Resonant tunneling between two continua

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We consider the tunneling between two monomode continua through a monomode structure with the symmetry of two orthogonal mirror planes. We derive in closed form the conditions for selective transfer of a single propagating state from one continuum to the other, leaving all other neighbor states unaffected. We illustrate the results of this analysis by analytical solutions for a simple structure made out of one-dimensional photonic waveguides. These theoretical results are confirmed by experiments using coaxial cables.
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For the last ten years, complete channel drop tunneling between one-dimensional continua has been studied extensively for electrons[1] as well as for electromagnetic waves.[2,3] This selective transfer of propagating state from one continuum to the other, leaving the other neighbor states unaffected, may occur when the continua are coupled through a coupling element that supports localized resonant states. The understanding of channel drop tunneling between one-dimensional continua is of fundamental and practical interest, as well as for monoenergy electrons as for single-frequency photons. Applications of such transfer processes are important for wavelength demultiplexing in optical communications[4,5] and for electron spectroscopy.[1] In a recent theoretical work,[2] Fan *et al.* discussed the criteria for a complete transfer on the basis of symmetry arguments. They demonstrated the relevance of their analysis by numerical simulations, using a finite-difference time domain scheme, of the propagation of electromagnetic waves in a two-dimensional defective photonic crystal.

In this paper, we pursue the appealing possibility of devising simple structures exhibiting resonant coupling at a specific frequency. We propose a model system and give in closed form conditions for complete channel drop tunneling for monomode coupling structures having the symmetry of two orthogonal mirror planes. We then illustrate the results of this analytical study on the example of an original structure made out of monomodes photonic waveguides. However the system model is general and may be easily transposed to other excitations such as electrons, magnons, or acoustic waves. Simple experiments using coaxial cables confirming the theoretical results for the above original structure are also reported.

Let us consider the generic system schematically presented in Fig. 1(a). This system is supposed to have the symmetry of two mirror planes. The two continua are the two infinite lines passing by, respectively, points (1,2) and (3,4). It is also convenient to consider the finite system (1,2,3,4) obtained by removing the four semi-infinite lines at points 1, 2, 3, and 4. When a propagating state is excited in the line attached to point 1, the corresponding reflection R and transmission coefficients T_{1j}, $j=2,3,4$ are easily found to be related to the elements of the Green's function of this system by the following relations:

$$R=|1+2i\alpha g(11)|^2, \tag{1a}$$

$$T_{1j}=|2i\alpha g(1j)|^2, \quad j=2,3,4. \tag{1b}$$

In Eqs. (1), the parameter α is defined, for monomode electromagnetic waves, as $\alpha=\omega\sqrt{\varepsilon}/c$, where ω is the frequency, c the speed of light, ε the relative dielectric constant of all

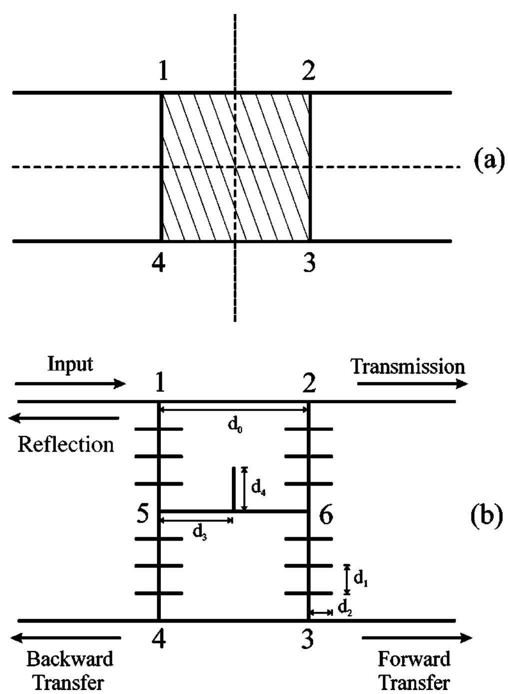

FIG. 1. (a) The general system under consideration. (b) The peculiar system for application.

