Localization of phonons in a two-component superlattice with random-thickness layers

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(Received 26 October 1998; revised manuscript received 30 March 1999)

A two-component superlattice film of $2N$ layers is considered and a dimensionally quantized spectrum of phonons is found. The problem of localization of phonons in the superlattice with random-thickness layers is investigated. The Landauer resistance of the transport of phonons and the correlation length is calculated exactly. For short range disorder the numerical analyses shows that, at frequency $\omega=0$, there is a delocalized state and the correlation length index ν is equal to 2. [S0163-1829(99)05233-9]

I. INTRODUCTION

The interest in superlattice structure in condensed matter physics is based on the possibility of manipulation of the physical properties of devices by changing of characteristic lattice parameters.

The growth techniques can be used to prepare specimens consisting of alternating layers of thickness d_1 of constituent A and thickness d_2 of constituent B. Samples can be prepared so that d_1 and d_2 have any value from two or three atomic spacings up to one hundred atomic spacings. The entities A and B can be materials with different acoustic, electronic, and magnetic properties and one can consider semiconductor, metal, insulator, and superconductor constituents, attempting to change values of expectable physical variables in a desirable regime.

The technical advance in fabrication of superlattices motivated an intensive study of various physical properties of these systems, especially electronic and vibration spectra, optical and magnetic properties, etc. The calculations[1,2] show that the spectrum of quasiperiodic systems are intermediate between periodic and random ones.

Along with electronic properties, study of elastic waves in bulk superlattices has been a subject of interest in the past decades.[3–14] A one-dimensional theory of acoustic vibrations in layered material was given long ago by Rytov.[15] Elastic waves have also been investigated in semi-infinite superlattices.[16–19]

The consideration of the superlattice films instead of the massive ones provides additional opportunities for controlling the elastic and electronic parameters of the superlattices. A sufficiently complete experimental knowledge about the oscillator spectra of binary laminated semiconductor systems is available today.[20] Numerous and generally mutually compatible results about the frequencies of long-wave phonons in InAs-GaSb, Ge-GaAs are presented in the literature.[21]

Together with strongly periodic superlattices the effects of localization and tunneling of the electrons were studied in short range disorder superlattices.[22–25]

The problem of localization of electrons in random potential and hopping parameters in low dimensional spaces are of continuous interest to physicists after Anderson's remarkable article.[26] Originally Mott and Twose[27] conjectured that all states are localized in one-dimensional (1D) systems for any degree of disorder. It was argued[28] that in the case of full randomness all states are localized in dimensions equal to or less than 2. However, recent investigations show[29–31] that delocalized states can appear in the case of correlated disorder.

In the early 1970s Landauer[32] proposed that the dc conductance G of noninteracting (spinless) electrons in a disordered medium in strictly one dimension is given by

$$G=\frac{e^2}{2\pi\hbar}\frac{|\tau|^2}{1-|\tau|^2}, \tag{1}$$

where $|\tau|$ is the transmission amplitude. This expression is attractive for at least two reasons. First, as it was demonstrated in Ref. 33, the disorder here can be taken into account exactly. Second, the dimensionless expression $(2\pi\hbar/e^2)G$ of conductance is assumed to be the only relevant variable in a scaling theory treatment of the localization problem.[28]

In Refs. 32 and 34 the dimensionless conductance was obtained by dividing the current I of electrons by the chemical potential difference $\Delta\mu$ between the left and right sides of the sample

$$\frac{2\pi\hbar}{2e^2}G=\frac{2\pi\hbar I}{2e\Delta\mu}, \tag{2}$$

which can be expressed as the ratio of transmitted intensity of electrons over reflected ones. The differences between Landauer conductance and the Kubo formula were analyzed in Ref. 35.

It is easy to see that we can define the Landauer conductance also for phonons, where I/e represents the phonons number current, while the difference of chemical potential is caused by the variation of density of the matter in the left and right sides of the bunch of layers. The acoustic phonons current, or in other words the sound waves, can be created by the deformation of the density of the sample.

The transport of phonons within the Landauer approach was considered also in Ref. 36, where the quantization of thermal conductance was predicted.

