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Crossover effects for critical currents within Landau-Ginzburg phenomenology incorporating Josephson weak links in superconducting thin films of $YBaCu_2O_{7-\delta}$

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Following a number of investigations into the nature of critical currents in superconducting thin films of $YBaCu_2O_{7-\delta}$ (YBCO), we provide a theoretical analysis of the possible origins of crossover effects. The framework within which our analysis is based involves local condensates described by the Landau-Ginzburg (LG) model and coupled to their neighbors via Josephson weak links. Within the LG phenomenology critical current behavior depends on the presence and strength of the various expansion coefficients. We show that the corresponding critical exponent for j_c varies between $\frac{3}{2}$ and 1. We then incorporate the oxygen deficiency effect, which scales the critical temperature as well as other model parameters. In the last part of the paper we investigate the effect of weak links on the critical current's exponent and again show that the latter may vary between the LG and Ambegaokar-Baratoff regimes of $\frac{3}{2}$ and 1, respectively. Combining this analysis with an appropriate choice of model parameters we obtain excellent agreement with the recent experimental data of Darhmaoui and Jung [Phys. Rev. B **53**, 14 621 (1996)]. [S0163-1829(99)00838-3]

I. INTRODUCTION

The importance of critical current behavior in elucidating the various aspects of superconductivity is well recognized. Superconducting current density is limited by depairing and magnetic flux depinning processes.[1,2] The depairing critical current is determined by the pair-breaking strength and results in a supression of the superconducting order parameter. The depinning critical current, on the other hand, is determined by the interplay of the magnetic flux motion and its pinning forces. The superconducting phase remains stable when the current flows through it in the presence of flux lines immobilized by pinning forces. Important theoretical work concerning critical currents was published by Ambegaokar and Baratoff,[3] DeGennes,[4] and Likharev.[5]

Recently, measurements of the temperature dependence of the critical current $I_c(T)$ in YBCO thin films revealed three different types of behavior of I_c on temperature: (a) a ''convex'' curvature[6,7] characteristic of an Ambegaokar-Baratoff-type mechanism, (b) a quasilinear behavior,[6–10] and (c) a ''concave'' curvature characteristic of the Ginzburg-Landau-like mechanism.[6,7,10–14] Mannhart *et al.*[6] showed that the transition from a quasilinear temperature dependence of I_c to a concave temperature dependence is a result of increasing the strength of applied magnetic fields from 0 to 1.0 T. The crossover from quasilinear to concave temperature dependence was also seen by Jones *et al.*[10] upon reduction of the oxygen content.

On the one hand, homogeneous superconducting order parameter systems have been successfully modeled using Landau-Ginzburg theories, which lead to $j_c \sim (T-T_c)^{3/2}$ behavior close to T_c.[15–18] On the other hand, granular superconductors quite often can be described within the Josephson weak link picture, which typically results in a linear scaling of j_c with temperature: $j_c \sim (T-T_c)$ close to T_c.[19] In many high-temperature superconductors a crossover can be observed in the scaling dependence of j_c versus $T-T_c$ as the oxygen deficiency δ is varied, changing from a weak-link-like behavior for small values of the oxygen deficiency δ to Landau-Ginzburg for higher values. Simultaneously, the critical temperature is lowered in a highly nonlinear fashion, as δ is increased.

In our paper we intend to fully explore the range of critical current behavior within the Landau-Ginzburg (LG) model and its extensions in order to provide insight into the observed experimental results for YBCO thin films.[20] Our first theoretical observation will be concerned with a crossover effect within the standard LG model with a sixth power free-energy expansion in the order parameter. We then account for a shift in the critical temperature resulting from the amount of oxygen deficiency.[21] Our final contribution to this analysis incorporates weak links between superconducting grains that are modeled using a combination of local domains of LG-like superconductivity and Josephson proximity effects.[19,22,23] It will be shown that a mapping exists between this fairly complicated picture of a flux lattice[24,23] and the homogeneous model with redressed model parameters that directly motivates our next section.

II. CROSSOVER PHENOMENA WITHIN THE LG FORMALISM

The LG theory of critical phenomena has been very successful over the years in providing a physical interpretation at a phenomenological level, of a host of phase transitions within solid-state physics[15] and elsewhere. In superconductors the order parameter $\Psi(\mathbf{r})$ is a complex function of the spatial coordinate $\mathbf{r}$ and where modulus squared $|\Psi|^2$ represents the condensate density.[16] In the absence of a magnetic field, the free-energy density expansion takes the form

$$G_s(\mathbf{r})=G_n(\mathbf{r})+A(T)|\Psi(\mathbf{r})|^2+\tfrac{1}{2}C|\Psi(\mathbf{r})|^4 + \frac{\hbar^2}{2m^*}|\nabla\Psi(\mathbf{r})|^2, \qquad (2.1)$$

where m^* is the Cooper pair's mass (effective mass), the subscript s refers to the ordered (superconducting) phase, n to the disordered (normal) phase, $A(T)$ is assumed proportional to $(T-T_c)$, i.e., $A(T)=a(T-T_c)$ with $a>0$ and $C>0$ for stability reasons.

