying between them. The fraction of phonons absorbed from each mode could then be measured. In practice there are no sources of monochromatic phonons which operate in high magnetic fields and, similarly, no

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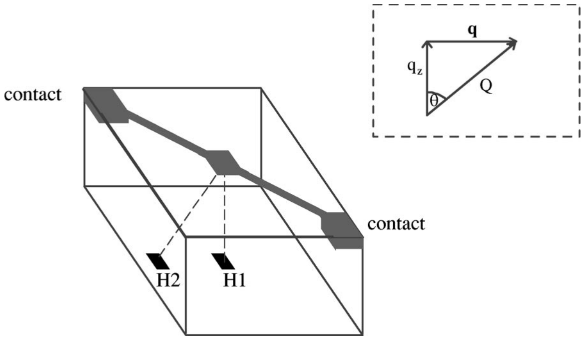

FIG. 1. Schematic view of the devices used in phonon spectroscopy experiments showing the 2DES connected by long lines to 3D contact pads and two heaters, $H1$ and $H2$ mounted on the back face. The inset shows the 2D and 3D wave-vector conventions used.

spectrally sensitive phonon detectors. The best that one can hope to do at present is to inject a pulse of phonons with a distribution of modes and detect it on the far side of the 2DES with a bolometer. In fact, even this has yet to be done. What is actually done at present involves the use of the 2DES itself as a bolometer.

A schematic picture of the samples used in the experiment is shown in Fig. 1. The basic device consists of a GaAs/Al$_x$Ga$_{1-x}$As heterojunction grown on top of a thick ($\sim$2 mm) substrate of semi-insulating GaAs. The active area of the 2DES is defined lithographically together with long connections to heavily doped (3D) contacts. The active area is patterned into a long meander line of 2DES so that electrically it is a long, thin, Hall bar with an aspect ratio of $\sim$30:1. This patterning increases the sensitivity of the device as a bolometer by increasing the longitudinal resistance. To compensate for this, exceptional care is taken to reduce the capacitance of the sample and the leads with respect to the ground so that the time constant of the measuring circuit is not too large.[3] Small area metallic films are evaporated onto the back face of the device and separate pairs of contacts made to each of these to allow them to act as heaters. The application of a current pulse to a heater leads to Joule heating within the film. Phonons are emitted from the film into the GaAs with a blackbody distribution of energies characterized by a pulse temperature T_ϕ, which can be accurately predicted from the power dissipated in the film using acoustic mismatch theory.[5]

Acoustic phonons in a semiconductor material such as GaAs travel ballistically in the bulk of the material for frequencies up to $\sim$1 THz (this limit is set by the onset of Rayleigh scattering from isotopic impurities) which is much higher than the frequencies of interest here. The constant frequency surfaces in Q space are significantly nonspherical, so that the phonon group and phase velocities are markedly different except along high-symmetry directions. In fact, the curvature of the constant frequency surfaces changes sign so that phonon wave packets with a range of Q vectors tend to be focused into preferred directions.[6] Another consequence of the elastic anisotropy of the semiconductors is that phonon modes are only strictly transverse or longitudinal along high-symmetry directions.

Throughout the experiment a constant current is passed through the 2D electron system and the two-terminal voltage monitored. The experiments are conducted at the center of the quantum Hall plateau at $\nu = 1/m$. At zero temperature the two-terminal resistance would be identical to the quantized Hall resistance $R_2 = 1/\overline{\sigma_H}$, $\overline{\sigma_H} = e^2/mh$. At a finite temperature the resistance is

$$R_2(T_e) \sim \frac{\overline{\sigma_H} + n\,\delta\sigma_{xx}(T_e) + \delta\sigma_{xy}(T_e)}{[\overline{\sigma_H} + n\,\delta\sigma_{xx}(T_e)]^2 + [\delta\sigma_{xy}(T_e)]^2} \sim \frac{1}{\overline{\sigma_H}}\left(1 + \frac{n\,\delta\sigma_{xx}(T_e)}{\overline{\sigma_H}} - \frac{\delta\sigma_{xy}(T_e)}{\overline{\sigma_H}}\right),$$

where $\delta\sigma_{\mu\nu}(T_e)$ is the finite-temperature correction to quantized conductance tensor for a square 2DES, and n is the number of squares in series in the device. One expects that at the plateau center $\delta\sigma_{xy}(T_e)$ will be small and hence, given that $n \sim 30$, the change in the two-terminal resistance is dominated by $\delta\sigma_{xx}(T_e)$.

Before the application of the pulse to the heater, the voltage across the 2DES is constant and has the value characteristic of the base temperature T_0 of the system. When the current is applied to the heater, a pulse of phonons is injected into the substrate which travels ballistically across until it hits the 2DES, only when this occurs does the voltage change. The substrate is sufficiently thick that the component of the phonon pulse which contains the fast LA modes spatiotemporally separates from the slower component containing the TA modes. Some of the phonons from the pulse(s) are absorbed by the electron sheet leading to a change in the two-terminal resistance which is recorded. Once the phonons have passed the 2DES they hit the top face of the device, where they undergo a variety of processes including mode conversion, diffuse scattering, and reflection. The top face of the device is usually very close to the 2DES, so that this occurs virtually simultaneously with the encounter with the 2DES. The reflected pulse travels back towards the rear face of the sample where it again undergoes a variety of scattering/reflection processes. What is left of the ballistic component of the pulse, having suffered two boundary reflections, heads back towards the 2DES. A clear response to the twice-reflected phonons can be seen in the experiments. This is an experimental verification of the fact that the phonons of importance in these experiments really do propagate ballistically. At longer times the ballistic component is essentially gone and the time evolution of the two-terminal voltage is simply due to the general heating of the lattice by the energy dissipated in the heater; at very long times the temperature control of the refrigeration system extracts this energy, returning the system to its base temperature. The contacts that feed current to the 2DES are deliberately placed far from it so that the phonon pulses do not hit the contacts until long after they have passed the 2DES, since the contacts will have a large response to the phonons.

III. ELECTRONS AND PHONONS IN HETEROSTRUCTURES

The electrons which accumulate at a GaAs/Al$_x$Ga$_{1-x}$As heterojunction at low temperatures form a two-dimensional electron system.[7] Each electron has the same wave function for motion in the direction perpendicular to the junction (z

direction) which we will denote by $f(z)$. The acoustic phonons in GaAs are well described by the anisotropic Debye approximation in which each phonon mode is labeled by a branch index $j=1$, 2, and 3 (roughly speaking 1 LA mode and 2 TA modes) and a 3D wave vector $\underline{Q}$. A given mode has frequency

$$\omega_j(\underline{Q})=c_j(\underline{Q}/Q)Q \tag{1}$$

and polarization $\xi^j(\underline{Q}/Q)$, where $Q=|\underline{Q}|$, and $c_j(\hat{\underline{n}})$ is a direction-dependent speed of sound. In this paper we will neglect focussing effects and use the isotropic approximation that replaces $c_j(\underline{Q}/Q)$ by a constant. This oversimplification will be removed in subsequent numerical works. The main quantitative errors introduced by this approximation are expected to be in the relative magnitudes of the features associated with TA and LA phonons and their detailed dependance on phonon angle of incidence.

There are two mechanisms by which electrons and phonons interact in GaAs.[8] Deformation-potential coupling arises because the presence of a long-wavelength lattice distortion locally alters the band parameters, particularly the energy of the conduction-band edge. This is described by a phenomenological expression for the potential energy experienced by an electron,

$$V_{dp}(\underline{R})=\Xi_{\mu\nu}S_{\mu\nu}(\underline{R}), \quad \mu,\nu=x,y,z, \tag{2}$$

where $\Xi_{\mu\nu}$ is the deformation potential tensor and $S_{\mu\nu}(\underline{R})$ is the lattice strain tensor. Another coupling mechanism arises from the fact that, in materials whose lattice structure does not possess a center of symmetry, elastic distortions give rise to local charge rearrangements which are characterized by an electric polarization field

$$P_\mu(\underline{R})=\frac{1}{\kappa}h_{\mu\nu\lambda}S_{\nu\lambda}(\underline{R}), \tag{3}$$

where κ is the relative permittivity of the material and $h_{\mu\nu\lambda}$ is the piezomodulus tensor. This polarization field also gives rise to a one-electron potential. In GaAs the deformation potential tensor is actually a simple scalar,

$$\Xi_{\mu\nu}=\Xi_0\delta_{\mu\nu}, \tag{4}$$

with $\Xi_0\sim7$ eV, while the piezomodulus tensor has the form

$$h_{\mu\nu\lambda}=\begin{cases} h_{14} & \mu\neq\nu\neq\lambda \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

with $h_{14}\sim0.14\,\mathrm{C\,m}$.

