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Effect of a Static Electric Field on the Propagation of Electromagnetic Waves in Indium Antimonide*

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The effect of a weak longitudinal static electric field on the propagation of transverse circularly polarized electromagnetic waves traveling along the magnetostatic field in indium antimonide plasma has been studied theoretically. Using the dispersion relationship, the variation of the amplitude constant α, the phase constant β of both the left- and right-hand circularly polarized waves with the wave frequency f, the static magnetic field $\vec{B}_0$, and the static electric field $\vec{E}_0$ are examined in detail for three cases: case (i), intrinsic indium antimonide at room temperature; case (ii), n-type indium antimonide at liquid-nitrogen temperature; and case (iii), p-type indium antimonide at liquid-nitrogen temperature. The amplitude constant α may be either negative or positive for a positive β depending upon the particular combination of the system parameters f, $\vec{B}_0$, and $\vec{E}_0$. When α is negative, the wave decays spatially and the decay rate $|\alpha|$ tends to decrease while $\beta>0$ tends to increase with $|\vec{E}_0|$ in general. In the presence of carrier drift, i.e., $|E_0|\neq 0$, it is possible for the transverse wave to grow spatially ($\alpha\beta>0$) in indium antimonide for all three cases considered, provided the combination of parameters is proper. For example, in case (i) the product $(\alpha_l\beta_l)$ for the left-hand wave is positive when $B_0=13$ kG, $f<0.04$ GHz, $E_0=80$ V/cm or $B_0=13$ kG, $f=0.1$ GHz, and $E_0>120$ V/cm. In case (ii) the product $(\alpha_l\beta_l)$ for the left-hand wave is positive when $B_0=10$ kG, $f<0.7$ GHz, $E_0=36$ V/cm or $B_0=10$kG, $f=0.5$ GHz, and $E_0>30$ V/cm. In case (iii), $(\alpha_l\beta_l)$ is positive when $2.4<B_0<54$ kG, $f=0.1$ GHz, $E_0=120$ V/cm or $B_0=5$ kG, $f<0.2$ GHz, and $E_0=120$ V/cm. The influence of the presence of E_0 on the propagation of the helicon wave in case (ii) and the microwave Faraday rotation at $f=35$ GHz in the range $130\le B_0\le 150$ kG is also considered.

I. INTRODUCTION

The propagation of transverse electromagnetic waves through indium antimonide in a static magnetic field parallel to the direction of propagation has been studied by various workers.[1-3] This configuration has also been used to produce Faraday rotation of electromagnetic waves propagating through a free-carrier plasma in a semiconductor under the influence of static magnetic fields. The microwave Faraday rotation in indium antimonide plasma has also been investigated.[3-6]

When a static longitudinal electric field is added to the Faraday configuration (i.e., with the propagation vector $\vec{k}$ parallel to an external magnetic field), an electron and a hole in the semiconductor are forced to drift. Carrier drift, under proper conditions, may cause instabilities possibly leading to amplification of the electromagnetic waves.

The question of the existence of instabilities of a transverse wave in a solid-state magnetoplasma in the presence of carrier drift has been considered by various authors.[7-11] The related problem of microwave emission has also been investigated. For example, the experimental observations of the microwave emission from indium antimonide subjected to static electric and magnetic fields have been reported.[12-16] In these works cited above, the emphasis has been placed upon either the condition for the occurrence of instabilities or the threshold condition for emission. However, it is also desirable to know the effect of static electric fields on the propagation characteristic of a transverse wave in the vicinity of the threshold condition.

The purpose of this paper, therefore, is to describe the effect of a static electric field $\vec{E}_0$ on the amplitude and phase of transverse electromagnetic waves, propagating along the static magnetic field $\vec{B}_0$ in indium antimonide plasma.

