Vortex core spectroscopy and vortex cooling: A photovoltaic effect

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Resonant optical-absorption spontaneous emission of photons by quasiparticles inside of a vortex core in type-II superconductors results in a net momentum transfer to the vortex (scattering force). Similar to the case of atomic physics (laser cooling), this scattering force is *quantum* in nature, nonlinear in the vector potential of the photon, and can be used to *cool* or *heat* vortices. The problem here is not as simple as in the case of an atom since the vortex is a self-consistent field embedded in a superfluid background. To illustrate this *cooling* or *heating* effect we calculate the difference between the total dissipation with and without the scattering force as a function of frequency as well as the voltage change due to the presence of the scattering force. Possible experimental connections with $NbSe_2$, $Nb_{1-x}Ta_xSe_2$, and $Nd_{2-x}Ce_xCuO_4$ are also made.
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I. INTRODUCTION

For a few years it has been known both theoretically[1–3] and experimentally[4,5] that atoms can be cooled and manipulated by electromagnetic radiation. It is simple to understand why. An atom of mass M traveling through a medium at rest (air or vacuum), with a given velocity $\mathbf{v}$ and an atomic transition frequency ω_{ge} between the ground state (g) and an excited state (e) can absorb photons with energy ω and momentum $\hbar\mathbf{k}$, provided that the selection rules for absorption are right. The photon absorption is therefore directional and along the $\mathbf{k}$ vector. Since the atom is moving, the atomic transition seen by the photon is Doppler shifted by an amount equal to $\mathbf{v}\cdot\mathbf{k}$. Therefore, following the absorption of a photon with frequency $\omega=\omega_{ge}+\mathbf{v}\cdot\mathbf{k}$, the atom changes its velocity from $\mathbf{v}$ to $\mathbf{v}+\Delta\mathbf{v}$, where $\Delta\mathbf{v}=\hbar\mathbf{k}/2M$. The spontaneously emitted photon, on the other hand, has frequency $\omega'=\omega_{ge}+\mathbf{k}'\cdot(\mathbf{v}-\Delta\mathbf{v})$ and a momentum $\hbar\mathbf{k}'$. In the spontaneous emission process, the photon is emitted with equal probabilities in all directions, therefore no average momentum change is felt by the atom. As a consequence there is a net scattering force of *quantum* nature acting on the atom, which accounts for a net momentum transfer from the absorption spontaneous emission process.[6] If light is treated classically there is no such effect, since there is no spontaneous emission in a classical theory. The existence of this scattering force may dramatically change the atoms kinetic energy because the frequency of the emitted light ω' is different from that of the absorbed light ω: the atom may be *cooled* when $\omega'>\omega$ by losing its kinetic energy to radiation field or *heated* when $\omega'<\omega$ by gaining kinetic energy from the radiation field. Notice that when the photon beam and the atoms are counterpropagating ω' is always larger than ω, hence *cooling* takes place.

It is important to emphasize that a large reduction or increase in the atoms kinetic energy is only possible at resonance. To achieve resonant conditions when the speed of the atom $|\mathbf{v}|$ is an appreciable fraction of the speed of light c it is necessary to take into account the effects of the Doppler shift in the atomic transition. In this case, an appreciable reduction (cooling) or increase (heating) of the atoms kinetic energy can be achieved either by tuning continuously the laser frequency to the Doppler shifted atomic transition or by keeping the laser frequency fixed and tuning the atomic transition to the laser frequency via the dynamical Zeeman effect. On the other hand, when $|\mathbf{v}|/c\ll 1$, the Doppler shift is not so important and a reasonable change in the atoms kinetic energy can still be achieved. In any case, the reduction or increase of the atoms kinetic energy is strongest at perfect resonance.

Thus it is natural to ask the question: is it possible to cool or put into motion a single vortex in a superconductor via a resonant optical absorption and spontaneous emission process? It is the purpose of this paper to provide a more detailed answer to this question, which was first outlined a few years ago.[7] The proposed cooling or heating effects have some analogies with the atomic physics picture, but also some striking differences. Among the analogies, vortices have core states that may couple with light and a resonant absorption spontaneous emission process may take place. Among the differences, a vortex is a self-consistent object that moves on a superfluid background, hence its dynamics is quite different than that of a moving atom in free space. It is our purpose here to show that vortices may be cooled or heated by the presence of a quantum light field, despite the additional complications in the vortex motion caused by the superfluid background. This quantum light field may resonantly excite vortex core states in an absorption spontaneous emission process, thus transferring a net momentum to the vortex core due to the existence of a scattering force. In addition, the scattering force is nonlinear in the vector potential of the photon and may be used to cool or heat vortices. To illustrate this cooling or heating effect proposed here, we calculate the frequency dependence of the total dissipation and the total voltage changes due to the presence of the scattering force.

In order to provide a meaningful answer to the proposed question we organize the rest of this paper as follows. In Sec. II, we choose a physical system, establish the existence of vortex core states, describe the quasiparticle wave functions, and analyze the coupling of core states to photons. In Sec. III, we discuss the nature of the scattering force due to light, the selection rules, and the resonance condition. In Sec. IV,

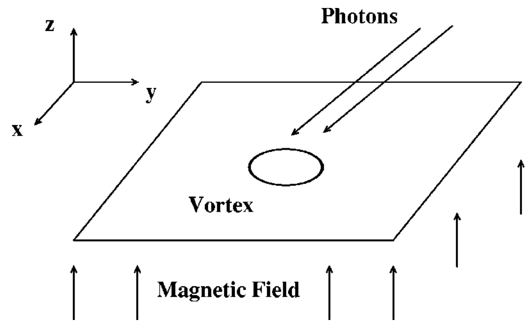

FIG. 1. The geometry is indicated in this figure. The magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$ and the vortex moves along the xy plane.

we describe the effects of the scattering force on the dissipation (cooling and heating) and induced voltages (photovoltaic effect) caused by the vortex motion. In Sec. V, we make experimental connections to $NbSe_2$, $Nb_{1-x}Ta_xSe_2$, and $Nd_{2-x}Ce_xCuO_4$. Finally, in Sec. VI, we summarize our main results.

II. CHOICE OF SYSTEM

We begin then with the choice of a system. We consider the situation corresponding to a superconducting thin film of thickness d (z direction) and infinite dimensions in the xy plane. We further assume that the superconductor is in the clean limit, that the superconducting state is s wave, and that a weak-coupling theory is valid. The electrons are considered to have effective masses m along the xy planes, m_z along the z direction, and coherence lengths ξ_0 along the xy plane and ξ_z along the z direction. In addition, we consider this system to be at zero temperature ($T=0$) for magnetic fields (H) perpendicular to the xy plane ($\mathbf{H}\|\hat{\mathbf{z}}$). The superconductor is assumed to be of extreme type II, where the penetration depth $\lambda \gg \max[\xi_0,\xi_z]$. We also assume that H is slightly above the lower critical field H_{c_1}, i.e., $H_{c_1}\approx H \ll H_{c_2}$ such that the single vortex approximation is justified.[8] In addition, we neglect the quantization effects of the magnetic field and treat H in the semiclassical regime (eikonal approximation). The penetration depth λ is assumed to be much larger then the film thickness d, such that the current distribution is uniform along the z direction. An illustration of the geometry considered can be seen in Fig. 1.

The choices made here are perhaps closer to experiments performed in the s-wave superconductors $NbSe_2$ (Ref. 9) and $Nb_{1-x}Ta_xSe_2$,[10] where the existence of vortex core states is well established. This means that it may be possible for the scattering force to act on these core states. Another system which can be potentially studied are the n-type high-temperature superconductors $Nd_{2-x}Ce_xCuO_4$ (Ref. 11) which are believed to be s wave. Although we are interested here in s-wave superconductors, it is important to mention recent experimental studies in $YBa_2Cu_3O_{7-\delta}$, which is believed to be a d-wave superconductor and where vortex core states have been observed.[12] These experimental findings motivated several theoretical papers dealing with d-wave vortex core states.[13–16] It is thus possible that the effects proposed here for s-wave superconductors may also be encountered in the cuprate oxides like $YBa_2Cu_3O_{7-\delta}$, but that remains to be investigated.

Having made the choice of an s-wave system with the assumptions described above, we are ready to move on to the analysis of core states, which shall be discussed next.

A. Vortex core states

For the s-wave system in consideration, under all the previous assumptions, the eigenenergies and eigenfunctions for the core states of a static vortex are obtained from the Bogoliubov-deGennes (BdG) equation.[17,18] When $\xi_z \gg d$, the gap function is essentially uniform along the z direction and the BdG equation is separable in cylindrical coordinates (ρ,ϕ,z). When the quasiparticles are confined within the film $(0\leqslant z\leqslant d)$, the eigenfunctions along the z direction are $F_n(z)=\sqrt{(2/d)}\sin(n\pi z/d)$, where n is an integer. Exploring the azimuthal symmetry of the problem we can write the BdG equation in polar coordinates $\mathbf{r}\equiv(\rho,\phi)$, with the eigenfunctions

$$\psi_\nu(\mathbf{r})=\begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix}, \tag{1}$$

where $u_\nu(\mathbf{r})$ and $v_\nu(\mathbf{r})$ are the Bogoliubov quasiparticle amplitudes. It is best, though, to express $\psi_\nu(\mathbf{r}) = \mathbf{R}_\nu^{(\sigma_z)}(\rho)\exp[i(\nu-\sigma_z\phi/2)]$. With this decomposition the radial part of the BdG equation becomes

$$\hat{\mathbf{H}}_\nu(\rho)\mathbf{R}_\nu^{(\sigma_z)}(\rho)=E_\nu\mathbf{R}_\nu^{(\sigma_z)}(\rho), \tag{2}$$

with

$$\hat{\mathbf{H}}_\nu(\rho)=\sigma_z\frac{\hbar^2}{2m}\left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\lambda_\nu(\rho)\right]+\sigma_x\Delta(\rho), \tag{3}$$

where $\lambda_\nu(\rho)=\beta_\nu^2(\rho)/\rho^2-k_\rho^2$ and $\beta_\nu(\rho)=\nu-\sigma_z A_\phi e\rho/\hbar c$. The index ν is an angular momentum quantum number that takes half integer values. The matrices σ_x and σ_z are Pauli matrices and k_ρ is given by $k_\rho^2=2m\mu_F/\hbar^2-(n\pi/d)^2(m/m_z)$, where μ_F is the chemical potential.

