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Phase Reversal and Modulated Flux Motion in Superconducting Thin Films*

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It is shown that a small magnetic field alternating at audio frequencies causes an amplitude modulation of the electric field associated with the flux motion oscillating at microwave frequencies in superconducting thin films. The phase of the modulation component of this electric field can be changed almost 180° by reversing the sweep of the external magnetic field. These phenomena can be explained by invoking the influence of boundary currents on the motion of the flux lattice.

I. INTRODUCTION

If a constant homogeneous magnetic field $\vec{H}$ is applied along the normal of a superconducting film carrying a current $\vec{J}$ exceeding a critical value, a flux flow results in the direction perpendicular to both $\vec{J}$ and $\vec{H}$.[1] This critical current is not the critical current for destroying the superconductivity but is related to the pinning of flux in the sample. The vector relation among $\vec{J}$, $\vec{H}$, and the flux-flow velocity $\vec{v}$, is shown in Fig. 1(a). It has been established both experimentally[1] and theoretically[2]

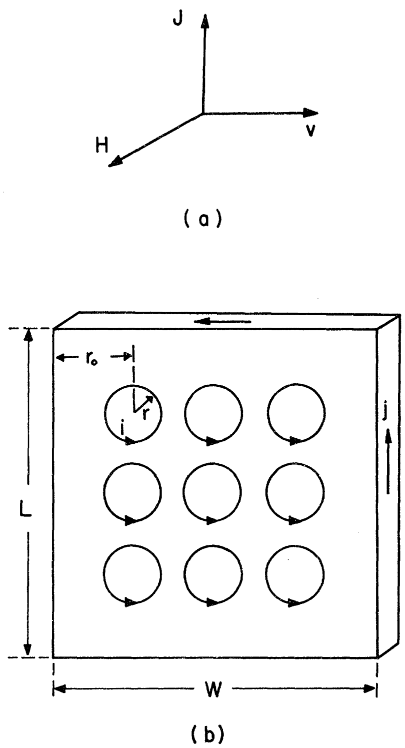

FIG. 1. (a) Vector relation among the external magnetic field $\vec{H}$, the microwave current $\vec{J}$, and the flux-flow velocity $\vec{v}$. (b) Geometry used in the text to estimate the force between the boundary current j and the flux lattice. The flux lines are represented by ring currents i of radius r. W is the width of the sample along the y-direction. L is the length.

that the flux flow induces an electric field $\vec{E}$ given by

$$\vec{E} = (1/c)\vec{v}\times\vec{B}\ , \qquad (1)$$

where $\vec{B}$ is the magnetic induction in the sample. Thus, the induced electric field is in the direction of the current $\vec{J}$. Experimentally, the induced electric field can be studied by measuring the induced voltage across the sample.

The flux-flow model has been successfully used to describe experimental results not only for the case where $\vec{J}$ is constant but even for the case where $\vec{J}$ varies at microwave frequencies.[3] Reference 3 also shows that the effect of flux pinning in a thin superconducting film is negligible for x-band microwave frequencies (8–12 GHz). Therefore, it is advantageous to use microwave techniques to study properties of flux flow which are not attributable to the pinning effect.

In this paper, we wish to describe the effect of a low-frequency magnetic field on the flux motion which is induced by a microwave current. We are concerned with the situation where the amplitude of this low-frequency field is extremely small (of the order of 0.1 G or less). This is distinct from recent experiments in which a low-frequency magnetic field of large amplitude (of the order of 10 G or more) was used.[4] We will show that the effect of this low-frequency field depends strongly on its amplitude. For the large-amplitude case, the dominant effect is to modulate the flux density in the interior of the sample. For the small-amplitude case, its effect is primarily to change the boundary current, proposed by Bean,[5] without noticeable modulation of the flux density inside the sample. This boundary current perturbs the microwave-induced flux motion in the sample to cause an amplitude modulation of the microwave electric field.

In Sec. II, we will describe the experimental procedure and the results. A theoretical model will be proposed in Sec. III. Finally, Sec. IV will be devoted to the discussion of the results.