The aim of the present article is twofold. First we found the spectrum of transversal phonons in two-component superlattice film with boundaries with an arbitrary finite number of slices. The results, obtained here, are also applicable for longitudinal waves, which moves in the transversal to the layers direction. Second, we consider random distribution of the thicknesses of the superlattice compounds and calculate the Landauer resistance[32] (R/T, the inverse of the conductance) of the acoustic phonons exactly. The continuous model is used, in which the layers are considered as macroscopic elastic bodies.

II. BOUNDARY CONDITIONS IN THE SUPERLATTICE FILM, TRANSFER MATRIX, AND THE SPECTRUM

We follow here the notations and derivations of the book in Ref. 37.

Let us consider superlattice, elementary cells which consist of two layers of various materials A and B with the thickness d_1, d_2 and modulus of rigidity μ_1, μ_2 (Fig. 1). The number of pairs in the film is N.

We consider transversal elastic waves [$\mathrm{div}\,\vec{u}=0$, $\vec{u}(x)$ is the vector of elastic displacement of the matter at the space point x] propagating inside a superlattice in arbitrary direction. It can be shown that all results are reproducible for longitudinal waves if they propagate in the perpendicular to layers direction.

Let us choose a coordinate system such that waves are propagating in the (x,y) plane [x represents perpendicular to layers direction, while (y,z) plane is parallel to the layers plane], with the wave vectors $(k_1,q_1,0)$ and $(k_2,q_2,0)$ in the A and B materials correspondingly. Without loss of generality one can take $u_x=u_y=0$ and $u_z=u_i$, where i numerates the layers.

The wave equation for transversal waves is

$$\frac{\partial^2 u_i}{\partial t^2}-c_t^2\Delta u_i=0, \tag{3}$$

where the velocity of sound c_t is defined by the density of matter ρ and modulus of rigidity μ as

$$c_t^2=\frac{\mu}{\rho}. \tag{4}$$

The solutions of this wave equation with frequency ω, which fulfills transversality condition $\mathrm{div}\vec{u}=0$, is the superposition of forward- and backward-traveling waves

$$u_{2n-1}(x,y)=(c_{2n-1}e^{ik_1x}+\overline{c}_{2n-1}e^{-ik_1x})e^{i(qy-wt)},$$
$$u_{2n}(x,y)=(c_{2n}e^{ik_2x}+\overline{c}_{2n}e^{-ik_2x})e^{i(qy-wt)}, \quad n=1,\dots,N, \tag{5}$$

where for $k_i, i=1,2$ we have

$$k_i^2+q^2=\frac{\omega^2}{c_{it}^2}. \tag{6}$$

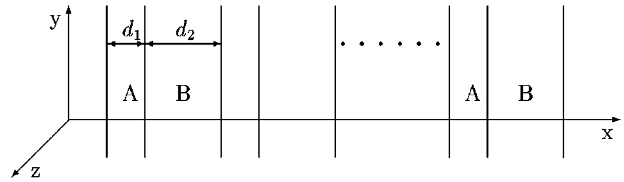

FIG. 1. The two-component superlattice in the coordinate system.

In formulas (5) $2n$ (correspondingly $2n-1$) numerates the layers of B (or A) type and $c_{1t}(c_{2t})$ are the velocities of sound in those materials.

We should now impose the boundary conditions on the displacements u_{2n} and u_{2n-1}. Let us consider now free boundaries of the film, which means the use of Neumann boundary conditions

$$u_1'(x_0)=u_{2N}'(x_{2N})=0, \tag{7}$$

where x_0 and x_{2N} are the boundary coordinates.

On the boundary x_i of the A and B layers one should use the continuity condition for the displacements

$$u_{2n}(x_{2n-1})=u_{2n-1}(x_{2n-1}), \quad n=1,2,\dots,N \tag{8}$$

as well as for the forces

$$F_i=\sigma_{ik}ds_k, \tag{9}$$

where σ_{ik} is the stress tensor (see Ref. 37).