In order to find out the eigenvalues and eigenfunctions of the states inside the vortex core we choose a gauge where the gap function $\Delta(\rho)$ is real. We also take the explicit form

$$\Delta(\rho)=\Delta_\infty\left[\frac{\rho}{\xi_0}+\left(1-\frac{\rho}{\xi_0}\right)\Theta(\rho-\xi_0)\right] \tag{4}$$

as an approximation of the gap function with $\xi_0\approx\hbar v_{F_\rho}/\Delta_\infty$ being the usual coherence length from weak-coupling theory, such that $k_{F_\rho}\xi_0\gg 1$. The eigenenergies $E_{n,\nu}=E_\nu+E_n$ are defined relative to the chemical potential μ_F. In the limit $|\nu| \ll k_{F_\rho}\xi_0$ the eigenenergies E_ν are given by

$$E_\nu\approx\frac{\nu}{k_F\xi_0}\Delta_\infty C, \tag{5}$$

where $C=0.9451$, while $E_n=(\hbar^2/2m_z)(n\pi/d)^2$. Notice that the vortex core spectrum $E_{n,\nu}$ is symmetric as a consequence of particle-hole symmetry in weak coupling. Notice in addition that the energies in Eq. (5) can also be written in a very simple form depending only on ξ_0, i.e., $E_\nu=\nu\hbar^2 C/m\xi_0^2$. This expression is qualitatively similar to E_n, and its form could have been guessed by considering a simple quantummechanical estimate, i.e., the energy of a quasiparticle of mass m in a box of characteristic length ξ_0. The eigenenergies then become

$$E_{n,\nu}=E_\nu[1+(n^2\pi^2/2\nu C)\times(m/m_z)\times(\xi_0/d)^2].$$

From now on we will focus on the limit where $m/m_z\ll 1$, such that the correction to the energy E_ν is small even for large n, i.e., such that

$$|n|\ll[(2|\nu|C)^{1/2}/\pi](m_z/m)^{1/2}(d/\xi_0).$$

For instance, when $|\nu|=1/2$, $d/\xi_0=3$ and $m_z/m=10^4$, $|n|\ll 942$.[19]

Hence in the limit $m/m_z\to 0$, the characteristic low-energy spectrum of quasiparticles inside of a vortex core is governed by E_ν, which can be rewritten as $E_\nu=2\nu C R_y(a_0/\xi_0)^2$ given in eV. Here $R_y=13.61$ eV is the Rydberg and $a_0=0.529$ Å is the Bohr radius. For large coherence length superconductors $\xi_0\approx 2000$ Å the quasiparticle excitation spectrum lies on the microwave range $E_{1/2}\approx 0.9\times\mu$eV and can be accessible by a maser. On the other hand, for short coherence length superconductors $\xi_0\approx 30$ Å, $E_{1/2}\approx 4.0\times$ meV, so that the spectrum may be explored by far-infrared spectroscopy. Throughout this paper we will be interested only in the low-temperature limit where the positive energy levels ($E_\nu>0$) have essentially no thermal occupation. This low-temperature limit is defined by $T\ll\min[E_\nu]$, where $\min[E_\nu]=CR_y(a_0/\xi_0)^2$. For instance, for $\xi_0=30$ Å this means $T\ll 46.4$ K and for $\xi_0=2000$ Å it implies $T\ll 10$ mK. Thus large coherence lengths impose a much more stringent requirement on the meaning of the low-temperature limit.

Vortex core states in s-wave systems were observed at low temperatures, first in $NbSe_2$ (Ref. 9) and later in $Nb_{1-x}Ta_xSe_2$ (Ref. 10) using scanning tunneling microscopy (STM). In addition, vortex core states were also observed in the cuprate oxide $YBa_2Cu_3O_{7-\delta}$ using two different spectroscopic techniques: far-infrared spectroscopy[20] and STM.[12] The work on far-infrared spectroscopy by Karrai *et al.*[20] renewed the theoretical interest in the optical properties of vortex core states.[21,22] Vortex core states for s-wave superconductors were first studied many years ago,[17,18] where it was noticed that the physical properties (e.g., optical absorption) associated with vortex core states are not sensitive only to the quasiparticle energies but also sensitive to the quasiparticle wave functions, which we shall briefly study next.

B. Quasiparticle wave functions

Following the pioneering work of Caroli, Matricon, and de Gennes[17] now identify the radial wave functions corresponding to the energies defined in Eq. (5) with the choice of $\Delta(\rho)$ given in Eq. (4). The radial wave functions $\mathbf{R}_\nu^{(\sigma_z)}(\rho)\equiv g_\nu^{(\sigma_z)}(\rho)$ have a characteristic spatial extension given by the parameter $\rho_{b,\nu}\ll\xi_0$, provided that $|\nu|\ll k_F\xi_0$. For $\rho\ll\rho_{b,\nu}$, the radial wave functions are

$$g_\nu^{(\sigma_z)}(\rho)\approx A_{\sigma_z}J_{\nu-\sigma_z/2}(k_F\rho), \tag{6}$$

where A_{σ_z} is the normalization constant and $J_r(z)$ is the Bessel function of order r and argument z. The corresponding radial wave functions for $\rho\gg\rho_{b,\nu}$ are

$$g_\nu^{(\sigma_z)}(\rho)=B_{\sigma_z}F_\nu(\rho)\exp[-\Lambda(\rho)]\chi^{(\sigma_z)}(\rho), \tag{7}$$

where B_{σ_z} is a normalization constant and

$$\chi^{(+)}(\rho)=\cos\left[k_F\rho+\frac{\eta^2}{2k_F\rho}-\frac{\pi\nu}{2}-\frac{\Omega(\rho)}{2}\right], \tag{8}$$

$$\chi^{(-)}(\rho)=\sin\left[k_F\rho+\frac{\eta^2}{2k_F\rho}-\frac{\pi\nu}{2}+\frac{\Omega(\rho)}{2}\right]. \tag{9}$$

The auxiliary functions appearing in Eq. (7) are

$$F_\nu(\rho)=2\left[\frac{2}{\pi\sqrt{k_F^2\rho^2-\eta^2}}\right]^{1/2}, \tag{10}$$

with $\eta^2=\nu^2+1/4$,

$$\Lambda(\rho)=\frac{1}{\hbar v_F}\int_0^\rho\Delta(\rho')d\rho', \tag{11}$$

and

$$\Omega(\rho)=\int_\rho^\infty\exp[2\Lambda(\rho)-2\Lambda(\rho')]\left[\frac{2E_\nu}{\hbar v_F}+\frac{\nu}{k_F\rho'}\right]d\rho'. \tag{12}$$

The wave functions defined by Eqs. (6) and (7) are going to be used to estimate matrix elements, as we shall see soon, when an additional electromagnetic field that couples with the quasiparticles is introduced.

C. Coupling of core states to photons

The theory of optical properties of vortex core states was explored a few years ago,[21,22] where the incoming light was treated as classical electromagnetic waves. For us, though, it is important to establish the existence of a scattering force of quantum nature acting on a vortex core, thus the quantization of the electromagnetic field is indispensable. We consider the geometry illustrated in Fig. 1 and assume the vortex is initially at rest (with respect to the laboratory frame) in a given position along the film. The film is supposed to be illuminated with light of frequency ω, momentum $\hbar\mathbf{k}$, and an arbitrary polarization. The angle that the photon momentum forms with respect to the direction of the magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$ is defined to be θ_f.

We also assume the highly anisotropic limit $m/m_z\ll 1$ $(m/m_z\to 0)$ and that the photon electromagnetic field wavelength is much larger than the film thickness, i.e., $(|\mathbf{k}|d\ll 1)$. In this case, the quantum electromagnetic field couples to electrons (first order in the vector potential) as

$$H_{int}=\int d\mathbf{r}\Psi^\dagger(\mathbf{r},\sigma)\hat{\mathbf{O}}(\mathbf{r},t)\Psi(\mathbf{r},\sigma), \tag{13}$$

where $\Psi(\mathbf{r},\sigma)$ is the electron annihilation operator at position $\mathbf{r}\equiv(\rho,\phi)$ and spin σ, which is related to the quasiparticle operators by the Bogoliubov transformation $\Psi(\mathbf{r},\sigma)$

$=\Sigma_\nu[u_\nu(\mathbf{r})\gamma_{\nu,\sigma}-\text{sgn}(\sigma)v_\nu^*(\mathbf{r})\gamma_{\nu,-\sigma}^\dagger]$. In addition, the coupling to the electrons is mediated by the operator

$$\hat{\mathbf{O}}(\mathbf{r},t)=\frac{i\hbar e}{2mc}\sum_\alpha[\mathbf{A}_{ext}^{(\alpha)}(\mathbf{r},t)\cdot\nabla+\nabla\cdot\mathbf{A}_{ext}^{(\alpha)}(\mathbf{r},t)], \quad (14)$$

and the vector potential

$$\mathbf{A}_{ext}^{(\alpha)}(\mathbf{r},t)=\kappa[a_{k\alpha}(0)\epsilon^{(\alpha)}\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)+\text{H.c.}] \quad (15)$$

corresponds to a monochromatic light of photon energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$, where $\kappa=c\sqrt{\hbar/2\omega V}$. Here, $a_{k\alpha}$ is the annihilation operator of a photon with momentum $\hbar\mathbf{k}$ and polarization component α, while $\epsilon^{(\alpha)}$ is the α component of the polarization vector.

In the quasiparticle representation H_{int} can be separated in two parts. The first one involves quasiparticle-quasihole excitations,

$$H_1=\sum_{\nu\nu'\sigma}[H_{uu}\gamma_{\nu',\sigma}^\dagger\gamma_{\nu,\sigma}+H_{vv}\gamma_{\nu',\sigma}\gamma_{\nu,\sigma}^\dagger], \quad (16)$$

where the matrix elements $H_{uu}=H_{uu}(\nu',\nu)$ and $H_{vv}=H_{vv}(\nu',\nu)$ are given by

$$H_{uu}(\nu',\nu)=\int d\mathbf{r}u_{\nu'}^*(\mathbf{r})\hat{\mathbf{O}}(\mathbf{r},t)u_\nu(\mathbf{r}), \quad (17)$$

$$H_{vv}(\nu',\nu)=\int d\mathbf{r}v_{\nu'}(\mathbf{r})\hat{\mathbf{O}}(\mathbf{r},t)v_\nu^*(\mathbf{r}). \quad (18)$$

The second part involves quasiparticle-quasiparticle and quasihole-quasihole excitations,

$$H_2=-\sum_{\nu\nu'\sigma}\text{sgn}(\sigma)[H_{vu}\gamma_{\nu',-\sigma}\gamma_{\nu,\sigma}+H_{uv}\gamma_{\nu',\sigma}^\dagger\gamma_{\nu,-\sigma}^\dagger], \quad (19)$$

where the matrix elements $H_{vu}=H_{vu}(\nu',\nu)$ and $H_{uv}=H_{vu}(\nu',\nu)$ are given by

$$H_{vu}(\nu',\nu)=\int d\mathbf{r}v_{\nu'}(\mathbf{r})\hat{\mathbf{O}}(\mathbf{r},t)u_\nu(\mathbf{r}), \quad (20)$$

$$H_{uv}(\nu',\nu)=\int d\mathbf{r}u_{\nu'}^*(\mathbf{r})\hat{\mathbf{O}}(\mathbf{r},t)v_\nu^*(\mathbf{r}). \quad (21)$$

The terms H_1 and H_2 of the interaction Hamiltonian are responsible for the existence of the scattering force, which we shall discuss next.

