Effects of interplay of dipole-dipole interactions and single-ion easy-plane anisotropy on two-dimensional ferromagnets

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(Received 20 October 1998)

Using the nearest-neighbor Heisenberg model as a starting point, we provide a theoretical analysis on the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy on the magnetism of two-dimensional ferromagnets. The respective influence of the dipole-dipole interactions and single-ion easy-plane anisotropy on the stability of the long-range magnetic order at finite temperature in two-dimensional ferromagnets is investigated analytically, and the low-temperature spin-wave excitation spectrum, the temperature dependence of the spontaneous magnetization, and the transition temperature are calculated numerically. [S0163-1829(99)09637-X]

I. INTRODUCTION

There has been much interest recently in the field of magnetism in ultrathin films for reasons of both fundamental physics and technical applications.[1] These works are partly motivated by the possible integration of the semiconductor microelectronic technology with magnetic elements[2] and is simulated by the success in the growth of magnetic ultrathin films on top of semiconductor surfaces. Although in the past some theoretical work has been devoted to low-dimensional magnetic systems, it is only in the past few years that technical progress has offered the possibility of comparing theoretical predictions with experimental measurements. This in turn will lead to more realistic understanding of magnetic behavior in low-dimensional magnetic systems. Initial theoretical work on the particular case of a single monolayer film, i.e., a two-dimensional system, indicated that such a system cannot present any spontaneous magnetization at finite temperature. This was first pointed out by Bloch[3] and his conjecture was later rigorously proved by Mermin and Wagner,[4] whose theorem shows that any long-range magnetic order at finite temperature cannot exist in an infinite two-dimensional spin system coupled by isotropic short-range exchange interactions. But recent experiments have established that spontaneous magnetization at finite temperature does exist in ultrathin films (including single monolayer films), and both perpendicular and in-plane spontaneous magnetization have been observed in magnetic ultrathin films(including single monolayer films).[5–12] This apparent contradiction between theoretical predictions and experimental results arises from the fact that an isotropic short-range exchange-coupled Heisenberg model does not take into account all those important factors that may affect significantly the properties of a magnetic system, for example, the finite-size of specimens,[13] the magnetocrystalline anisotropy,[14–16] and the dipole-dipole interactions.[17–18] Especially for magnetic ultrathin films, the magnetocrystalline anisotropy and the dipole-dipole interactions may play the crucial role in determining their magnetic properties.[18–29] Due to these reasons, in the study of magnetism in ultrathin films, a clear understanding of the effects of various kinds of magnetic anisotropies and their interplay with dipole-dipole interactions is both theoretically and experimentally much desirable. A very common form of the magnetocrystalline anisotropy is the single-ion anisotropy, taking usually the form of favoring an easy magnetization axis (single-ion easy-axis anisotropy) or favoring an easy magnetization plane (single-ion easy-plane anisotropy). It has since been shown in the case of a single-ion easy-axis anisotropy favoring an easy magnetization axis, the long-range magnetic order at finite temperature in two-dimensional ferromagnets is stabilized by an anisotropy-induced energy gap at the bottom of the spin-wave excitation spectrum which removes the two-dimensional (2D) divergence. However, in the case of single-ion easy-plane anisotropy favoring an easy magnetization plane, since the spins can rotate freely in the easy plane, no such gap should arise (the Goldstone theorem). In such cases, the stability of the long-range magnetic order at finite temperature in two-dimensional ferromagnets cannot be accounted for by the formation of energy gap at the bottom of the spin-wave excitation spectrum, and the long-range magnetic order at finite temperature should result from the interplay of the anisotropy and dipole-dipole interactions. While the effects of the interplay of the single-ion easy-axis anisotropy and dipole-dipole interactions have been intensively studied by many authors in recent years,[19–29] up to now there have been no theoretical investigations on the more subtle effects of the interplay between the single-ion easy-plane anisotropy and dipole-dipole interactions. Their respective influence on the magnetism of two-dimensional ferromagnets still remains to be clarified. In this paper, using the nearest-neighbor Heisenberg model as a starting point, we present a theoretical analysis on the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisot-

ropy on the magnetism of two-dimensional ferromagnets, including the stability of the long-range magnetic order at finite temperature, the low-temperature spin-wave excitation spectrum, the temperature dependence of the spontaneous magnetization, and the transition temperature.

The paper is organized as follows: In Sec. II, we first establish a self-consistent Green's-function formalism to describe the off-diagonal quantum mixing effects of the anisotropy in single-ion easy-plane ferromagnets with any spin quantum number. Due to the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy, the theoretical treatment of single-ion easy-plane ferromagnets is more difficult than that of single-ion easy-axis ferromagnets.[30–41] In the case of isotropic or single-ion easy-axis ferromagnets, it is well known that the most remarkable merit of the Green's-function method is its approximate validity at both low and high temperature except in the vicinity of the critical point,[42,43] which the other approaches such as spin-wave theory, mean-field approximation, and high-temperature series expansion method did not possess. But due to the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy, the usual Green's-function formalism that are applicable to isotropic or single-ion easy-axis ferromagnets cannot be applied to single-ion easy-plane ferromagnets, and significant modifications are needed. In Sec. II, we will establish a modified form of the usual Green's-function formalism so that it can be applied to single-ion easy-plane ferromagnets. By making comparison with some other previous theoretical approaches for single-ion easy-plane ferromagnets, we will show that just like the usual Green's-function formalism for isotropic or single-ion easy-axis ferromagnets, this modified Green's-function formalism can also present an approximately valid description for single-ion easy-plane ferromagnets at both low and high temperature, except in the critical region. In Sec. III, we will apply this modified Green's-function formalism to investigate the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy on the magnetism of two-dimensional ferromagnets.

II. GREEN'S-FUNCTION APPROACH FOR THE OFF-DIAGONAL QUANTUM MIXING EFFECTS OF SINGLE-ION EASY-PLANE ANISOTROPY

In this section, we establish a self-consistent Green's-function formalism to describe the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy. For clarity, we will neglect the dipole-dipole interactions for the moment. But as will be shown in Sec. III, it is easier to include the dipole-dipole interactions in this Green's-function formalism.

A spin-S ferromagnet with single-ion anisotropy is described by the following Hamiltonian:

$$H=-J\sum_{\langle ij\rangle} S_i\cdot S_j+D\sum_{\mathbf{i}} (S_i^x)^2, \tag{1}$$

where S_i are Heisenberg spin operators with spin quantum number S, J is the nearest-neighbor exchange integral, the symbol $\langle ij\rangle$ indicates that the sums are restricted to nearest-neighbor pairs. Since the main purpose of this paper is to investigate the effects of the dipolar interactions and the single-ion easy-plane anisotropy, for simplicity and clarity, only the nearest-neighbor exchange couplings of spins will be taken into account in the following. The second term in Hamiltonian (1) is the single-ion anisotropy term. If $D\langle 0$, the anisotropy is the easy-axis type, and the x axis is the easy magnetization axis. If $D>0$, the anisotropy is the easy-plane type, and the y-z plane is the easy magnetization plane, this is the case which we will discuss in this paper. In this case, the spins will be forced into the easy-plane (the y-z plane) by the anisotropy. By means of the identity $S^-S^+=S(S+1)-S^z-(S^z)^2$, the Hamiltonian can be rewritten as

$$H=-J\sum_{\langle ij\rangle} S_i\cdot S_j+\frac{1}{4}D\sum_{\mathbf{i}} [(S_i^+)^2+(S_i^-)^2]$$
$$-\frac{1}{2}D\sum_i (S_i^z)^2+\frac{1}{2}NDS(S+1), \tag{2}$$