the photonic waveguides, and $i=\sqrt{-1}$. Taking into account the symmetry of the two mirror planes shown by dashed lines in Fig. 1(a), we obtain

$$g(11)=Z_1+Z_2+Z_3+Z_4, \tag{2a}$$

$$g(12)=Z_1+Z_2-Z_3-Z_4, \tag{2b}$$

$$g(13)=Z_1-Z_2+Z_3-Z_4, \tag{2c}$$

$$g(14)=Z_1-Z_2-Z_3+Z_4, \tag{2d}$$

where

$$Z_n=1/[4\alpha(y_n-i)], \quad n=1,2,3,4. \tag{3}$$

In Eq. (3), the y_n are purely real quantities determined by the finite structure contained in the shaded square (1,2,3,4). One finds then that in order to have a complete transfer of a propagating state at a given frequency ω_0 from site 1 to site 3 (namely, $T_{13}=1$ and $R=T_{12}=T_{14}=0$), one must fulfill the following conditions:

$$y_1=y_3=-1/y_2=-1/y_4. \tag{4}$$

If one wants this frequency ω_0 to be in the middle of a frequency domain for which only direct transmission exists (namely, $T_{12}=1$ and $R=T_{13}=T_{14}=0$), one must fulfill in this domain the following conditions:

$$y_1=y_2=-1/y_3=-1/y_4. \tag{5}$$

In other words, these conditions (5) require that the system that couples the two continua must have a gap in the above-defined frequency domain. Conditions (4) imply that the complete system must have one resonant frequency ω_0 inside this gap. These conclusions are equivalent to the ones obtained by Fan *et al.*[2] for such a system. Conditions (4) and (5) enable us to determine completely from closed form expressions the geometrical parameters of the system.

In what follows, we illustrate these general results by choosing one very simple system, shown schematically in Fig. 1(b). This system is built from the two infinite monomode waveguides passing, respectively, by the points (1,2) and (3,4). The distance between 1 and 2, called d_0, is the same as that between 3 and 4. Four identical monomode photonic structures are branched between points (1,5), (5,4), (2,6), and (6,3). These structures have N equidistant (distance d_1) sites. Stars of N' side branches of length d_2 are grafted onto the $(N-2)$ internal sites. In Fig. 1(b), N and N' are equal to 5 and 2, respectively. Such photonic structures were shown[6] to have giant gaps. They enable us also to adjust the frequency range of these gaps to almost any desired domain by tuning the distance d_2, d_1 and the numbers N and N'. Between points 5 and 6 is fixed one waveguide of length $2d_3$ with a side branch of length d_4 in its middle.

We give now the analytical expressions of the y_n defined in Eq. (3). Let us define the quantities

$$A_m=-1/\tan(\alpha d_m), \tag{6a}$$

$$B_m=1/\sin(\alpha d_m), \tag{6b}$$

with $m=0,1,2,3,4$.

A_0 and B_0 are associated with the finite parts of the infinite continua situated between points (1,2) and (3,4). We define also the terms A_5 and B_5 related to the structures with large gaps grafted between points (1,5), (5,4), (2,6), and (6,3) as

$$A_5=-N'A_2-A_1-B_1\sin(Nkd_1)/\sin[(N-1)kd_1], \tag{7a}$$

and

$$B_5=B_1\sin(kd_1)/\sin[(N-1)kd_1], \tag{7b}$$

where k is defined by

$$\cos(kd_1)=-\frac{1}{B_1}\left(A_1+\frac{N'}{2}A_2\right). \tag{7c}$$

Expression (7c) is the dispersion relation of such an infinite star waveguide, when the electric field vanishes at the free extremities of the side branches.

The properties of the structure grafted between points 5 and 6 in Fig. 1(b) are related to

$$B_6=-B_3^2/(2A_3+A_4), \tag{8a}$$

and

$$A_6=A_3+B_6. \tag{8b}$$

The definition of these quantities leads to the following expressions for the y_n associated with the final system depicted in Fig. 1(b). They are given as

$$y_1=y_2-2B_5^2/(2A_5+A_6+B_6), \tag{9a}$$

with

$$y_2=A_0+B_0+A_5, \tag{9b}$$

$$y_3=A_0-B_0+A_5, \tag{9c}$$

$$y_4=y_3-2B_5^2/(2A_5+A_6-B_6). \tag{9d}$$

Now we are able to precise these system parameters for a complete channel drop tunneling between the two continua at a frequency ω_0 falling in the middle of a given frequency band $\Delta\omega$. First the condition $y_2y_3=-1$ [Eqs. (4) and (5)] is satisfied for

$$A_5(\omega_0)=0. \tag{10}$$

We determine the length d_0 such that

$$y_2(\omega_0)=-y_3(\omega_0)=1. \tag{11}$$

by choosing

$$\tan(\alpha_0 d_0/2)=1, \tag{12}$$

where $\alpha_0=\omega_0\sqrt{\varepsilon}/c$.