In second quantized form, the combination of these mechanisms gives rise to an electron-phonon coupling Hamiltonian of the form

$$H=-\sum_{j,\underline{Q}} M_j(\underline{Q})Z(q_z)\rho_{-\mathbf{q}}[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})], \tag{6}$$

where $\hat{a}_j^\dagger(\underline{Q})$ creates a phonon in the given mode, $\hat{\rho}_\mathbf{q}$ is the Fourier component of the (2D) electron density, $M_j(\underline{Q})$ is the matrix element for phonon absorption and emission and $Z(q_z)$ is a form factor for the finite thickness of the 2DES. Details of the derivation of this form are presented in Appendix A along with the detailed expressions for $M_j(\underline{Q})$. The simplified forms for the electron phonon couplings that emerge from the isotropic Debye approximation are discussed in Appendix B.

IV. FRACTIONAL QUANTUM HALL LIQUID

The fractional quantum Hall effect was the first experimentally observed manifestation of the incompressible quantum liquid state that occurs in 2D electron systems subject to a strong magnetic field B, such that the Landau level filling factor $\nu=2\pi l_c^2\overline{\rho}$ ($\overline{\rho}$ is the area density of electrons, and $l_c=\sqrt{\hbar/eB}$ is the magnetic length) is close to a low-order, odd-denominator fraction.[9–12] Laughlin[13] proposed a trial wave function for the ground state of this liquid at filling factors of the form $\nu=1/(2m+1)$, which clearly gave these fractions an especially low energy. Laughlin also described the charged quasiparticle excitations above this ground state as fractionally charged objects that obey fractional statistics. These quasiparticles require a finite energy for their creation, even in the thermodynamic limit, which accounts for the incompressibility of the state. The energy gap for the creation of these objects, Δ_{tr}, determines the longitudinal conductivity of the system, which has an activated temperature dependence $\sigma_{xx}\sim\sigma_0 e^{-\Delta_{\mathrm{tr}}/T}$ in ideal systems. In real systems this form is not observed at the lowest temperatures because of the Anderson localization of the quasiparticles. Instead, conduction is supposed to occur via variable range hopping.

Girvin, MacDonald and Platzman[14] turned their attention to the possibility of low-energy collective excitations of the Laughlin liquid. They employed a modification of the technique used by Feynman[15] to describe the collective modes of superfluid helium-4. They proposed neutral excited states characterized by a 2D wave vector $\mathbf{q}$ of the form

$$|\mathbf{q}\rangle=\mathcal{N}_\mathbf{q}^{-1/2}\overline{\rho_\mathbf{q}}|\Psi_L\rangle, \tag{7}$$

where $|\Psi_L\rangle$ is Laughlin's ground state and $\mathcal{N}_\mathbf{q}^{-1/2}$ is a normalization factor. The operator $\overline{\rho_\mathbf{q}}$ is the Fourier component of the electronic density, projected onto the lowest Landau level (see Ref. 16 for details of this projection). The expectation value of the energy in this state was expressed exactly in terms of the lowest-Landau-level-projected static structure factor $\overline{s(\mathbf{q})}$. This quantity has an exact relation to the ordinary static structure factor which is known from Monte Carlo hypernetted chain calculations on the Laughlin wave function. The dispersion curve that arose from that analysis[14] is presented schematically in Fig. 2. This dispersion $\Delta(q)$ has a number of important features. The collective mode has a gap at all wave vectors; unlike superfluid helium, there is no linearly dispersing, gapless phonon regime. This is a manifestation of the incompressibility of the state. The gap is a minimum, Δ^*, at a finite wave vector q^*, which corresponds approximately to a wavelength equal to the average interparticle spacing. By analogy with superfluid helium, the excitations close to q^* are referred to a magnetorotons.

The supposition that the trial states $|\mathbf{q}\rangle$ are true eigenstates is equivalent to using the single mode approximation for the dynamic structure factor[17]

$$S(q,\omega)=\overline{s}(q)\delta[\omega-\Delta(q)], \quad \omega\ll\omega_c, \tag{8}$$

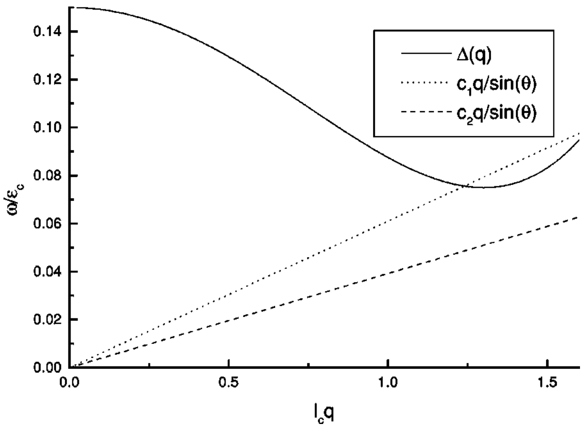

FIG. 2. Schematic graph illustrating the dispersion relation for magnetorotons with the dispersion curves for $\theta = 30°$ phonons.

where $\omega_c = eB/m^*$ is the cyclotron frequency which determines the Landau-level spacing.

At large q the collective mode is more properly thought of as an exciton composed of a pair of oppositely charged Laughlin quasiparticles separated by a distance $l_c^2 q/\nu$.[14] The energy of the exciton will be essentially the mutual Coulomb energy of the charged constituents

$$\Delta(q) \sim \Delta_\infty - \frac{\nu^3 e^2}{4\pi\varepsilon_0 \kappa l_c^2 q}, \quad l_c q \to \infty, \tag{9}$$

where Δ_∞ is the energy needed to create an infinitely separated pair of quasiparticles, i.e., $\Delta_\infty = \Delta_{\text{tr}}$.

Since, as we shall argue, the interaction between bulk acoustic phonons and the Laughlin liquid is dominated by the creation of magnetorotons with wave vectors close to q^*, we employ the standard roton form for the energy in the vicinity of the minimum:[18]

$$\Delta(q) \sim \Delta^* + \frac{(q - q^*)^2}{2\mu}, \quad q \to q^*, \tag{10}$$

$$\mu^{-1} = \left(\frac{d^2\Delta(q)}{dq^2} \right)_{q = q^*}. \tag{11}$$

From now on we will use units which are most natural for the system in question by taking $\hbar = l_c = \mathcal{E}_c = 1$ where $\mathcal{E}_c$ is the Coulomb energy scale for the system which in *Systeme International* units has the form

$$\mathcal{E}_c = \frac{e^2}{4\pi\varepsilon_0 \kappa l_c}. \tag{12}$$

The unit of velocity is then $v_c = l_c \mathcal{E}_c/\hbar$ which is conveniently independent of magnetic field and density. In these reduced units the average speeds of sound are $c_1 \approx 0.0305$ and $c_2 = c_3 \approx 0.0196$, while the magnetoroton minimum for $\nu = 1/3$ is (in the absence of any finite thickness, disorder, and Landau-level mixing effects) $\Delta^* \approx 0.075$, and $q^* \approx 1.3$. In a real system the expectation is that Δ^* will be reduced by all these effects.

V. PHONON TRANSPORT

We can treat the propagation of a phonon pulse across a device semiclassically. A formal solution of the phonon Boltzman equation is not difficult, but the absence of elastic scattering at the frequencies of interest renders even this level of sophistication unnecessary. We assume that the heater emits phonons isotropically in $\underline{Q}$-space. This assumption is known to be incorrect in detail,[5] but will not lead to qualitative errors here. In fact the most obvious effect of the anisotropic emission is the suppression of phonon emission at large angles to the normal to the interface between heater and substrate, these phonons would ‘‘miss’’ the 2DES system anyway (in a more careful treatment they would miss by an even greater margin because of phonon focusing). The anisotropy in the emissivity for LA phonons, $e_1(\theta,\phi)$, is generally weak. The anisotropy in the emissivity for the TA modes, $e_2(\theta,\phi)$ and $e_3(\theta,\phi)$, is stronger, but the total TA emissivity $e_2 + e_3$ is only weakly anisotropic so that, at least within the isotropic Debye approximation used here, we do no real harm by neglecting it.