where N is the total number of spins in the system and we have chosen the z axis to be along the direction of the spontaneous magnetization. From this form of the Hamiltonian, we can see that the single-ion easy-plane anisotropy induces an off-diagonal quantum mixing between the $|m-2\rangle,|m\rangle,|m+2\rangle$ single-ion eigenstates of S^z. Because of the complexities caused by this off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy, the theoretical treatment of single-ion easy-plane anisotropy is much more difficult than that of single-ion easy-axis anisotropy, even the description of the ground states of such spin systems is very nontrivial. Many conventional theoretical methods that are applicable to isotropic or single-ion easy-axis magnets cannot be applied to single-ion easy-plane magnets. For instance, the well-known spin-Bose operator transformations by Holstein and Primakoff[44] and by Dyson[45] and Maleev[46] cannot be applied in the description of low-temperature spin-wave excitations in single-ion easy-plane magnets.[30–33] Due to these complexities, the mean-field approximation is commonly used in the calculations of the thermodynamic quantities of single-ion easy-plane magnets. However, in the mean-field approximation, both quantum and thermal fluctuation correlations have been neglected. Several approaches which can improve the mean-field approximation have been proposed: the method of matching of matrix elements (MME) (Refs. 30–33) and the method of characteristic angle (CA) transformation[34] for describing the low-temperature spin-wave excitations in easy-plane ferromagnets, the zero-temperature series expansion method[35] and the coupled-cluster method[36–38] for studying the ground-state energy and the ground-state magnetization of easy-plane spin-1 ferromagnets, the linked-cluster series expansion method (LCE) (Refs. 39–41) for studying the thermodynamic quantities of easy-plane spin-one ferromagnets. Though there have been such approaches for easy-plane ferromagnets, there are some shortcomings in these approaches: the MME method and the CA transformation method can describe the low-temperature spin-wave excitations in easy-plane ferromagnets, but these methods that based on the transformations of spin operators into simpler

Bose operators are valid only at low temperatures (far below the Curie temperature); the zero-temperature series expansion method and the coupled-cluster method are appropriate for studying the ground-state energy and the ground-state magnetization of easy-plane ferromagnets, but they cannot describe the low-temperature magnetic excitations in such spin systems, moreover, it is difficult to extend these method to easy-plane ferromagnets with spin greater than 1; the linked-cluster series expansion method (LCE) is also valid only for studying the thermodynamic quantities and cannot describe the low-temperature spin-wave excitations in easy-plane ferromagnets, it is also very difficult to extend this method to easy-plane ferromagnets with any spin quantum number. In addition, if the dipole-dipole interactions are included, the applications of these methods will become much more complicated. To overcome these shortcomings of these theoretical methods, we establish a self-consistent Green's-function formalism to describe the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy. As will be shown in the following, like the usual Green's-function formalism for isotropic or single-ion easy-axis ferromagnets, this self-consistent Green's-function formalism can also present an approximately valid description for single-ion easy-plane ferromagnets with any spin quantum number at both low- and high-temperature except in the vicinity of the critical point. At low temperature, it is similar to the method of matching of matrix elements (MME) and can present a correct description of the low-temperature spin-wave excitations in single-ion easy-plane ferromagnets; at high temperature, it can improve the mean-field theory substantially except in the critical region. Moreover, unlike those previous theoretical methods,[30–41] this Green's-function formalism can be easily extended to include the dipole-dipole interaction.

Considering the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy, unlike the case of isotropic ferromagnets or ferromagnets with easy-axis anisotropy, we introduce two kinds of retarded Green's functions, $G_1^{(n)}(i,j;t,t')$ and $G_2^{(n)}(i,j;t,t')$, to describe the propagation of magnetic excitations in such spin systems:

$$G_1^{(n)}(i,j;t,t') = \langle\langle S_i^+(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle = -i\Theta(t-t') \times\langle[S_i^+(t),(S_j^-(t'))^n(S_j^+(t'))^{n-1}]_-\rangle, \quad (3)$$

$$G_2^{(n)}(i,j;t,t') = \langle\langle S_i^-(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle = -i\Theta(t-t') \times\langle[S_i^-(t),(S_j^-(t'))^n(S_j^+(t'))^{n-1}]_-\rangle, \quad (4)$$

where $n=1, 2, \ldots, 2S$, and $\langle A\rangle = \mathrm{Tr}(e^{-\beta H}A)/\mathrm{Tr}(e^{-\beta H})$, $A(t)=e^{iHt}Ae^{-iHt}$, $[A,B]_-=AB-BA$. In the case of isotropic ferromagnets or ferromagnets with easy-axis anisotropy [without the off-diagonal terms like $(S^+)^2$ and $(S^-)^2$ in Hamiltonian (2)], the second kind of Green's functions $G_2^{(n)}$ should vanish exactly due to the conservation of the z component of the total angular momentum. The off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy term breaks down this conservation condition, thus the second kind of Green's functions will be nonvanishing. To derive the Green's functions $G_1^{(n)}$ and $G_2^{(n)}$, we follow the method of equation of motion. The equation of motion for $G_1^{(n)}$ and $G_2^{(n)}$ gives

$$i\frac{d}{dt}G_1^{(n)}(i,j;t,t') = \delta(t-t')\,\delta_{ij}\langle g_1^{(n)}(S^z)\rangle - 2J\sum_{\mathbf{a}}\langle\langle S_i^z(t)S_{i+a}^+(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle + 2J\sum_{\mathbf{a}}\langle\langle S_{i+a}^z(t)S_i^+(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle + \frac{1}{2}D\langle\langle S_i^z(t)S_i^+(t)+S_i^+(t)S_i^z(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle + \frac{1}{2}D\langle\langle S_i^z(t)S_i^-(t)+S_i^-(t)S_i^z(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle, \quad (5)$$

$$i\frac{d}{dt}G_2^{(n)}(i,j;t,t') = \delta(t-t')\,\delta_{ij}\langle g_2^{(n)}(S^-,S^z)\rangle + 2J\sum_{\mathbf{a}}\langle\langle S_i^z(t)S_{i+a}^-(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle, - 2J\sum_{\mathbf{a}}\langle\langle S_{i+a}^z(t)S_i^-(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle - \frac{1}{2}D\langle\langle S_i^z(t)S_i^+(t)+S_i^+(t)S_i^z(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle - \frac{1}{2}D\langle\langle S_i^z(t)S_i^-(t)+S_i^-(t)S_i^z(t);(S_j^-(t'))^n(S_j^+(t'))^{n-1}\rangle\rangle, \quad (6)$$

where **i** and **i**+**a** denote two nearest-neighboring lattice sites and

$$g_1^{(n)}(S^z)=[S^+,(S^-)^n(S^+)^{n-1}] =(2nS^z+n^2-n)\Phi^{(n-1)}(S^z), \tag{7}$$

$$g_2^{(1)}(S^-,S^z)=[S^-,S^-]=0, \tag{8}$$

$$g_2^{(2)}(S^-,S^z)=[S^-,(S^-)^2S^+]=-2(S^-)^2S^z, \tag{9}$$

$$g_2^{(n)}(S^-,S^z)=[S^-,(S^-)^n(S^+)^{n-1}] =-[(n-1)(n-2)(S^-)^2+2(n-1) \times(S^-)^2S^z]\Phi^{(n-2)}(S^z), \quad (n>2), \tag{10}$$

in which

$$\Phi^{(n)}(S^z)=(S^-)^n(S^+)^n =\Pi_{p=1}^n[S(S+1)-(n-p)(n-p+1) -(2n-2p+1)S^z-(S^z)^2]. \tag{11}$$