The intrinsic indium antimonide at room temperature ($T=290\,^\circ$K) and the extrinsic indium antimonide (both n and p type) at liquid-nitrogen temperature are considered. E_0 under consideration is within the range of validity of Ohm's law. Using the dispersion relationships, the variations of the amplitude constant $\alpha(B_0, E_0, f)$ and the phase constant $\beta(B_0, E_0, f)$ of both the right- and left-hand circularly polarized waves are examined in detail. The effect of a static electric field on Faraday rotation is also investigated.

Particular attention is given to consideration of the behavior of α and β in the ranges of parameters B_0, E_0, and f, over which the product $\alpha\beta$ can change its algebraic sign from negative to positive. This change in the algebraic sign of $\alpha\beta$ has a special physical significance from the point of view of power exchange between the transverse electromagnetic wave and the drifting plasma, which is discussed in Sec. V. It is shown, within the framework of a linear small-signal analysis, that the case $\alpha\beta < 0$ implies that the power is being transferred from the transverse electromagnetic wave to the drifting plasma so that the wave should suffer an attenuation (e.g., when $\beta > 0$ and $\alpha < 0$). On the other hand, the case $\alpha\beta > 0$ implies that the power is being transferred from the drifting plasma to the transverse electromagnetic wave so that the wave should experience spatial growth (e.g., when $\beta > 0$ and $\alpha > 0$), provided that sufficient energy is transferred.

II. DISPERSION RELATIONSHIP

The interaction of a transverse electromagnetic wave and the drifting plasma can be described mathematically by the set of Maxwell's equations governing the behavior of electromagnetic waves, coupled to the momentum equation governing the behavior of the drifting plasma, in the magnetohydrodynamic approach. Each of the variables, electric field intensity $\vec{E}$, magnetic flux density $\vec{B}$, velocity v, and carrier density n, is assumed to be the sum of an equilibrium, or the time-invariant part (subscript 0) and a small time-varying part (subscript 1), e.g., $n = n_0 + n_1 e^{i(\omega t - \vec{k}\cdot\vec{r})}$. Here ω and $\vec{k}$ denote the wave angular frequency and the propagation vector, respectively. The linearized momentum and continuity equations for a given carrier type are expressed, respectively, as

$$[i(\omega - \vec{k}\cdot\vec{v}_0)\tau + 1]\vec{v}_1 = \mu\vec{E}_1 + \mu\vec{v}_0\times\vec{B}_1 + \mu\vec{v}_1\times\vec{B}_0 - (D/n_0)\nabla n_1 \quad (1)$$

and

$$\vec{v}_0\cdot\nabla n_1 + n_0\nabla\cdot\vec{v}_1 + i\omega n_1 = 0 \ , \quad (2)$$

where $\vec{B}_1 = -(\nabla\times\vec{E}_1)/i\omega$, $1/\tau$ is the carrier-lattice collision frequency, μ is the carrier mobility (assumed to be isotropic), and $D = |\kappa T/e|\mu$ is the diffusion constant, where κ denotes the Boltzmann constant. The static electric field $\vec{E}_0$ is an implicit part of $\vec{v}_0$ through $\vec{v}_0 = \mu\vec{E}_0 + \mu\vec{v}_0\times\vec{B}_0$ (mks units are used throughout). Equation (1) is valid if the wavelengths of interest are much larger than a mean free path $kl < 1$. This condition implies that

$$|\vec{k}\cdot\vec{v}_0|\tau = |(\vec{v}_0/v_T)\cdot\vec{k}l| \ll 1 \ , \quad (3)$$

because the ratio of drift to thermal velocity (v_0/v_T) is generally small compared to unity. Equations (1) and (2) are also valid for a large magnetic field, defined by $\omega_c\tau \gg 1$, where ω_c is the carrier cyclotron frequency,[17] provided that the wavelength is large compared to the Larmor radius $\lambda \gg v_T/\omega_c$. Since this requirement can be rewritten as $kl/\omega_c\tau \ll 1$, it is sufficient to require that $kl < 1$.