III. SCATTERING FORCE

The general procedure to calculate the scattering force is quite standard[1] and it is based on the density-matrix approach. Here, only the scattering force due to absorption spontaneous emission will be studied,[6] for which we just quote the final result obtained using perturbation theory. For an incoming photon with energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$, the scattering force acting on the vortex core is uniform over the film thickness d and is simply given by

$$\mathbf{F}_s=\Gamma_{ab}\langle\hbar\mathbf{k}\rangle_{ab}+\Gamma_{em}\langle\hbar\mathbf{k}\rangle_{em}, \quad (22)$$

where Γ_{ab} is the absorption transition rate (number of photons absorbed per unit time) and Γ_{em} is the emission transition rate (number of photons emitted per unit time). The averages involving the momentum $\hbar\mathbf{k}$ are over the directions of absorbed and emitted photons. The momentum of the emitted photon has no preferred direction, hence $\langle\hbar\mathbf{k}\rangle_{em}=0$. As a consequence, the scattering force is determined solely by the absorption. Therefore to calculate $\mathbf{F}_s$ explicitly we need only to determine Γ_{ab}, which is given by

$$\Gamma_{ab}=\Gamma_1+\Gamma_2, \quad (23)$$

where Γ_1 is the absorption rate due to H_1 (quasiparticle-quasihole processes), and Γ_2 is the absorption rate due to H_2 (quasiparticle-quasiparticle, quasihole-quasihole processes).

Let us focus for the moment on Γ_1, which is given by

$$\Gamma_1=\frac{2\pi}{\hbar}\sum_{\nu\nu'\sigma}|M_1(\nu\sigma,\nu'\sigma)|^2f_1(\nu,\nu')N_1(\omega), \quad (24)$$

where $M_1(\nu\sigma,\nu'\sigma')$ is just the matrix element for absorption due to H_1,

$$M_1(\nu\sigma,\nu'\sigma')=\langle H_{uu}(\nu',\nu)-H_{vv}(\nu,\nu')\rangle_{ab}, \quad (25)$$

where the $\langle\cdots\rangle_{ab}$ involves the absorption of a photon, while $f_1=f(E_\nu)[1-f(E_{\nu'})]$ where $f(\zeta)$ is the Fermi function of energy argument ζ and $N_1(\omega)=N_1(\nu,\nu',\omega)$ given by

$$N_1(\nu,\nu',\omega)=\frac{1}{\pi}\frac{\hbar/\tau_1}{(E_{\nu'}-E_\nu-\hbar\omega)^2+(\hbar/\tau_1)^2}, \quad (26)$$

with $\tau_1=\tau_1(\nu,\nu')$ being the lifetime associated with the transition.

Now let us turn our attention to Γ_2, which is given by

$$\Gamma_2=\frac{2\pi}{\hbar}\sum_{\nu\nu'\sigma}|M_2(\nu\sigma,\nu'-\sigma)|^2f_2(\nu,\nu')N_2(\omega), \quad (27)$$

where $M_2(\nu\sigma,\nu'-\sigma)$ is just the matrix element for absorption due to H_2,

$$M_2(\nu\sigma,\nu'-\sigma)=-\text{sgn}(\sigma)\langle H_{vu}(\nu',\nu)\rangle_{ab}, \quad (28)$$

where the $\langle\cdots\rangle_{ab}$ involves the absorption of a photon, while $f_2=[1-f(E_\nu)][1-f(E_{\nu'})]$ where $f(\zeta)$ is the Fermi function of energy argument ζ and $N_2(\omega)=N_2(\nu,\nu',\omega)$ given by

$$N_2(\nu,\nu',\omega)=\frac{1}{\pi}\frac{\hbar/\tau_2}{(E_{\nu'}+E_\nu-\hbar\omega)^2+(\hbar/\tau_2)^2}, \quad (29)$$

with $\tau_2=\tau_2(\nu,\nu')$ being the lifetime associated with the transition.

As can be seen from Eqs. (24) and (27), the scattering force depends strongly on three factors: on the matrix element of the transition and its selection rules, on the occupations of the quasiparticle states, and finally on the frequency match between the photon and the transition (resonance).

A. Selection rules

Let us concentrate for the moment on the selection rules for the matrix elements M_1 and M_2, although they are not the main subject of our discussion they are necessary to the existence of the scattering force. The short presentation given here parallels and complements previous works.[21] We will establish the selection rules and the magnitude of the matrix elements at the simple dipole approximation which takes $\mathbf{k}=0$, i.e., the photon wavelength is much larger than the characteristic size of the vortex: $|\mathbf{k}|\xi_0 \ll 1$.[23]

The matrix element M_1 is obtained from

$$\langle H_{uu}(\nu',\nu)\rangle_{ab}=\frac{i\hbar e}{2mc}[I^{(-)}+I^{(+)}] \tag{30}$$

with the coefficients

$$I^{(-)}=A\,\delta_{\nu',\nu-1}[I_1+(\nu-1/2)I_2], \tag{31}$$

$$I^{(+)}=A^*\delta_{\nu',\nu+1}[I_1-(\nu-1/2)I_2]; \tag{32}$$

and from

$$\langle H_{vv}(\nu,\nu')\rangle_{ab}=\frac{i\hbar e}{2mc}[K^{(-)}+K^{(+)}] \tag{33}$$

with the coefficients

$$K^{(-)}=A\,\delta_{\nu',\nu-1}[K_1-(\nu'+1/2)K_2], \tag{34}$$

$$K^{(+)}=A^*\delta_{\nu',\nu+1}[K_1+(\nu'+1/2)K_2], \tag{35}$$

via Eq. (25). $A=\Sigma_\alpha[A_x^{(\alpha)}+iA_y^{(\alpha)}]$ contains the polarized components (α index) of the incoming light projected in the lab coordinate system (xyz), with the polarization axis being defined by the direction of the magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$, while

$$I_1=\int_0^\infty \rho g_{\nu'}^{(+)*}(\rho)\frac{\partial}{\partial\rho}g_\nu^{(+)}(\rho)d\rho, \tag{36}$$

$$I_2=\int_0^\infty g_{\nu'}^{(+)*}(\rho)g_\nu^{(+)}(\rho)d\rho, \tag{37}$$

$$K_1=\int_0^\infty \rho g_\nu^{(-)}(\rho)\frac{\partial}{\partial\rho}g_{\nu'}^{(-)*}(\rho)d\rho, \tag{38}$$

$$K_2=\int_0^\infty g_\nu^{(+)}(\rho)g_{\nu'}^{(-)*}(\rho)d\rho. \tag{39}$$

From Eqs. (30) and (33) we can see that despite the initial choice of an incident photon with a given arbitrary polarization, only the circular components A and A^* can be absorbed. These selection rules are just a manifestation of the conservation of angular momentum in the dipole approximation: $\nu'=\nu\pm1$, which describes the process by which an incoming photon transfers a single unit of angular momentum to the vortex core causing a transition between different states which are compatible with the selection rule. These selection rules belong to the class of the so called *chiral* selection rules, so common in atomic physics when a magnetic field is present, due to the breaking of time-reversal symmetry, e.g., Zeeman effect.

The matrix element M_2 is obtained from

$$\langle H_{uv}(\nu',\nu)\rangle_{ab}=\frac{i\hbar e}{2mc}[T^{(-)}+T^{(+)}] \tag{40}$$

with the coefficients

$$T^{(-)}=A\,\delta_{\nu',-\nu-1}[T_1-(\nu+1/2)T_2], \tag{41}$$

$$T^{(+)}=A^*\delta_{\nu',-\nu+1}[T_1+(\nu+1/2)T_2], \tag{42}$$

via Eq. (28). The coefficients T_1 and T_2 are

$$T_1=\int_0^\infty \rho g_{\nu'}^{(+)}(\rho)\frac{\partial}{\partial\rho}g_\nu^{(-)}(\rho)d\rho, \tag{43}$$

$$T_2=\int_0^\infty g_{\nu'}^{(+)}(\rho)g_\nu^{(-)}(\rho)d\rho. \tag{44}$$

Again, only the circularly polarized components of light are absorbed and we have a simple dipole selection rule: $\nu'=-\nu\pm1$, where the incoming photon transfers a single unit of angular momentum to create two quasiparticles in the absorption process.

Now let us turn our attention to the occupation factors f_1 and f_2. Here we focus only in the $T=0$ limit, where $f_1=[1-\Theta(E_\nu)]\Theta(E_{\nu'})$ and $f_2=\Theta(E_\nu)\Theta(E_{\nu'})$. This indicates that absorption is only possible for the Γ_1 processes when $E_{\nu'}>0$ and $E_\nu<0$, i.e., $\nu'>0$ and $\nu<0$, thus creating a quasiparticle-quasihole excitation. This condition combined with the selection rule $\nu'=\nu\pm1$, restricts the absorption at $T=0$ only to $\nu'=1/2$ and $\nu=-1/2$ and only the circularly polarized component of light with projected helicity $l_z=+1$ is absorbed. On the other hand, for the Γ_2 processes, absorption is only possible when $E_{\nu'}>0$ and $E_\nu>0$, i.e., $\nu'>0$ and $\nu>0$, thus creating quasiparticle-quasiparticle excitation. Combining this condition with the selection rule $\nu'=-\nu\pm1$, restricts the absorption at $T=0$ only to $\nu'=1/2$ and $\nu=+1/2$ and only the circularly polarized component of light with projected helicity $l_z=+1$ is absorbed.

It is important here to elucidate the meaning of the projected helicity l_z. Recall first that the applied magnetic field is parallel to the z axis, thus defining a natural polarization axis. As a consequence, for a photon absorbed along the z direction $l_z=+1$ means a helicity $l=+1$ in the photon's own reference frame, on the other hand for a photon absorbed along the $-z$ direction $l_z=+1$ implies a helicity $l=-1$ in the photon's own reference frame. In addition, for a photon with angles of incidence θ_f measured from the z axis and ϕ_f measured from the x axis, the light must be elliptically polarized is its own reference frame in order to carry one unit of angular momentum projected along the z axis.