To constitute a set of closed equations for $G_1^{(n)}$ and $G_2^{(n)}$, as usual, we decouple the higher-order Green's functions in the right-hand sides of Eqs. (5) and (6) according to the random phase approximation (RPA):[42]

$$S_i^z(t)S_j^{\pm}(t)\approx\langle S^z\rangle S_j^{\pm}(t), \quad i\neq j. \tag{12}$$

However, it should be noted the above ordinary RPA decoupling scheme is not appropriate if $i=j$. In this case, an appropriate decoupling scheme is that of Callen and Anderson:[43]

$$S_i^z(t)S_i^{\pm}(t)+S_i^{\pm}(t)S_i^z(t)\approx 2\Gamma\langle S^z\rangle S_i^{\pm}(t), \tag{13}$$

$$\Gamma=1-\frac{1}{2S^2}[S(S+1)-\langle(S^z)^2\rangle]. \tag{14}$$

Using the decoupling schemes of Eqs. (12) and (13), and making the usual Fourier transformations, we obtain two closed equations for the Fourier components $G_1^{(n)}(k,\omega)$ and $G_2^{(n)}(k,\omega)$:

$$[\omega-\varepsilon_k-D\Gamma\langle S^z\rangle]G_1^{(n)}(k,\omega)-D\Gamma\langle S^z\rangle G_2^{(n)}(k,\omega) =\langle g_1^{(n)}(S^z)\rangle, \tag{15}$$

$$[\omega+\varepsilon_k+D\Gamma\langle S^z\rangle]G_2^{(n)}(k,\omega)+D\Gamma\langle S^z\rangle G_1^{(n)}(k,\omega) =\langle g_2^{(n)}(S^-,S^z)\rangle, \tag{16}$$

where

$$\varepsilon_k=2JZ\langle S^z\rangle(1-\gamma_k), \tag{17}$$

$$\gamma_k=\frac{1}{Z}\sum_{\mathbf{a}} e^{i\mathbf{k}\cdot\mathbf{a}}, \tag{18}$$

in which Z is the number of the nearest neighbors and **a** denotes the lattice vectors between two nearest neighbors. From Eqs. (15) and (16), we get

$$G_1^{(n)}(k,\omega)=\frac{1}{2E(k)[\omega-E(k)]}\{D\Gamma\langle S^z\rangle\langle g_2^{(n)}(S^-,S^z)\rangle +\langle g_1^{(n)}(S^z)\rangle[E(k)+\varepsilon_k+D\Gamma\langle S^z\rangle]\} -\frac{1}{2E(k)[\omega+E(k)]}\{D\Gamma\langle S^z\rangle\langle g_2^{(n)}(S^-,S^z)\rangle -\langle g_1^{(n)}(S^z)\rangle[E(k)-\varepsilon_k-D\Gamma\langle S^z\rangle]\}, \tag{19}$$

$$G_2^{(n)}(k,\omega)=-\frac{1}{2E(k)[\omega-E(k)]}\{D\Gamma\langle S^z\rangle\langle g_1^{(n)}(S^z)\rangle -\langle g_2^{(n)}(S^-,S^z)\rangle[E(k)-\varepsilon_k-D\Gamma\langle S^z\rangle]\} +\frac{1}{2E(k)[\omega+E(k)]}\{D\Gamma\langle S^z\rangle\langle g_1^{(n)}(S^z)\rangle +\langle g_2^{(n)}(S^-,S^z)\rangle[E(k)+\varepsilon_k+D\Gamma\langle S^z\rangle]\}, \tag{20}$$

where

$$E(k)=\sqrt{(\varepsilon_k+D\Gamma\langle S^z\rangle)^2-(D\Gamma\langle S^z\rangle)^2}, \tag{21}$$

which is just the magnon dispersion relation.

From Eqs. (7)–(11), (19), and (20), we can see that $G_1^{(n)}(k,\omega)$ and $G_2^{(n)}(k,\omega)$ are functions of two kinds of unknown variables: $\langle(S^z)^n\rangle$ and $\langle(S^-)^2(S^z)^{n-1}\rangle$, $(n=1,2,\ldots)$. The nonvanishing of the second kind of variables is a direct consequence of the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy. These two kinds of unknown variables must be determined by some self-consistent procedures. Through the use of the Zubarev's equation,[47] we can get

$$\begin{aligned}\langle(S^-)^n(S^+)^n\rangle &=\frac{1}{N}\sum_k i\int\frac{d\omega}{2\pi}\frac{G_1^{(n)}(k,\omega+i0^+)-G_1^{(n)}(k,\omega-i0^+)}{e^{\beta\omega}-1}\\ &=\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{\beta E(k)}-1]}\{D\Gamma\langle S^z\rangle\langle g_2^{(n)}(S^-,S^z)\rangle\\ &\quad+\langle g_1^{(n)}(S^z)\rangle[E(k)+\varepsilon_k+D\Gamma\langle S^z\rangle]\}\\ &\quad-\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{-\beta E(k)}-1]}\{D\Gamma\langle S^z\rangle\langle g_2^{(n)}(S^-,S^z)\rangle\\ &\quad-\langle g_1^{(n)}(S^z)\rangle[E(k)-\varepsilon_k-D\Gamma\langle S^z\rangle]\},\end{aligned} \tag{22}$$

$$\langle (S^-)^n (S^+)^{n-1} S^- \rangle$$
$$= \frac{1}{N}\sum_k i\int \frac{d\omega}{2\pi} \frac{G_2^{(n)}(k,\omega+i0^+) - G_2^{(n)}(k,\omega-i0^+)}{e^{\beta\omega}-1}$$
$$= -\frac{1}{N}\sum_k \frac{1}{2E(k)[e^{\beta E(k)}-1]} \{D\Gamma\langle S^z\rangle \langle g_1^{(n)}(S^z)\rangle$$
$$- \langle g_2^{(n)}(S^-, S^z)\rangle [E(k) - \varepsilon_k - D\Gamma\langle S^z\rangle]\}$$
$$+ \frac{1}{N}\sum_k \frac{1}{2E(k)[e^{-\beta E(k)}-1]} \{D\Gamma\langle S^z\rangle \langle g_1^{(n)}(S^z)\rangle$$
$$+ \langle g_2^{(n)}(S^-, S^z)\rangle [E(k) + \varepsilon_k + D\Gamma\langle S^z\rangle]\}, \quad (23)$$

in which $n = 1,2, \ldots, 2S$, and

$$\langle (S^-)^n (S^+)^n \rangle = \langle \Phi^{(n)}(S^z)\rangle,$$
$$\langle (S^-)^n (S^+)^{n-1} S^- \rangle = \langle (S^-)^2 \rangle \quad \text{for } n = 1$$

and

$$\langle (S^-)^n (S^+)^{n-1} S^- \rangle$$
$$= \langle (S^-)^2 [S(S+1) + S^z - (S^z)^2] \Phi^{(n-2)}(S^z)\rangle$$

for $n > 1$. [The expression for $\Phi^{(n)}(S^z)$ is given in Eq. (11).] From Eqs. (7)–(11), we can see that in Eqs. (22) and (23), both the left-hand sides and the right-hand sides are functions of the two kinds of unknown variables: $\langle (S^z)^n \rangle$ and $\langle (S^-)^2 (S^z)^{n-1}\rangle$, $(n = 1,2,\ldots)$. Noticing that for any spin quantum number S, the following relations are satisfied:

$$\Pi_{m=-S}^{m=S}(S^z - m)|\psi\rangle = 0, \quad (24)$$

$$(S^-)^{2S+2}|\psi\rangle = 0, \quad (25)$$

where $|\psi\rangle$ is any state vector. From these relations, we have

$$\langle \Pi_{m=-S}^{m=S}(S^z - m)\rangle = 0, \quad (26)$$

$$\langle (S^-)^{2S+2}(S^+)^{2S}\rangle = \langle (S^-)^2 \Phi^{(2S)}(S^z)\rangle = 0. \quad (27)$$