In Eq. (9) $ds_k=n_k ds$ is the normal vector to the boundary and equal to a small area in modulo. Hence we have

$$\sigma_{ik}^1 n_k=\sigma_{ik}^2 n_k. \tag{10}$$

On the boundaries of the film the forces are equal to zero

$$\sigma_{ik}n_k=0. \tag{11}$$

By use of the expression for σ_{ik} (see Ref. 37) and from the boundary conditions (7),(8) and (10),(11) one can easily obtain the following set of equations for the displacements u:

$$u_{2N}'(x_{2N})=u_1'(x_0)=0,$$

$$\mu_2 u_{2n}'(x_{2n-1})=\mu_1 u_{2n-1}'(x_{2n-1}),$$

$$u_{2n}(x_{2n-1})=u_{2n-1}(x_{2n-1}), \quad n=1,2,\dots N. \tag{12}$$

These equations transform into the following equations for the coefficients of the forward- and backward-traveling waves:

$$c_{2N}e^{ik_2(d_1+d_2)n}-\overline{c}_{2N}e^{-ik_2(d_1+d_2)n}=0$$

$$\vdots$$

$$\begin{aligned}\mu_2k_2c_{2n}&e^{ik_2[(d_1+d_2)(n-1)+d_1]}-\mu_2k_2\overline{c}_{2n}e^{-ik_2[d_1n+d_2(n-1)]}\\&=\mu_1k_1c_{2n-1}e^{ik_1[d_1n+d_2(n-1)]}-\mu_1k_1\overline{c}_{2n-1}e^{-ik_1[d_1n+d_2(n-1)]}c_{2n}e^{ik_2[d_1n+d_2(n-1)]}+\overline{c}_{2n}e^{-ik_2[d_1n+d_2(n-1)]}\\&=c_{2n-1}e^{ik_1[d_1n+d_2(n-1)]}+\overline{c}_{2n-1}e^{-ik_1[d_1n+d_2(n-1)]}\end{aligned} \tag{13}$$

$$\vdots$$

$$c_1-\overline{c}_1=0.$$

We will solve this set of linear equations by use of the transfer matrix method.[3]

Let us define now

$$\psi_{2n}=\begin{pmatrix}c_{2n}\\ \overline{c}_{2n}\end{pmatrix}. \tag{14}$$

Then half of the set of equations (14) can be reformulated as follows:

$$A_{2n}\psi_{2n}=B_{2n-1}\psi_{2n-1}, \tag{15}$$

with

$$A_{2n}=\begin{pmatrix}e^{ik_2(nd_1+(n-1)d_2)}, & -e^{-ik_2(nd_1+(n-1)d_2)}\\ e^{ik_2(nd_1+(n-1)d_2)}, & e^{-ik_2(nd_1+(n-1)d_2)}\end{pmatrix} \tag{16}$$

and

$$B_{2n-1}=\begin{pmatrix}\frac{\mu_1k_1}{\mu_2k_2}e^{ik_1(nd_1+(n-1)d_2)}, & -\frac{\mu_1k_1}{\mu_2k_2}e^{-ik_1(nd_1+(n-1)d_2)}\\ e^{ik_1(nd_1+(n-1)d_2)}, & e^{-ik_1(d_1n+(n-1)d_2)}\end{pmatrix}. \tag{17}$$

Equation (15) can be rewritten as

$$\psi_{2n}=A_{2n}^{-1}B_{2n-1}\psi_{2n-1}. \tag{18}$$

Similarly, the other half of the equations (13) appears as

$$\psi_{2n-1}=A_{2n-1}^{-1}B_{2n-2}\psi_{2n-2}, \tag{19}$$

with

$$A_{2n-1}=\begin{pmatrix}e^{ik_1((n-1)d_1+(n-1)d_2)}, & -e^{-ik_1((n-1)d_1+(n-1)d_2)}\\ e^{ik_1((n-1)d_1+(n-1)d_2)}, & e^{-ik_1((n-1)d_1+(n-1)d_2)}\end{pmatrix} \tag{20}$$

and

$$B_{2n-2}=\begin{pmatrix}\frac{\mu_2k_2}{\mu_1k_1}e^{ik_2[(n-1)d_1+(n-1)d_2]}, & -\frac{\mu_2k_2}{\mu_1k_1}e^{-ik_2[(n-1)d_1+(n-1)d_2]}\\ e^{ik_2[(n-1)d_1+(n-1)d_2]}, & e^{-ik_2[(n-1)d_1+(n-1)d_2]}\end{pmatrix}. \tag{21}$$

Recursion equations (18) and (19) allow us to connect ψ_{2n} with ψ_1 in the following way:

$$\psi_{2n}=A_{2n}^{-1}B_{2n-1}A_{2n-1}^{-1}B_{2n-2}\cdots A_2^{-1}B_1\psi_1. \tag{22}$$