B. Resonance conditions

Let us now turn our attention to the resonance conditions and the frequency match between the photon and the vortex core transitions. A substantial scattering force can only be achieved near resonance, therefore it is important to know how large the lifetimes τ_1 and τ_2 are. Since we are not considering in our interaction Hamiltonian H_{int} given Eq. (13)

any other scattering process like phonons or impurities, the lifetime τ_j are intrinsically given by the spontaneous lifetimes $\tau_{j_{sp}}$ of the excited states. In order to estimate the mean lifetimes $\tau_{j_{sp}}$ we must recall that they are equal to the sum of the transition probabilities due to spontaneous emission into all possible final states allowed by the selection rules and energy conservation. These considerations lead to

$$\frac{1}{\tau_{1_{sp}}(\nu,\nu')}=\frac{2}{3}\frac{\alpha_h\omega^3}{c^2}|R_{uu}(\nu',\nu)-R_{vv}(\nu,\nu')|^2, \quad (45)$$

$$\frac{1}{\tau_{2_{sp}}(\nu,\nu')}=\frac{1}{3}\frac{\alpha_h\omega^3}{c^2}|R_{uv}(\nu',\nu)|^2, \quad (46)$$

where $\alpha_h=1/137$ is the hyperfine constant, ω is the frequency of the spontaneously emitted photon, and c is the speed of light, while

$$R_{uu}(\nu',\nu)=\int_0^\infty \rho^2 g_{\nu'}^{(+)*}(\rho)g_\nu^{(+)}(\rho)d\rho, \quad (47)$$

$$R_{vv}(\nu,\nu')=\int_0^\infty \rho^2 g_{\nu}^{(-)}(\rho)g_{\nu'}^{(-)*}(\rho)d\rho, \quad (48)$$

and

$$R_{uv}(\nu',\nu)=\int_0^\infty \rho^2 g_{\nu'}^{(-)}(\rho)g_\nu^{(+)}(\rho)d\rho. \quad (49)$$

From Eqs. (45) and (46) we can estimate

$$\tau_{1_{sp}}(-1/2,1/2)\approx\frac{3}{2}(k_F\xi_0)^2k_F^2\left[\frac{c^2}{\alpha_h\omega^3}\right], \quad (50)$$

with $\omega=(E_{1/2}-E_{-1/2})/\hbar$ and

$$\tau_{2_{sp}}(1/2,1/2)\approx 3(k_F\xi_0)^2k_F^2\left[\frac{c^2}{\alpha_h\omega^3}\right], \quad (51)$$

with $\omega=(E_{1/2}+E_{1/2})/\hbar$. Notice the dependence on $k_F\xi_0$, which in the limit $k_F\xi_0\gg 1$ enhances the spontaneous lifetimes by a large factor, in addition recall that ω here depends on ξ_0 through $E_{1/2}$ and $E_{-1/2}$.

For a particle density $n_{2D}\approx 1.6\times 10^{17}$ cm^2 ($k_F=10^9$ cm^{-1}) and $\xi_0\approx 30$ Å we get lifetimes $\tau_{1_{sp}}\approx 10^7$ sec and $\tau_{2_{sp}}\approx 2\times 10^7$ sec which are extremely large in comparison to the lifetime of the $2p\rightarrow 1s$ transition in the hydrogen atom: $\tau_{sp}(2p\rightarrow 1s)\approx 1.6\times 10^{-9}$ sec. As a consequence, the broadenings $\hbar/\tau_{j_{sp}}$ are extremely small and the resonance enhancement can be much larger than in typical atoms, provided that the effects of phonons and impurities are ignored. In a real system, though, we must consider the effects of impurities and phonons. Thus, when the inverse lifetimes due to impurities ($1/\tau_{j_{im}}$) and phonon ($1/\tau_{j_{ph}}$) scattering are also taken into account, the total inverse lifetime $1/\tau_j=1/\tau_{j_{sp}}+1/\tau_{j_{im}}+1/\tau_{j_{ph}}$. We shall return to this point, when we analyze the scattering acceleration in Sec. IV.

Due to this resonance aspect the scattering force $\mathbf{F}_s=\mathbf{F}_s^{(1)}+\mathbf{F}_s^{(2)}$ may be extremely large despite the smallness of the incoming photon momentum $\hbar\mathbf{k}$. To estimate the strength of such a force consider the incident photon at an angle θ_f with the magnetic field and elliptic polarization in the photon reference frame with major axis vector potential given by $A_{(+)}$ such that the projected helicity $l_z=\pm 1$ satisfies the angular momentum selection rule. As a result, the absorption rates on resonance are given by

$$\Gamma_1=\frac{4}{\hbar^2}[|M_1(1/2\uparrow,-1/2\uparrow)|^2\tau_1(-1/2,1/2)], \quad (52)$$

$$\Gamma_2=\frac{4}{\hbar^2}[|M_2(1/2\uparrow,1/2\downarrow)|^2\tau_2(1/2,1/2)], \quad (53)$$

where

$$|M_j|=\epsilon\times|\alpha_j|\times|\cos\theta_f|, \quad (54)$$

with

$$\epsilon=\frac{1}{\sqrt{2}C}|e|\frac{\omega_b}{c}|A_{(+)}|\xi_0 \quad (55)$$

being the characteristic strength of the matrix element $|M_j|$, $\omega_b=\hbar C/m\xi_0^2$ being the frequency associated with the transition in the vortex core, and α_j being a numerical factor of $O(1)$ which depends on the details of the quasiparticle wave functions. Notice that $|M_j|$ is proportional to $|\cos\theta_f|$. This proportionality is a consequence of the fact that the elliptically polarized light in the photon's own reference frame must be circularly polarized when projected along the xy plane, perpendicular to the magnetic field axis (z direction) in order to satisfy the angular momentum dipole selection rules previously discussed. The factor $\cos\theta_f$ is the geometrical factor that accounts for this projection, a remanent of the Malus law. Notice in addition that $|M_j|$ vanishes at $\theta_f=\pi/2$, since the angular momentum carried by the photon is now perpendicular to the magnetic field and has zero component along the z axis, thus not satisfying the dipole selection rules. This leads to vanishing absorption rates Γ_j and vanishing scattering forces $\mathbf{F}_s^{(j)}$, when $\theta_f=\pi/2$.

Before estimating the absorption rates and the scattering force, it is important to put bounds to the intensities of the electromagnetic field that can be used to obtain such a force using perturbation theory. The perturbative approach is valid when $|M_j|/\hbar\omega_b\ll 1$, thus limiting

$$|A_{(+)}|\times|\alpha_j|\times|\cos\theta_f|\ll\frac{\hbar c}{|e|}\frac{\sqrt{2}C}{\xi_0} \quad (56)$$

which gives $|A_{(+)}|\times|\alpha_j|\times|\cos\theta_f|\ll[1/\xi_0]\times 2.64\times 10^3\times$ V $\times$Å, when ξ_0 is given in Å.

Within the bounds of the perturbative approach, now we are ready to estimate the strength of the scattering force. For $\xi_0=30$ Å, $n_{2D}=1.6\times 10^{17}$ cm^2, $|A_{(+)}|=\lambda\times$V and considering only the spontaneous lifetimes $\tau_{j_{sp}}$ the absorption rates are $\Gamma_j>\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 7.1\times 10^{29}\times$ photons/sec. These absorption rates can be used to estimate the scattering force $\mathbf{F}_s\equiv(F_{s_x},F_{s_y},F_{s_z})$ providing the following lower bounds for the maximal scattering force

$$|\mathbf{F}_{s_x}^{(j)}|_{max}=|F_s^{(j)}|_{max}\times\sin\theta_f\times\cos\phi_f, \quad (57)$$

$$|\mathbf{F}_{s_y}^{(j)}|_{max}=|F_s^{(j)}|_{max}\times\sin\theta_f\times\sin\phi_f, \tag{58}$$

$$|\mathbf{F}_{s_z}^{(j)}|_{max}=|F_s^{(j)}|_{max}\times\cos\theta_f, \tag{59}$$

where $|F_s^{(j)}|_{max}>\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.0\times 10^5$ dyn. Depending on the angles (θ_f,ϕ_f) and λ the maximal scattering force here can be much larger than the characteristic scattering force for the $(3s\rightarrow 2p)$ transition in sodium, where $|\mathbf{F}_s|_{max}\approx 10^{-15}$ dyn.

The possible large scattering force acting on the vortex core immediately raises the questions: how large is the acceleration imposed on the vortex core by the scattering force? Can it be used to cool or heat a single vortex, i.e., how does this acceleration affects the dynamics of a single vortex? To answer these final questions we must study how the scattering force modifies the equation of motion of a single vortex, which we shall discuss next.

IV. EQUATION OF MOTION

We shall consider a vortex initially at rest, but with freedom to move with velocity $\mathbf{v}_l$ in the plane of the film (xy plane) and in a superfluid background with velocity $\mathbf{v}_s$. Also assume the incident photon with momentum $\hbar\mathbf{k}$ that *hits* the vortex core at angles (θ_f,ϕ_f). The photon is assumed to have elliptic polarization in its own reference frame, such that the light field is circularly polarized when the polarization vectors are projected in the xy plane. The vortex is considered to move at velocities $|\mathbf{v}_l|/c\ll 1$ such that the existing scattering force $\mathbf{F}_s$ probes the vortex core essentially with zero Doppler shift. In addition, due to the constrained motion of the vortex to the xy plane, the z component of $\mathbf{F}_s$ does not affect the vortex motion.

Under these considerations the presence of $\mathbf{F}_s$ generalizes the de Gennes-Matricon-Hsu[25,22] equation of motion at $T=0$ to

$$\frac{d}{dt}\mathbf{v}_l(t)=\frac{d}{dt}\mathbf{v}_s(t)+\mathbf{a}_m(t)-\frac{1}{\tau}\mathbf{v}_l(t)+\mathbf{a}_s, \tag{60}$$

which describes the motion of a single vortex in the presences of the superfluid background and of the scattering force. In Eq. (60) $d\mathbf{v}_s(t)/dt$ is the acceleration due to the superfluid background; $\mathbf{a}_m(t)=\omega_b[\mathbf{v}_l(t)-\mathbf{v}_s(t)]\wedge\hat{\mathbf{z}}$, is the magnus acceleration, with $\omega_b=\hbar C/m\xi_0^2$ being the frequency associated with the characteristic core energy $E_b=\hbar^2C/m\xi_0^2$; $-\mathbf{v}_l(t)/\tau$ is the damping acceleration, where τ represents the average characteristic lifetime of the quasiparticle states inside the vortex core; and $\mathbf{a}_s=\mathbf{F}_{s_\perp}/M_v$ is the scattering acceleration in the xy plane, with $M_v=(k_F\xi_0)^2m/C$ being the kinetic vortex mass. Here $\mathbf{F}_{s_\perp}\equiv(F_{s_x},F_{s_y})$ and $\mathbf{a}_s\equiv(a_{s_x},a_{s_y})$ given by

$$a_{s_x}=a_s(\omega)\times\sin\theta_f\times\cos\phi_f, \tag{61}$$

$$a_{s_y}=a_s(\omega)\times\sin\theta_f\times\sin\phi_f, \tag{62}$$

where $a_s(\omega)=\Sigma_j a_s^{(j)}(\omega)$ with

$$a_s^{(j)}(\omega)=\frac{2\pi}{M_v}\frac{\omega}{c}|M_j|^2N_j(\omega), \tag{63}$$

where $|M_j|$ is defined in Eq. (54), $N_1(\omega)\equiv N_1(-1/2,1/2,\omega)$, $N_2(\omega)\equiv N_2(1/2,1/2,\omega)$ and $a_s(\omega)$ is the magnitude of the acceleration imposed by the scattering force on the vortex motion and ω is the frequency of the incoming photon.