From Eqs. (26) and (27), we can see that for any $n > 2S$, $\langle (S^z)^n \rangle$ can be expressed as a linear combination of $\langle S^z\rangle$, $\langle (S^z)^2\rangle, \ldots, \langle (S^z)^{2S}\rangle$, and $\langle (S^-)^2 (S^z)^{n-1}\rangle$ can be expressed as a linear combination of $\langle (S^-)^2\rangle$, $\langle (S^-)^2 S^z\rangle, \ldots, \langle (S^-)^2 (S^z)^{2S-1}\rangle$. Therefore both in the left-hand sides and in the right-hand sides of Eqs. (22) and (23), there are, in fact, only $4S$ independent variables: $\langle S^z\rangle$, $\langle (S^z)^2\rangle, \ldots, \langle (S^z)^{2S}\rangle$, and $\langle (S^-)^2\rangle$, $\langle (S^-)^2 S^z\rangle, \ldots, \langle (S^-)^2 (S^z)^{2S-1}\rangle$. These $4S$ unknown variables can be determined completely from the $4S$ simultaneous equations in Eqs. (22) and (23) by self-consistent procedure. After these unknown variables are determined by self-consistent procedure, the Green's functions $G_1^{(n)}(k,\omega)$ and $G_2^{(n)}(k,\omega)$ can be determined completely, then all the other calculations, such as the calculations of the spin-wave excitation spectrum, the temperature-dependent magnetization, and the Curie temperature, etc., can be carried out straightforwardly.

To test the validity of this Green's-function formalism, we first apply this Green's-function formalism to the case of three-dimensional single-ion easy-plane ferromagnets neglecting the dipole-dipole interactions (the dipole-dipole interactions are negligible for three-dimensional ferromagnets). In this case, there are several other theoretical approaches for comparison. In Fig. 1, we have plotted the low-temperature spin-wave excitation spectrums obtained by this Green's-function formalism and the MME method for three-dimensional easy-plane spin-one ferromagnets. As is well known, in the description of low-temperature spin-wave excitations, it is usually more convenient to transform the Hamiltonian of spin operators into a Hamiltonian of simpler Bose or Fermi operators. For example, the well-known transformations by Holstein and Primakoff[44] and by Dyson[45] and Maleev[46] are transformations of the spin operators to a series of Bose operators. But due to the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy, these well known transformations cannot be applied in the description of spin-wave excitations in such spin systems. For instance, a naive use of the well ordered Holstein-Primakoff transformation to Hamiltonian (1) will violate the Goldstone theorem and lead to imaginary values for the energies of the $k \sim 0$ modes no matter how small the anisotropy constant D is.[30–33] The Dyson-Maleev transformation is also not adequate in the presence of the single-ion easy-plane anisotropy, since it gives a non-Hermitian quadratic Hamiltonian where the kinematic condition, which requires that for spin $S = \frac{1}{2}$ the magnon energies should not be affected by the anisotropy, is not satisfied even at this order. The most appropriate way for overcoming these difficulties is the method of matching of matrix elements (MME).[30–33] Basically, the MME method provides a more convenient starting point for single-ion states by means of a perturbative treatment of the off-diagonal terms in the anisotropy, then a boson representation of the spin operators is achieved by equating the corresponding matrix elements in the respective spaces (matching of matrix elements, or MME). This method is very

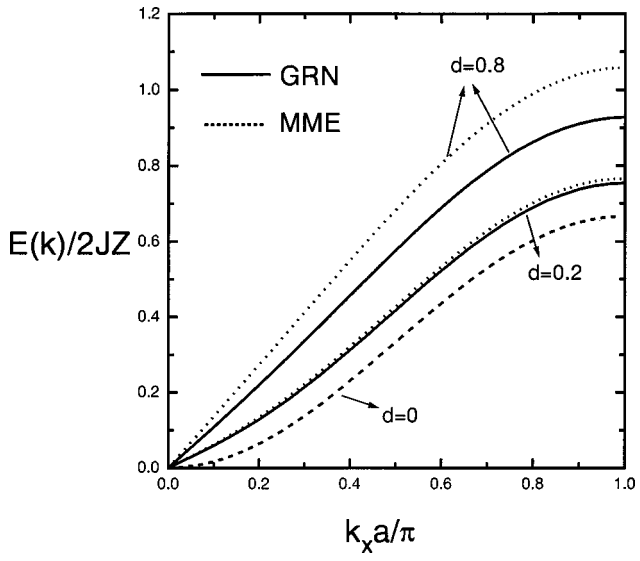

FIG. 1. The spin-wave excitation spectrums of 3D single-ion easy-plane spin-one ferromagnets for different values of the anisotropy constant. The solid lines represent the results of the present Green's-function formalism and the dotted lines represent the results of the MME method. In the case of $D = 0$, the two methods give exactly the same results. (All parameters are given in units of J. $T/J = 0.1$, $d = D/J$ is shown in the figure.)

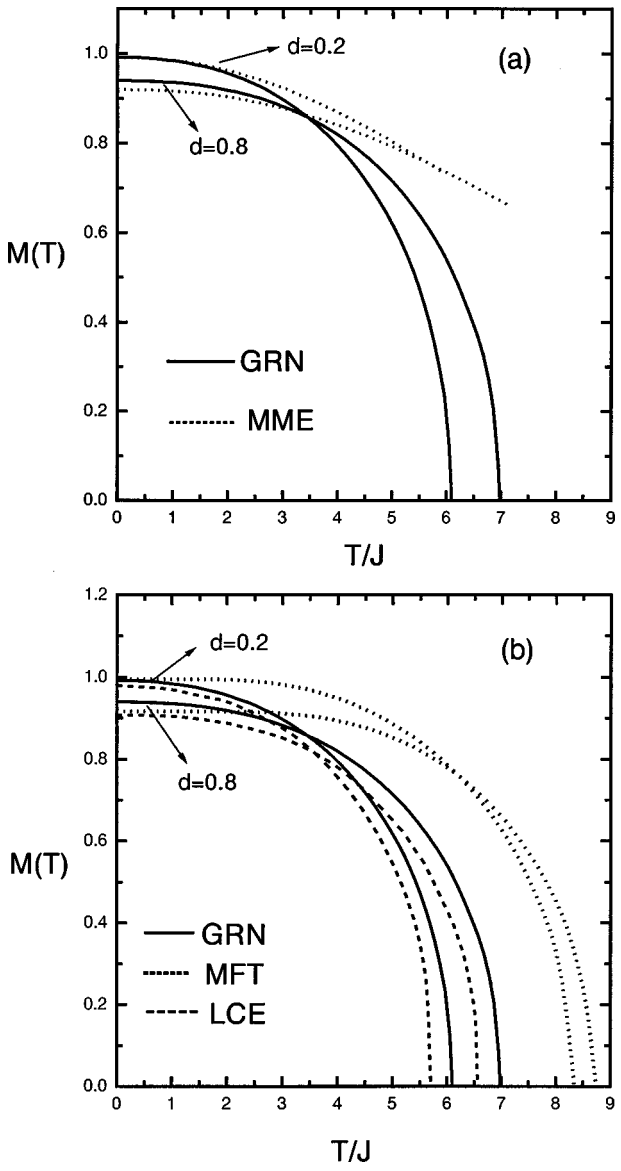

FIG. 2. (a) The comparison between the temperature dependence of magnetization of 3D single-ion easy-plane spin-one ferromagnets obtained by the present Green's-function formalism and the MME method. (b) The comparison between the temperature dependence of magnetization of 3D single-ion easy-plane spin-one ferromagnets obtained by the present Green's-function formalism (GRN), the mean-field theory (MFT), and the LCE method, respectively. (The parameters are shown in the figure.)