A subsequent minimization of $G_s(\mathbf{r})$ with respect to Ψ^* gives rise to the nonlinear equation below:

$$A(T)\Psi+C|\Psi|^2\Psi-\frac{\hbar^2}{2m^*}\nabla^2\Psi=0. \qquad (2.2)$$

Since the order parameter in the argument-modulus form is[17]

$$\Psi=\eta\exp(i\phi), \qquad (2.3)$$

the real part of Eq. (2.2) is

$$A(T)\eta+C\eta^3-\frac{\hbar^2}{2m^*}\nabla^2\eta+\frac{\hbar^2}{2m^*}\eta(\nabla\phi)^2=0, \qquad (2.4)$$

and the imaginary part is

$$\nabla\cdot[\eta^2\nabla\phi]=0. \qquad (2.5)$$

The latter equation represents a continuity equation and it can be satisfied assuming that $\eta^2\nabla\phi=j_s$, where j_s is the superconducting current density:

$$j_s\equiv\frac{-ie^*\hbar}{2m^*}(\psi^*\nabla\psi-\psi\nabla\psi^*)=\frac{e^*\hbar}{m^*}\eta^2\nabla\phi. \qquad (2.6)$$

Combining Eqs. (2.6) and (2.4) gives

$$A(T)\eta+C\eta^3-\frac{\hbar^2}{2m^*}\nabla^2\eta+\frac{m^*j_s^2}{2(e^*)^2\eta^3}=0. \qquad (2.7)$$

This equation has been solved exactly in one-dimensional space[33] with the spatial coordinate x, where it can be integrated once to yield

$$\left(\frac{d\eta}{dx}\right)^2=\frac{m^*}{\hbar^2}\left[2A(T)\eta^2+C\eta^4-\frac{m^*j_s^2}{(e^*)^2\eta^2}\right]+c_0=P(\eta), \qquad (2.8)$$

where c_0 is an arbitrary integration constant. We solve this equation using a geometrical method where we plot, based on Eq. (2.8), $(d\eta/dx)^2$ versus η in a series of diagrams with different values of j_s and assuming that $T<T_c$, i.e., $A(T)<0$. On increasing the value of the current density, nonsingular solutions of this equation become less and less stable and eventually, at $j=j_c$, they disappear completely. This can be seen in Fig. 1, where Eq. (2.8) is represented graphically and each straight line cutting horizontally through the diagram represents a solution at a given value of the integration constant c_0 representing the level with respect to the η axis. Lines having only one intersection point with the potential curve represent singular solutions, while those with two such points represent nonsingular solutions. From Fig. 1

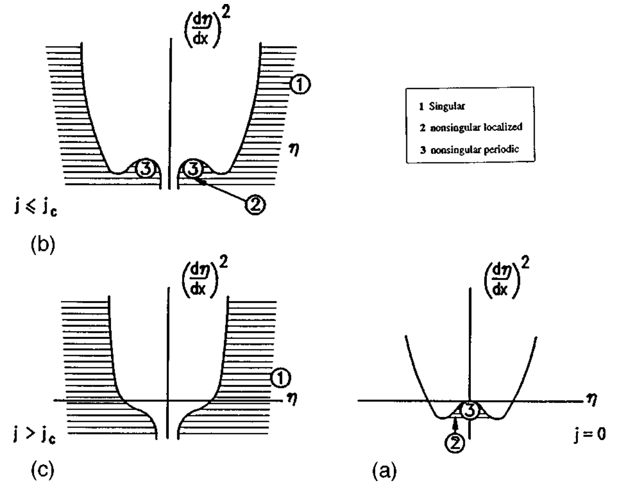

FIG. 1. Graphical illustration of the solutions to Eq. (2.8) with $T<T_c$ and (a) $j=0$, (b) $j\leqslant j_c$, and (c) $j>j_c$.

it is clear that the critical current j_c corresponds to an inflection point of $P(\eta)$, i.e., the first such situation where no nonsingular solutions are allowed (complete destruction of superconductivity or pair breaking). In simple mathematical terms the inflection point is characterized by the coincidence of the two equations below:

$$0=\frac{dP}{d\eta}=\left[2A(T)+2C\eta^2+\frac{m^*j_c^2}{(e^*)^2\eta^4}\right]2\eta \qquad (2.9)$$

and

$$0=\frac{d^2P}{d\eta^2}=4A(T)+12C\eta^2-\frac{6m^*j_c^2}{(e^*)^2\eta^4}. \qquad (2.10)$$

Simple algebra enables one to obtain j_c from Eqs. (2.9) and (2.10) as

$$j_c=\frac{2e^*a}{3C}\sqrt{2a/3m^*}|T-T_c|^{3/2}. \qquad (2.11)$$

This result is in perfect agreement with the well-known $\frac{3}{2}$ exponent for standard type-I superconductors[18] close enough to T_c. The above problem, i.e., the minimization of Eq. (2.1) in the presence of a vector potential, becomes much more complicated as one must minimize with respect to Ψ^* and the vector potential. Nonetheless, exact solutions have been obtained analytically in one and two dimensions[25] and a numerical analysis of this problem has been implemented for YBCO ceramics.[26]

We now wish to examine the same property for superconductors that may undergo a first-order phase transition between the normal and superconducting phases. For conventional superconductors the LG free energy containing a quartic order parameter expansion and the presence of a vector potential does allow for the existence of both first- and second-order phase transitions. This is a result of the competition between the coherence length ξ and the magnetic penetration depth λ, representing two separate length scales. For standard superconductors the ratio of the two, $K=\lambda/\xi$, delineates the boundary between type-I and type-II superconductors in parameter space, with $K<1/\sqrt{2}$ resulting in type-I and $K>1\sqrt{2}$ in type-II superconductivity. Therefore there is no need for standard superconductors to introduce higher-order terms in the free energy. However, for ceramic superconductors the situation is far from clear. Attempts at developing an LG description for high-T_c superconductors have been made in the past (see, for example, Refs. 27–29) and they indicate a strongly type-II layered behavior calling for the inclusion of Lawrence-Doniach coupling between superconducting islands. This indeed is the line of attack we adopted in the main thrust of our paper (see Sec. IV). There is, however, a mathematically much more straightforward way of accomplishing a crossover in the value of the critical exponents and indeed in changing the order of the transition. We will later see that a direct mapping may be made between this simple to analyze approach and the more physically justified multigrain picture developed in Sec. IV. We, therefore, begin by simply adding a sixth power to the free energy expansion so that