Now we can estimate the lower bound of the maximal accelerations $|a_s^{(j)}|_{max}$ for $k_F=10^9$ cm^{-1} and $\xi_0=30$ Å leading to $|a_s^{(j)}|_{max}>\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.5\times 10^{24}$ cm/sec^2, which may be enormous $\approx\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.5\times 10^{21}g$! Therefore it is expected that the scattering force contributes significantly to the dynamics of a single vortex (cooling or heating), i.e., to the total dissipation and induced voltages produced by the vortex motion. It is important to emphasize that such large scattering accelerations (forces) occur only in highly idealized systems where the energy broadenings $\hbar/\tau_j$ include only the spontaneous lifetimes $\tau_{j_{sp}}$ of the quasiparticle states. In real systems, the effects of impurity and phonon scattering have to be taken into account, i.e., $1/\tau_j=1/\tau_{j_{sp}}+1/\tau_{j_{im}}+1/\tau_{j_{ph}}$. This procedure for $1/\tau_1=10^{-9}$ sec reduces $|F_s^{(j)}|_{max}=\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.0\times 10^{-11}$ dyn and leads to a maximal acceleration $|a_s^{(j)}|_{max}>\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.5\times 10^8$ cm/sec^2, which is still very large $(\approx\lambda^2\times|\alpha_j|^2\times\cos^2\theta_f\times 2^{(j-1)}\times 3.5\times 10^5 g)$, where g is the acceleration due to gravity. These values are comparable to the maximal acceleration achieved in the $(3s\rightarrow 2p)$ transition for sodium atoms $|a_s|_{max}\approx 10^8$ cm/sec^2, i.e., $\approx 10^5 g$!

Notice, though, that when the incoming photon momentum $\hbar\mathbf{k}$ is along the magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$ direction $(\theta_f=0,\pi)$, the scattering force is either parallel $(\theta_f=0)$ or antiparallel $(\theta_f=\pi)$ to $\hat{\mathbf{z}}$. In this case, $\mathbf{F}_s$ $(\mathbf{F}_{s_\perp}=0)$ does not contribute to the equation of motion (60) since the vortex is constrained to move in the xy plane, i.e., along the thin film. As a consequence we can put $\mathbf{a}_s=0$ in Eq. (60) and the resulting expression describes the classical dynamics of a single vortex. On the other hand, when the incoming photon momentum $\hbar\mathbf{k}$ is not along $\mathbf{H}$ $(\theta_f\neq 0,\pi)$ and when the matrix element $|M_j|$ does not vanish identically $(\theta_f\neq\pi/2)$ the scattering force $\mathbf{F}_s$ $(\mathbf{F}_{s_\perp}\neq 0)$ does affect the vortex dynamics. As a consequence, $\mathbf{F}_s$ may contribute strongly to dissipation and induced voltages, thus producing vortex cooling (decreased dissipation) or heating (increased dissipation). Hence for the existence of such effects it is fundamental to have the incoming light to be at an angle θ_f with the external magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$. It is also important to emphasize again that the scattering force considered here is *quantum* in nature, in the sense that it results from the absorption spontaneous emission of photons by the vortex core (recall that spontaneous emission is a *quantum* process due to vacuum fluctuations in the quantized electromagnetic field).

A. Solutions of the equation of motion

Now we turn our attention to the solution of Eq. (60) in the presence of the scattering force. This solution will be used (as we shall see) to calculate the total dissipation and induced voltages in order to characterize the proposed cooling or heating effects. For a given superfluid velocity $\mathbf{v}_s \equiv (v_{s_x}, v_{s_y})$, we can calculate $\mathbf{v}_l \equiv (v_{l_x}, v_{l_y})$. The superfluid velocity has the form

$$\mathbf{v}_s(t) = \mathbf{v}_{s_c} + \mathbf{v}_{s_{ext}}(t), \tag{64}$$

where $\mathbf{v}_{s_c}$ is caused by a uniform and time independent (dc) current applied along the xy plane and $\mathbf{v}_{s_{ext}}(t) = \mathbf{v}_s(0)\exp[i\omega t]$ is the contribution to the total superfluid velocity caused by the incident light, which can be used to obtain

$$\mathbf{v}_l(t) = \mathbf{v}_l^{(0)}(t) + \delta\mathbf{v}_l(t), \tag{65}$$

where $\mathbf{v}_l^{(0)}(t)$ is the solution of Eq. (60) without the scattering acceleration $\mathbf{a}_s$.

Equation (60) become more transparent when we rewrite it as

$$\frac{d}{dt} v_{l,\alpha}(t) = \omega_\alpha v_l(t) + \frac{d}{dt} v_{s,\alpha}(t) + i\alpha\omega_b v_{s,\alpha}(t) + a_{s,\alpha}, \tag{66}$$

where $\omega_\alpha = -(i\alpha\omega_b + 1/\tau)$, the vortex velocity $v_{l,\alpha} = v_{l_x} + i\alpha v_{l_y}$, the superfluid velocity $v_{s,\alpha} = v_{s_x} + i\alpha v_{s_y}$, and the scattering acceleration $a_{s,\alpha} = a_{s_x} + i\alpha a_{s_y}$, with $\alpha = \pm$. Under the assumption that the vortex is initially at rest $[\mathbf{v}_l(0) = 0 \rightarrow \mathbf{v}_{l,\alpha}(0) = 0]$ the solution of Eq. (66) is

$$v_{l,\alpha}(t) = v_{l,\alpha}^{(0)}(t) + \delta v_{l,\alpha}(t). \tag{67}$$

The first contribution to $v_{l,\alpha}(t)$ is independent of the scattering force,

$$v_{l,\alpha}^{(0)}(t) = L_\alpha(\omega,t) v_{s,\alpha}(0) + L_\alpha(0,t) v_{s_c,\alpha}, \tag{68}$$

where the function

$$L_\alpha(\omega,t) = K_\alpha(\omega)[\exp(i\omega t) - \exp(-i\alpha\omega_b t - t/\tau)] \tag{69}$$

contains the time dependent terms with the prefactor

$$K_\alpha(\omega) = \frac{i\omega + i\alpha\omega_b}{i\omega + i\alpha\omega_b + 1/\tau}. \tag{70}$$

Notice in Eq. (67) the steady-state behavior $(t \rightarrow \infty)$ where $v_{l,\alpha}^{(0)}(t \rightarrow \infty)$ has a purely oscillatory part associated with $v_{s,\alpha}(0)$, which is the amplitude of the superfluid velocity induced by the light field, and a constant part associated with v_{s_c}, which is due to the to the dc current applied in the xy plane. When $v_{s_c} = 0$, the light field component of the superfluid velocity $v_{s_{ext}}(t)$ induces a circular motion of the vortex with a definite handness, thus transferring some angular momentum to the vortex center of mass. On the other hand, when $v_{s_ext}(t) = 0$ the constant part of the superfluid velocity v_{s_c} provides a constant steady-state velocity for the vortex center of mass, thus transferring linear momentum to the vortex center of mass. When both v_{s_c} and $v_{s_{ext}}$ are nonzero, the resulting vortex motion is a two-dimensional spiral.

The second contribution to $v_{l,\alpha}(t)$ is the additional change in the vortex velocity,

$$\delta v_{l,\alpha}(t) = \frac{a_{s,\alpha}(\omega)}{i\alpha\omega_b + 1/\tau}[1 - \exp(-i\alpha\omega_b t - t/\tau)], \tag{71}$$

due to the presence of the scattering force. Notice that in the steady-state limit $(t \rightarrow \infty)$, $\delta v_{l,\alpha}(t \rightarrow \infty)$ has no oscillatory part. As a result, the scattering force transfers linear momentum to the vortex center of mass.

To characterize vortex cooling or heating, let us recall that in atomic physics the cooling and heating processes are identified with the change in the average kinetic energy of the atom: when the average kinetic energy is increased we have heating of the atom and when it is decreased we have cooling. Such situations occur when the scattering acceleration is antiparallel (cooling) or parallel (heating) to the atom's velocity. Hence as a measure of the vortex cooling or heating effects let us look at the time-averaged vortex velocity in the steady-state limit $\langle v_{l,\alpha}(t \rightarrow \infty)\rangle$, which has two contributions, the first one being $\langle v_{l,\alpha}^{(0)}(t \rightarrow \infty)\rangle = L_\alpha(0,t \rightarrow \infty) v_{s_c}$ and the second one being $\langle \delta v_{l,\alpha}(t \rightarrow \infty)\rangle = a_{s,\alpha}(\omega)/(1\alpha\omega_b + 1/\tau)$. Thus to characterize vortex cooling or heating we can monitor the total average vortex velocity $\langle v_{l,\alpha}(t \rightarrow \infty)\rangle$ as a function of the direction of the incoming scattering acceleration, when the total average velocity is reduced, we have cooling (reduction of the vortex kinetic energy), when the total average velocity is increased, we have heating (increase of the vortex kinetic energy). Now it is important to turn our attention to how the scattering force affects the dissipation due to vortex motion, and thus vortex cooling or vortex heating.

B. Dissipation effects: Cooling versus heating

To characterize the cooling or heating effect for vortices it is useful to calculate directly measurable quantities. For instance, it is important to compare the total dissipation with and without the scattering force. To calculate the total dissipation $W_t(\omega,t) \equiv \mathrm{Re}[\mathbf{J}^*(t) \cdot \mathbf{E}(t)]$ we need to make use of the expression for the total current,

$$\mathbf{J}(t) = n_s e \mathbf{v}_s(t) + N_v \frac{\hbar k_F^2}{2\omega^*} \frac{e}{m}[\mathbf{v}_l(t) - \mathbf{v}_s(t)], \tag{72}$$

where $\omega^* = \beta\omega_b$,[24] and for the total electric field,

$$\mathbf{E}(t) = \frac{m}{e}\frac{d}{dt}\mathbf{v}_s(t) - N_v \frac{h}{2e}\mathbf{v}_l(t) \wedge \hat{\mathbf{z}}, \tag{73}$$

where the superfluid acceleration is

$$\frac{d}{dt}\mathbf{v}_s = -\frac{e}{mc}\frac{d}{dt}\mathbf{A}_{ext}(t) \tag{74}$$

and $\mathbf{A}_{ext}(t)$ is the vector potential due to the light field projected in the xy plane.