The drifting plasma is characterized by the convection-current density and the total time-varying convection-current density $\vec{J}_1$ is given by

$$\vec{J}_1 = \sum_{\text{CT}} (qn_0\vec{v}_1 + qn_1\vec{v}_0) \ , \quad (4)$$

where q is the charge of the carrier, and the summation is extended over all carrier types (CT).

Suppose that the transverse electromagnetic wave under consideration is taken to be propagating in the z direction, with the phase factor $e^{i(\omega t - kz)}$ of an infinite medium having a relative dielectric constant ϵ_l. There is a static magnetic field $\vec{B}_0$ parallel to the direction of propagation. A static electric field $\vec{E}_0$ may be either parallel to or antiparallel to $\vec{B}_0$ so that the plasma is drifting along the z direction (i.e., $\vec{B}_0 = \vec{a}_z B_0$ and $\vec{v}_0 = \vec{a}_z v_0$, with $\vec{a}_z$ being a unit vector).

It should be noted that $\vec{J}_1$ in Eq. (4) can be expressed in terms of $\vec{E}_1$, with the aid of Eqs. (1) and (2). Moreover, $\vec{E}_1$ can be written

$$\vec{E}_1 = \vec{E}_\perp + \vec{E}_\parallel = \vec{a}_\perp E_\perp + \vec{a}_z E_z \ ,$$

where $\vec{a}_\perp$ and $\vec{a}_z$ are the transverse and longitudinal unit vectors, respectively. Since in the present study the transverse electromagnetic wave is of primary interest, it is assumed that $E_z = 0$ and also that the effect of diffusion is unimportant.

By adopting the procedure used by Meyer and Van Duzer[11] it can be easily shown that

$$\vec{v}_1 = \frac{\mu'(1-\xi)}{1+\eta^2}(\vec{E}_\perp + \vec{E}_\perp\times\vec{\eta}) \ , \quad (5)$$

where

$$\xi = \frac{kv_0}{\omega} \ , \quad \mu' = \frac{\mu}{1+i\omega\tau}, \quad \text{and} \quad \vec{\eta} = \mu'\vec{B}_0 \ .$$

From Eq. (2),

$$n_1 = n_0 \frac{\vec{k}\cdot\vec{v}_1}{\omega - \vec{k}\cdot\vec{v}_0} \ . \quad (6)$$

In view of the fact that the purely transverse wave is being considered, $\vec{k}\cdot\vec{v}_1 = 0$ so that $n_1 = 0$ and Eq. (4) becomes

$$\vec{J}_1 = \sum_{\text{CT}} qn_0\vec{v}_1 \ . \quad (7)$$

The transverse electric field $\vec{E}_\perp$ is governed by the following wave equation which resulted from combining two curl equations of Maxwell's equations:

$$-\nabla^2\vec{E}_\perp = (\omega^2/c^2)\epsilon_l\vec{E}_\perp - i\omega\mu_0\vec{J}_1 \ , \quad (8)$$

where c is the speed of light in free space.

By introducing the rotating unit vector for the right-hand polarized wave $\vec{a}_r = \vec{a}_+$ and for the left-hand polarized wave $\vec{a}_l = \vec{a}_-$, which are defined as

$$\vec{a}_\pm = (1/\sqrt{2})(\vec{a}_x \mp i\vec{a}_y) \ , \quad (9)$$

$\vec{E}_\perp$ can be expressed as

$$\vec{E}_\perp = \vec{a}_\pm E_\pm \quad (10)$$

so that

$$\vec{E}_\perp \times \vec{\eta} = \mp i\eta \vec{a}_\pm E_\pm \ , \tag{11}$$

where the upper sign and the lower sign are taken for the right- and left-hand polarized waves, respectively.