$$G_s(r)=G_n(r)+A(T)|\Psi(r)|^2+\tfrac{1}{2}C|\Psi(r)|^4+\tfrac{1}{3}B|\Psi(r)|^6 + \frac{\hbar^2}{2m^*}|\nabla\Psi(\mathbf{r})|^2. \tag{2.12}$$

Following an identical procedure to the one presented above we arrive at an analogue of Eq. (2.8), namely,

$$\left(\frac{d\eta}{dx}\right)^2=\frac{m^*}{\hbar^2}\left\{2A(T)\eta^2+C\eta^4+\tfrac{2}{3}B\eta^6-\frac{m^*j_s^2}{(e^*)^2\eta^2}\right\}+d_0 = R(\eta), \tag{2.13}$$

where d_0 is another integration constant. Before we present a general result that is slightly more involved, let us consider a simple special case, i.e., the vicinity of the tricritical point where a line of first-order phase transitions merges with one for second order and requires that $C=0$. In this case the critical current is found via

$$0=\frac{dR}{d\eta}=2\eta\left[2A+2B\eta^4+\frac{m^*j_c^2}{(e^*)^2\eta^4}\right] \tag{2.14}$$

and

$$0=\frac{d^2R}{d\eta^2}=4A+20B\eta^4-\frac{6m^*j_c^2}{(e^*)^2\eta^4}. \tag{2.15}$$

Solving these two simultaneous equations for j_c yields

$$j_c=\frac{ae^*|T-T_c|}{\sqrt{2Bm^*}}. \tag{2.16}$$

Thus, the tricritical point exponent for the superconducting current density is unity. We infer that a crossover must occur on going from the critical to the tricritical points.

In the general case when the coefficients A, B, and C are not zero, the result for j_c is much more complicated but nevertheless can be expressed analytically as

$$j_c^2=\frac{2e^{*2}}{m^*}\left\{\pm\frac{27C^4}{512B^3}\left(1-\frac{32}{9}\frac{AB}{C^2}\right)^{3/2} + \frac{1}{4B}\left(A^2-\frac{9}{8}\frac{AC^2}{B}+\frac{27}{128}\frac{C^4}{B^2}\right)\right\}. \tag{2.17}$$

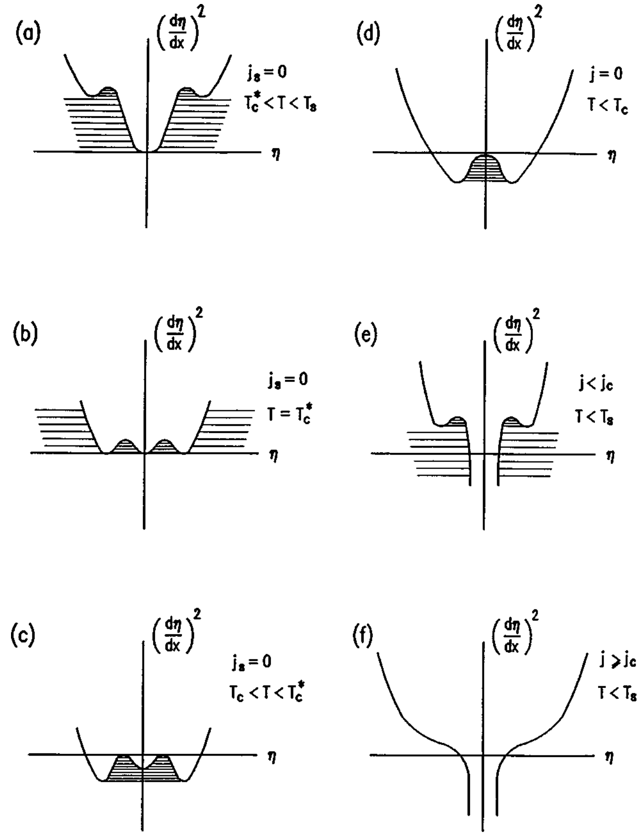

FIG. 2. Graphical illustration of the solutions to Eq. (2.13) when (a) $j=0$; $T_C^*<T<T_s$, (b) $j=0$; $T=T_C^*$, (c) $j=0$; $T_c<T<T_c^*$, (d) $j=0$; $T<T_c$, (e) $j<j_c$; $T<T_s$, and (f) $j\geqslant j_c$; $T<T_s$. Here, $T_c^*=T_c+3B^2/16aC$; $T_s=T_c+B^2/4aC$.