In calculating the total dissipation $W_t(\omega,t)$ we will not be concerned with transient effects, instead we look at the steady-state $(t \rightarrow \infty)$ time average $\langle W_t(\omega,t \rightarrow \infty)\rangle \equiv W_t(\omega)$, which can be separated in two contributions,

$$W_t(\omega) = W_s(\omega) + W_v(\omega), \tag{75}$$

the first coming from the supercurrent density

$$W_s = -N_v(\pi\hbar n_s)[1-\gamma]\mathrm{Re}\langle \mathbf{v}_s^*(t)\cdot[\mathbf{v}_l(t)\wedge\hat{\mathbf{z}}]\rangle \quad (76)$$

and the other coming from the vortex motion

$$W_v = N_v \frac{\hbar k_F^2}{2\omega^*}\mathrm{Re}\left\langle \mathbf{v}_l^*(t)\cdot\frac{d}{dt}\mathbf{v}_s(t)\right\rangle, \quad (77)$$

where $\gamma=(N_v/n_s)[\hbar k_F^2/2m\omega^*]$. Notice also that in the limit $k_F\xi_0 \gg 1$ considered here the superfluid density n_s can be approximated by the total particle density $n_{2D}=k_F^2/2\pi$, thus allowing us to write $\gamma=\omega_0/\omega^*$, with $\omega_0=N_v h/2m$. The numerical value of the dimensionless constant γ is very small and can be estimated as follows. First we identify ω_0 with the cyclotron frequency $\omega_c=eH/mc$ and consider fields such that $H_{c_1}\approx H \ll H_{c_2}$. Further, consider the thin film of thickness d to have a disc geometry with radius R. Thus using $H_{c_1}=(\phi_0/4\pi\lambda^2)\ln[\lambda/\xi_0](d/R)$, where λ is the London penetration depth and ϕ_0 is the flux quantum, we may finally write $\gamma\approx(1/4\beta C)(\xi_0/\lambda)^2\ln[\lambda/\xi_0](d/R)$. Since we have made the assumption of extreme type-II superconductivity, $\xi_0/\lambda \ll 1$ and that the film thickness is much smaller than the disc radius, $d/R\ll 1$, the result $\gamma \ll 1$ immediately follows.

To compare the total dissipation with the scattering force and without the scattering force it is useful to rewrite the total dissipation

$$W_t(\omega)=W_t^{(0)}(\omega)+\Delta W_t(\omega), \quad (78)$$

where $W_t^{(0)}(\omega)$ is the dissipation due to $\mathbf{v}_l^{(0)}$ alone, i.e., without the contribution of the scattering force and $\Delta W_t(\omega)$ is the additional contribution due to $\delta\mathbf{v}_l$ alone, i.e., due to the scattering force alone. Notice that when $\Delta W_t(\omega)<0$ we have vortex cooling (decreased dissipation) and when $\Delta W_t(\omega)>0$ we have vortex heating (increased dissipation). Notice, in addition, that $W_t^{(0)}(\omega)$ has the same functional form as $W_t(\omega)$ with the replacement $\mathbf{v}_l\to\mathbf{v}_l^{(0)}$ and so does $\Delta W_t(\omega)$ with the replacement $\mathbf{v}_l\to\delta\mathbf{v}_l$. This is a consequence of the linear dependence of $W_t(\omega)$ on the vortex velocity $\mathbf{v}_l$.

To calculate $W_t^{(0)}$ and ΔW_t explicitly we need to use the expression for the superfluid velocity $\mathbf{v}_s$ in Eq. (64). We first analyze the contribution to dissipation without the scattering force, which results in

$$W_t^{(0)}(\omega)=W_s^{(0)}(\omega)+W_v^{(0)}(\omega). \quad (79)$$

The superfluid contribution may be further separated in two parts,

$$W_s^{(0)}(\omega)=W_{s_c}^{(0)}(\omega)+W_{s_{ext}}^{(0)}(\omega), \quad (80)$$

with the part due to static field ($\mathbf{v}_{s_c}$) being

$$W_{s_c}^{(0)}(\omega)=-P\,\mathrm{Re}[\mathbf{v}_{s_c}^*\cdot\langle\mathbf{v}_l^{(0)}(t\to\infty)\rangle\wedge\hat{\mathbf{z}}], \quad (81)$$

and with the part due to the light field ($\mathbf{v}_{s_{ext}}$) being

$$W_{s_{ext}}^{(0)}(\omega)=-P\,\mathrm{Re}\langle\mathbf{v}_{s_{ext}}^*(t\to\infty)\cdot\mathbf{v}_l^{(0)}(t\to\infty)\wedge\hat{\mathbf{z}}\rangle, \quad (82)$$

where $P=N_v(\pi\hbar n_s)[1-\gamma]$. Analogously, the vortex contribution $W_v^{(0)}$ can be further separated in two parts,

$$W_v^{(0)}(\omega)=W_{v_c}^{(0)}(\omega)+W_{v_{ext}}^{(0)}(\omega), \quad (83)$$

with the part due to static field ($\mathbf{v}_{s_c}$) being

$$W_{v_c}^{(0)}(\omega)=0, \quad (84)$$

and with the part due to the light field ($\mathbf{v}_{s_{ext}}$) being

$$W_{v_{ext}}^{(0)}(\omega)=D\,\mathrm{Re}\left\langle \mathbf{v}_l^{(0)*}(t\to\infty)\cdot\frac{d}{dt}\mathbf{v}_{s_{ext}}(t\to\infty)\right\rangle, \quad (85)$$

where $D=N_v(\hbar k_F^2/2\omega^*)$. As a result of the time averaging in the steady-state limit we may finally write

$$W_t^{(0)}=W_{s_c}^{(0)}+W_{s_{ext}}^{(0)}+W_{v_{ext}}^{(0)}. \quad (86)$$

Notice that $W_t^{(0)}(\omega)$ depends on the light polarization via $\mathbf{v}_{s_{ext}}$ and has in addition a contribution, which is independent of the light polarization via $\mathbf{v}_{s_c}$. Some analysis of $W_t^{(0)}(\omega)$ without the static field ($\mathbf{v}_{s_c}$) may be found in Ref. 22. Our results for $W_t^{(0)}(\omega)$, on the other hand, generalize Hsu's expressions in two important ways: the first one being the presence of the uniform and static (dc) current that creates $\mathbf{v}_{s_c}$ and the second one being that $\mathbf{v}_{s_{ext}}$ is generated by elliptically polarized light, which is circularly polarized when viewed along the direction of the magnetic field. We shall further discuss these results below.

For the moment, let us turn our attention to the contributions to $\Delta W_t(\omega)$, which can be further described by

$$\Delta W_t(\omega)=\Delta W_s(\omega)+\Delta W_v(\omega). \quad (87)$$

The superfluid contribution may, in addition, be separated in two parts,

$$\Delta W_s(\omega)=\Delta W_{s_c}(\omega)+\Delta W_{s_{ext}}(\omega), \quad (88)$$

with the part due to the static field ($\mathbf{v}_{s_c}$) being

$$\Delta W_{s_c}(\omega)=-P\,\mathrm{Re}[\mathbf{v}_{s_c}^*\cdot\langle\delta\mathbf{v}_l(t\to\infty)\rangle\wedge\hat{\mathbf{z}}], \quad (89)$$

and with the part due to the light field ($\mathbf{v}_{s_{ext}}$) being

$$\Delta W_{s_{ext}}(\omega)=0. \quad (90)$$

Analogously, the vortex contribution ΔW_v can be further separated in two parts,

$$\Delta W_v(\omega)=\Delta W_{v_c}(\omega)+\Delta W_{v_{ext}}(\omega), \quad (91)$$

with the part due to the static field ($\mathbf{v}_{s_c}$) being

$$\Delta W_{v_c}(\omega)=0, \quad (92)$$

and with the part due to the light field ($\mathbf{v}_{s_{ext}}$) being

$$\Delta W_{v_{ext}}(\omega)=0. \quad (93)$$

Since $\Delta W_v(\omega)=0$ due to steady-state averages, we may finally write

$$\Delta W_t(\omega) = \Delta W_{s_c}(\omega). \tag{94}$$

As a consequence, the cooling ($\Delta W_t < 0$) or heating ($\Delta W_t > 0$) effects are caused only by $\delta \mathbf{v}_l$ and $\mathbf{v}_{s_c}$, i.e., when both $\langle \delta \mathbf{v}_l(t\to\infty)\rangle \neq 0$ and $\mathbf{v}_{s_c} \neq 0$.

Hence we may finally write

$$W_t^{(0)} = W_{s_{ext}}^{(0)} + W_{v_{ext}}^{(0)} + W_{s_c}^{(0)} + \Delta W_{s_c}. \tag{95}$$

A direct comparison of $W_{s_c}^{(0)}(\omega)$ and $\Delta W_{s_c}(\omega)$ leads to the following picture in the steady-state limit: when the change in the vortex velocity $\langle \delta \mathbf{v}_l(t\to\infty)\rangle$ is antiparallel to the vortex velocity in the absence of the scattering force $\langle \mathbf{v}_l^{(0)}(t\to\infty)\rangle$ there is a reduction in the total dissipation and hence vortex cooling, when $\langle \delta \mathbf{v}_l(t\to\infty)\rangle$ is parallel $\langle \mathbf{v}_l^{(0)}(t\to\infty)\rangle$ there is an increase in the total dissipation and hence vortex heating. These conclusions follow from the fact that the average electric field felt by the vortex core in the absence of the scattering force $\langle \mathbf{E}_v^{(0)}(t\to\infty)\rangle = -N_v(h/2e)\langle \mathbf{v}_l^{(0)}(t\to\infty)\rangle \wedge \hat{\mathbf{z}}$ is reduced (cooling) or enhanced (heating) by the action of the scattering force via $\langle \delta \mathbf{v}_l(t\to\infty)\rangle$, while the corresponding current contribution to the total dissipation is unaffected by the scattering force. In particular, notice that when $\langle \delta \mathbf{v}_l(t\to\infty)\rangle = -\langle \mathbf{v}_l^{(0)}(t\to\infty)\rangle$ the contribution to the total dissipation from ΔW_t exactly cancels the contribution from $W_t^{(0)}$, given that the total average electric field felt by the vortex $\langle \mathbf{E}_v(t\to\infty)\rangle = -N_v(h/2e)\langle \mathbf{v}_l(t\to\infty)\rangle \wedge \hat{\mathbf{z}}$ vanishes! In this case the electric field created by the scattering force exactly cancels the electric field caused by the applied dc current, thus bringing the vortex total average center of mass velocity $\langle \mathbf{v}_l(t\to\infty)\rangle$ to *zero*. The cooling effect, on the other hand, is not perfect ($W_t \neq 0$), i.e., $W_t = W_{s_{ext}}^{(0)} + W_{v_{ext}}^{(0)}$, despite the vanishing of $\langle \mathbf{v}_l(t\to\infty)\rangle$, the reason being that the vortex still performs a circular motion $\mathbf{v}_l(t\to\infty) \neq 0$ which is in phase with the velocity field due to light $\mathbf{v}_{s_{ext}}(t\to\infty)$ and thus produces dissipation through the terms $W_{s_{ext}}^{(0)}$ and $W_{v_{ext}}^{(0)}$. From this we conclude that the scattering force considered here (under the assumptions of this paragraph) cannot completely stop the vortex motion since it only affects its center of mass average velocity!