Using Eqs. (5), (7), and (11), Eq. (8) becomes

$$k_\pm^2 E_\pm = \frac{\omega^2}{c^2}\epsilon_l E_\pm - i\omega\mu_0 \sum_{\text{CT}} \frac{q_0 n_0 \mu'(1-\xi)}{1 \pm i\eta} E_\pm \ . \tag{12}$$

Since $|E_\pm| \neq 0$, the desired dispersion relation for the transverse electromagnetic wave is given by

$$k_\pm^2 = \frac{\omega^2}{c^2}\epsilon_l - i\omega\mu_0 \left(\frac{q_e n_e \mu_e'(1-\xi_e)}{1 \pm i\eta_e} + \frac{q_h n_h \mu_h'(1-\xi_h)}{1 \pm i\eta_h} \right) . \tag{13}$$

The subscripts e and h are used for the quantities associated with the electron and those with the hole, respectively. Equation (13) can be rewritten in the familiar form[18]

$$\frac{c^2 k_\pm^2}{\omega^2} = \epsilon_l - \frac{\omega_{pe}^2(\omega - k_\pm v_{0e})}{\omega^2[(\omega \mp \omega_{ce}) - i\nu_e]} - \frac{\omega_{ph}^2(\omega - k_\pm v_{0h})}{\omega^2[(\omega \pm \omega_{ch}) - i\nu_h]} \tag{14}$$

by using the following facts:

$$\tau_e = \frac{1}{\nu_e}, \quad \mu_e = -\frac{e}{m_e \nu_e}, \quad \mu_e B_0 = -\frac{\omega_{ce}}{\nu_e},$$
$$q_e n_e \mu_e = \frac{\omega_{pe}^2}{\nu_e}\epsilon_0, \quad \tau_h = \frac{1}{\nu_h}, \quad \mu_h = \frac{e}{m_h \nu_h}, \tag{15}$$
$$\mu_h B_0 = \frac{\omega_{ch}}{\nu_h}, \quad q_h n_h \mu_h = \frac{\omega_{ph}^2}{\nu_h}\epsilon_0 \ .$$

In the above expressions the symbols used are m for the effective mass, ν for the collision frequency, ω_p for the plasma frequency, ω_c for the cyclotron frequency of the electron (with subscript e) and of the hole (with subscript h). The dielectric constant of free space is ϵ_0, and e is the magnitude of electronic charge. The drift velocity of electron v_{0e} and that of hole v_{0h} are given by

$$v_{0e} = \mu_e E_0 = -\frac{e}{m_e \nu_e} E_0$$

and

$$v_{0h} = \mu_h E_0 = \frac{e}{m_h \nu_h} E_0 \ .$$

For convenience of discussion Eq. (14) can be written in the following normalized form:

$$\tilde{K}_\pm^2 = \tilde{K}_0^2 + \delta\tilde{A}\,\tilde{K}_\pm \ , \tag{16}$$

where

$$\tilde{K} = c\tilde{k}/\omega = \bar{\beta} + i\bar{\alpha} \ ,$$
$$\tilde{A} = A_1 + iA_2 \ , \tag{17a}$$
$$\tilde{K}_0^2 = D_1 - iD_2 \ ,$$

with

$$A_1 = X\theta_1(p_1 - q_1/a^2 b) \ ,$$
$$A_2 = X\theta_1(p_2 - q_2/a^2 b) \ ,$$
$$D_1 = \epsilon_l - X\theta_1(p_1 + q_1/a) \ ,$$
$$D_2 = X\theta_1(p_2 + q_2/a) \ ,$$
$$p_1 = \frac{(\theta_1 \mp \theta_{ce})}{(\theta_1 \mp \theta_{ce})^2 + 1}, \quad q_1 = \frac{d(\theta_1 \pm \theta_{ce}/a)}{(\theta_1 \pm \theta_{ce}/a)^2 + b^2} \ ,$$
$$p_2 = \frac{1}{(\theta_1 \mp \theta_{ce})^2 + 1}, \quad q_2 = \frac{db}{(\theta_1 \pm \theta_{ce}/a)^2 + b^2} \ ,$$

in which

$$\theta_1 = \omega/\nu_e \ , \quad \theta_{ce} = \omega_{ce}/\nu_e \ , \quad a = m_h/m_e \ ,$$
$$b = \nu_h/\nu_e \ , \quad d = n_h/n_e \ , \quad X = (\omega_{pe}/\omega)^2 \ , \tag{17b}$$
$$\delta = v_{0e}/c \ .$$