We describe the phonons in terms of wave packets whose spatial scale determines a normalization volume V. When the phonons are created at the heater they are characterized by a distribution function

$$n_j^{\text{heater}}(\underline{Q}) = n_B[\omega_j(\underline{Q})/T_\phi]\vartheta(q_z), \tag{13}$$

where n_B is the usual Bose-Einstein distribution function, and the step function simply accounts for the fact that only phonons propagating into the sample are created (the sample is mounted in a vacuum). The determination of which wave packets have reached the location of the 2DES at time t is a geometric problem. In principle the distribution of phonons in contact with the 2DES is given by

$$n_j(\underline{Q};t) = \frac{n_B(\omega_j(\underline{Q})/T_\phi)}{|v_z^j(\underline{Q})|} \vartheta\left(t - \frac{L}{v_z^j(\underline{Q})} \right) \vartheta\left(t_p + \frac{L}{v_z^j(\underline{Q})} - t \right) \times \int d^2\mathbf{r}\, \chi_2(\mathbf{r}) \chi_h\left(\mathbf{r} - \frac{\mathbf{v}^j(\underline{Q})}{v_z^j(\underline{Q})} L \right), \tag{14}$$

where L is the thickness of the substrate, the group velocity associated with mode j, $\underline{Q}$ is

$$\underline{V}^j(\underline{Q}) = [\mathbf{v}^j(\underline{Q}), v_z^j(\underline{Q})],$$

$$V_\mu^j(\underline{Q}) = \frac{\partial \omega_j(\underline{Q})}{\partial Q_\mu},$$

t_p is the duration of the pulse, and χ_2 and χ_h are characteristic functions for the 2DES and the heater [i.e., $\chi_2(\mathbf{r})$ is unity if $\mathbf{r}$ is within the area of the 2DES and zero otherwise, etc.]. The two step functions ensure that the front of the pulse has reached the far side of the device but the back of the pulse has not, while the integral ensures that the group velocity points from somewhere on the heater to somewhere on the 2DES; the energy distribution is still that of a blackbody because no inelastic processes have occurred. In general $n_j(\underline{Q})$ is a complicated object to calculate because the group velocity is a nontrivial function of the wave vector. In the isotropic approximation used here we have

$$V_\mu^j(\underline{Q}) = c_j \frac{Q_\mu}{Q},$$

so that

$$n_j(\underline{Q};t) = n_B(c_j Q/T_\phi)\,\frac{Q}{|c_j q_z|}\,\vartheta\left(t - \frac{QL}{c_j q_z}\right)\vartheta\left(t_p + \frac{QL}{c_j q_z} - t\right) \times \int d^2\mathbf{r}\,\chi_2(\mathbf{r})\,\chi_h\left(\mathbf{r} - \frac{\mathbf{q}}{q_z}L\right) = n_B(c_j Q/T_\phi)\,\chi_j(\mathbf{q},q_z;t), \tag{15}$$

which defines the characteristic function $\chi_j(\mathbf{q},q_z;t)$ for the phonon pulse. In practice, what will matter for the purposes of this paper are the maximum and minimum angles of incidence $\theta = \tan^{-1}(|\mathbf{q}|/q_z)$, for which χ_j is nonzero at any instant.

VI. PHONON ABSORPTION IN THE FRACTIONAL QUANTUM HALL REGIME

Let us now suppose that at some instant the phonons in contact with the 2DES are characterized by a nonequilibrium distribution function $n_j(\underline{Q})$ as described above, and that the 2DES is in internal equilibrium at some temperature $T_e \ll T_\phi$. In Appendix C it is shown that the rate at which energy is transferred from the phonon pulse to the incompressible liquid is given by the golden rule expression

$$P = 2\pi\bar{\rho}\Omega \sum_{j,\underline{Q}} \omega_j(\underline{Q})|M_j(\underline{Q})|^2|Z(q_z)|^2 S[\mathbf{q},\omega_j(\underline{Q})] \times \{n_j(\underline{Q}) - e^{-\omega_j(\underline{Q})/T_e}[1+n_j(\underline{Q})]\}, \tag{16}$$

where $M_j(\underline{Q})$ is the electron-phonon matrix element discussed above, and $S(\mathbf{q},\omega)$ is the dynamic structure factor of the electron liquid. In Appendix D it is shown that in the regime $T_e \ll \Delta^*$, the leading asymptotics of P can be obtained by using the zero-temperature form for the structure factor for which we will use the single-mode approximation.[14] Hence we find

$$P = 2\pi\bar{\rho}\Omega \sum_{j,\underline{Q}} \omega_j(\underline{Q})|M_j(\underline{Q})|^2|Z(q_z)|^2\overline{s}(\mathbf{q}) \times \{n_j(\underline{Q}) - e^{-\omega_j(\underline{Q})/T_e}[1+n_j(\underline{Q})]\}\,\delta[\omega_j(\underline{Q}) - \Delta(q)], \tag{17}$$

which has the straightforward interpretation that the processes involved are the direct conversion between phonons and magnetorotons with the matrix element $|M_j(\underline{Q})|^2|Z(q_z)|^2\overline{s}(\mathbf{q})$. The two terms within the brackets involving the phonon occupation numbers represent phonon absorption and emission, respectively. We are concentrating on the regime $T_e, T_\phi \ll \Delta^*$, which ensures that $n_j(\underline{Q}) \ll 1$ for all modes that have sufficient energy to couple to the collective mode so that stimulated emission is always negligible.

The phonon-magnetoroton conversion process is subject to the conservation of in-plane wave vector and energy, so that when a phonon with wave vector $\underline{Q} = (\mathbf{q}, q_z)$ is absorbed, a magnetoroton of wave vector $\mathbf{q}$ is created only when $\omega(\underline{Q}) = \Delta(q)$. It is often very useful to visualize such processes in terms of the crossing of dispersion curves. Here there is the subtlety that the magnetoroton energy depends on the 2D wave vector q, while the phonon energy depends on the 3D wave vector $\underline{Q}$. We can, however, create such a picture by concentrating on phonons with a given angle of incidence θ (see the inset to Fig. 1). For these phonons the energy depends on the in-plane component of the wave vector via

$$\omega = c_j Q = \frac{c_j q}{\sin\theta}. \tag{18}$$

In Fig. 2 the dispersion relation for the magnetorotons (in the ideal case for the case $\nu = 1/3$) is plotted along with the dispersion curves for LA and TA phonons incident at $\theta = 30°$. In this case the LA phonon with the quoted angle of incidence and $q = 1.25$ or 1.65 couple to the electron liquid but no TA phonons at this angle can. [The dispersion curves will eventually cross because the $\Delta(q)$ curve flattens out at larger q, but this crossing will happen at such large q that the projected static structure factor, $\overline{s}(q)$, which decays as e^{-q^2} for $ql_c \gg 1$, will be negligibly small.] The actual location of the crossing point, of course, depends on the energy at the magnetoroton minimum. In real systems there are three effects which all act to reduce the size of the magnetoroton gap: finite thickness modifications to the effective interelectron interaction, Landau-level mixing, and disorder.[20]

VII. ASYMPTOTIC EXPRESSIONS FOR THE ENERGY-TRANSFER RATE

Let us concentrate on the regime $T_\phi,\ T_e \ll \Delta^*$. In this case we can neglect stimulated phonon emission and replace the Bose-Einstein factor $n_B(\omega/T_\phi)$ by the exponential $e^{-\omega/T_\phi}$ for all phonons that can couple to the electron liquid. Hence, converting the sum over wave vectors into an integral, we obtain

$$P \sim \bar{\rho}\Omega \frac{V}{4\pi^2}\sum_j \int d^3\underline{Q}\,\omega_j(\underline{Q})|M_j(\underline{Q})|^2|Z(q_z)|^2\overline{s}(q) \times (e^{-\omega_j(\underline{Q})/T_\phi}\chi_j(\mathbf{q},q_z) - e^{-\omega_j(\underline{Q})/T_e})\,\delta[\omega_j(\underline{Q}) - \Delta(q)]. \tag{19}$$

At this point we will make a drastic simplification which will surely lead to quantitative error, but will allow swift analytic progress and qualitative understanding: we will make the isotropic Debye approximation and set $\omega_j(\underline{Q}) = c_j Q$. As can be seen in Appendix B this greatly simplifies the form of $|M_j(\underline{Q})|^2$, leading to

$$P=P_{LA}^{+}-P_{LA}^{-}+P_{TA}^{+}-P_{TA}^{-},$$

$$\begin{aligned}P_s^{\pm}\sim\overline{\rho}\Omega\frac{Vc_s}{4\pi^2}\int_0^{\infty}dQQ^3e^{-c_sQ/T_{\pm}}\\ \times\int_0^{\pi}d\theta\sin\theta|Z(Q\cos\theta)|^2\overline{s}(Q\sin\theta)\\ \times\delta[c_sQ-\Delta(Q\sin\theta)]\int_0^{2\pi}d\phi\Gamma_s(Q,\sin\theta,\phi)\\ \times\chi_j^{\pm}(Q,\sin\theta,\phi),\end{aligned}\tag{20}$$

where s is either LA or TA, and where $\chi_j^{+}=\chi_j$, $\chi_j^{-}=1$, $T_{+}=T_{\phi}$, and $T_{-}=T_e$. The ϕ integral averages the electron phonon coupling over available in-plane directions; hence we define