Let us now turn our attention to the explicit evaluation of $\Delta W_t(\omega)$ in order to elucidate its dependence on the frequency (ω) and on the angles (θ_f, ϕ_f) that the incoming photon momentum makes with the xyz laboratory reference frame. Since the film is considered to be isotropic in the xy plane we can choose the direction of the applied velocity $\mathbf{v}_{s_c}$ to be along the x axis, i.e., $\mathbf{v}_{s_c} = v_{s_c}\hat{\mathbf{x}}$. In this case, $\Delta W_t = -P v_{s_c}\langle \delta v_{l_y}(t\to\infty)\rangle$, which can be rewritten as

$$\Delta W_t(\omega) = -PK(\omega_b) v_{s_c} \tau [a_{s_y}(\omega) - \omega_b \tau a_{s_x}(\omega)], \tag{96}$$

where

$$K(\omega_b) = \frac{1}{(\omega_b^2 \tau^2 + 1)}. \tag{97}$$

Notice in Eq. (96) that when $a_{s_y}(\omega) > \omega_b \tau a_{s_x}(\omega)$ ($\tan\phi_f > \omega_b\tau$), $\Delta W_t(\omega) < 0$, i.e., we have vortex cooling and when

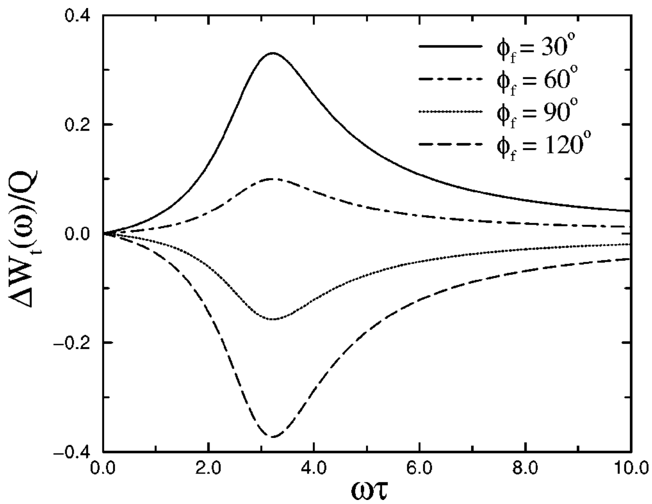

FIG. 2. The contribution to the total dissipation due to the scattering force alone, for $\omega_b\tau = 3.0$ and $\theta_f = 45°$. Notice the resonant effect for $\omega\tau \approx 3.0$.

$a_{s_y}(\omega) < \omega_b \tau a_{s_x}(\omega)$ ($\tan\phi_f < \omega_b\tau$), $\Delta W_t(\omega) > 0$, i.e., we have vortex heating. The cooling and heating effects are strongly dependent upon the angles (θ_f, ϕ_f) that the scattering force makes with the fixed lab reference frame. This angular dependence appears in Eq. (96) through the accelerations a_{s_x} and a_{s_y} defined in Eqs. (61) and (62). To emphasize the angular dependences we may write $\Delta W_t(\omega)$ in the dimensionless form

$$\frac{\Delta W_t(\omega)}{Q} = -S(\omega)[\sin\phi_f - \omega_b\tau\cos\phi_f]\cos^2\theta_f \sin\theta_f, \tag{98}$$

where the coefficient Q is given by $Q = P v_{s_c} v_{eff}$, where $v_{eff} = [\pi/(k_F\xi_0)^2][|e|^2|A_{(+)}|^2/m^2c^3]$ and the function $S(\omega)$ is defined as

$$S(\omega) = \omega_b \tau K(\omega_b) U(\omega), \tag{99}$$

with $U(\omega) = \Sigma_j |\alpha_j|^2 N_j(\omega)\hbar\omega$ leading to

$$U(\omega) = \sum_j |\alpha_j|^2 \frac{1}{\pi} \frac{\omega\tau_j}{(\omega\tau_j - \omega_b\tau_j)^2 + 1}. \tag{100}$$

In Eq. (98) many symmetry properties become evident. For instance, the $\cos^2\theta_f$ appearing in Eq. (98) comes from the matrix element $|M_j|^2$ and is a remanent of the Malus law as previously discussed, while the terms $\sin\theta_f$, $\sin\phi_f$, and $\cos\phi_f$ come from the projections of the photon momentum $\hbar\mathbf{k}$ along the lab xyz reference frame. Also notice that the symmetry property $\Delta W_f(\omega,\theta_f,\phi_f) = -\Delta W_f(\omega,\theta_f,\pi-\phi_f)$ holds, which means that by changing the azimuthal direction of the incoming photons (changing ϕ_f with fixed θ_f) by a 180° the dissipation due to the scattering force alone changes sign. In addition, for fixed ϕ_f the dissipation $\Delta W_t(\omega)$ is maximal when $\sin\theta_f = 1/\sqrt{3}$, i.e., $\theta_f \approx 35.26°$ or $\theta_f \approx 144.74°$, and vanishes when $\theta_f = 0, 90°, 180°$.

In Fig. 2, the results for $\Delta W_f(\omega)$ expressed in Eq. (98) are presented. The dissipation $\Delta W_f(\omega)$ is given in units of Q, for $\omega_b\tau = 3.0$, for a fixed angle of incidence $\theta_f = 45°$ but for a varying azimuthal (xy) angle ϕ_f. The dissipation $\Delta W_f(\omega)$ is also a function of the parameters $|\alpha_j|$ and τ_j that depend on the detailed structure of the quasiparticle wave functions defined in Eqs. (6) and (7). Here, in order of magnitude, $|\alpha_j| \approx 1$, $\tau_1 \approx \tau$, $\tau_2 \approx \tau/2$, and $\beta \approx 1/2$. The frequencies ω are expressed in units of τ^{-1}. Notice in Fig. 2, in particular, the regions in frequency space for various azimuthal angles ϕ_f where there is vortex cooling, i.e., $\Delta W_t(\omega) < 0$. In addition notice the resonant effect when $\omega \approx \omega_b$. This resonant effect is much more pronounced when $\omega_b \tau \gg 1$, i.e., when the maximal energy broadening due to impurities or phonons $\max[\hbar/\tau_{im}, \hbar/\tau_{ph}] \ll \hbar\omega_b$.

C. Photovoltaic effect

Now we move on to the calculation of the total induced voltage and of the induced voltage change due to the presence of the scattering force. Consider two points at positions $\mathbf{d}_1$ and $\mathbf{d}_2$ in the lab xyz reference frame and define $\mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1$. Since the total electric field $\mathbf{E}(t)$ is spatially uniform, the total induced voltage is $V(\mathbf{d},t) = \mathbf{E}(t) \cdot \mathbf{d}$, which can be further separated in two parts,

$$V(\mathbf{d},t) = V^{(0)}(\mathbf{d},t) + \Delta V(\mathbf{d},t), \tag{101}$$

in analogy to the calculation of the total dissipation $W(t)$. The voltage

$$V^{(0)}(\mathbf{d},t) = \frac{m}{e}\frac{d}{dt}\mathbf{v}_s(t)\cdot\mathbf{d} - N_v \frac{h}{2e}\mathbf{v}_l^{(0)}(t) \wedge \hat{\mathbf{z}} \cdot \mathbf{d} \tag{102}$$

is the voltage in the absence of the scattering force, while

$$\Delta V(\mathbf{d},t) = -N_v \frac{\pi\hbar m}{e^2}\mathrm{Re}[\delta\mathbf{v}_l(t) \wedge \hat{\mathbf{z}}]\cdot\mathbf{d} \tag{103}$$

is the additional contribution due to the scattering force.

Again, we are not interested in transient effects, instead we look at the steady-state ($t \to \infty$) time averages $\langle V(\mathbf{d},t \to \infty)\rangle = V(\omega)$. Notice that the contributions to $V(\omega)$ coming from $\mathbf{v}_{s_{ext}}$, which are linear in the photon vector potential $\mathbf{A}_{ext}$, vanish. Hence $V(\omega)$ can be further expressed as a sum of the voltages measured along the x axis (V_x) and along the y axis (V_y),

$$V(\omega) = V_x(\omega) + V_y(\omega), \tag{104}$$

where

$$V_x(\omega) = -N_v \frac{\pi\hbar m}{e^2} d_x \ \mathrm{Re}[\langle v_{l_y}(t \to \infty)\rangle], \tag{105}$$

$$V_y(\omega) = +N_v \frac{\pi\hbar m}{e^2} d_y \ \mathrm{Re}[\langle v_{l_x}(t \to \infty)\rangle]. \tag{106}$$

Notice here that when $\langle v_{l_y}(t \to \infty)\rangle = 0$ the voltage $V_x = 0$ and analogously when $\langle v_{l_x}(t \to \infty)\rangle = 0$ the voltage $V_y = 0$. Using the decomposition of velocities $\mathbf{v}_l = \mathbf{v}_l^{(0)} + \delta\mathbf{v}_l$ we can write $V_n(\omega) = V_n^{(0)}(\omega) + \Delta V_n(\omega)$, where $n = x,y$, from which we immediately realize that $V_x = 0$ occurs when $\delta v_{l_y} = -v_{l_y}^{(0)}$ and that $V_y = 0$ occurs when $\delta v_{l_x} = -v_{l_x}^{(0)}$.