So we study the quantity $A_5(\omega)$ given by Eq. (7a) and choose N, N', d_1, and d_2 in order that Eq. (10) shall be satisfied inside a gap corresponding to the required frequency band $\Delta\omega$. If $\Delta\omega$ is required to be even larger than the giant gaps reported for such star structures, one can improve on this point by using different dielectric constants in

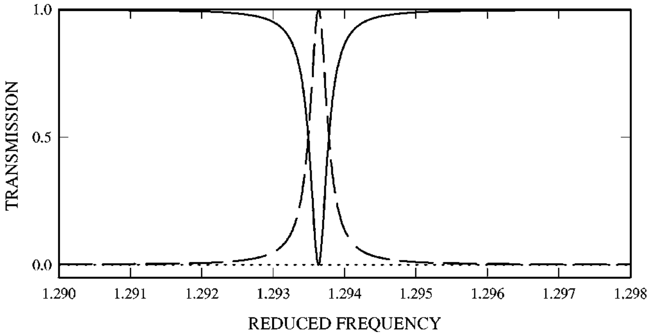

FIG. 2. Variation of the intensity of the transmitted signal from site 1 to site 2 (solid line), and of the forward signal (T_{13}) (long dashed line), in the structure shown in Fig. 1(b) versus the reduced frequency. The dots represent the signal intensity in the backward direction (T_{14}). These theoretical results were obtained for $N=5$, $N'=2$, $d_0=1.214\,189d_1$, $d_2=0.4d_1$, $d_3=1.2138d_1$, and $d_4=0.0003d_1$. The resonant reduced frequency is $\Omega_0=1.293\,634$.

branches 1 and 2 and/or by putting two or more such structures in series. However, the quality factor of the transferred frequency peak increases when the gap $\Delta\omega$ increases, as well as the requirements on the precision on the distances d_m. The remainder of the conditions given by Eqs. (4) are then satisfied for

$$A_6(\omega_0)=0 \tag{13}$$

and

$$B_6(\omega_0)=B_5^2(\omega_0). \tag{14}$$

These last two conditions lead to

$$\tan(\alpha_0 d_3)=1/B_5^2(\alpha_0), \tag{15}$$

and

$$\tan(\alpha_0 d_4)=-\frac{1}{2}\tan(2\alpha_0 d_3), \tag{16}$$

which define the distances d_3 and d_4 for a given ω_0.

So, as announced above, our model system enables us to determine completely and in closed form the system parameters for a complete channel transfer once a frequency ω_0 is chosen.

In order to illustrate the results of the above analytic theory, we present in Fig. 2, the variations of the transmission coefficients T_{12}, T_{13}, and T_{14} versus the reduced frequency $\Omega=\omega\sqrt{\varepsilon}d_1/c$. Figure 2 shows together the dip (solid line) in the direct transmission from site 1 to site 2 (T_{12}) and the forward drop (long dashed line) from site 1 to site 3 (T_{13}). The backward transferred signal from site 1 to site 4 (T_{14}) is completely absent over the entire frequency range and is represented by the dots in Fig. 2. This application was done for $N=5$, $N'=2$, $d_0=1.214\,189d_1$, $d_2=0.4d_1$, $d_3=1.2138d_1$, and $d_4=0.0003d_1$. The boundary condition at the free ends of all the side branches is $E=0$ (vanishing of the electric field). With these parameters, the reduced resonant frequency Ω equals 1.293 634. One can easily check that with these values, Eqs. (11) and (16) are verified. The quality factor of the sharp peaks defined as the ratio between the central frequency and the full width at half maximum is of the order of 4300. A more complete study shows that this quality factor depends strongly on the characteristic lengths as well as on the integers N and N'.