$$\widetilde{\Gamma}_s^{\pm}(Q,\sin\theta)=\frac{1}{2\pi}\int_0^{2\pi}d\phi\Gamma_s(Q,\sin\theta,\phi)\chi_j^{\pm}(Q,\sin\theta,\phi),$$

leaving us with the simplified expression

$$\begin{aligned}P_s^{\pm}=\overline{\rho}\Omega\frac{Vc_s}{2\pi}\int_0^{\infty}dQQ^3e^{-c_sQ/T_{\pm}}\\ \times\int_0^{\pi}d\theta\sin\theta\widetilde{\Gamma}_s^{\pm}(Q,\sin\theta)|Z(Q\cos\theta)|^2\overline{s}(Q\sin\theta)\\ \times\delta[c_sQ-\Delta(Q\sin\theta)],\end{aligned}\tag{21}$$

which can be recast into the form (see Appendix E)

$$\begin{aligned}P_s^{\pm}\sim\frac{\overline{\rho}\Omega V}{2\pi c_s^2}\int_0^{\infty}dqq\int_{c_sq}^{\infty}d\omega\frac{\omega^2}{q_s^z(\omega,q)}\\ \times\widetilde{\Gamma}_s^{\pm}\left(\frac{\omega}{c_s},\frac{c_sq}{\omega}\right)|Z[q_s^z(\omega,q)]|^2\\ \times\overline{s}(q)e^{-\omega/T_{\pm}}\delta[\omega-\Delta(q)],\end{aligned}\tag{22}$$

where we have used the convenient shorthand notation

$$q_s^z(\omega,q)=\frac{\sqrt{\omega^2-c_s^2q^2}}{c_s}.\tag{23}$$

Carrying out the trivial ω integration then gives

$$P_s^{\pm}\sim\int F_s^{\pm}(q)\overline{s}(q)e^{-\Delta(q)/T_{\pm}}dq,\quad T_{+}=T_{\phi},\quad T_{-}=T_e,\tag{24}$$

$$\begin{aligned}F_s^{\pm}(q)=\frac{V\overline{\rho}\Omega}{2\pi}\left(\frac{\Delta(q)}{c_s}\right)^2\widetilde{\Gamma}_s^{\pm}\left(\frac{\Delta(q)}{c_s},\frac{c_sq}{\Delta(q)}\right)|Z[q_s^z(\Delta(q),q)]|^2\\ \times q\frac{\vartheta(\Delta(q)-c_jq)}{q_s^z(\omega,q)},\end{aligned}\tag{25}$$

which is still a rather formidable integral. However, we can make use of the fact that we are in the regime T_e, $T_{\phi}\ll\Delta^*$ and use the method of steepest descents[19] to establish the leading behavior of the integral. Naively, we would expect the integrals to be dominated by the minimum in the magnetoroton dispersion curve at q^* but this is only true for the absorption integral provided

$$\widetilde{\Gamma}_s^{\pm}([\Delta(q^*)/c_s],[c_sq^*/\Delta(q^*)])\neq 0,$$

in other words that there are phonons in the pulse in contact with the electrons that can couple at the magnetoroton minimum. If we characterize the pulse by maximum and minimum angles of incidence $\theta_{\max}$ and $\theta_{\min}$ then we can distinguish three cases.

(1) In the case that $c_jq^*/\sin\theta_{\max}<\Delta(q^*)<c_jq^*/\sin\theta_{\min}$ then there are phonons in the pulse that can couple to the magnetoroton minimum and the absorption integral will be dominated by the point q^*: we refer to this as type-I absorption.

(2) In the case that $\Delta(q^*)<c_jq^*/\sin\theta_{\max}<c_jq^*/\sin\theta_{\min}$ there are no phonons in the pulse that can couple to the magnetoroton minimum and the absorption integral is dominated by coupling to the collective mode with the lowest energy which can couple to phonons in the pulse, these have wave vector given by the lowest-energy solution of

$$\Delta(q)\sin\theta_{\max}=c_jq;\tag{26}$$

this solution will always have $q<q^*$: we refer to this as type-II absorption.

(3) In the case that $c_jq^*/\sin\theta_{\max}<c_jq^*/\sin\theta_{\min}<\Delta(q^*)$ there are simply no phonons in the pulse that can couple to the electronic system and there is no absorption (type III).

For the emission integral the same cases apply except that $\theta_{\max}=\pi/2$ and $\theta_{\min}=0$ [so that case (3) does not arise]. This of course ignores the possibility that for some directions (θ,ϕ) the characteristic function χ for the pulse may be nonzero but the coupling to the electrons may vanish because of a zero in the $A(\theta)$ or $B(\phi)$ coefficients defined in Appendix B.

We thus have two integrals to estimate:

$$\int_0^{\infty}F_j^{\pm}(q)e^{-\Delta(q)/T}dq\tag{27}$$

and

$$\int_0^{q_0}F_j^{\pm}(q)e^{-\Delta(q)/T}dq,\quad q_0<q^*.\tag{28}$$

In the first case we expand around the minimum and find

$$\begin{aligned}P_j^{+}&\sim\int F_j^{\pm}(q)e^{-\Delta(q)/T_{\pm}}dq\\ &\sim F_j^{\pm}(q^*)e^{-\Delta^*/T_{\pm}}\int_{-\infty}^{\infty}e^{-x^2/2\mu T_{\pm}}dx\\ &=\sqrt{2\pi\mu T_{\pm}}F_j^{\pm}(q^*)e^{-\Delta^*/T_{\pm}},\quad T_{\pm}\to 0.\end{aligned}\tag{29}$$

In the second case the integral is dominated by the upper end point and we must expand around here:

$$\Delta(q)\sim\Delta(q_0)+v_q(q-q_0),\tag{30}$$

where, because $q_0<q^*$, we will have

$$v_q=\left(\frac{d\Delta(q)}{dq}\right)_{q=q_0}<0. \tag{31}$$

Hence

$$\begin{aligned}P_j^{+}&\sim\int_0^{q_0}F_j^{\pm}(q)e^{-\Delta(q)/T_{\pm}}dq\\&\sim F_j^{\pm}(q_0)e^{-\Delta(q_0)/T_{\pm}}\int_0^{\infty}e^{-|v_{q_0}|x/T_{\pm}}dx\\&=\frac{T_{\pm}}{|v_{q_0}|}F_j^{\pm}(q_0)e^{-\Delta(q_0)/T_{\pm}},\quad T_{\pm}\to 0.\end{aligned} \tag{32}$$

From the above analysis we can see that, at least in principal, phonon absorption experiments have the potential to reveal detailed spectroscopic information about the magnetoroton dispersion curve, at least in the vicinity of the minimum. By choosing the geometry of the experiment such that one is the situation where the phonon band delimited by $\theta_{\max}$ and $\theta_{\min}$ does not include the magnetoroton minimum, the rate of energy transfer should be dominated by phonons with the maximum angle of incidence.

In order to illustrate these ideas let us consider the sample geometry shown in Fig. 1, and estimate the response to the angled heater $H2$, taking the maximum and minimum angles of incidence to be $\theta_{\min}=\pi/5$ and $\theta_{\max}=2\pi/7$. Suppose that the magnetoroton gap is somewhat smaller than its ideal value, say $\Delta^*=0.05\mathcal{E}_c$. Figure 3 shows the extreme LA lines on top of the magnetoroton dispersion curve, while Fig. 4 shows the extreme TA lines. From these we can see that there will be no TA absorption (type III) while the LA absorption is (just) type II with q_0 very close to q^*. Of course the details of which type of process dominates in a given geometry will depend on the actual phonon dispersion relations including the effects of anisotropy.

VIII. ELECTRON HEATING

Let us now consider in more detail the experimental situation. In the experiments the quantity that is directly measured is the two-terminal resistance of the device as a function of time. We appeal to the wide separation between the time scales for the electron system ($\sim 10^{-14}$ s) and that of the experiment ($\sim 10^{-8}$ s), and the relatively weak coupling to the phonons to assert that on experimentally resolvable time scales the electronic system is always in local equilibrium at some well-defined temperature T_e. We suppose then, that the effect of the phonons is to heat up the electron gas and all that we observe is a consequence of the time variation of the electron temperature $T_e(t)$. The resistance of the sample as a function of temperature can be obtained in a straightforward transport experiment in which the equilibrium temperature of the whole device is varied while the current is kept constant. Hence the results of the phonon experiments can be converted directly into a plot of $T_e(t)$.