II. EXPERIMENTAL PROCEDURES AND RESULTS

Our samples were thin films of Pb or Sn evaporated in a vacuum of 5×10^{-6} Torr onto glass or quartz substrates. The films were rectangular in form approximately 0.5 by 1.0 cm and several hundred angstroms thick. The sample was placed close to the center of an x-band cylindrical cavity with its surface parallel to the axis. The cavity was designed in such a way that it was excited in its TM_{210} mode. Therefore, the microwave electric field was parallel to the plane of the sample. Both the constant and the modulating magnetic fields were oriented perpendicular to the plane of the film. The microwave system employed in the present experiment was a Varian x-band electron-spin-resonance spectrometer which was modified for the present work. The cavity with the sample was locked to the klystron by means of automatic frequency control so that the dispersion mode signal could be eliminated. The microwave reflected from the cavity containing the sample was detected by using a standard amplitude-modulation (AM) linear detection technique in which a microwave crystal diode was dc biased in its linear region. Therefore, the detected voltage is proportional to the amplitude of the microwave E field. The block diagram of the microwave spectrometer is shown in Fig. 2. The microwave carrier frequency was chosen to be 9.6 GHz. The modulating magnetic field used in the experiment had the form $h = h_0 \sin\omega t$, where the frequency $\omega/2\pi$ was a few hundred Hertz and the amplitude h_0 was 0.1 G or less. The AM component of the microwave due to the modulating field was then obtained by measuring the ac component of the output voltage of the detector. Typical examples of this time-dependent voltage $V(t)$ for $\omega/2\pi = 400$ Hz are shown in Figs. 3(a)

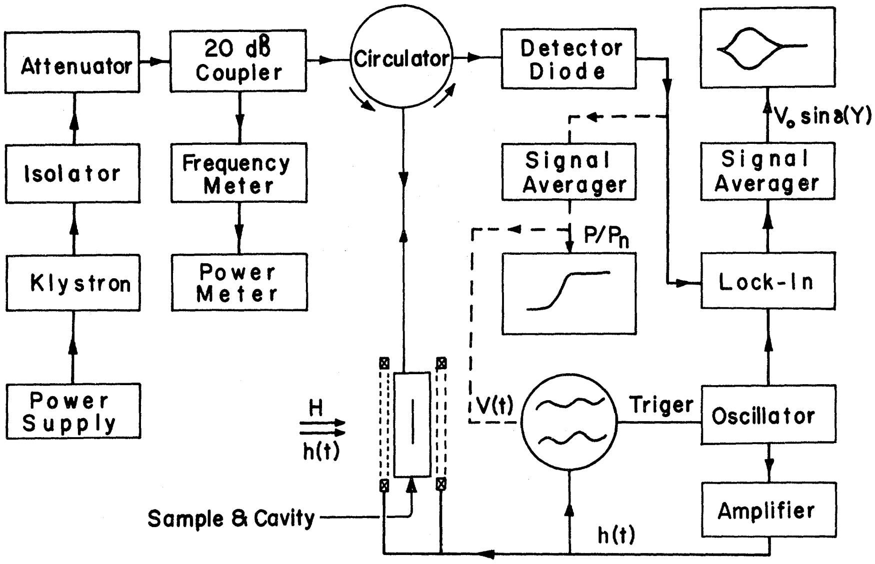

FIG. 2. Block diagram for the microwave electronic system.

and 3(b), where both traces were obtained at one and the same value H_0 of the external magnetic field.[6] However, H_0 corresponding to Fig. 3(a) was reached by increasing the magnetic field, whereas H_0 corresponding to Fig. 3(b) was obtained by decreasing the field. Figure 3(c) simply represents $h_0 \sin\omega t$. Traces (a) and (c), as well as (b) and (c), were obtained simultaneously so that their relative phases can be compared. Examination of these experimental traces reveals the striking result that the relative phase of the time-dependent voltages is determined by the way in which H_0 is reached. Specifically, the phase difference between traces (a) and (b) is approximately 180° for all values of H_0 up to almost H_{c2}.