Let us now define the transfer matrices as

$$T_1=B_{2n-1}A_{2n-1}^{-1}=\begin{pmatrix}\frac{\mu_1k_1}{\mu_2k_2}\cos k_1d_1 & i\frac{\mu_1k_1}{\mu_2k_2}\sin k_1d_1\\ i\sin k_1d_1 & \cos k_1d_1\end{pmatrix} \tag{23}$$

and

$$T_2 = B_{2n} A_{2n}^{-1} = \begin{pmatrix} \frac{\mu_2 k_2}{\mu_1 k_1} \cos k_2 d_2 & i \frac{\mu_2 k_2}{\mu_1 k_1} \sin k_2 d_2 \\ i \sin k_2 d_2 & \cos k_2 d_2 \end{pmatrix}. \tag{24}$$

Then Eq. (22) becomes

$$\psi_{2n} = B_{2n}^{-1} T^n A_1 \psi_1 = U \psi_1, \tag{25}$$

where

$$T = T_2 T_1 = \begin{pmatrix} \cos k_1 d_1 \cos k_2 d_2 - \frac{\mu_1 k_1}{\mu_2 k_2} \sin k_1 d_1 \sin k_2 d_2, & i \cos k_1 d_1 \sin k_2 d_2 + \frac{i \mu_1 k_1}{\mu_2 k_2} \sin k_1 d_1 \cos k_2 d_2 \\ i \sin k_1 d_1 \cos k_2 d_2 + \frac{i \mu_1 k_1}{\mu_2 k_2} \cos k_1 d_1 \sin k_2 d_2 & -\frac{\mu_2 k_2}{\mu_1 k_1} \sin k_1 d_1 \sin k_2 d_2 + \cos k_1 d_1 \cos k_2 d_2 \end{pmatrix}. \tag{26}$$

The first equation of Eq. (13) for $n = N$ can be written as

$$\overline{\psi}_{2N} \psi_{2N} = 0, \tag{27}$$

where

$$\overline{\psi}_{2N} = (e^{i k_2 (N d_1 + N d_2)}, \quad -e^{-i k_2 (N d_1 + N d_2)}). \tag{28}$$

For another boundary of the superlattice film, where $n = 1$, we have

$$c_1 = \overline{c_1} = c, \tag{29}$$

which means that

$$\psi_1 = c \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{30}$$

From Eqs. (25) and (27) we can obtain the following equation for the spectrum of transversal phonons in the superlattice film of $2N$ layers:

$$\mathrm{Tr}[C T^N] = 0, \tag{31}$$

where

$$C_\beta^\alpha = (A_1 \psi_1)_\beta (\overline{\psi}_{2N} B_{2N}^{-1})^\alpha = \begin{pmatrix} 0 & 0 \\ 2 \frac{\mu_1 k_1}{\mu_2 k_2} & 0 \end{pmatrix}. \tag{32}$$

To proceed further we need to calculate the $N-1$ degree of the transfer matrix, which can be achieved simply by diagonalizing T. Obviously

$$T^N = W^{-1} \begin{pmatrix} \lambda^N, & 0 \\ 0, & \overline{\lambda}^N \end{pmatrix} W, \tag{33}$$

where λ and $\overline{\lambda}$ are eigenvalues of T, and W is the diagonalizing matrix. One easily can find the eigenvalues or the transfer matrix T as

$$\lambda = e^{\pm i\Theta} = \left(\cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2} \left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1} \right) \sin k_1 d_1 \sin k_2 d_2 \right) \pm i \sqrt{1 - \left(\cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2} \left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1} \right) \sin k_1 d_1 \sin k_2 d_2 \right)^2}, \tag{34}$$

where

$$\cos\Theta=\cos k_1d_1\cos k_2d_2-\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}+\frac{\mu_2k_2}{\mu_1k_1}\right)\sin k_1d_1\sin k_2d_2. \tag{35}$$

Further, a simple calculations shows that Eq. (31) reduces to

$$\text{Im}\lambda^N=0, \tag{36}$$

which means that

$$\Theta=\pi\frac{Q}{N},\quad Q=1,\ldots,N. \tag{37}$$

Finally we obtain the following equation for the spectrum of transversal phonons:

$$\cos k_1d_1\cos k_2d_2-\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}+\frac{\mu_2k_2}{\mu_1k_1}\right)\sin k_1d_1\sin k_2d_2=\pm\cos\pi\frac{Q}{N}, \tag{38}$$

where

$$k_i^2=\frac{\omega^2}{c_i^2}-q^2. \tag{39}$$

We see that this equation is coinciding with the equation for the spectrum of phonons in the bulk,[15,3] but the momentums perpendicular to the layers direction are quantized due to dimensional restriction of the film.