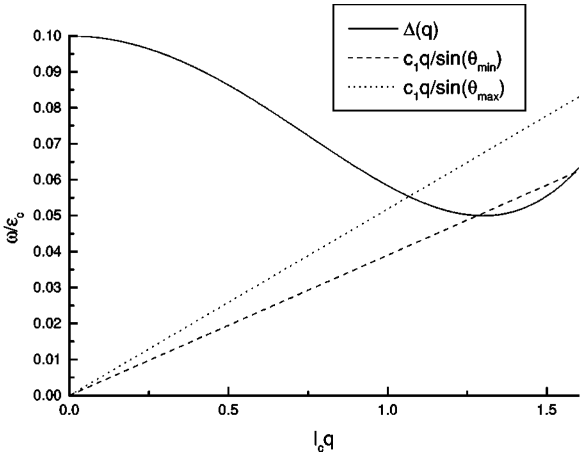

FIG. 3. Extreme LA lines (corresponding to $\theta_{\min}$ and $\theta_{\max}$) superposed on the magnetoroton dispersion corresponding to $\Delta^*=0.05\mathcal{E}_c$.

The time dependence of the electron temperature is given by

$$\frac{dT_e(t)}{dt}=\frac{P[T_\phi,T_e(t)]}{C[T_e(t)]}, \tag{33}$$

where P is the rate of energy transfer to the 2DES, and $C(T_e)$ is the heat capacity of the electronic system at temperature T_e. We will focus on the early-time behavior when $T_e \ll T_\phi \ll \Delta^*$ and suppose that the geometry of the heater and active device are such that the absorption is type I. If we suppose that we have a uniform system in the $\nu=1/m$ (m odd) state then we have

$$C(T_e)=C_0e^{-\Delta^*/T_e}$$

for the electron gas itself. However, the electron gas will be in thermal contact with metallic contacts with heat capacity $A_{\mathrm{contact}}T_e$ so that we assume

$$\frac{dT_e(t)}{dt}\sim\frac{\tilde{F}\sqrt{T_\phi}e^{-\Delta^*/T_\phi}}{C_0e^{-\Delta^*/T_e(t)}+A_{\mathrm{contact}}T_e(t)}.$$

At early times the denominator will be dominated by the linear term, so that we expect

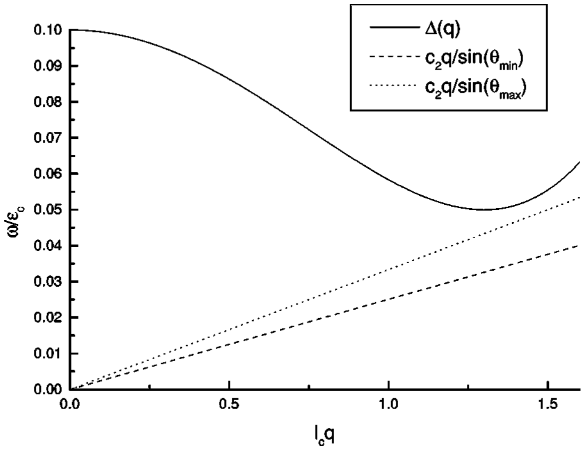

FIG. 4. Extreme TA lines (corresponding to $\theta_{\min}$ and $\theta_{\max}$) superimposed on the magnetoroton dispersion corresponding to $\Delta^*=0.05\mathcal{E}_c$.

$$T_e(t) \sim \sqrt{T_0^2 + \left(2\frac{\tilde{F}}{A}\sqrt{T_\phi}e^{-\Delta^*/T_\phi} \right) t}$$

$$\sim T_0\left[1 + \left(\frac{\tilde{F}}{A}\sqrt{T_\phi}e^{-\Delta^*/T_\phi} \right) t \right], \quad t \to 0.$$

In a real high mobility device the 2DES is subject to potential fluctuations which are slow on the scale of the magnetic length l_c so that their effect can be described within the Thomas-Fermi approximation.[21–24] This predicts that, in the quantum Hall regime with $\nu = 1/2m+1$, the bulk of the sample will consist of a percolating ''lake'' of incompressible fluid. Embedded within the lake will be isolated ''islands'' of compressible fluid (perhaps themselves containing fully depleted regions). Similarly at the boundary of the active area there will be strips of compressible fluid between the incompressible lake and the fully depleted region outside the device. Let η be the fraction of the area occupied by the electrons which is covered by the percolating incompressible liquid, then we write

$$P[T_\phi, T_e(t)] = \eta P_i[T_\phi, T_e(t)] + (1-\eta)P_c[T_\phi, T_e(t)], \tag{34}$$

$$C[T_e(t)] = \eta C_i[T_e(t)] + (1-\eta)C_c[T_e(t)], \tag{35}$$

where P_i and C_i are the energy-transfer rate and the heat capacity of the incompressible fluid and P_c and C_c the corresponding quantities for the compressible regions. In the samples used in the experiments η may not be so small, because the active 2DES is patterned into a meander line to increase its overall resistance and hence its sensitivity as a bolometer. In the previous sections we have developed a theory for P_i. Its counterpart in the compressible regions[25] will have the form

$$P_c[T_\phi, T_e(t)] \sim U_0\{T_\phi^n - [T_e(t)]^n\}, \tag{36}$$

where the power n depends on the dominant coupling process and on the nature of the correlations in the compressible state. The heat capacity of the compressible regions will also be metallic with

$$C_c(T_e) = A_c T_e. \tag{37}$$

Hence we have that

$$\frac{dT_e(t)}{dt} = \frac{\eta F\sqrt{T_\phi}e^{-\Delta^*/T_\phi} + (1-\eta)U_0\{T_\phi^n - [T_e(t)]^n\}}{\eta C_0 e^{-\Delta^*/T_e} + [(1-\eta)A_c + A_{\text{contact}}]T_e}. \tag{38}$$

Thus we see that the presence of compressible regions can greatly complicate the interpretation of results.

IX. ELECTRON EQUILIBRATION

All of the above is based on the supposition that there is a wide separation between the time scale for experimental measurements of the resistance and the internal (energy) relaxation time for the 2DES. Normally one supposes that electronic relaxation times are of the order of 10^{-15} s, while the instrumental resolution of current phonon experiments is of the order of a nanosecond. Our confidence in the belief that the electronic systems is always in local equilibrium at temperature T_e is based on recent systematic experiments which show that the results obtained in the phonon absorption experiments do not have any significant dependence on the duration of the phonon pulse t_p.[26] We will, however, at least consider the equilibration processes involved.

Equilibration within the compressible regions will be dominated by quasiparticle scattering and therefore rapid except at the very lowest temperatures. Equilibration within the incompressible regions is more problematic. One could imagine a process whereby the creation of magnetorotons by the absorption of phonons from the high-energy tail of the Bose distribution leads directly to the formation of unbound charged quasiparticles. This process would involve a magnetoroton being scattered into a higher-energy state by the absorption of a low-energy phonon from the bulk of the Bose distribution, and that this gives rise to a steady ''heating'' of a magnetoroton until its size exceeds some screening length, at which point it dissociates into a pair of oppositely charged quasiparticles. A simple estimate[27] for the rate of this kind of process was made which simply assumed a magnetoroton ''density of states'' with the result that the growth in the number of quasiparticles was sublinear ($\sim \sqrt{t}$). A more careful analysis of this process including phase space restrictions on magnetoroton-phonon scattering,[28] however, shows that this process will not heat the magnetorotons to sizes in excess of $\sim 3l_c$ and hence will not lead to any free quasiparticles. This process could of course still dominate in ''dirty'' situations in which conservation of magnetoroton momentum is not necessary due to the effects of short-range correlated disorder. The dominant mechanism for equilibration is therefore likely to involve collisions between magnetorotons. This rate should be proportional to $e^{-2\Delta^*/T_e}$ and so could become rather slow at low electron temperatures. Equilibration between the percolating sea of Laughlin liquid and the compressible regions (including the contacts) is presumably by conversion processes in which a ballistically propagating magnetoroton encounters a compressible region and is converted into a particle-hole excitation.

X. SUMMARY

In this paper we have developed a basic framework for the interpretation of phonon absorption experiments in the fractional quantum Hall regime. Our basic picture is the following. Phonons with a blackbody distribution of energies are injected into the GaAs substrate of a device containing a 2DES at a GaAs/Al$_x$Ga$_{1-x}$As heterojunction. The phonons propagate ballistically with spatiotemporal separation of the LA and TA phonons. Those phonons that have the appropriate group velocities eventually come into contact with the 2DES. The transfer of energy from the part of the phonon pulse that meets the electron gas is controlled by the process in which phonons are destroyed and magnetorotons created. The energy that is transferred to the electronic system via this bottleneck rapidly equilibrates, with metallic contacts (and possibly islands of compressible fluid) acting as thermal reservoirs, leaving the electronic system at a raised temperature with a consequent increase in the two-terminal resistance. We predict that the rate at which the electronic system absorbs energy from the phonon pulse will have a dependence on the heater temperature of the form $\sqrt{T_\phi}e^{-\Delta^*/T_\phi}$, where Δ^* is the gap at the magnetoroton minimum, provided there are phonons in the pulse that can couple at the minimum (type-I absorption). If this is not the case then the energy absorption rate will be dominated by the lowest-energy magnetorotons to which the phonons can couple and will have the form $T_\phi e^{-\Delta(q_0)/T_\phi}$, where q_0 is the lowest energy solution of Eq. (26). Hence, by judiciously positioning heaters on the bottom face of a device so that a range of angles can be sampled, it should be possible to map the dispersion relation $\Delta(q)$ for $q \lesssim q^*$ at appropriate discrete q values. It has also been argued that the presence of compressible regions within the active area of the 2DES will introduce additional phonon absorption with a power-law dependence on the heater temperature.