Note that since A is linearly dependent on $(T-T_c)$, the critical exponent will smoothly vary between the asymptotic limits of 1 for $B=0$ and $\frac{3}{2}$ for $C=0$ depending on the relative strength of the quartic coefficient C. A graphical illustration of the types of behavior in Eq. (2.13) for first-order transitions ($C<0$) is shown in Fig. 2, which is analogous to Fig. 1 for second-order transitions. Figure 3 illustrates the scaling of j_c with temperature for various strengths and signs of the quartic coefficient C. It is worth commenting in this regard on the role of the sixth-order contribution to the free energy. While such terms incorporated in lattice Hamiltonians are deemed irrelevant by the results of the renormalization-group technique,[30] since they only redress lower-order contributions, this is not so in mean-field models as was first pointed out by Ginzburg, Levanyuk, and Sobyanin[31] for the exponents β, γ, and δ. It turns out that adding a sixth-power term to a standard Landau free energy allows one to continuously shift the values of the critical exponents β, γ, and δ from the critical point, or so-called ''classical values,'' to the tricritical point values depending on the ratio of the quartic to the sixth-power expansion coefficients. What we have demonstrated in this section of our paper is an analogous crossover effect taking place for the critical current's exponent, which is, therefore, of great relevance to the subject matter of this paper.

Returning to Eq. (2.17) it is very convenient for both a theoretical discussion and fitting to experiment to notice that if we define a parameter μ by

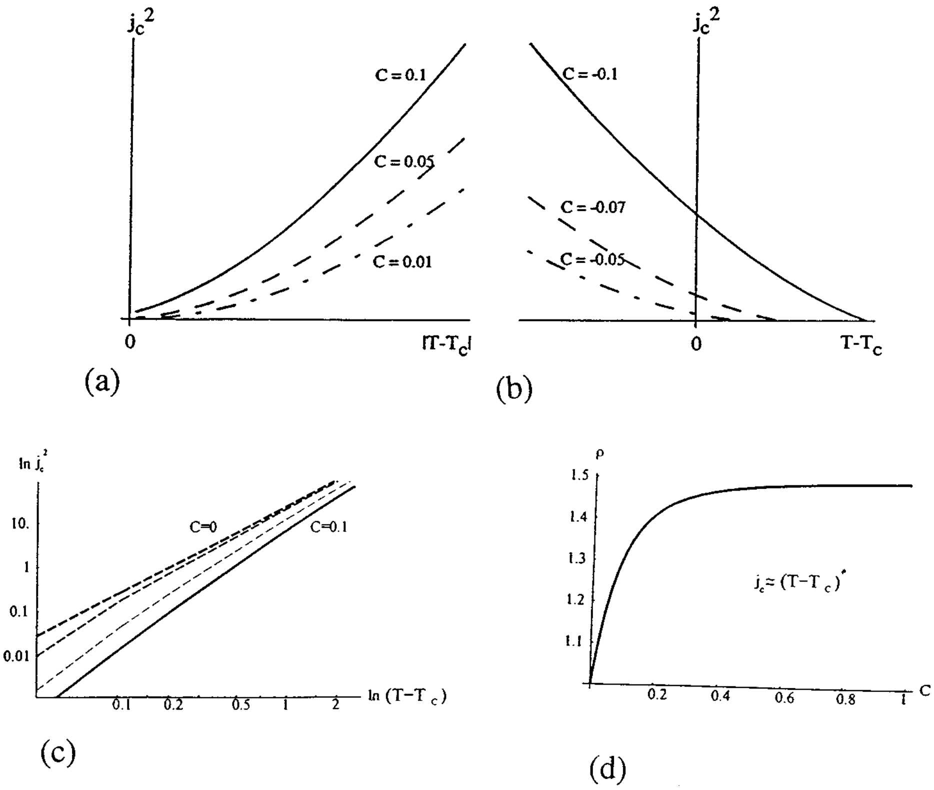

FIG. 3. Plots of $j_c^2(T)$ for different values of C and thus μ following Eqs. (2.19) and (2.18).

$$\mu = AB/C^2 = aB(T-T_c)/C^2, \tag{2.18}$$

then this expression for j_c^2 may be written as

$$j_c^2 = \frac{2e^2}{m^*}\frac{C^4}{512B^3}\left\{\pm 27\left(1-\frac{32\mu}{9}\right)^{3/2} + 128\left(\mu^2-\frac{9}{8}\mu+\frac{27}{128}\right)\right\}. \tag{2.19}$$

A normalized j_c^2 for example, $j_c^2(T)/j_c^2(T=10\,\mathrm{K})$, can thus be described with one parameter only, namely, μ. Close to $\mu \simeq 0$ the term $(1-32\mu/9)^{3/2}$ may be expanded to third order in μ to give

$$\left(1-\frac{32\mu}{9}\right)^{3/2} \simeq 1-\frac{48\mu}{9}+\frac{128}{27}\mu^2-\frac{2048}{729}\mu^3, \tag{2.20}$$

and the critical current, in this approximation [taking the minus sign in Eq. (2.19)], becomes

$$j_c^2 \simeq \frac{8e^2}{27m^*}\frac{a^3}{C^2}(T-T_c)^3. \tag{2.21}$$

In this case j_c, as a function of temperature, has an LG-type of behavior. On the other hand, when $\mu \ll -1$ the term in μ^2 in Eq. (2.19) will dominate to give

$$j_c^2 \simeq \frac{e^2a^2(T-T_c)^2}{2m^*B}. \tag{2.22}$$

The extended LG model so far, therefore, exhibits a crossover in the temperature dependence of j_c from $(T-T_c)^{3/2}$ to $(T-T_c)$. While we do not expect the sixth-power expansion to be a realistic representation for the YBCO thin films we study in this paper, the calculations provided in this section give an important building block of the theoretical model. We will show in Sec. IV that the equations for a more realistic free-energy expansion can still be mapped on the simple model described here. It should also be added, however, that the simultaneous presence of several degrees of freedom in high-T_c materials may indeed require an effective free-energy expansion with a sixth-power term. This, for example, has been the case with the coexistence of superconductivity and magnetism.[32] In high-T_c superconductors one should indeed develop a phenomenology that includes not only the superconducting order parameter but also the order parameters for antiferromagnetic and the structural reordering due to the nature of the phase diagram. Trying to maintain the level of mathematical complexity to a minimum we turn our attention now to what we view as the key departure from a standard LG-type of behavior, namely, the presence of oxygen deficiencies.