III. THE LANDAUER RESISTANCE OF PHONONS IN THE SUPERLATTICE WITH RANDOM DISTRIBUTION OF THICKNESSES OF THE LAYERS

The problem of elastic waves in superlattice is essentially one dimensional. One-dimensional problems are especially attractive because of their possible exact integrability. In the article by Erdos and Herndon[33] the problem of the transport of particles in the one-dimensional space for a wide class of disorders was considered in the transfer matrix approach and general results were obtained. It was proved that transfer matrix of the one-dimensional problem belongs to $SL(2,R)$ group and randomness can be exactly taken into account for such quantities as Landauer resistance.[32]

Some exact results for the Kronig-Penney model in the case of nondiagonal disorder by other methods were obtained in Ref. 38.

It is easy to see from the formulas (26) for the transfer matrix T that here we also have a representative of the $SL(2,R)$ group. One can make a link between transfer matrices of the Kronig-Penney model and phonons in the superlattice.

In this section we will consider transversal phonons propagating in the perpendicular to layers direction, i.e., $q=0$. Following Refs. 32, 34, and 33 let us define dimensionless resistance as a ratio of reflection to transmission coefficients, which, by use of formula (25), is equal to

$$\rho=\frac{1-|\tau|^2}{|\tau|^2}=U_{12}U_{12}^{*}=U_2^1(U^+)_1^2, \tag{40}$$

where U_2^1 is the 1,2 matrix element of the evolution matrix U

$$U=B_{2N}^{-1}(T_2T_1)^NA_1. \tag{41}$$

It is clear from this definition that the Landauer resistance can be measured as a ratio of reflection from the SL layers intensity of acoustic waves over transmitted intensity:

$$\rho=\frac{I_{\text{reflected}}}{I_{\text{transmitted}}}. \tag{42}$$

We are going to consider random distribution of thicknesses of the layers and take the average of Landauer resistance. For further convenience we will normalize $T_1(T_2)$ transfer matrices on order to have a unit determinant. It will not change Eq. (41) because the normalization factors for T_1 and T_2 cancel each other. Hence we will consider

$$T_{2i}=\begin{pmatrix}\left(\frac{\mu_2k_2}{\mu_1k_1}\right)^{1/2}\cos k_2(x_{2i}-x_{2i-1}) & i\left(\frac{\mu_2k_2}{\mu_1k_1}\right)^{(1/2)}\sin k_2(x_{2i}-x_{2i-1})\\ i\left(\frac{\mu_1k_1}{\mu_2k_2}\right)^{1/2}\sin k_2(x_{2i}-x_{2i-1}) & \left(\frac{\mu_1k_1}{\mu_2k_2}\right)^{1/2}\cos k_2(x_{2i}-x_{2i-1})\end{pmatrix} \tag{43}$$

for the even slices. The similar expression for odd slices T_{2i-1} can be found simply by permuting variables k and μ for 1 and 2.

Now let us analyze the direct product of the evolution matrices, the $(U\otimes U^+)_{2,1}^{1,2}$ matrix element which defines Landauer resistance. For this purpose we should calculate first the simplest constituent block of that expression, namely the direct product $T_i\otimes T_i^+$ of U_i's. In the article in Ref. 33 it was demonstrated that this direct product can be represented as $1\oplus(3\times3)=4\times4$ matrix. It happened because of the fact that T_i matrices are spinor representations of the $SL(2,R)$; hence the direct product of two 1/2 representations can be expanded as a sum of scalar and vector representations. In the language of the group elements $T\in SL(2,R)$ this expansion looks like

$$(T_i)_{\alpha'}^{\alpha}(T_i^{-1})_{\alpha}^{\beta'}=\frac{1}{2}\delta_{\beta}^{\alpha}\delta_{\alpha'}^{\beta'}-\frac{1}{2}(\sigma^{\mu})_{\alpha'}^{\beta'}\Lambda_i^{\mu\nu}(\sigma^{\nu})_{\beta}^{\alpha}, \tag{44}$$