In order to study the effect of a static electric field on the propagation characteristics of the wave, it is necessary to investigate the variation of the complex propagation constant $\tilde{k} = (\beta + i\alpha)$ with the variation of E_0, by solving Eq. (16) for $\tilde{K} = c\tilde{k}/\omega$. Since Eq. (16) is a quadratic equation in $\tilde{K}$ it can easily be solved to give

$$\tilde{K}_\pm = \tfrac{1}{2}\{\delta\tilde{A} \mp [(\delta\tilde{A})^2 + 4\tilde{K}_0^2]^{1/2}\} \ . \tag{18}$$

Upon specifying the parameters in Eq. (17b), the coefficients A_1, A_2, D_1, and D_2, given in Eq. (17a), are determined, and $\tilde{K}_\pm$ can be calculated to give $\alpha_\pm$ as well as $\beta_\pm$. The subscripts + (or r) and − (or l) are introduced here to emphasize the fact that the quantity under consideration is for the right- and left-hand polarized waves, respectively. For example, $\alpha_+ = \alpha_r$ denotes the amplitude constant of the right-hand wave.

III. VARIATIONS OF AMPLITUDE AND PHASE CONSTANTS

Physical parameters for indium antimonide used in the present sample calculation are taken as follows:

$$\epsilon_l = 17.5 \ , \quad m_e = 0.013 m_0 \ , \quad m_h = 0.40 m_0 \ .$$

Case (i): Intrinsic indium antimonide at room temperature, $T = 290\ ^\circ K$:

$$\mu_e = -7.7 \times 10^4\ \text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1} \ ,$$
$$\mu_h = 7.5 \times 10^2\ \text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1} \ ,$$
$$n_i = n_e = n_h = 1.6 \times 10^{16}\ \text{cm}^{-3} \ .$$

Case (ii): n-type indium antimonide at liquid-nitrogen temperature, $T = 77\ ^\circ K$:

$$\mu_e = -5 \times 10^5\ \text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1} \ ,$$
$$\mu_n = 10^4\ \text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1} \ ,$$
$$n_i = 10^{10}\ \text{cm}^{-3} \ ,$$
$$n_e = 2.8 \times 10^{14}\ \text{cm}^{-3} \ .$$