We have also found that the amplitude of $V(t)$ depends on H. Both the amplitude V_0 and the phase δ (relative to the phase of h) of $V(t)$ have been measured simultaneously as a function of H by using a phase-sensitive detection technique. The results are illustrated in Fig. 4(a), where $V_0 \sin\delta$ has been plotted vs magnetic field for the cases where H is increasing (upper curve) or decreasing (lower curve). We have verified that δ is +90° for the entire upper curve and −90° for the entire lower curve, so that Fig. 4(a) merely shows $\pm V_0$ as a function of H.

As we just mentioned, the relative phase of $V(t)$ is determined by the way in which a specific magnetic field H_0 is reached. Hence we can change δ from +90° to −90° simply by reducing the field by an amount ΔH, as indicated by the dashed line in Fig. 4(a). Similarly, we can change δ from −90° to +90° by increasing the field. The minimum ΔH required to affect these phase changes turns out to be quite small and to depend on the magnetic field. A typical H dependence of the minimum ΔH is shown in Fig. 4(b). For purpose of comparison we have also displayed in Fig. 4(c) the resistive transition curve and its mathematical derivative for the same sample. The resistive transition curve is a plot of the dc output voltage of the detector as a function of sweeping magnetic field without modulation. It should be noted that the peak

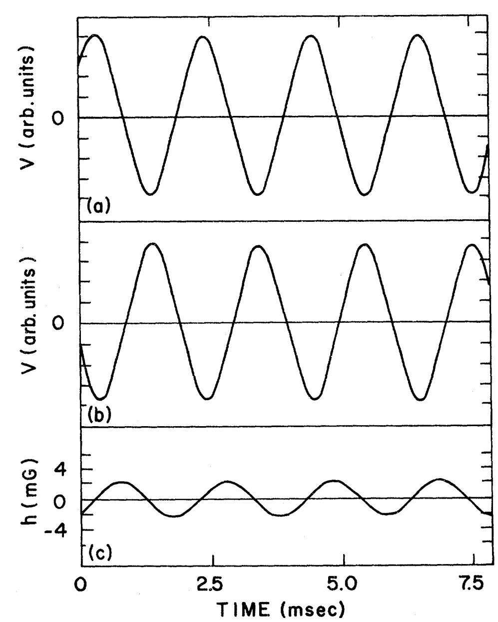

FIG. 3. Induced time-dependent voltage $V(t)$ at field $H_0 = 600$ G, where in (a) H_0 was reached from above and in (b) H_0 was reached from below. (c) The modulating magnetic field h vs time.

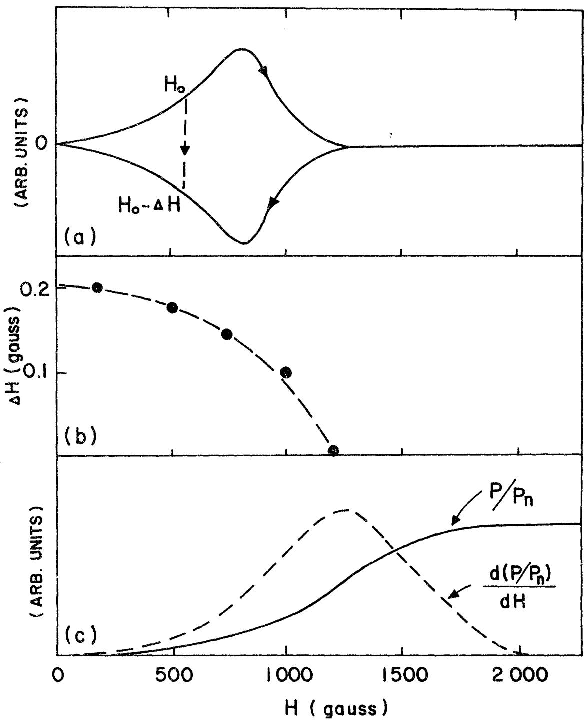

FIG. 4. (a) Phase-sensitive detector output $V_0 \sin\delta$ vs H. The dashed line indicates the path in which the phase is reversed by changing the field by an amount ΔH. (b) The minimum ΔH required to change the phase of $V(t)$. (c) Solid line: resistive transition obtained by measuring the microwave absorption. Dashed line: mathematical derivative of the solid line.

of the curve shown in Fig. 4(a) occurs at a considerably lower field than the peak of the derivative curve in Fig. 4(c). We have found that the AM component depends strongly on the amplitude of the modulating magnetic field. The dependence is illustrated in Fig. 5 where $V_0 \sin\delta$, obtained by using a phase-sensitive detector, is shown as a function of H for several different values of modulation amplitude h_0. If h_0 is 10 G or more, $V_0 \sin\delta$ becomes the true derivative of the resistive transition curve. This amplitude dependence will be discussed in Sec. IV.