III. OXYGEN DEFICIENCY EFFECTS

In order to make contact with the experimental results of Ref. 20, we must account for the existence of samples with different oxygen deficiencies δ. The effect of the oxygen deficiency on the critical current profiles turned out to be more complex than first anticipated. Three distinct influences of δ on the model parameters have been found. First, following Ref. 21, we included the shift of the critical temperature T_c as a result of using different δ's. A simple formula was found that approximates the functional dependence of T_c on δ for the low and intermediate values of δ. It is given by

$$T_c = 73 - 18\tanh[12(\delta - 0.26)] \tag{3.1}$$

(see Fig. 4). When this is included in the formula for the critical current we notice an obvious spreading of the curves

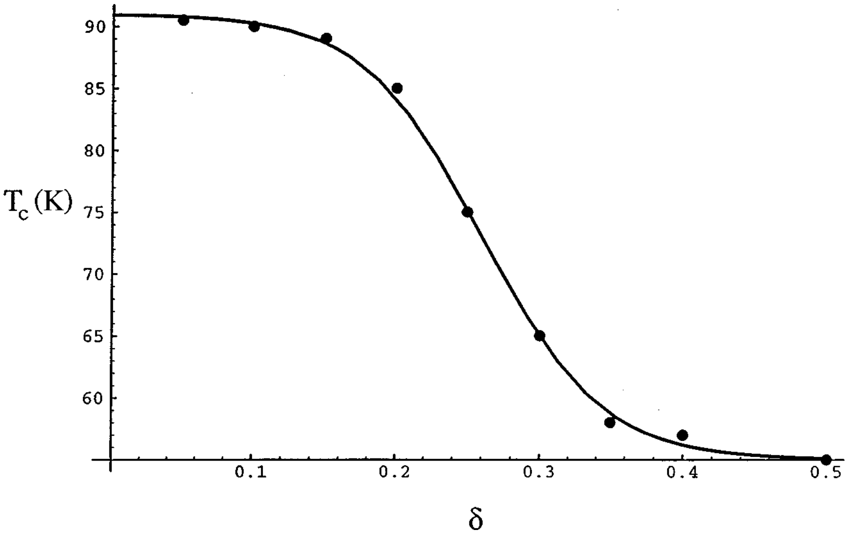

FIG. 4. Diagrammatic representation of the variation of T_c with δ following the results of Jorgensen *et al.* (Ref. 21).

for different δ's [see Fig. 5(a)]. Comparison of these plots with those of the experiment [see Fig. 5(b)] shows a certain degree of similarity but is still quantitatively inadequate. This raises the question of the second effect of δ, namely, a redressing of the remaining model parameters. Since the formula for j_c^2 is expressed in terms of a constant prefactor and a variable μ, we first focus our attention on the role of μ.

There is a suggestive similarity between our empirical formula. Eq. (3.1), and the Ambegaokar-Baratoff formula,[5]

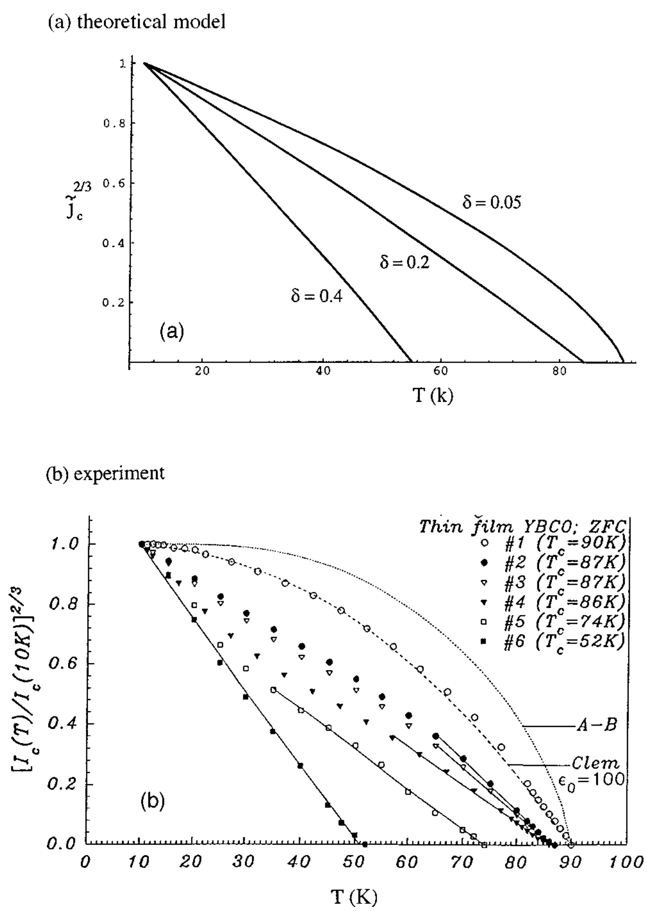

FIG. 5. Theoretical (a) and experimental (b) plots of $j_c^{2/3}$ as a function of temperature for a number of values of δ.