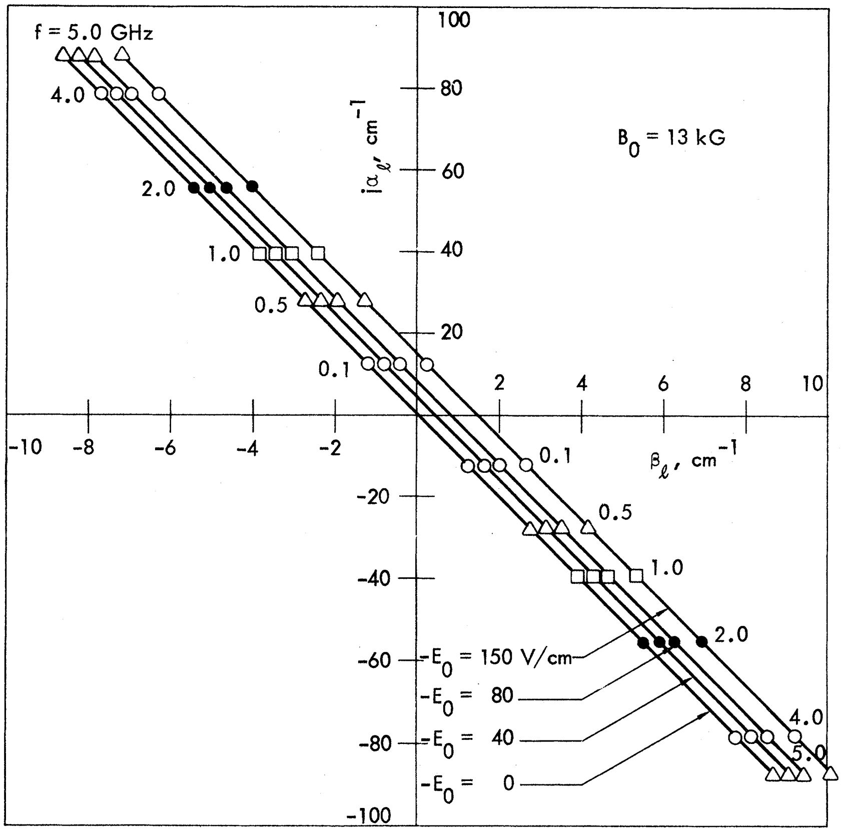

FIG. 1. Complex propagation constant $\tilde{k}_l$ vs wave frequency f for different values of static electric field E_0 for an intrinsic indium antimonide at room temperature T = 290 °K [case (i)] and B_0 = 13 kG.

Case (iii): p-type indium antimonide at liquid-nitrogen temperature, $T = 77$ °K:

$$\mu_e = -2\times10^5 \text{ cm}^2\,\text{V}^{-1}\,\text{sec}^{-1},$$

$$\mu_h = 9\times10^3 \text{ cm}^2\,\text{V}^{-1}\,\text{sec}^{-1},$$

$$n_i = 10^{10} \text{ cm}^{-3},$$

$$n_h = 3\times10^{14} \text{ cm}^{-3}.$$

In most semiconductors, the relative dielectric constant ϵ_l of the lattice is between 10 and 20. Although no direct measurement of the static dielectric constant ϵ_l has been performed, it is often assumed to be 17.5. The effective mass m_e of the electron and of the hole m_h is expressed in terms of the free-electron mass m_0. Cyclotron resonance measurement of the effective mass has shown that at $T = 290$ °K, $m_e \simeq 0.013 m_0$.[19] There is some evidence that the ratio m_e/m_0 may decrease slightly with increasing temperature. On the other hand, the cyclotron resonance studies on p-type material have shown that the light-hole effective mass is $(0.021 \pm 0.005) m_0$. The heavy-hole effective mass is anisotropic, with a mean value of the order of $0.4 m_0$.[20]

The carrier conductivity mobilities μ_e and μ_h used are those of typical values found at $T = 290$ and 77 °K.[21,22] At 290 °K the intrinsic concentration n_i of indium antimonide is $n_i = 1.6\times10^{16}$ cm^{-3}.[23] Owing to the steep dependence of the intrinsic concentration upon temperature at low temperatures, the exact value at 77 °K has not been established. It is probably in the 10^{10}-cm^{-3} range.

It is well known that the carrier concentrations n_e and n_h in thermal equilibrium satisfy the mass-action law,[24] $n_e n_h = n_i^2$, which is equally valid in either an extrinsic or intrinsic semiconductor. Thus, by knowing the number of electrons in a material, the number of holes can be determined, or vice versa.

The values of the static electric field used for the present calculation are so chosen that the current density and the electric field intensity satisfy Ohm's law. The range of E_0 for which Ohm's law is valid depends on the temperature and the type of materials.[21] At 290 °K in intrinsic material, the deviation from Ohm's law is observed in a pulsed field of the order of 150 V/cm and above. At 77 °K, very small deviation from Ohm's law has been observed in n-type material below 30 V/cm. At 77 °K in p-type material, electron-hole pair creation occurred at 700 V/cm.