To establish explicitly the conditions under which the voltages V_n vanish we must compare $V_n^{(0)}$ with ΔV_n. For $\mathbf{v}_{s_c} = v_{s_c}\hat{x}$, we may write

$$V_x^{(0)}(\omega) = -B_{x_0}\omega_b \tau K(\omega_b) v_{s_c}, \tag{107}$$

$$V_y^{(0)}(\omega) = +B_{y_0}\omega_b^2 \tau^2 K(\omega_b) v_{s_c}, \tag{108}$$

where $B_{n_0} = N_v d_n(\pi\hbar m/e^2)$, with $n = x,y$. On the other hand, we write

$$\Delta V_x = -B_{x_0}\tau K(\omega_b)[a_{s_y} - \omega_b \tau a_{s_x}], \tag{109}$$

$$\Delta V_y = +B_{y_0}\tau K(\omega_b)[a_{s_x} + \omega_b \tau a_{s_y}]. \tag{110}$$

Using the expressions for a_{s_x} and a_{s_y} in Eqs. (61) and (62), it is easy to establish that $V_x = 0$ occurs when

$$\tau a_s(\omega)\sin\theta_f[\sin\theta_f - \omega_b\tau\cos\phi_f] = -\omega_b \tau v_{sc} \tag{111}$$

and that $V_y = 0$ occurs when

$$\tau a_s(\omega)\sin\theta_f[\cos\phi_f + \omega_b\tau\sin\theta_f] = -\omega_b^2\tau^2 v_{sc}. \tag{112}$$

Notice in addition that the simultaneous vanishing of the voltages V_n can occur only when $\phi_f = -\pi/2$, i.e., when the scattering acceleration is applied along the $-\hat{\mathbf{y}}$ direction. Here, the scattering acceleration is perpendicular both to the applied dc current ($\|\hat{\mathbf{x}}$) and to the applied magnetic field ($\|\hat{\mathbf{z}}$). Only under these conditions the total average electric field felt by the vortex core, vanishes, i.e., the average electric field induced by the dc current on the vortex core can be exactly canceled by the electric field produced by the scattering acceleration.

Since in the steady-state limit the time averaged contributions to V_n coming from $\mathbf{v}_{ext}$ vanish, the measurement of voltages V_x and V_y is a direct evidence of the effects of the scattering force. The measurements for V_n can be performed using a crossed four point contact: two leads along the x direction, i.e., aligned with the applied dc current $\mathbf{J}_{s_c} = J_x\hat{\mathbf{x}}$, where $J_x = n_s e v_{s_c}$ can be used to determine V_x, while two leads along the y direction, i.e., perpendicular to $\mathbf{J}_{s_c}$ can be used to determine V_y.

Since the voltages $V_{n_0}^{(0)}$ due to the dc current are independent of ω, we shall concentrate on the contribution $\Delta V_n(\omega)$, due to the scattering acceleration, which can be written in the dimensionless forms

$$\frac{\Delta V_x(\omega)}{V_{x_0}} = -S(\omega)[\sin\phi_f - \omega_b\tau\cos\phi_f]\cos^2\theta_f \sin\theta_f, \tag{113}$$

$$\frac{\Delta V_y(\omega)}{V_{y_0}} = +S(\omega)[\cos\phi_f + \omega_b\tau\sin\phi_f]\cos^2\theta_f \sin\theta_f, \tag{114}$$

where $V_{n_0} \equiv B_{n_0} v_{eff}$, with $n = x,y$.

Notice the interesting symmetry property

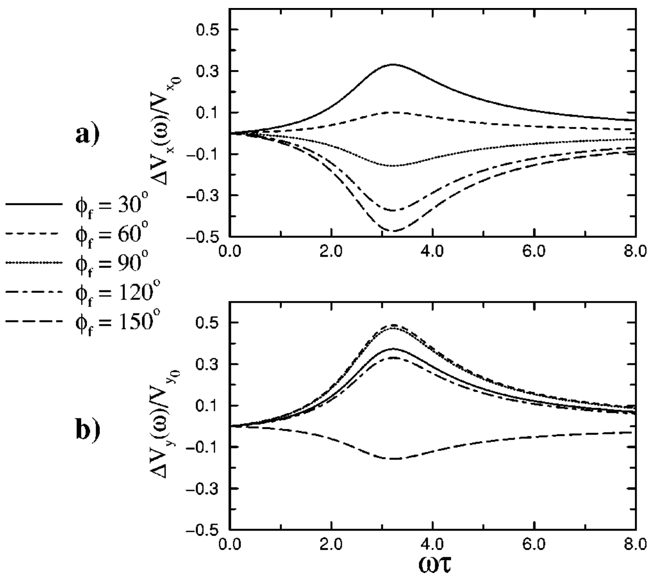

FIG. 3. Contributions to induced voltages due to the scattering force alone. It is shown in (a) the contribution along the x direction, while it is shown in (b) the contribution along the y direction. Curves are plotted for $\omega_b\tau = 3.0$ and $\theta_f = 45°$. Notice the resonant effect for $\omega\tau \approx 3.0$.

$$\Delta V_x(\omega,\theta_f,\phi_f+\pi/2)/V_{x_0} = \Delta V_y(\omega,\theta_f,\phi_f)/V_{y_0}, \tag{115}$$

which relates the measured voltages along the dc current (x direction) and perpendicular to the dc current (y direction). Notice also that $\Delta V_n(\omega,\theta_f,\phi_f) = -\Delta V_n(\omega,\theta_f,\pi-\phi_f)$, for $n = x,y$, which means that the induced voltage due to the scattering force alone changes sign when the azimuthal direction of the incoming photons is rotated by 180°. In addition, for a fixed ϕ_f, ΔV_{t_n} is maximal when $\sin\theta_f = 1/\sqrt{3}$ just like ΔW_t. In Fig. 3 we plot the results for ΔV_x and ΔV_y for the same set of parameters of Fig. 2.

V. EXPERIMENTAL CONSIDERATIONS

Several experimental systems are potential candidates for the observation of vortex cooling (decreased dissipation) and heating (increased dissipation) and the photovoltaic effect proposed here. Among many possible systems, let us mention s-wave systems where vortex core states have been already experimentally observed: $NbSe_2$,[9] $Nb_{1-x}Ta_xSe_2$.[10] The theory developed here may be applicable to these systems, but it may be applicable also the to n-type high-temperature superconductor $Nd_{2-x}Ce_xCu\,O_4$,[11] where to the best of our knowledge no one has yet attempted to observe vortex core states. Furthermore, it is important to mention that vortex core states have also been observed in $YBa_2\,Cu_3\,O_{7-\delta}$,[12] which is believed to be a d-wave superconductor. Although in this manuscript we have not discussed the vortex cooling and the photovoltaic effect for the d-wave case, it is quite natural to explore this possibility in the immediate future.

The theoretical suggestions presented here rely on the fact that experiments must be performed at low temperatures and in clean systems. The meaning of low temperatures here is defined by $T \ll \min[E_\nu]$, where $\min[E_\nu] = CR_y(a_0/\xi_0)^2$ is the energy of the lowest-lying vortex core state. For instance, for $\xi_0 = 30$ Å this means $T \ll 46.4$ K and for $\xi_0 = 2000$ Å it implies $T \ll 10$ mK. Thus large coherence lengths impose a much more stringent requirement on the meaning of the low-temperature limit. In addition, the cooling effect relies on resonant momentum transfer from photons to the vortex core. Thus it is quite important that the energy broadenings due to disorder (impurities), for instance, are small, i.e., it is important that $\hbar/\tau_{imp} \ll 2E_{1/2}$, where $\tau_{imp} = l/v_F$, and $E_{1/2} = C\hbar v_F/[(2k_F\xi_0)\xi_0]$ is the energy of the lowest-lying vortex core state. This condition is equivalent to say that the system must be in the clean limit $l \gg (k_F\xi_0)\xi_0$, where l is the electronic mean free path.

Studies performed in $Nb_{1-x}Ta_xSe_2$ (Ref. 10) have indicated that increasing disorder (x) leads to a broadening and suppression of the STM conductance peak associated with vortex core states. Thus good experimental candidates must satisfy both the low-temperature and the low disorder conditions. These conditions seem to be satisfied in the short coherence length superconductor $YBa_2Cu3O_{7-\delta}$, where vortex core states were recently observed.[12] As result of that, it also possible that the effects discussed here may be observable in this system, which is believed to be a d-wave superconductor. Lastly we would like to propose another possible candidate for the experimental observations of vortex cooling and the photovoltaic effect. The candidate is the s-wave short coherence length superconductors $Nd_{2-x}Ce_xCuO_4$,[11] where the low-temperature and low disorder conditions seem to be satisfied.

VI. SUMMARY

In summary, we have considered a thin film of an s-wave short coherence length superconductor. The superconductor was assumed to be type II, isotropic, and under an applied magnetic field slightly over the lower upper critical field, where a single vortex approximation is meaningful. We considered a single vortex with a characteristic core size given by the superconducting coherence length ξ_0 and the corresponding vortex core states. We have proposed the existence of the scattering force acting on a vortex core due to the absorption spontaneous emission of photons involving vortex core states. This process resulted in a net momentum transfer to the vortex core and affected the vortex dynamics. Despite the smallness of the photon momentum a large resonant enhancement provided the possibility of enormous scattering forces. The absorption spontaneous emission process considered is quantum in nature since there is no spontaneous emission when the electromagnetic field is treated classically.

We have proposed a vortex cooling or heating effect similar to the case in atomic physics, where atoms can be cooled or heated via the scattering force. We emphasized, though, that due to the presence of the superfluid background, the dynamics of a vortex core in the presence of the scattering force is very different than that of a moving atom in the presence of the same force. In addition, we emphasized the cases where the scattering force is absent in a two-dimensional vortex core. For instance, one such a case occurs when the net momentum transfer to the core is perpendicular to the vortex motion xy plane, i.e., when the incoming photon is along the z axis. To characterize the cases where the scattering force was present and its cooling or heating effects, we have calculated the additional dissipation due to the presence of the scattering force. When the additional dissipation was negative, we had vortex cooling and when it was positive, vortex heating. Furthermore, we have analyzed the induced voltages due the scattering force in order to fully characterize this interesting photovoltaic effect. And finally we have proposed $Nd_{2-x}Ce_xCuO_4$ as a possible s-wave short coherence length superconductor where the effects proposed here may be experimentally observed.

ACKNOWLEDGMENTS

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[24] Quite generally, $\omega^* = \beta\omega_b$ where β is $O(1)$ depending on the precise forms of the quasiparticle wave functions. The parameter β^{-1} originates from the spatial averaging of the current operator $\mathbf{J}_p(\mathbf{r},t) = (i\hbar e/2m)\Sigma_\sigma \Psi^\dagger_\sigma(\mathbf{r},t)\nabla\Psi_\sigma(\mathbf{r},t) + \text{H.c.}$, where $\Psi^\dagger_\sigma(\mathbf{r},t)$ creates an electron at position $\mathbf{r}$ and time t.

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