where

$$\Lambda_i^{\mu\nu}=\frac{1}{2}\mathrm{Tr}(T_i\sigma^\mu T_i^{-1}\sigma^\nu) \tag{45}$$

is the spin-one part of the direct product. But for Landauer resistance we need to calculate $T\otimes T^+$. It is easy to see from the formula (43) that

$$\sigma_1 T^{-1}\sigma_1=T^+, \tag{46}$$

therefore, by multiplying the expression (44) in the left and right by σ_1 we will have

$$(T_i)^\alpha_{\alpha'}(T_i^+)^{\beta'}_\beta=\frac{1}{2}(\sigma_1)^\alpha_\beta(\sigma_1)^{\beta'}_{\alpha'}-\frac{1}{2}(\sigma^\mu\sigma_1)^{\beta'}_{\alpha'}\Lambda_i^{\mu\nu}(\sigma^\nu\sigma_1)^\alpha_\beta. \tag{47}$$

Now the calculation of the direct product $U\otimes U^+$ is straightforward. The product $\Pi_{i=1}^{2N}T_i$ of T_i's transforms into product of $\Lambda_i^{\mu\nu}$'s. Finally we will obtain

$$(U)^\alpha_{\alpha'}(U^+)^{\beta'}_\beta=(B_{2N}^{-1})^\alpha_\gamma(A_1^+)^{\beta'}_{\delta'}\Bigg[\frac{1}{2}(\sigma_1)^\gamma_\delta(\sigma_1)^{\delta'}_{\gamma'}-\frac{1}{2}(\sigma^\mu\sigma_1)^{\delta'}_{\gamma'}\left(\prod_{i=1}^{N}\Lambda_{2i-1}\Lambda_{2i}\right)^{\mu\nu}(\sigma^\nu\sigma_1)^\gamma_\delta\Bigg]\times(A_1)^{\gamma'}_{\alpha'}(B_{2N}^{-1})^{+\delta}_\beta. \tag{48}$$

Substituting this expression, together with the expressions for B_{2N}^{-1} and A_1 [from Eqs. (21) and (20) correspondingly], into Eq. (40), after some simple algebra for Landauer resistance ρ we will have

$$\rho=\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}\right)\Bigg[-1+(\Lambda^N)^{11}\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}+\frac{\mu_2k_2}{\mu_1k_1}\right)+i(\Lambda^N)^{12}\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}-\frac{\mu_2k_2}{\mu_1k_1}\right)\Bigg], \tag{49}$$

where $(\Lambda^N)^{11}$ [correspondingly $(\Lambda^N)^{12}$] is the 11(12) matrix element of the matrix $\Lambda=\Lambda_1\Lambda_2$, which is a product of Λ's of the I and II slices.

The average over any type of random distributions of the parameters of the model can be calculated now exactly. We consider random distribution of thicknesses of the slices, keeping boundaries fixed $x_0=0,x_{2N}=L$. We see from formula (46) that T_i depends only on the thickness of the slice x_i-x_{i-1}. The only restriction we have is the condition that

$$\sum_{i=1}^{2N}\Delta x_i=L. \tag{50}$$

Therefore, the average of the Λ^N, with the probability distribution $g(y)[\int_0^\infty g(y)dy=1]$, is defined in the following way:

$$\left\langle\prod_{i=1}^{N}\Lambda_{2i-1}\Lambda_{2i}\right\rangle=\int_0^\infty dy_1\cdots dy_{2N}g(y_1)\cdots g(y_{2N})\times\delta\left(\sum_{j=1}^{2N}y_j-L\right)\prod_{i=1}^{N}\Lambda_{2i-1}(y_{2i-1})\Lambda_{2i}(y_{2i})=\int_{-\infty}^\infty dpe^{-ipL}(\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle)^N, \tag{51}$$

where

$$\langle\Lambda_{1,2}(p)\rangle=\int_0^\infty dye^{ipy}g(y)\Lambda_{1,2}(y). \tag{52}$$

The average Landauer resistance is now equal to

$$\langle\rho\rangle=\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}\right)\Bigg\{-1+\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}+\frac{\mu_2k_2}{\mu_1k_1}\right)\int_{-\infty}^\infty dpe^{ipL}[(\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle)^N]^{11}+\frac{1}{2}\left(\frac{\mu_1k_1}{\mu_2k_2}-\frac{\mu_2k_2}{\mu_1k_1}\right)\int_{-\infty}^\infty dpe^{ipL}[(\langle\Lambda_1(p)\rangle\times\langle\Lambda_2(p)\rangle)^N]^{12}\Bigg\}. \tag{53}$$

It is obvious that in a case of homogeneous media (two components of the superlattice are coinciding) we restore the expression for the Landauer resistance of electrons, obtained in Ref. 33.