In order to illustrate graphically the variation of α and β, the complex plots of $k(f, E_0, B_0)$ as the function of f are shown in Figs. 1 and 2(a), respectively, for cases (i) and (ii). The variation

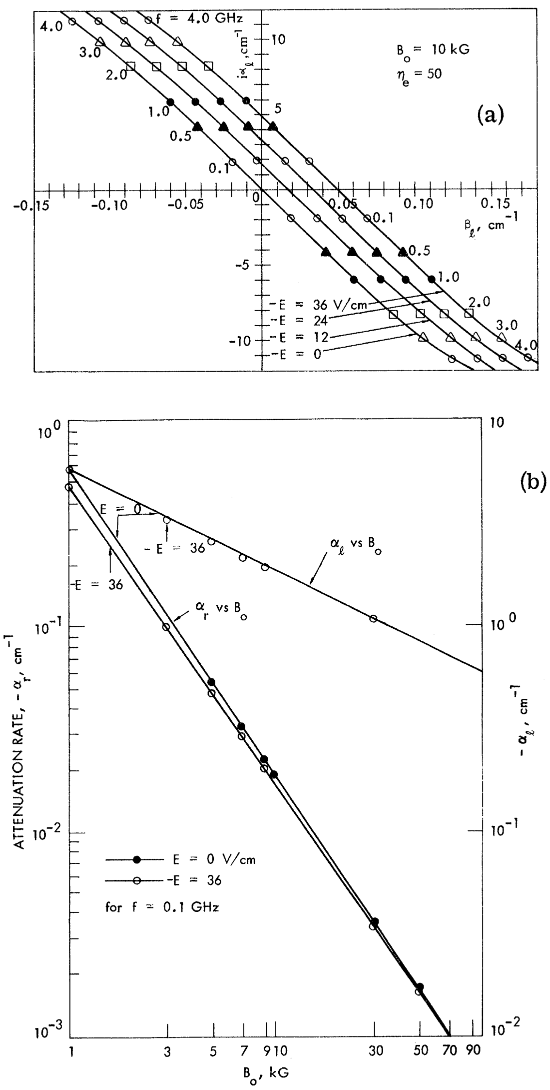

FIG. 2. (a) Complex propagation constant $\tilde{k}_l$ vs wave frequency f for different values of static electric E_0 for n-type indium antimonide at liquid-nitrogen temperature $T = 77\,°\mathrm{K}$ [case (ii)] and $B_0 = 10$ kG. (b) Amplitude constants α_r and α_l vs static magnetic field B_0 for case (ii) at $f = 0.1$ GHz.

of the amplitude constants α_r and α_l with the static magnetic field B_0 are illustrated in Figs. 2(b) and 3(a) for cases (ii) and (iii), respectively.

IV. FARADAY ROTATION

It is well known that a linearly polarized electromagnetic wave, propagating along $\vec{B}_0$ in a plasma, can be decomposed into two counter-rotating circularly polarized waves with different phase velocities. This difference in the phase velocities of the left- and right-hand waves is responsible for a rotation of the plane of polarization. As the wave travels in a distance L in the plasma, the angle of rotation φ of the plane of polarization can be given by[5,6]

$$\varphi = \tfrac{1}{2} L (\beta_l - \beta_r) = (L\omega/2c)\,\mathrm{Re}(\tilde{K}_- - \tilde{K}_+)\,, \tag{19}$$

provided that the amplitudes of the left- and right-hand waves are comparable; i.e., the attenuation