All of the detailed calculations in the preceding sections have been carried out in the asymptotic regime $T_e, T_\phi \ll \Delta^*$, and have made isotropic approximations for the propagation of the phonons. Numerical calculations are in progress to rectify these deficiencies and these will appear as a subsequent publication. Another shortcoming of the present work is that we have based everything on the single-mode approximation of Girvin, MacDonald and Platzman which is only reliable for fractional states with filling factors $\nu = 1/3$, 1/5, etc. Work is in progress to investigate the use of other frameworks for the collective modes of fractional quantum Hall systems based on composite fermion ideas.[29]

ACKNOWLEDGMENTS

This work was supported by the EPSRC (U.K.) under Grant No. GR/K41168. We are grateful to the following for many useful discussions: M. Brownlie, A. J. Kent, U. Zeitler, L. J. Challis, W. Dietsche, V. I. Fal'ko, D. Lehmann, A Devitt, J. E. Digby, and R. H. Eyles.

APPENDIX A: ELECTRON-PHONON COUPLING

The two mechanisms by which the electrons and phonons couple lead to similar forms of Hamiltonian; fortunately, they appear $\pi/2$ out of phase so that they do not interfere with one another quantum mechanically. We will first consider the deformation-potential coupling and then the piezoelectric coupling; however, a number of preliminary quantities are used in both. The second quantized Hamiltonian for electrons subject to a one-body potential $V(\underline{R})$ has the form

$$H=\int V(\underline{R})\hat{\rho}(\underline{R})d^3\underline{R}, \tag{A1}$$

where $\hat{\rho}(\underline{R})$ is the electron density. In order that we only have to deal with two-dimensional quantities when referring to the 2DES, it is useful to take the expectation value with respect to the vertical (z) part of the electron wave function so that

$$H=\int \hat{\rho}(\mathbf{r})\left\{\int V(\mathbf{r},z)|f(z)|^2dz\right\}d^2\mathbf{r}=\int \hat{\rho}(\mathbf{r})U(\mathbf{r})d^2\mathbf{r}, \tag{A2}$$

where $\hat{\rho}(\underline{\mathbf{r}})$ is the area density of electrons in the lowest vertical subband. It is convenient to work in the Fourier representation

$$\hat{\rho}(\mathbf{r})=\frac{1}{\Omega}\sum_{\mathbf{q}}\hat{\rho}_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}},\quad \rho_{\mathbf{q}}=\int \rho(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d^2\mathbf{r}, \tag{A3}$$

where Ω is the area of the 2DES. In this connection we will often require the form factor

$$Z(q_z)=\int |f(z)|^2e^{iq_zz}dz. \tag{A4}$$

It is common to use the Fang-Howard form

$$f(z)=\frac{1}{\sqrt{2a^3}}ze^{-z/2a}\vartheta(z) \tag{A5}$$

for a heterojunction (with the Al$_x$Ga$_{1-x}$As on the $z<0$ side) where a is a measure of the "thickness" of the 2DES in which case

$$Z(q_z)=(1+iaq_z)^{-3}. \tag{A6}$$

The elastic displacement at position $\underline{R}$ is written in terms of phonon operators as

$$u_\mu(\underline{R})=\sum_{j,\underline{Q}}u_\mu^j(\underline{Q})e^{i\underline{Q}\cdot\underline{R}}, \tag{A7}$$

$$u_\mu^j(\underline{Q})=\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}\,\xi_\mu^j(\underline{Q})[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})], \tag{A8}$$

where N_0 is the density of the material (5.14 kg m^{-3} for GaAs) and V is a normalization volume. The strain field is then given by

$$\mathcal{S}_{\mu\nu}(\underline{R})=\frac{1}{2}\left(\frac{\partial u_\mu(\underline{R})}{\partial R_\nu}+\frac{\partial u_\nu(\underline{R})}{\partial R_\mu}\right) \tag{A9}$$

$$=\sum_{j,\underline{Q}}\mathcal{S}_{\mu\nu}^j(\underline{Q})e^{i\underline{Q}\cdot\underline{R}}, \tag{A10}$$

$$\begin{aligned}\mathcal{S}_{\mu\nu}^j(\underline{Q})=\frac{i}{2}\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}[\xi_\mu^j(\underline{Q})Q_\nu+Q_\mu\xi_\nu^j(\underline{Q})]\\ \times[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})].\end{aligned} \tag{A11}$$

1. Deformation potential coupling

The deformation potential due to a lattice strain $\mathcal{S}_{\mu\nu}$ is

$$V_{dp}(\underline{R})=\Xi_{\mu\nu}\mathcal{S}_{\mu\nu}(\underline{R}), \tag{A12}$$

so that, in second quantization, we have

$$\hat{H}_{dp}=\int U_{dp}(\mathbf{r})\hat{\rho}(\mathbf{r})d^2\mathbf{r} \tag{A13}$$

$$=\Xi_{\mu\nu}\int \hat{\rho}(\mathbf{r})\int |f(z)|^2\hat{\mathcal{S}}_{\mu\nu}(\mathbf{r},z)dzd^2\mathbf{r} \tag{A14}$$

$$=i\sum_{j,\underline{Q}} \Xi_j(\underline{Q})Z(q_z)\hat{\rho}_{-\mathbf{q}}[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})], \tag{A15}$$

where

$$\Xi_j(\underline{Q})=\frac{1}{2}\Xi_{\mu\nu}\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}[\xi_\mu^j(\underline{Q})Q_\nu+Q_\mu\xi_\nu^j(\underline{Q})], \tag{A16}$$

which, for GaAs, reduces to

$$\Xi_j(\underline{Q})=\Xi_0\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}[\xi^j(\underline{Q})\cdot\underline{Q}]. \tag{A17}$$

The conventional value for the deformation potential parameter is $\Xi_0\approx 7$ eV but the variance between different measurements is quite large.

2. Piezoelectric coupling

The polarization due to a lattice strain $\mathcal{S}_{\mu\nu}$ is

$$P_\mu(\underline{R})=\frac{1}{\kappa}h_{\mu\nu\lambda}\mathcal{S}_{\nu\lambda}(\underline{R}). \tag{A18}$$

The electrostatic potential associated with this polarization field is $\phi(\underline{R})$, which corresponds to an electric field $\underline{E}=-\nabla\phi$. These are related by the Maxwell equation

$$\nabla\cdot\underline{D}=\nabla\cdot(\varepsilon_0\underline{E}+\underline{P})=0, \tag{A19}$$

so that

$$\varepsilon_0\nabla^2\phi=\nabla\cdot\underline{P}. \tag{A20}$$

Now

$$\nabla\cdot\underline{P}(\underline{R})=\frac{i}{\kappa}h_{\mu\nu\lambda}\sum_{j,\underline{Q}} Q_\mu\mathcal{S}_{\nu\lambda}^j(\underline{Q})e^{i\underline{Q}\cdot\underline{R}}, \tag{A21}$$

so that

$$\phi(\underline{R})=\sum_{\underline{Q}} \phi(\underline{Q})e^{i\underline{Q}\cdot\underline{R}}, \tag{A22}$$

where

$$\phi(\underline{Q})=-ih_{\mu\nu\lambda}\frac{Q_\mu\Sigma_j\mathcal{S}_{\nu\lambda}^j(\underline{Q})}{\kappa\varepsilon_0Q^2}. \tag{A23}$$