appropriate in the case of small anisotropy, but it will become worse if the anisotropy is large due to its perturbative treatment of the off-diagonal terms in the anisotropy. In the present Green's-function formalism, since the off-diagonal quantum mixing effects of the single-ion easy-plane anisotropy have been taken into account in a self-consistent way, those difficulties encountered by such as Holstein-Primakoff (HP) transformation and Dyson-Maleev transformation are overcome naturally. From the magnon dispersion relation [see Eq. (21)], after some simple algebra, we can easily verify that: (i) the magnon energy $E(k)$ is always positive for the $k \neq 0$ modes (no imaginary values will occur for the energies of the $k \sim 0$ modes); (ii) $E(k) \rightarrow 0$ in the limit of $k \rightarrow 0$ (satisfying correctly the Goldstone theorem); (iii) for spin $S = \frac{1}{2}$ the magnon energies will not be affected by the anisotropy (satisfying correctly the kinematic condition). Moreover, as is shown in Fig. 1, in the case of small anisotropy, the results obtained by the present Green's-function formalism are very close to the corresponding results of the MME method, which is correct in the case of small anisotropy, and for very small anisotropy, the results obtained by these two method will become actually the same.

In Figs. 2(a) and (b) we have plotted the temperature dependences of magnetization of three-dimensional single-ion easy-plane spin-one ferromagnets for different values of the anisotropy constant D, obtained by this Green's-function formalism (GRN), the method of matching of matrix elements (MME), the mean-field theory (MFT), and the linked-cluster series expansion method (LCE), respectively. From Fig. 2(a) we see again that at low temperature, the results of the present Green's-function formalism are very close to that of the MME method, which is valid at low temperature, and the smaller the anisotropy is, the smaller the difference between the corresponding results of the two methods. But at high temperature, the MME method will break down completely. From Fig. 2(b) we can see that at high temperature, compared with the mean-field theory, the results of the present Green's-function formalism are very close to the corresponding results of the LCE method which provides the most accurate calculations for the thermodynamic quantities at high temperature. The reason for this is clear. The mean-field theory neglects the quantum and thermal fluctuation correlations completely. In contrast, the present Green's-function formalism and the LCE method both include the effects of the quantum and thermal fluctuation correlations. The values of magnetization at high temperature and the Curie temperature can be reduced substantially from the mean-field values by both quantum and thermal fluctuation correlations, as is shown in Fig. 2(b). From these comparisons, we can see that this self-consistent Green's-function formalism can present an approximately valid description for single-ion easy-plane ferromagnets at both low and high temperature. At low temperature, it is similar to the method of matching of matrix elements (MME) and can present a correct description of the spin-wave excitations in single-ion easy-plane ferromagnets; at high temperature, it can improve the mean-field theory substantially. But it should be noted that, due to the mean-field nature of the decoupling procedure in Eqs. (12) and (13), like the mean-field theory, this Green's-function approach cannot present a correct description of the critical behavior in the vicinity of the critical point. This is also the shortcoming of the usual Green's-function approach for isotropic or single-ion easy-axis ferromagnets. Since in this paper we will not discuss the critical behavior in the critical region, this self-consistent Green's-function formalism can be applied at both low and high temperature. In Sec. III, we will apply this Green's-function formalism to investigate the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy on the magnetism of two-dimensional ferromagnets.

III. EFFECTS OF INTERPLAY OF DIPOLE-DIPOLE INTERACTIONS AND SINGLE-ION EASY-PLANE ANISOTROPY ON TWO-DIMENSIONAL FERROMAGNETS

In this section, we apply the Green's-function formalism established in Sec. II to study the effects of the interplay of

the dipole-dipole interactions and single-ion easy-plane anisotropy on the magnetism of two-dimensional ferromagnets, including the stability of the long-range magnetic order at finite temperature, the spin-wave excitation spectrum, the temperature dependence of the spontaneous magnetization, and the transition temperature. While there have been recently many theoretical investigations on the effects of the interplay of the single-ion easy-axis anisotropy and dipole-dipole interactions on 2D ferromagnets,[19–29] up to now, there have been no theoretical investigations on the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy on two-dimensional ferromagnets, their respective influence on the magnetism of two-dimensional ferromagnets still remains to be clarified.

In the presence of the dipole-dipole interactions, a two-dimensional ferromagnet with single-ion easy-plane anisotropy can be described by the following Hamiltonian:

$$H=-J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j+D\sum_{\mathbf{i}}(S_i^x)^2-h\sum_i S_i^z + \frac{\Omega}{2}\sum_{ij}\frac{1}{|\mathbf{l}_i-\mathbf{l}_j|^3}\left(\mathbf{S}_i\cdot\mathbf{S}_j-3\frac{[\mathbf{S}_i\cdot(\mathbf{l}_i-\mathbf{l}_j)][\mathbf{S}_j\cdot(\mathbf{l}_i-\mathbf{l}_j)]}{|\mathbf{l}_i-\mathbf{l}_j|^2}\right), \tag{28}$$

where $\mathbf{l}_i$ is the two-dimensional lattice vector in the film plane (the y-z plane), Ω is the dipole-dipole interaction constant, $D(>0)$ is the single-ion easy-plane anisotropy constant, h is proportional to the external field. As in Sec. II, for simplicity, only the nearest-neighbor exchange couplings of spins are taken into account. By the same procedure as in Sec. II, it can be easily verified that after including the term of the dipole-dipole interactions, the Fourier components of the Green's function $G_1^{(n)}$ and $G_2^{(n)}$ defined in Eqs. (3) and (4), can be expressed as

$$G_1^{(n)}(k,\omega)=\frac{1}{2E(k)[\omega-E(k)]}\{\langle g_1^{(n)}(S^z)\rangle[E(k)+F_1(k)] +F_2(k)\langle g_2^{(n)}(S^-,S^z)\rangle\} + \frac{1}{2E(k)[\omega+E(k)]}\{\langle g_1^{(n)}(S^z)\rangle[E(k)-F_1(k)] -F_2(k)\langle g_2^{(n)}(S^-,S^z)\rangle\}, \tag{29}$$

$$G_2^{(n)}(k,\omega)=\frac{1}{2E(k)[\omega-E(k)]}\{\langle g_2^{(n)}(S^-,S^z)\rangle \times[E(k)-F_1(k)]-F_2(k)\langle g_1^{(n)}(S^z)\rangle\} + \frac{1}{2E(k)[\omega+E(k)]}\{\langle g_2^{(n)}(S^-,S^z)\rangle \times[E(k)+F_1(k)]+F_2(k)\langle g_1^{(n)}(S^z)\rangle\}, \tag{30}$$

and correspondingly, the self-consistent Eqs. (22) and (23) can be rewritten as

$$\langle(S^-)^n(S^+)^n\rangle=\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{\beta E(k)}-1]} \times\{F_2(k)\langle g_2^{(n)}(S^-,S^z)\rangle +\langle g_1^{(n)}(S^z)\rangle[E(k)+F_1(k)]\} -\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{-\beta E(k)}-1]} \times\{F_2(k)\langle g_2^{(n)}(S^-,S^z)\rangle -\langle g_1^{(n)}(S^z)\rangle[E(k)-F_1(k)]\}, \tag{31}$$