which links the superconductor-insulator-superconductor coupling constant J_{ij} with the average energy gap $\Delta(T)$:

$$J_{ij}\ \alpha\ \Delta(T)\tanh[\Delta(T)/2kT]. \tag{3.2}$$

The ordering temperature for spin systems is proportional to J_{ij} and if the energy gap, as a first-order approximation, is a linear function of oxygen deficiency, δ, this may well give an explanation of its form

$$\mu = \frac{aB}{C^2}(T-T_c), \tag{3.3}$$

and we feel it is both prudent and justified to expand both B and C linearly in terms of δ, i.e.,

$$B \simeq B_0 + \delta B_1, \quad C \simeq C_0 + \delta C_1. \tag{3.4}$$

Thus, to first order we get

$$\mu \simeq \frac{aB_0}{C_0^2}\left[1+\left(\frac{B_1}{B_0}-2\frac{C_1}{C_0}\right)\delta\right](T-T_c). \tag{3.5}$$

Since B is the coefficient of the Ψ^6 term in our model, it is most likely to be purely electronic (Coulomb interaction) in origin. We, therefore, postulate that this interaction is reduced in magnitude as a result of introducing holes and thus

$$B_0 > 0, \quad B_1 < 0. \tag{3.6}$$

We have incorporated the linear expansion in δ for μ [see Eq. (3.5)] in the calculation of j_c and a sampling of graphical results is given in Fig. 5.

Having corrected the initial temperature and the fitting parameter μ for the presence of oxygen deficiency, we have set out to obtain precise fits to the experimental plots of $j_c(T)$ published in Ref. 20. However, we encountered another obstacle especially for the plots exhibiting an inflection point such as sample 1 of Ref. 20 in zero field. What we wish to address in the remainder of this paper is the need for further improving the model such that the effects of granularity are properly taken into account. Until now we have not accounted for the presence of superconducting islands in the thin film structure under consideration and the methods used

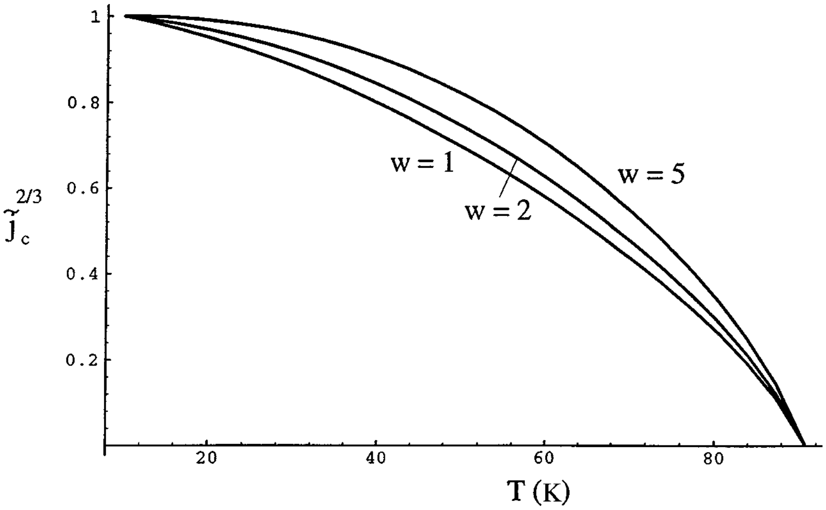

FIG. 6. Plots of $j_c^{2/3}$ versus T based on Eq. (4.18) using several values of parameter w.

are strictly speaking only applicable to a homogeneous distribution of superconducting phase.

IV. WEAK LINKS ADDED TO THE LG MODEL

In this section we wish to explore the effect of the granular nature of the new high-temperature superconductors, which has been seen using a number of experimental techniques, including anomalous voltage excursions, as a function of temperature and magnetic field,[23] logarithmic time decays of magnetization,[34] flux trapping,[35] and tails of sensitivity versus temperature due to boundary resistance between grains.[36] What we wish to do is to maintain the simplicity of a phenomenological approach but incorporate in the free-energy density a contribution that partially describes the role played by oxygen vacancies in creating superconducting grains. The depletion or excess of oxygen ions first leads to the creation of additional charge carries.[37] Since the latter are holes with a high effective mass compared with electrons, we expect a much greater degree of localization than in the low-temperature standard superconductors. For the inclusion of an appropriate free-energy density we follow the ideas of Deutscher and Müller[24] and visualize regions smaller than the mean grain size. These correspond to domains of superconductivity and may be separated by regions of normal phase. We suppose, at least initially, that these islands are well separated and interactions between them are rather weak so that they may be modeled using the weak-link approximation, as arrays of Josephson junctions.[38] In the vicinity of the critical temperature we represent the free-energy density as a sum of individual LG contributions for each of the superconducting islands with interactions between them in the Lawrence-Doniach form.[40] We also assume that there is no magnetic field present. Thus following our earlier work,[41] we write the free energy as

$$G_s=\int [G_n+G_1+G_2]d\mathbf{r}, \tag{4.1}$$

where

$$G_1=\sum_n \left\{A_2|\psi_n|^2+A_4|\psi_n|^4+A_6|\psi_n|^6+\frac{\hbar^2}{2m^*}|\nabla_n\psi_n|^2\right\} \tag{4.2}$$

and

$$G_2=\tfrac{1}{2}\sum_{\langle n\neq l\rangle} b_{nl}|\psi_n-\psi_l|^2. \tag{4.3}$$

In the expressions for G_1 and G_2, n and l denote superconducting islands whose size is on the order of the coherence length and we denote the order parameter for each island by ψ_n. The notation $\langle n\neq l\rangle$ expresses the fact that we only include interactions with *nearest*-neighbor islands. The parameter b_{nl} is a distance-dependent interisland coupling constant. As we did earlier, we write $A_2=\overline{a}(T-T_c)$ and assume, as for A_4, that it is independent of the particular island.