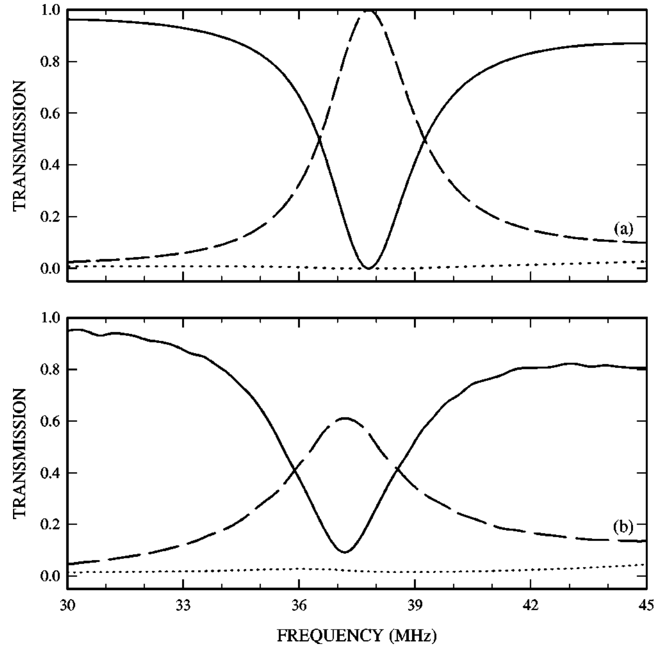

FIG. 3. Comparison between theoretical (a) and experimental (b) results for $\varepsilon=2.3$, $N=3$, $N'=1$, $d_0=1.3m$, $d_1=1m$, $d_2=0.4m$, $d_3=1.10m$, and $d_4=0.15m$. The boundary condition is $E=0$ at the free ends of all the side branches. Solid line, direct transmission (T_{12}); long dashed line, forward signal (T_{13}); and dotted line, backward signal (T_{14}).

In order to check the validity of the above theoretical model, we have examined, with a simple experimental setup, the behavior of the transmission coefficients in the frequency range of a few tens of MHz. In these experiments, both the one-dimensional waveguides and the side branches are constituted by standard 50-Ω coaxial cables and the transmission measurement is realized by using a tracking generator coupled to a spectrum analyzer. The top panel of Fig. 3 shows the theoretical variations of the transmission coefficients while the bottom panel presents the experimental measurements for the $E=0$ boundary condition and for $N=3$, $N'=1$, $\varepsilon=2.3$, $d_0=1.3$ m, $d_1=1$ m, $d_2=0.4$ m, $d_3=1.10$ m, $d_4=0.15$ m. One notes that the behavior of the theoretical transmission coefficients in Figs. 2 and 3(a) is quite similar except for the weaker quality factor in Fig. 3(a). This is mainly explainable by the lower values of the integers N and N' in Fig. 3(a) than in Fig. 2. The comparison between Figs. 3(a) and 3(b) reveals a good agreement between the theoretical predictions and the experimental spectra. Nevertheless the experimental resonant frequency is slightly different (≈ 0.6 MHz) from the theoretical value of 37.8 MHz. An uncertainty on the length of the coaxial cables may explain this discrepancy. Another point of contention between experiment and theory stands in the amplitude of the peaks. For example, the experimental maximum of the forward signal drops to 62% compared to the 100% maximum obtained theoretically. This probably results from the attenuation of the signal occurring in the coaxial cables.[6] It is also possible that one part of the input signal has been reflected at point 1 of the system [see Fig. 1(b)]. The reflection coefficient R is strictly equal to zero in the theoretical calculation. An experimental measurement of this coefficient is not allowed with our experimental setup. One notes also that in the frequency range of Fig. 3, the intensity of the backward signal reaches a very low value. In spite of these slight disagreements, one can notice that our experimental measurements, made with a common experimental setup, validate fairly well the theoretical prediction. This is also, to our knowledge, the first experimental observation of the existence of resonant tunneling of electromagnetic waves.

In summary, we have investigated the tunneling between two monomodes continua coupled by a monomode structure. Inspired by the work of Fan *et al.*,[2] we have conceived a model system allowing a complete analytical and experimental study of the conditions for a selective transfer of propagating state from one continuum to the other leaving all the other neighbor states unaffected. Our theoretical model enables us to determine completely the geometric parameters of the monomode structure for a complete channel transfer at a specific frequency, once this frequency has been chosen. The frequency domain where the channel drop tunneling occurs only depends on the characteristic lengths of the constituents of the model system. Our theoretical model is then universal and valid in all the frequency domains of the electromagnetic spectrum. Moreover, the above formalism developed for electromagnetic waves can be easily transposed to electrons and magnons, as well as acoustic waves. This will be the subject of future work.

We think that our model system may have potential applications for making filtering or multiplexing devices. For example, a succession of a few ''resonant tunneling structures'' with different characteristics may allow us to extract several specific frequencies out of an input signal of large frequency band. Finally, one can notice that recent experimental works have shown that one-dimensional structures with dangling side branches can be fabricated at the submicrometer scale using high-resolution electron beam lithography.[7,8] We hope then that the present work should lead to the manufacturing of optical devices having this geometry.

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