$$\langle(S^-)^n(S^+)^{n-1}S^-\rangle=-\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{\beta E(k)}-1]} \times\{F_2(k)\langle g_1^{(n)}(S^z)\rangle -\langle g_2^{(n)}(S^-,S^z)\rangle[E(k)-F_1(k)]\} +\frac{1}{N}\sum_k\frac{1}{2E(k)[e^{-\beta E(k)}-1]} \times\{F_2(k)\langle g_1^{(n)}(S^z)\rangle +\langle g_2^{(n)}(S^-,S^z)\rangle[E(k)+F_1(k)]\}, \tag{32}$$

where

$$E(k)=\sqrt{[F_1(k)]^2-[F_2(k)]^2}, \tag{33}$$

$$F_1(k)=\varepsilon_k+\frac{\Omega}{2}\langle S_z\rangle p_{xx}(k)+\frac{\Omega}{2}\langle S_z\rangle p_{yy}(k) -\Omega\langle S^z\rangle p_{zz}(0)+h+D\Gamma\langle S^z\rangle, \tag{34}$$

$$F_2(k)=\frac{\Omega}{2}\langle S_z\rangle p_{xx}(k)-\frac{\Omega}{2}\langle S_z\rangle p_{yy}(k)+D\Gamma\langle S^z\rangle, \tag{35}$$

$$\varepsilon_k=2JZ\langle S^z\rangle(1-\gamma_k), \tag{36}$$

$$\gamma_k=\frac{1}{Z}\sum_{\mathbf{a}}e^{i\mathbf{k}\cdot\mathbf{a}}, \tag{37}$$

in which Z is the number of the nearest neighbors, $\mathbf{a}$ denotes lattice vectors between two nearest-neighbor lattice sites, and $p_{\alpha\beta}(k)$ is defined as follows:

$$p_{\alpha\beta}(\mathbf{k})=\sum_j\frac{1}{|\mathbf{l}_i-\mathbf{l}_j|^3}\left(\delta_{\alpha\beta}-3\frac{(\mathbf{l}_i-\mathbf{l}_j)_\alpha(\mathbf{l}_i-\mathbf{l}_j)_\beta}{|\mathbf{l}_i-\mathbf{l}_j|^2}\right)e^{i\mathbf{k}\cdot(\mathbf{l}_j-\mathbf{l}_i)}, \quad(\alpha,\beta=x,y,z). \tag{38}$$

Due to the translational invariance, these Fourier components are independent of the subscript i, and $p_{\alpha\beta}(\mathbf{k})=0$ for $\alpha\neq\beta$. The lattice sums in Eq. (38) converge very slowly in an oscillatory manner, direct and accurate computation of the lattice sums is difficult, but they can be converted into series which converge very rapidly using Ewald lattice summation method.[48] Following this method, $p_{\alpha\alpha}(\mathbf{k})$ can be expressed as

$$p_{xx}(\mathbf{k})=q_1(k_y,k_z)+q_2(k_y,k_z), \tag{39}$$

$$p_{yy}(\mathbf{k})=q_1(k_y,k_z)-2q_2(k_y,k_z), \tag{40}$$

$$p_{zz}(\mathbf{k})=q_2(k_y,k_z)-2q_1(k_y,k_z), \tag{41}$$

in which

$$q_1(k_y,k_z)=\frac{16}{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\left(n\pi+\frac{k_y}{2}\right)^2 \times\cos(mk_z)K_2\left(2m\left|n\pi+\frac{k_y}{2}\right|\right), \tag{42}$$

$$q_2(k_y,k_z)=\frac{16}{3}\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\left(n\pi+\frac{k_z}{2}\right)^2 \times\cos(mk_y)K_2\left(2m\left|n\pi+\frac{k_z}{2}\right|\right), \tag{43}$$

where $K_2(x)$ is the modified Bessel function of second order. Since for large values of x, $K_n(x)\sim e^{-x}/\sqrt{x}$, these series are very rapidly convergent, and they will be used in the numerical calculations. In the following, we need to consider the analytical form of the spin-wave excitation spectrum in the neighborhood of $k=0$, in this case it is more convenient to expand $p_{\alpha\alpha}(\mathbf{k})$ into a series of k in the neighborhood of $k=0$. Using a method similar to that of Ref. 18, which treat the dipole-dipole interactions within the long-wavelength approximation, we can get (in the limit of $k\rightarrow 0$)

$$p_{xx}(\mathbf{k})\approx\frac{8\pi f}{3}-2\pi k+O(k^2), \tag{44}$$

$$p_{yy}(\mathbf{k})\approx-\frac{4\pi f}{3}+\pi k(1-\cos 2\theta)+O(k^2), \tag{45}$$

$$p_{zz}(\mathbf{k})\approx-\frac{4\pi f}{3}+\pi k(1+\cos 2\theta)+O(k^2), \tag{46}$$

where $f=1.0782$ (for square lattice), θ is the angle between the wave vector k and the z axis.

As has been shown in Sec. II, both the left-hand sides and the right-hand sides in the self-consistent Eqs. (31) and (32) are functions of $4S$ independent variables: $\langle(S^z)^n\rangle$ and $\langle(S^-)^2(S^z)^{n-1}\rangle$, $(n=1,2,\ldots,2S)$. By the help of Eqs. (26) and (27), these $4S$ independent unknown variables can be determined completely from the $4S$ simultaneous Eqs. (31) and (32) through self-consistent procedures. Before we solve the self-consistent Eqs. (31) and (32) by numerical method, we should first investigate the stability of the long-range ferromagnetic order at finite temperature in an infinite two-dimensional lattice. Whether the long-range ferromagnetic order is stable at finite temperature is determined by the features of the long-wavelength thermal fluctuations. In the above Green's-function formalism, this stability depends on the convergence of the two-dimensional integrals of wave vector $\mathbf{k}$ at its lower bound in the self-consistent Eqs. (31) and (32). (The summation of wave vector $\mathbf{k}$ can be replaced by an integrals over wavevector $\mathbf{k}$ as $N\rightarrow\infty$.) The convergence or divergence of the integrals in Eqs. (31) and (32) is determined by the spin-wave excitation spectrum $E(\mathbf{k})$ in the neighborhood of $k=0$. In the neighborhood of $k=0$, we can expand $E(\mathbf{k})$ into a series of k. Using Eqs. (33)–(37) and (44)–(46), we arrive at the following conclusions: (i) If there is no external field ($h=0$), there will be no any energy gap at the bottom of the spin-wave excitation spectrum, i.e., $E(\mathbf{k})\rightarrow 0$ in the limit of $k\rightarrow 0$. This is required by the Goldstone theorem; (ii) If there is no dipolar interactions ($\Omega=0$), the spin-wave excitation spectrum $E(\mathbf{k})$ has the following dispersion relation in the neighborhood of $k=0$:

$$E(\mathbf{k})=\sqrt{C_0+C_2k^2+O(k^3)}, \tag{47}$$

in which

$$C_0=h^2+2hD\Gamma\langle S^z\rangle, \tag{48}$$

$$C_2=4J\langle S^z\rangle[h+D\Gamma\langle S^z\rangle]. \tag{49}$$

From this dispersion relation, we can see that if there is no external field ($h=0$), the integrals in Eqs. (31) and (32) diverge logarithmically at its lower bound. In this case, the self-consistent Eqs. (31) and (32) cannot have any definite solutions at finite temperature as $N\rightarrow\infty$ (i.e., an infinite two-dimensional lattice). This result shows that for an infinite two-dimensional ferromagnet, the single-ion easy-plane anisotropy alone cannot stabilize the long-range ferromagnetic order at finite temperature if there are no long-range dipole-dipole interactions. (If neglecting the term of the dipole-dipole interactions in Hamiltonian (28), the MME method can be applied, and the same conclusion as this result can be obtained.[49] But in the presence of the dipole-dipole interactions, just like the Holstein-Primakoff transformation, a naive application of the spin-Bose operator transformation derived by the MME method will also violate the Goldstone theorem[49]); (iii) In the presence of the long-range dipole-dipole interactions ($\Omega\neq 0$), the spin-wave excitation spectrum $E(\mathbf{k})$ has the following dispersion relation in the neighborhood of $k=0$:

$$E(\mathbf{k})=\sqrt{C_0+C_1k+O(k^2)}, \tag{50}$$

in which

$$C_0=h^2+4\pi f\Omega h\langle S^z\rangle+2hD\Gamma\langle S^z\rangle, \tag{51}$$

$$C_1=[h+4\pi f\Omega\langle S^z\rangle+2D\Gamma\langle S^z\rangle] \times\pi\Omega\langle S^z\rangle(1-\cos 2\theta)-2\pi\Omega h\langle S^z\rangle. \tag{52}$$