III. THEORETICAL MODEL

In this section we shall show that the phenomena just described can be satisfactorily accounted for by assuming that the boundary currents, originally introduced by Bean,[5] perturb the microwave-induced flux motion in the sample. According to Bean, a sample in a constant magnetic field has a boundary current whose sense of flow depends on whether the field has been reduced from $H_0+\Delta H$ to H_0 or increased from $H_0-\Delta H$ to H_0. The effect of this current has not been noticed in previous flux-flow measurements nor has it been taken into account theoretically. We assume that the small modulating field used in our experiment changes the boundary current but leaves the number and the distribution of the enclosed flux lines unchanged. Furthermore, we assume that the flux lattice is rigid and is driven by the microwave current.

First we will show that the force F_b per unit length of sample between the total boundary current and the flux lattice can be written

$$F_b = -Ky \tag{2}$$

for a small displacement y from its equilibrium position. The constant K will be estimated by confining the boundary current j to the edge of the sample and by assuming that the flux lines can be represented by ring currents of radius r. We shall find that K is then given by the simple expression

$$K = -\frac{jB}{cr}\left\{\left[1-\left(\frac{r}{r_0}\right)^2\right]^{-1/2} - 1\right\}, \tag{3}$$

where r_0 is the equilibrium distance between the edge of the flux lattice [which has been assumed to be square as shown in Fig. 1(b)] and the boundary current. For Eq. (3) to be valid, r must always be smaller than r_0.

In order to derive Eqs. (2) and (3) we make use of the geometry shown in Fig. 1(b). The y component of the force between the boundary current and a particular ring current whose center is a distance s from the left edge of the sample is classically given by

$$-2jBr\left\{\left[1-\left(\frac{r}{s}\right)^2\right]^{-1/2} - \left[1-\left(\frac{r}{W-s}\right)^2\right]^{-1/2}\right\}, \tag{4}$$

where B is the magnetic induction at the center of the ring current. If there are n flux lines per unit area, then the y component of the force per unit length due to the boundary current on the flux lattice is obtained by multiplying Eq. (4) by n and integrating over s from r_0+y to $W-r_0+y$. The width W of the sample is much larger then r_0 and r. By expanding the result of the integration in terms of y, and making use of the fact that for a square lattice $n=1/4r^2$, we obtain Eqs. (2) and (3). The sign of K is very important. For the geometry of Fig. 1(b), K is negative. However, if the sense of flow of j were reversed, then K would become positive. So, we can state that in Eq. (3), j is to be taken positive when its sense of flow is clockwise about $\vec{B}$, and negative otherwise.

Since the boundary current can be affected by changes in the externally applied magnetic field, it seems natural to assume that K will be modulated by the small modulating field h at the frequency $\omega/2\pi$. We therefore write K as the sum of a constant component K_0 and a small alternating component $k\cos\omega t$. Since the force per unit length on the flux lattice due to the microwave current is