For a large sample size ($N\gg 1$), as it was argued in Refs. 32 and 39, the resistance should behave as $e^{\gamma N}$, where Lyapunov exponent γ provides the phonons correlation length. By use of Eq. (53) and the definition of Lyapunov exponent $\gamma=\lim_{N->\infty}\ln(\rho/N)$ we can find an exact expression for correlation length

$$\xi^{-1}=\ln\lambda, \tag{54}$$

where λ is the closest to one eigenvalue of the matrix $\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle$. Whether excitations are localized or not depends on the behavior of ξ. If at some frequencies correlation length becomes infinite, we have a delocalized state and the expression (54) shows that the answer depends on the average value of $\Lambda^{\mu\nu}$. For further analysis let us consider the simplest case of the distribution, namely when there is equal probability for slices to have a thickness up to d_i (i=1,2)

$$g(y)=\begin{cases}\dfrac{1}{d_i}, & 0<y<d_i,\\ 0, & \text{otherwise.}\end{cases} \tag{55}$$

We have taken $Al_xGa_{1-x}As$ and GaAs as components of the superlattice with the parameters[20]

$$\mu_1=(3.25-0.09x)10^{11}\ \text{dyn/cm}^2,$$
$$\mu_2=3.2510^{11}\ \text{dyn/cm}^2,$$
$$\rho_1=(5.3176-1.6x)\ \text{g/cm}^3,\quad \rho_2=5.3176\ \text{g/cm}^3,$$
$$d_1=30\times(5.6532+0.0078x)\ \text{Å},\quad d_2=10\times 5.6532\ \text{Å} \tag{56}$$

and consider waves propagating in the perpendicular to the layers direction ($\vec{q}=0$).

For large enough N the asymptotics of ρ, and therefore the correlation length ξ, are defined by the closest to unity eigenvalues of $\langle\Lambda_1\rangle\langle\Lambda_2\rangle$. If it is λ, then

$$\xi(\omega)\sim 1/\ln\lambda(\omega). \tag{57}$$

Numerical calculations by use of Mathematica show that $\lambda(\omega=0)=1$, hence $\xi\rightarrow\infty$.

This result is easy to understand; $\omega=0$ means that we have constant displacement $\vec{u}$, which simply is the shift of the entire sample. Though this limiting value is not very interesting, the correlation length index ν from $\xi\sim\omega^{-\nu}$ is an important quantity, that defines the universality class of the model. In a case of correlated disorder some other delocalized states can appear and there is a necessity to compair the indexes around them. Obviously ν can be defined as a slope of the plot of $\ln\ln\lambda(\omega)$ versus $\ln\omega$ and, as presented in Fig. 2, appeared to be 2. All other states are localized.

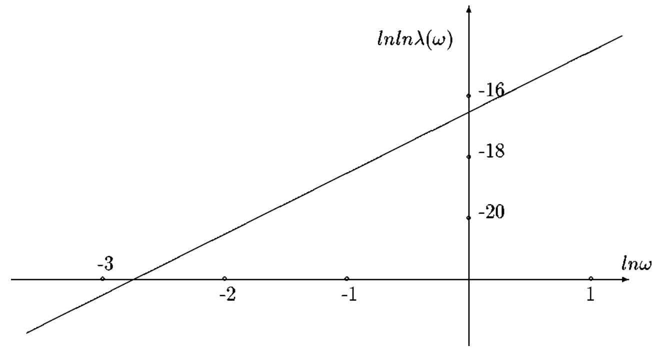

FIG. 2. The logarithm of the correlation length versus the logarithm of the energy near $\omega=0$. The slope of the curve defines the correlation length index.

ACKNOWLEDGMENTS

The authors acknowledge S. Badalyan and A. Khachatryan for many valuable discussions. The work of D.G.S. was partially supported by CRDF Grant No. 375100 and the work of A.G.S. by INTAS Grant No. 524.

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