From this dispersion relation, we can see that even if there is no external field ($h=0$ and hence $C_0=0$), the integrals in Eqs. (31) and (32) are convergent at its lower bound. In this case, the self-consistent Eqs. (31) and (32) have stable solutions as $N\rightarrow\infty$. This suggests that the long-range dipole-dipole interactions may itself stabilize the long-range magnetic order in an infinite two-dimensional lattice at finite temperature. Therefore, for two-dimensional single-ion easy-plane ferromagnets, the long-range dipole-dipole interactions play the crucial role in stabilizing the long-range magnetic order at finite temperature. This is significantly different from the case of two-dimensional easy-axis ferromagnets, in which the long-range magnetic order at finite temperature is stabilized by an anisotropy-induced energy gap at the bottom

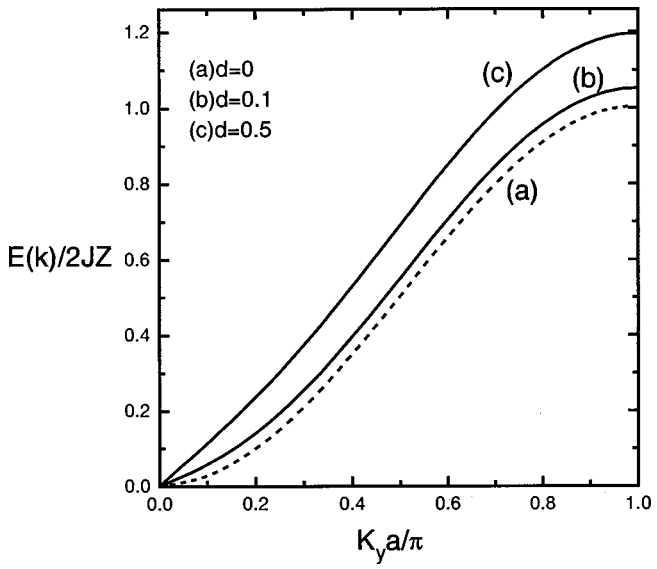

FIG. 3. The spin-wave excitation spectrums of 2D single-ion easy-plane spin-one ferromagnets. (All parameters are given in units of J. $\omega=\Omega/J=0.005$, $T/J=0.1$, $d=D/J$ is shown in the figure.)

of the spin-wave excitation spectrums and the effects of the dipole-dipole interactions are negligible compared with the effects of the easy-axis anisotropy; (iv) In the presence of external field ($h\neq 0$), there is always an energy gap at the bottom of the spin-wave excitation spectrum:

$$E(0)=\sqrt{h^2+4\pi f\Omega h\langle S^z\rangle+2hD\Gamma\langle S^z\rangle}. \tag{53}$$

This energy gap will increase greatly the stability of the long-range ferromagnetic order at finite temperature, and therefore two-dimensional easy-plane ferromagnets will have very sensitive dependence on the external field in the neighborhood of $h=0$.

We have discussed the stability of the long-range ferromagnetic order at finite temperature in an infinite two-dimensional ferromagnet with single-ion easy-plane anisotropy and have investigated the convergence of the two-dimensional integrals of wave vector $\mathbf{k}$ in the self-consistent Eqs. (31) and (32). We can see that the dipole-dipole interactions play a dominant role in stabilizing the long-range magnetic order at finite temperature in an infinite two-dimensional ferromagnet with single-ion easy-plane anisotropy. Once the convergence of the integrals in Eqs. (31) and (32) has been established, the numerical calculation can be performed safely. Some numerical results are shown in Figs. 3–5. In Fig. 3 we have plotted the low-temperature spin-wave excitation spectrums of two-dimensional single-ion easy-plane spin-one ferromagnets for different values of the anisotropy constant D. In Fig. 4(a) and (b) we have plotted the temperature dependences of the spontaneous magnetization for different values of the anisotropy constant D and the dipole-dipole interaction constant Ω, respectively. In Figs. 5(a) and (b), we have shown the relations between the transition temperature and the anisotropy constant D and the dipole-dipole interaction constant Ω, respectively. From Fig.

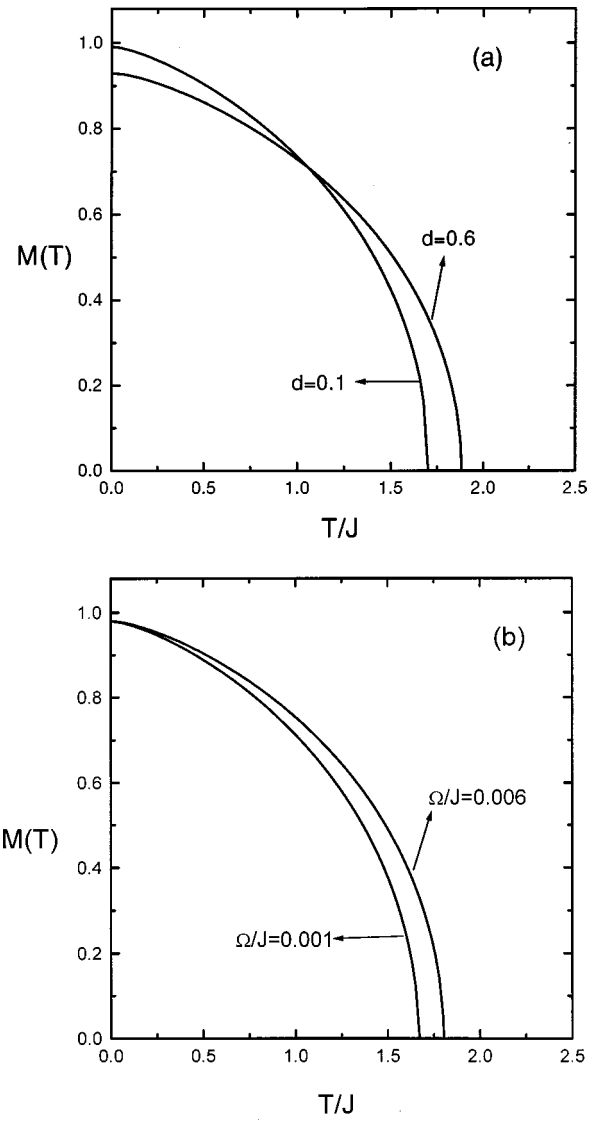

FIG. 4. (a) The temperature dependence of the spontaneous magnetization of 2D single-ion easy-plane spin-one ferromagnets for different values of the anisotropy constant. The parameters: $\omega=\Omega/J=0.005$, $d=D/J$ is shown in the figure. (b) The temperature dependence of the spontaneous magnetization for different values of the dipole-dipole interaction constant. The parameters: $d=0.2$, ω is shown in the figure.