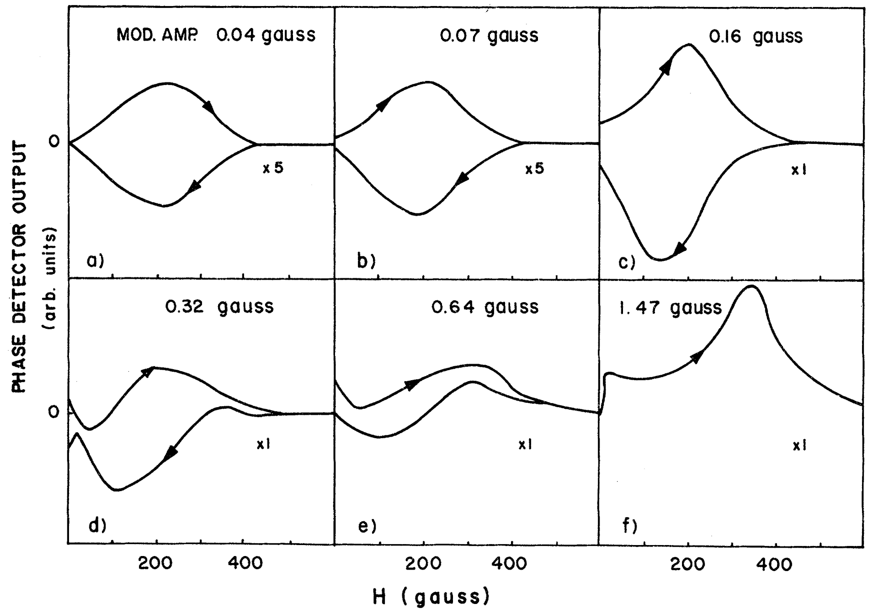

FIG. 5. $V_0 \sin\delta$ vs H for several different values of h_0. (a) $h_0 = 0.04$ G, (b) 0.07 G, (c) 0.16 G, (d) 0.32 G, (e) 0.64 G, (f) 1.47 G. In (a) and (b) V_0 has been multiplied by a factor of 5.

given by $(\vec{J}\times\vec{B})/c$, we may finally write the equation of motion for the flux lattice in the y direction as

$$m\ddot{y} + \eta\dot{y} + (K_0 + k\cos\omega t)y = (BJ_0/c)\cos\Omega t. \quad (5)$$

Here, m is the total mass (per unit length)[7] of the flux lattice, η is the viscosity coefficient. J_0 and Ω are the amplitude and the frequency of the microwave current, respectively. In Eq. (5) we have ignored the influence of pinning, which has been verified to be a good approximation at microwave frequencies.[3]

We shall discuss some of the properties of the solution of Eq. (5). It should be noted that Eq. (5) is the inhomogeneous damped Mathieu equation whose particular solution has been studied to some extent.[8] However, since k is much smaller than K_0, we can obtain an approximate particular solution in a rather simple way by writing $y = y_0 + \delta y$, where y_0 is the well-known particular solution of Eq. (5) for $k = 0$, and δy is the particular solution of the linearized equation

$$m\,\delta\ddot{y} + \eta\,\delta\dot{y} + K_0\delta y = -ky_0\cos\omega t. \quad (6)$$

Equation (6) can be solved by elementary methods. In order to obtain the induced electric field in the sample we need, according to Eq. (1), to solve for $\dot{y}$. We find, to first order in k, η, and ω,

$$E = (J_0B^2\Omega/|K_0|c^2)[1 - \epsilon\cos(\omega t - \phi)]\sin(\Omega t - \alpha), \quad (7)$$

where $\epsilon = |k/K_0|$, $\tan\phi = 2\eta\omega/K_0$, and $\tan\alpha = \eta\Omega/K_0$. Equation (7) is valid for $k \ll |K_0|$, $\omega \ll \Omega$, and $|K_0| \gg m\Omega^2$. In deriving Eq. (7) we have neglected a term which is a factor ω/Ω smaller than the ϵ term. Since in our experiment ω/Ω is of the order of 10^{-7}, this is completely justified.

Equation (7) indicates that the induced E field is indeed amplitude modulated. The ϵ term is proportional to the AM component observed in our experiment (see Fig. 3). Careful analysis reveals that the sign of $\cos\phi$ can be changed by changing the sign of K_0. On the other hand, the sign of $\sin\phi$ does not depend on the sign of K_0. The phase ϕ of the AM component can therefore be shifted by almost 180°, if $2\eta\omega \ll |K_0|$, by changing the sign of K_0. According to Eq. (3), the sign of K_0 can be changed by reversing the sense of flow of the boundary current. In our experiment this is achieved by changing the field H by an amount ΔH.

Since the AM component is proportional to k, we expect qualitatively that its amplitude will increase with h_0. This is actually observed when $h_0 \ll \Delta H$. Furthermore, since E is proportional to B^2, the induced voltage will initially increase with H. Figure 4(a) shows that this is also observed experimentally. When H becomes too large [i.e., larger than 800 G as apparent from Fig. 4(a)], the flux structure becomes probably too complex for our simple model to remain valid.