The electron phonon Hamiltonian is then

$$\begin{aligned}\hat{H}_{pa}&=-e\int \hat{\phi}(\underline{R})\hat{\rho}(\underline{R})d^3\underline{R}\\&=-e\sum_{\underline{Q}} \hat{\phi}(\underline{Q})\int \hat{\rho}(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d^2\mathbf{r}\int |f(z)|^2e^{iq_zz}dz\\&=\frac{ieh_{\mu\nu\lambda}}{\kappa\varepsilon_0}\sum_{j\underline{Q}} \frac{Q_\mu\hat{\mathcal{S}}^j_{\nu\lambda}(\underline{Q})}{Q^2}\hat{\rho}_{-\mathbf{q}}Z(q_z)\\&=-\sum_{j\underline{Q}} \Lambda_j(\underline{Q})Z(q_z)\hat{\rho}_{-\mathbf{q}}[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})],\end{aligned} \tag{A24}$$

where

$$\Lambda_j(\underline{Q})=\frac{-eh_{\mu\nu\lambda}}{2\kappa\varepsilon_0}\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}\times\frac{Q_\mu[\xi_\nu^j(\underline{Q})Q_\lambda+Q_\nu\xi_\lambda^j(\underline{Q})]}{Q^2},$$

which for GaAs reduces to

$$\Lambda_j(\underline{Q})=\frac{-eh_{14}}{\kappa\varepsilon_0}\sqrt{\frac{\hbar}{2N_0V\omega_j(\underline{Q})}}\times\frac{Q_xQ_y\xi_z^j(\underline{Q})+Q_yQ_z\xi_x^j(\underline{Q})+Q_zQ_x\xi_y^j(\underline{Q})}{Q^2}. \tag{A25}$$

In conventional work on phonons in semiconductors it is customary to regularize the $1/Q$ dependence by including screening of the interaction in the Thomas-Fermi approximation, leading to

$$|\Lambda_j(\underline{Q})|^2\sim\frac{|\gamma_j^{pa}|^2}{Q+Q_{TF}}, \tag{A26}$$

where Q_{TF} is the usual inverse screening length. We shall not do this here, partly because there is no coupling to small Q phonons because of the presence of a gap, but mostly on the grounds that we are directly employing an approximation (and a very good one in the regime of interest) for the dynamic structure factor which includes all screening effects. In practice, of course, the incompressible nature of the fractional quantum Hall state means that potentials such as this are hardly screened at all.

Hence we can write the overall electron-phonon coupling Hamiltonian as

$$=-\sum_{j\underline{Q}} M_j(\underline{Q})Z(q_z)\rho_{-\mathbf{q}}[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})], \tag{A27}$$

where

$$M_j(\underline{Q})=\Lambda_j(\underline{Q})-i\Xi_j(\underline{Q}). \tag{A28}$$

If we choose a basis for single-particle electron states $\{|m\rangle\}$, then we can write

$$\hat{\rho}(\mathbf{r})=\sum_{mn} \hat{c}_m^\dagger\hat{c}_n\langle m|\mathbf{r}\rangle\langle\mathbf{r}|n\rangle, \tag{A29}$$

$$\hat{\rho}_{\mathbf{q}}=\sum_{mn} \Gamma_{mn}(\mathbf{q})\hat{c}_m^\dagger\hat{c}_n\,, \tag{A30}$$

where

$$\Gamma_{mn}(\mathbf{q})=\langle m|e^{i\mathbf{q}\cdot\hat{r}}|n\rangle, \tag{A31}$$

so that

$$H_{e\phi}=-\sum_{j\underline{Q},mn}\Gamma_{mn}(\mathbf{q})M_j(\underline{Q})\hat{c}_m^\dagger\hat{c}_n[\hat{a}_j^\dagger(\underline{Q})+\hat{a}_j(-\underline{Q})], \tag{A32}$$

which bears the usual interpretation of a phonon absorption/emission causing an electron to scatter from state n to state m.

APPENDIX B: ISOTROPIC DEBYE APPROXIMATION

In the isotropic Debye approximation we neglect the anisotropy in the speeds of sound and set $\omega_j(\underline{Q})=c_jQ$, where $c_1=c_{LA}$ and $c_2=c_3=c_{TA}$. The phonon polarizations become strictly transverse and longitudinal. A suitable set of orthonormal polarization vectors would be

$$\underline{\xi}^1(\underline{Q})=\underline{Q}/Q=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta),$$

$$\underline{\xi}^2(\underline{Q})=(-\sin\phi,\cos\phi,0),$$

$$\underline{\xi}^3(\underline{Q})=(\cos\theta\cos\phi,\cos\theta\sin\phi,-\sin\theta),$$

but any combination of the form

$$\underline{\xi}^{TA_1}(\underline{Q})=\cos(\alpha)\underline{\xi}^2(\underline{Q})+\sin(\alpha)\underline{\xi}^3(\underline{Q}),$$

$$\underline{\xi}^{TA_2}(\underline{Q})=-\sin(\alpha)\underline{\xi}^2(\underline{Q})+\cos(\alpha)\underline{\xi}^3(\underline{Q})$$

is equally good. Fortunately in final expressions for energy-transfer rates, all dependence on α disappears.

In this approximation the deformation potential coupling has the form

$$\Xi_j(\underline{Q})=\gamma^{dp}\delta_{j1}\sqrt{Q},$$

where

$$\gamma^{dp}=\Xi_0\sqrt{\frac{\hbar}{2N_0Vc_j}},$$

so that the electrons only couple to the LA phonons via the deformation potential, and the coupling is isotropic. Similarly the piezoelectric couplings have the forms

$$\Lambda_1(\underline{Q})=\frac{\gamma_{LA}^{pa}}{\sqrt{Q}}A_1(\theta)B_1(\phi),$$

$$\Lambda_j(\underline{Q})=\frac{\gamma_{TA}^{pa}}{\sqrt{Q}}A_j(\theta)B_j(\phi),\quad j=2\ \text{and}\ 3,$$

where θ and ϕ are the polar angles of the Q vector, and

$$A_1(\theta)=3\sin^2\theta\cos\theta,\quad B_1(\phi)=\tfrac{1}{2}\sin(2\phi),$$

$$A_2(\theta)=\sin\theta\cos\theta,\quad B_2(\phi)=\cos(2\phi),$$

$$A_3(\theta)=\sin\theta(3\sin^2\theta-2),\quad B_3(\phi)=\tfrac{1}{2}\sin(2\phi).$$

Hence, in the isotropic approximation, we have

$$|M_1(Q,\theta,\phi)|^2=\frac{(\gamma_{LA}^{pa})^2}{Q}[A_1(\theta)B_1(\phi)]^2+(\gamma^{dp})^2Q,$$

$$|M_j(Q,\theta,\phi)|^2=\frac{(\gamma_{TA}^{pa})^2}{Q}[A_j(\theta)B_j(\phi)]^2,\quad j=2\ \text{and}\ 3.$$

Since the only separate dependence on the two TA modes is in the A and B coefficients we define, using the fact that $A_j(\theta)$ can be written solely as a function of $\sin\theta$,

$$\Gamma_{LA}(Q,\sin\theta,\phi)=\frac{(\gamma_{LA}^{pa})^2}{Q}[A_1(\theta)B_1(\phi)]^2+(\gamma^{dp})^2Q,$$

$$\Gamma_{TA}(Q,\sin\theta,\phi)=\frac{(\gamma_{TA}^{pa})^2}{Q}\{[A_2(\theta)B_2(\phi)]^2+[A_3(\theta)B_3(\phi)]^2\}.$$

APPENDIX C: ENERGY-TRANSFER RATE

In order to derive an expression for the rate at which energy is transferred from the phonon pulse to the electron gas, we will use standard linear-response theory. We make the usual adiabatic switching-on assumption, and suppose that in the distant past ($t\rightarrow-\infty$) the electron and phonon systems were uncoupled with the electrons in equilibrium at temperature T_e. We will take the initial state of the phonons to be described by the density matrix

$$\varrho_\phi=\frac{1}{Z_\phi}\mathcal{P}_\chi e^{-H_\phi/T_\phi}\mathcal{P}_\chi, \tag{C1}$$

where $\mathcal{P}_\chi$ is a projection operator which excludes states corresponding to phonons that have not reached the vicinity of the 2DES, and H_ϕ is the phonon Hamiltonian.