3 we can see that the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy does not induce any energy gap at the bottom of the spin-wave excitation spectrum, but the single-ion easy-plane anisotropy can modify the spin-wave energies for the $k\neq 0$ modes significantly. (From Fig. 3 we can see that the increase of the anisotropy will increase the spin-wave energies for the $k\neq 0$ modes.) The numerical calculations also show that the dipole-dipole interactions have little influence on the spin-wave energies of the $k\neq 0$ modes, but as has been discussed above, the dipole-dipole interactions can modify significantly the magnon dispersion relation in the neighborhood of $k=0$ [see Eqs. (50)–(52)], and this modification plays the crucial role in the stabilization of the long-range magnetic order at finite temperature. From Fig. 4(a) we can see that the

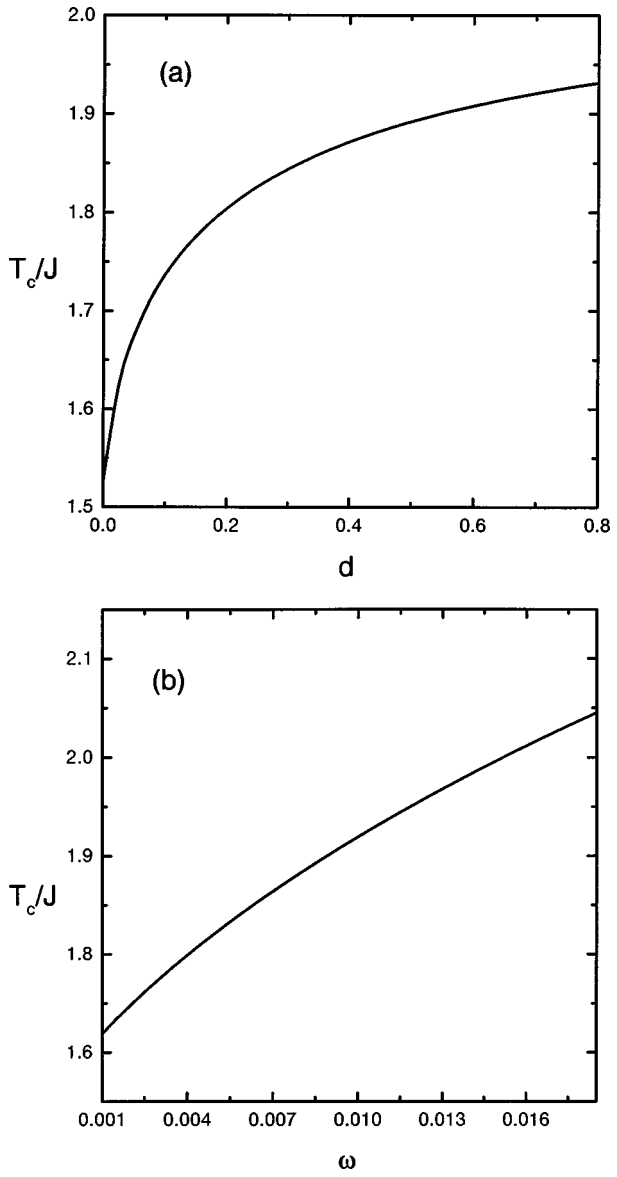

FIG. 5. (a) The relation between the transition temperature and the anisotropy constant for 2D single-ion easy-plane spin-one ferromagnets. The parameters: $\omega=\Omega/J=0.006$. (b) The relation between the transition temperature and the dipole-dipole interaction constant. The parameters: $D/J=0.2$.

increase of the anisotropy will decrease the ground-state magnetization but increase the stability of the magnetic order at high temperature. The reason is that the larger the anisotropy constant is, the stronger the quantum mixing between the $|m-2\rangle$, $|m\rangle$, $|m+2\rangle$ single-ion eigenstates of S^z, thus the smaller the ground-state magnetization is. But as is shown in Fig. 3, the increase of the anisotropy will also increase the magnon energies of the $k\neq 0$ modes, thus reduce the thermal fluctuations and increase the stability of the magnetic order at high temperature. From Fig. 4(b), we can see that the increase of the dipole-dipole interactions will also increase the stability of the magnetic order at high temperature, but the ground-state magnetization is almost independent of the dipole-dipole interaction constant Ω, i.e., the quantum mixing effects of the dipole-dipole interactions are negligible. From Figs. 5(a) and (b), we can see that the transition temperature decreases monotonically as the anisotropy constant decreasing or the dipole-dipole interaction constant decreasing, indicating that the magnetic order at finite temperature will become less and less stable as the anisotropy decreasing or the dipole-dipole interactions decreasing. In order to give a more quantitative idea of the above theoretical results, it may be helpful to make some contact with the realistic systems. Some experimental observations have found that, in some fcc (001) Co ultrathin films, the spontaneous magnetization lies in the film plane, and the in-plane spontaneous magnetizations can exist up to temperature as large as above 500 K.[6,7] In the above theory, if we choose the values of the model parameters to fit the transition temperature T_c of 3D easy-plane ferromagnets described by the Hamiltonian (1) to be about the magnitude of the Curie temperature of the bulk Co, then for the corresponding 2D easy-plane ferromagnets described by the Hamiltonian (28), the transition temperature T_c do can be as large as above 500 K in the presence of the dipole-dipole interactions, i.e., in the above theory, the dipolar interactions do can be responsible for the stabilization of the long-range ferromagnetic order up to a temperature of above 500 K. Of course, there may be some significant differences between the nearest-neighbor Heisenberg model used in the above theory and the realistic systems, but we think that the above qualitative features of the effects of the dipolar interactions and the easy-plane anisotropy should still remain if more realistic models are used in the calculations. This will be investigated in more detail in the future.

In conclusion, using the nearest-neighbor Heisenberg model as a starting point, we have provided a theoretical analysis on the effects of the interplay of the dipole-dipole interactions and single-ion easy-plane anisotropy on the magnetism of two-dimensional ferromagnets. The respective influence of the dipole-dipole interactions and single-ion easy-plane anisotropy on the stability of the long-range magnetic order at finite temperature in two-dimensional ferromagnets is investigated analytically. The results show that for two-dimensional ferromagnets with single-ion easy-plane anisotropy, the single-ion easy-plane anisotropy alone cannot stabilize the long-range magnetic order at finite temperature, and the dipole-dipole interactions play the crucial role in stabilizing the long-range magnetic order at finite temperature. The low temperature spin-wave excitation spectrums, the temperature dependence of the spontaneous magnetization, and the transition temperature, have been calculated numerically. Though the above theory is based on the nearest-neighbor Heisenberg model as a starting point, the theoretical method established in this paper can also be applied to some more realistic model systems. At the end, it should pointed out that, due to the mean-field nature of the decoupling scheme in Eqs. (12) and (13), this Green's-function formalism cannot present a correct description of the critical behavior in such magnetic systems. If one wants to investigate the critical behavior in such magnetic systems, one must resort to other theoretical approaches such as the linked-cluster series expansion method (LCE).[39–41] In addition, if the anisotropy constant D is very large (the anisotropy dominates over the exchange interactions, or approximately $D/zJ\sim 1$), the RPA decoupling scheme in Eqs. (12)

and (13) will break down due to the strong quantum mixing between single-ion energy levels induced by the single-ion easy-plane anisotropy, and hence the present Green's-function formalism will become inappropriate (the same is also true for some other theoretical methods). In such cases, more powerful decoupling schemes should be searched for.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation of China (No. 19774020), the National Education Commission (No. 97024610), and the Shanghai Center of Applied Physics.

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