It is imperative to see whether the conditions under which Eq. (7) has been obtained are feasible in practice. Under our experimental conditions, k and ω are always much smaller than $|K_0|$ and Ω, respectively. We estimate $r = 10^{-5}$ cm at $B = 500$ G. Since in equilibrium we expect that r_0 is only slightly greater than r, we take $r_0 = 1.01r$. Further, taking $j = 0.1$ A (obtained by using the critical current density of the film and by assuming the boundary to be 0.01 of the film width), we find from Eq. (3) that $|K_0| \simeq 4.5\times 10^6$ dyn/cm^2. For $m\Omega^2$ to be less than this value, m has to be less than 1.3×10^{-15} g/cm. This latter value roughly corresponds to 1500 electron masses per flux line in our sample. A mass of this magnitude is comparable to previously reported values.[3] Finally, the condition $2\eta\omega \ll |K_0|$ (which is necessary to obtain the nearly 180° phase change) is always satisfied in our experiment. In view of the fact that the magnitude of the parameters of our model at present is rather uncertain, it would be unrealistic to attempt to fit numerically experimental curves like those depicted in Figs. 3(a), 3(b), and 4(a).

We have also examined the stability of the solutions of Eq. (5) in the absence of the driving term $(BJ_0/c)\cos\Omega t$. According to the theory of the homogeneous Mathieu equation, stable solutions do exist for $-(k^2 2m\omega^2) < K_0 < \infty$. Since in our experiment $\Omega/\omega = 10^7$ and $|K_0| > m\Omega^2$, the condition $-(k^2/2m\omega^2) < K_0 < 0$ can certainly be satisfied if $k > 10^{-7}|K_0|$. This implies that for ΔH of the order of 0.1 G, stable solutions can be obtained for h_0 between 10^{-8} and 0.1 G.

IV. DISCUSSION

As we have mentioned in Sec. II, the AM component of the microwave E field behaves quite differently depending on the amplitude h_0 of the modulating field. Naively, one might expect that our measurement of this AM component would give the derivative of the resistance vs magnetic field transition curve. This is true only when a modulation of the external H results in solely a modulation of B in such a way that the change in B is a linear function of the change in H. However, according to Bean[5] and the subsequent work by Fink,[9] a minimum change ΔH of the external field is required to change B in the sample. It is conceivable that if the amplitude of the low-frequency modulating field is smaller than ΔH, the only effect due to this added oscillatory field is to change the boundary current while leaving the interior flux density almost unchanged. Experimentally, we indeed have chosen h_0 to be much smaller than the threshold field ΔH. Consequently, our AM component does not represent the derivative of the transition curve. On the other hand, using the same detection system and h_0 several times greater than ΔH we have found that the AM component closely represents the derivative of the transition curve. This is expected since for h_0 much larger than ΔH, the effect of the boundary current can be neglected and, therefore, the modulation of H is nearly proportional to the modulation of B in the sample.

In order to observe the effect of the boundary current on the flux lattice, it is necessary to have an oscillatory transport current, such as a microwave current, to displace the lattice from its equilibrium position. Recently, other investigators have also been interested in the effect of a low-frequency ac magnetic field on the properties of superconducting films.[4,10] Their experiments differ from ours in two respects: First, they used either a dc transport current or no current at all. Second, large-amplitude oscillatory magnetic fields were employed. Therefore, the phenomena obtained by these workers are sufficiently different from the effect described in the present paper.

A recent theoretical calculation[11] shows that the dissipation process associated with surface current depends on the ratio $h_0/\Delta H$, and this effect becomes especially important when $h_0/\Delta H \gg 2$. Our experimental condition $h_0/\Delta H \ll 2$ was therefore in the region where the energy dissipation due to the low-frequency ac field is negligible.

The intrinsic hysteresis caused by the surface current of type-II superconductors has been previously observed in connection with the microwave resistive transition[12] and with the magnetization.[13] The phenomenon we have described in the present paper is just another manifestation of the presence of the boundary current. In conclusion, we may say that we have presented a new technique to study the nature of the boundary current and its effect on the properties of flux lattice in a superconducting film.

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