The initial density matrix of the system is then

$$\varrho_0=\frac{1}{Z_e}e^{-H_e/T_e}\otimes\frac{1}{Z_\phi}\mathcal{P}_\chi e^{-H_\phi/T_\phi}\mathcal{P}_\chi, \tag{C2}$$

where H_e is the Hamiltonian for the electron liquid. The time evolution of the density matrix is governed by the von Neumann equation

$$i\frac{d\varrho(t)}{dt}=[H_e+H_\phi+H_{e\phi},\varrho(t)]. \tag{C3}$$

As usual it is convenient to change to the interaction picture of quantum mechanics from the usual Schrodinger one by defining

$$\tilde{O}(t)=e^{iH_0t}Oe^{-iH_0t}, \tag{C4}$$

$$H_0=H_e+H_\phi, \tag{C5}$$

so that

$$i\frac{d\tilde{\varrho}(t)}{dt}=[\tilde{H}_{e\phi}(t),\tilde{\varrho}(t)], \tag{C6}$$

which has the iterative solution (to lowest order in the electron phonon coupling)

$$\tilde{\varrho}(t)=\varrho_0-i\int_{-\infty}^{t}[\tilde{H}_{e\phi}(t'),\varrho_0]e^{\eta t'}dt', \tag{C7}$$

where η is the adiabatic switch-on rate. The rate of energy transfer to the electron gas is

$$P=\frac{d}{dt}\langle H_e\rangle=-\frac{d}{dt}\langle H_\phi\rangle=i\,\mathrm{tr}\{\tilde{\varrho}(t)[\widetilde{H_\phi}(t),\widetilde{H_{e\phi}}(t)]\}$$

$$=\int_{-\infty}^{t}\mathrm{tr}\{\varrho_0[[\widetilde{H_\phi}(t),\widetilde{H_{e\phi}}(t)],\tilde{H}_{e\phi}(t')]\}e^{\eta t'}dt'. \quad \text{(C8)}$$

Now

$$[H_\phi,H_{e\phi}]=-\sum_{j\underline{Q}}\omega_j(\underline{Q})[M_j(\underline{Q})\rho_{-q}a_j^\dagger(\underline{Q})$$

$$-M_j(-\underline{Q})\rho_q a_j(\underline{Q})], \quad \text{(C9)}$$

$$\langle\tilde{a}_j^\dagger(\underline{Q},t)\tilde{a}_{j'}(\underline{Q}',t')\rangle=\delta_{jj'}\delta_{\underline{Q},\underline{Q}'}n_j(\underline{Q})e^{i\omega_j(\underline{Q})(t-t')}, \quad \text{(C10)}$$

and

$$M_j(-\underline{Q})=M_j^*(\underline{Q}). \quad \text{(C11)}$$

Hence, after some rather tedious algebra and setting $\eta=0$, we find

$$P=2\pi\overline{\rho}\Omega\sum_{j\underline{Q}}\omega_j(\underline{Q})|M_j(\underline{Q})|^2\{n_j(\underline{Q})S[\mathbf{q},\omega_j(\underline{Q})]$$

$$-[1+n_j(\underline{Q})]S[-\mathbf{q},-\omega_j(\underline{Q})]\}, \quad \text{(C12)}$$

where

$$S(\mathbf{q},\omega)=\frac{1}{2\pi\overline{\rho}\Omega}\int\langle\rho_{-q}(t)\rho_{\mathbf{q}}(0)\rangle e^{i\omega t}dt \quad \text{(C13)}$$

is the dynamic structure factor. As usual (see Appendix D),[17]

$$S(-\mathbf{q},-\omega)=e^{-\omega/T_e}S(\mathbf{q},\omega), \quad \text{(C14)}$$

so that

$$P=2\pi\overline{\rho}\Omega\sum_{j\underline{Q}}\omega_j(\underline{Q})|M_j(\underline{Q})|^2S(\mathbf{q},\omega_j(\underline{Q}))$$

$$\times\{n_j(\underline{Q})-e^{-\omega_j(\underline{Q})/T_e}[1+n_j(\underline{Q})]\}. \quad \text{(C15)}$$

APPENDIX D: SPECTRAL ANALYSIS OF THE DYNAMIC STRUCTURE FACTOR

Let us consider the exact energy and electron number eigenstates of the electronic system $\{|\alpha\rangle\}$. The dynamic structure factor can then be written

$$S(\mathbf{q},\omega)=\frac{1}{2\pi\overline{\rho}\Omega}\int\frac{1}{Z_e}\sum_{\alpha\gamma}\langle\alpha|e^{-H_e/T_e}e^{iH_et}\rho_{-\mathbf{q}}e^{-iH_et}|\gamma\rangle$$

$$\times\langle\gamma|\rho_{\mathbf{q}}|\alpha\rangle e^{i\omega t}dt$$

$$=\frac{1}{\overline{\rho}\Omega}\frac{1}{Z_e}\sum_{\alpha\gamma}|\langle\gamma|\rho_{\mathbf{q}}|\alpha\rangle|^2e^{-E_\alpha/T_e}\delta[\omega-(E_\gamma-E_\alpha)], \quad \text{(D1)}$$

where $Z_e=\Sigma_\alpha e^{-E_\alpha/T_e}$ is the partition function for the electronic system. Now

$$S(-\mathbf{q},-\omega)=\frac{1}{\overline{\rho}\Omega}\frac{1}{Z_e}\sum_{\alpha\gamma}|\langle\gamma|\rho_{-\mathbf{q}}|\alpha\rangle|^2e^{-(E_\alpha-\mu N_\alpha)/T_e}$$

$$\times\delta[-\omega-(E_\gamma-E_\alpha)]; \quad \text{(D2)}$$

interchanging the dummy variables gives

$$S(-\mathbf{q},-\omega)=\frac{1}{\overline{\rho}\Omega}\frac{1}{Z_e}\sum_{\alpha\gamma}|\langle\alpha|\rho_{-\mathbf{q}}|\gamma\rangle|^2e^{-E_\gamma/T_e}$$

$$\times\delta[-\omega-(E_\alpha-E_\gamma)]$$

$$=\frac{1}{\overline{\rho}\Omega}\frac{1}{Z_e}\sum_{\alpha\gamma}|\langle\gamma|\rho_{\mathbf{q}}|\alpha\rangle|^2e^{-(E_\alpha+\omega)/T_e}$$

$$\times\delta[\omega-(E_\gamma-E_\alpha)]$$

$$=e^{-\omega/T_e}S(\mathbf{q},\omega) \quad \text{(D3)}$$

as required in Appendix C.

Now let us write

$$Z_e=\sum_\alpha e^{-E_\alpha/T_e}=e^{-E_0/T_e}\left(1+\sum_{\alpha\neq0}e^{-(E_\alpha-E_0)/T_e}\right), \quad \text{(D4)}$$

where E_0 is the ground-state energy. We know that $E_\alpha-E_0>\Delta^*\forall\alpha>0$, so that

$$Z\sim e^{-E_0/T_e}[1+O(e^{-\Delta^*/T_e})],\quad T_e\to0. \quad \text{(D5)}$$

Similarly

$$Z_eS(\mathbf{q},\omega)=\frac{1}{\overline{\rho}\Omega}\sum_{\alpha\gamma}|\langle\gamma|\rho_{\mathbf{q}}|\alpha\rangle|^2e^{-E_\alpha/T_e}\delta[\omega-(E_\gamma-E_\alpha)]$$

$$=e^{-E_0/T_e}\left\{\frac{1}{\overline{\rho}\Omega}\sum_\gamma|\langle\gamma|\rho_{\mathbf{q}}|0\rangle|^2\delta[\omega-(E_\gamma-E_0)]\right.$$

$$+\frac{1}{\overline{\rho}\Omega}\sum_{\alpha\neq0}e^{-(E_\alpha-E_0)/T_e}\sum_\gamma|\langle\gamma|\rho_{\mathbf{q}}|\alpha\rangle|^2$$

$$\left.\times\delta[\omega-(E_\gamma-E_\alpha)]\right\}, \quad \text{(D6)}$$

$$S(\mathbf{q},\omega)\sim\frac{1}{\overline{\rho}\Omega}\sum_\gamma|\langle\gamma|\rho_{\mathbf{q}}|0\rangle|^2\delta[\omega-(E_\gamma-E_0)]$$

$$+O(e^{-\Delta^*/T_e}),\quad T_e\to0,$$

where the leading term is simply the zero-temperature structure factor.

APPENDIX E: SIMPLIFICATION OF *P* INTEGRAL

We have an integral of the form

$$P_j=\int_0^\infty dQQ^2\int_0^\pi d\theta\sin\theta\,F(c_jQ,Q\sin\theta,Q\cos\theta). \quad \text{(E1)}$$

We introduce new variables $\omega=c_jQ$ and $q=Q\sin\theta$. Now

$$\left|\frac{\partial(\omega,q)}{\partial(Q,\theta)}\right| = \begin{vmatrix} c_j & 0 \\ \sin\theta & Q\cos \end{vmatrix} = c_j Q \cos\theta = \sqrt{\omega^2 - c_j^2 q^2}, \tag{E2}$$

from which

$$Q^2 \sin\theta dQ d\theta = \frac{\omega q}{c_j \sqrt{\omega^2 - c_j^2 q^2}} d\omega dq, \tag{E3}$$

so that

$$P_j = \int_0^\infty d\omega \int_0^\infty dq F\left(\omega, q, \frac{\sqrt{\omega^2 - c_j^2 q^2}}{c_j}\right) \frac{\omega q \vartheta(\omega - c_j q)}{c_j \sqrt{\omega^2 - c_j^2 q^2}}, \tag{E4}$$

which leads to the formula quoted in the text.

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