Minimizing the free-energy functional with respect to ψ_n^* we obtain

$$0=A_2\psi_n+A_4|\psi_n|^2\psi_n+A_6|\psi_n|^4\psi_n-\frac{\hbar^2}{2m^*}\nabla_n^2\psi_n + \tfrac{1}{2}\sum_{\langle l\neq n\rangle} b_{nl}(\psi_n-\psi_l). \tag{4.4}$$

Writing each other parameter ψ_n in modulus-argument form, $\psi_n=\eta\exp(i\phi_n)$ and assuming, for each island, $\eta_n=\eta_l=\eta$, we find for the real part of the above equation

$$0=A_2\eta+A_4\eta^3+A_6\eta^5-\frac{\hbar^2}{2m^*}[\nabla^2\eta-\eta(\nabla\phi_n)^2] + \tfrac{1}{2}\eta\sum_{\langle l\neq n\rangle} b_{nl}\{1-\cos(\phi_n-\phi_l)\}. \tag{4.5}$$

The corresponding imaginary component yields

$$0=2\nabla\eta\cdot\nabla\phi_n+\eta\nabla^2\phi_n+\tfrac{1}{2}\eta\sum_{\langle l\neq n\rangle} b_{nl}\sin(\phi_n-\phi_l). \tag{4.6}$$

A simple physical approximation now reduces these equations to a more manageable form in one dimension if we write $\phi_{n\mp1}\cong\phi_n\mp|\nabla\phi|a$, where a is related to the lattice spacing between the islands. The sine terms in the equation for the imaginary parts vanish and by putting $b_{nl}=b$ for every pair of nearest-neighbor islands we find

$$0=A_2\eta+A_4\eta^3+A_6\eta^5-\frac{\hbar^2}{2m^*}[\nabla^2\eta-\eta(\nabla\phi)^2]$$
$$+\eta b[1-\cos(a|\nabla\phi|)] \tag{4.7}$$

and

$$0=2\eta\nabla\eta\cdot\nabla\phi+\eta^2\nabla^2\phi=\nabla\cdot(\eta^2\nabla\phi), \tag{4.8}$$

where we have used the fact that $\nabla\phi\simeq(1/a)(\phi_n-\phi_{n-1})\simeq\nabla\phi_n$.

We can now argue similarly to the case with no weak links in one dimension to find a $Q(\eta)$ where

$$Q(\eta)=\left(\frac{d\eta}{dx}\right)^2=\frac{m^*}{\hbar^2}\left\{2A(T)\eta^2+C\eta^4+\tfrac{2}{3}B\eta^6-\frac{j_s^2}{\eta^2}\frac{m^*}{e^2}\right.$$
$$\left.+4b\int\eta\left[1-\cos\left(\frac{aj_s}{\eta^2}\right)\right]d\eta\right\}, \tag{4.9}$$

where we have made the identification

$$A_2\equiv A(T),\quad A_4\equiv C,\quad A_6\equiv B. \tag{4.10}$$

Putting $dQ/d\eta=0$ and $d^2Q/d\eta^2=0$ as before in order to find the conditions for the critical current we obtain analogous equations to those derived earlier, namely [see Eqs. (2.14) and (2.15) for comparison],

$$0=2\eta\left\{2A+2B\eta^4+2C\eta^2+\frac{m^*}{e^2}\frac{j_c^2}{\eta^4}+2b\left[1-\cos\left(\frac{aj_c}{\eta^2}\right)\right]\right\} \tag{4.11}$$

and

$$0=4A+12C\eta^2+20B\eta^4-6\frac{m^*}{e^2}\frac{j_c^2}{\eta^4}+4b\left[1-\cos\left(\frac{aj_c}{\eta^2}\right)\right.$$
$$\left.-\frac{2aj_c}{\eta^2}\sin\left(\frac{aj_c}{\eta^2}\right)\right], \tag{4.12}$$

except now island-island interaction terms are present in the square brackets of Eqs. (4.11) and (4.12).

Expanding cosine and sine functions in Eqs. (4.11) and (4.12) up to the lowest nonconstant terms clearly gives a dressing of the model parameters due to the weak links, and we find

$$0\simeq2\eta\left\{2A+2B\eta^4+2C\eta^2+\frac{j_c^2}{\eta^4}\left(\frac{m^*}{e^2}+ba^2\right)\right\} \tag{4.13}$$

and

$$0\simeq4A+12C\eta^2+20B\eta^4-6\frac{j_c^2}{\eta^4}\left(\frac{m^*}{e^2}+a^2b\right). \tag{4.14}$$

Solving the two equations above for j_c gives

$$j_c=\left(\frac{2e}{m^*a}\right)\eta^2\left[-\frac{24}{b}(A+\tfrac{3}{2}B\eta^2+2C\eta^4)\right]^{1/4}. \tag{4.15}$$

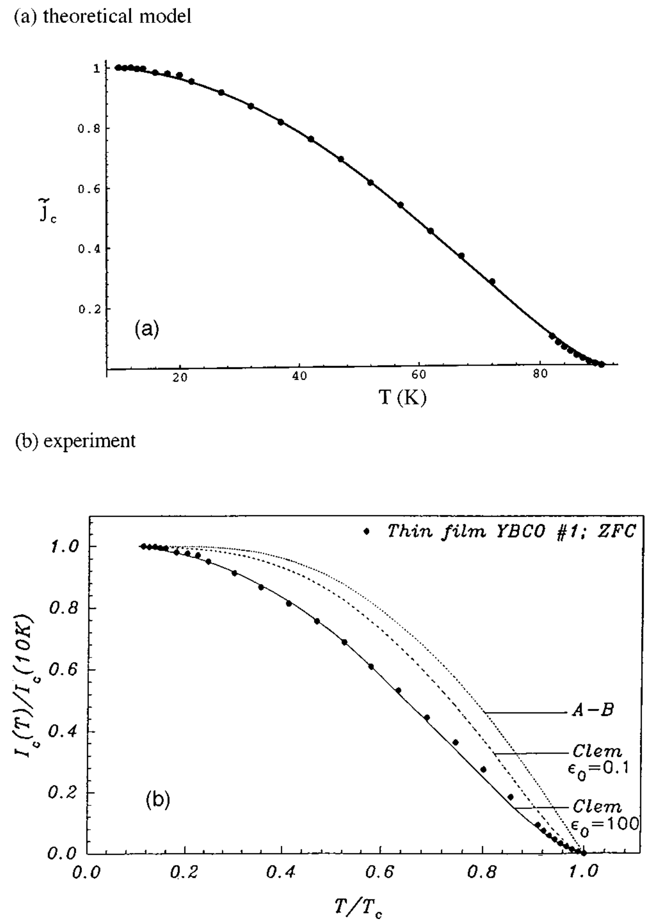

FIG. 7. The best fit of our theoretical curve for the normalized critical current $\tilde{j}_c$ to the experimental data points of Darhmaoui and Jung (Ref. 20).

To analyze the dominant scaling behavior of j_c with respect to $T-T_c$ we write the amplitude of the order parameter as $\eta\sim(T-T_c)^\beta$; then the dominant exponent from Eq. (4.15) will be

$$j_c\sim(T-T_c)^{3\beta}, \tag{4.16}$$

giving a value between 1 and $\frac{3}{2}$, since $\frac{1}{3}\leqslant\beta\leqslant\frac{1}{2}$, for all superconductors.[17,18] This is still in quantitative agreement with our discussion early in the paper regarding the crossover effects between LG and Ambegaokar-Baratoff regimes in granular superconductors in spite of the fact that the earlier calculations ignored the effects of granularity.

With weak links present not only is the expansion coefficient A temperature dependent, but some authors have suggested that so also is the island-island interaction.[39] In order to gain an insight into this possible new effect we shall approximate and suppose it takes the form

$$b=\alpha\exp(-T/w), \tag{4.17}$$

where α and w are constants and T is the absolute temperature. Furthermore, to relate to our earlier work without weak links, we assume the sine and cosine terms in Eqs. (4.11) and (4.12) may be expanded and only terms up to $1/\eta^4$ need be retained. We find that our equations *with* weak links are now of the same form except j_c^2 scales according to (see Figs. 6 and 7)

$$j_c^2 \rightarrow j_c^2 \left[1 + \frac{a^2 e^2}{m^*} \alpha \exp(-T/w) \right]. \qquad (4.18)$$

We have tried assessing the importance of scaling in Eq. (4.18) for a range of values of w as shown in Fig. 6 for $j_c^{2/3}(T)$. Note first that increasing the value of w enhances the curvature of the plot. When we incorporate this formula in our fitting procedure the values $w = 4.2$ and $\alpha = 30$ resulted in an almost perfect fit to the experimental data set (see Fig. 7).

It is worth noting that a simple explanation of the exponential dependence in Eq. (4.17) may be readily afforded from earlier work on the proximity effects in Josephson junctions, the Hamiltonian for which is[5,19,24,38]

$$H = -\sum_{ij} J_{ij} \cos(\phi_i - \phi_j - a_{ij}), \qquad (4.19)$$

where $a_{ij} = (2e/\hbar c)\int_i^j \mathbf{A} \cdot d\boldsymbol{l}$, $\boldsymbol{A}$ is the vector potential over which a path integral is taken. A nonzero interaction constant J_{ij} is proportional to $\exp[-d_{ij}/\xi(T)]$, where d_{ij} is the separation between islands and $\xi(T)$ is a coherence length in the normal matrix.[19,38] The coherence length $\xi(T)$ is typically proportional to a negative power of $T - T_c$. Hence $d_{ij}/\xi(T)$ becomes linear in temperature provided we are not too close to $T = T_c$. The constant can be considered to be absorbed into α in Eq. (4.17).

V. CONCLUSIONS

This paper has been concerned with the diverse origins of crossover effects in the temperature dependence of superconducting critical currents. The analysis presented here is entirely phenomenological, based on the Landau-Ginzburg model of superconductivity, which has been subsequently augmented by the presence of Josephson weak links via the Lawrence-Doniach term in the free energy. A major simplification in the analysis of the island-island problem was afforded by a direct mapping between a sixth-power LG expansion and a corresponding free energy with the presence of Lawrence-Doniach terms. We have then demonstrated analytically and numerically how the critical exponent for the superconducting current scales with the relative strength of the quartic- and sixth-power terms in the free energy. This is further modified by the oxygen deficiency parameter δ, together with the critical temperature's dependence on δ as revealed by the experiment. When weak links have been incorporated in our description in order to account for the granularity of analyzed samples, it was possible to make direct contact with the experimental data of Darhmaoui and Jung[20] by obtaining excellent fits to their plots of $j_c(T)$. Since this is a purely phenomenological model it would be desirable, in future work, to find an underlying microscopic Hamiltonian that would also shed light on the mechanism through which oxygen deficiency affects the superconducting critical current.

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