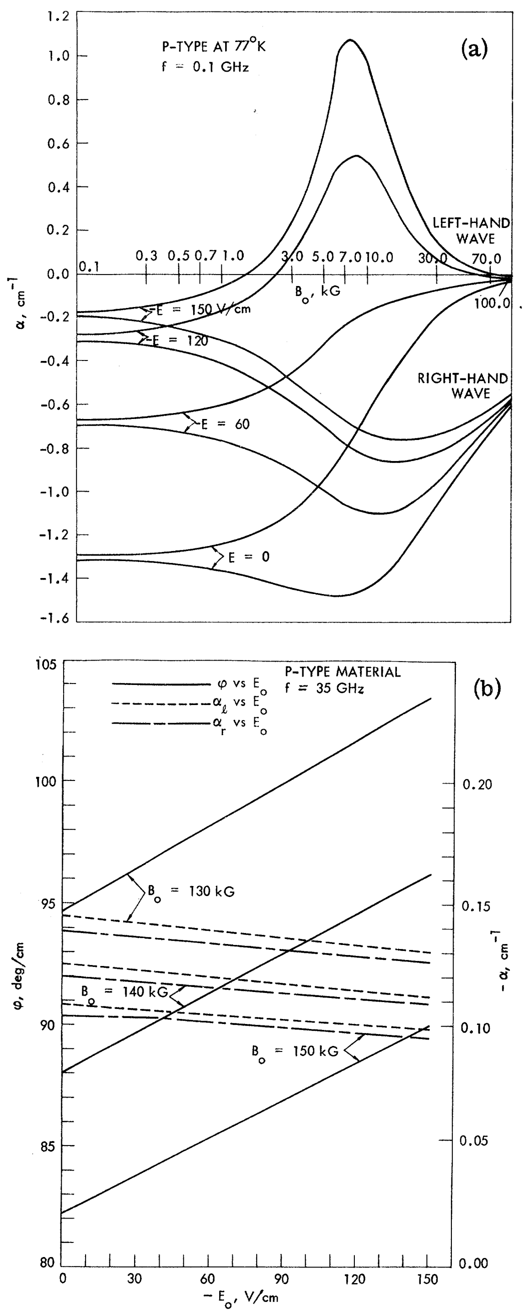

FIG. 3. (a) Amplitude constants α_r and α_l vs static magnetic field B_0 for case (iii) at $f = 0.1$ GHz. (b) Faraday rotation φ and amplitude constants α_r and α_l vs static electric field E_0 for different values of static magnetic field B_0, at $f = 35$ GHz for case (iii).

rates of both waves are small and comparable.

In order to see whether the static electric field E_0 has any effect on the Faraday rotation, it is first necessary to determine the range of parameters over which the attenuation rate of both circularly polarized waves are sufficiently small and comparable so that Eq. (19) is physically meaningful.

A careful investigation of the behavior of amplitude constants α_l and α_r given by Eq. (18), over a wide range of parameters f, B_0, and E_0 for all three cases under consideration, shows that for case (i), $|\alpha_l|$ and $|\alpha_r|$ are either both too large or one is much larger than the other. This suggests that the observation of Faraday rotational effect in an intrinsic indium antimonide at room temperature is not likely. As for case (ii), it is possible to observe the Faraday rotation provided that the magnetic field is sufficiently high. For example, microwave Faraday rotation in a nondrifting plasma has been studied experimentally as well as theoretically[6] for B_0 in the range 10–150 kG, at $f = 35$ GHz. In this range of parameter, the effect of static electric fields on the Faraday rotation is found to be negligible. However, in case (iii) there is a range of B_0 over which Eq. (19) is meaningful and the effect of E_0 may be significant. This is illustrated in Fig. 3(b) at $f = 35$ GHz.

V. POWER EXCHANGE BETWEEN WAVE AND DRIFTING PLASMA

It is well known that the integral of $P_r = \mathrm{Re}(\vec{\mathrm{J}}_1 \cdot \frac{1}{2}\vec{\mathrm{E}}_1^*)$ over a volume in the medium under consideration can be regarded as the net time-average power flow into the volume. When P_r is positive, the power is transferred to the medium, while when P_r is negative the power is extracted from the medium. In the present study, the medium under consideration is that of the drifting plasma. J_1 is given by Eq. (7) and $\vec{\mathrm{E}}_1$ is taken as $\vec{\mathrm{E}}_\perp$. The asterisk is used to denote the complex conjugation. Assuming that $\omega\tau \ll 1$, the complex power density $\tilde{P} = (\vec{\mathrm{J}}_1 \cdot \vec{\mathrm{E}}_1^*)$ can be computed with the aid of Eqs. (5) and (7) to give

$$\tilde{P} = \sum_{\mathrm{CT}} \sigma'(1-\tilde{\xi})\left[|E_\perp|^2 + \vec{\eta} \cdot (\vec{\mathrm{E}}_\perp^* \times \vec{\mathrm{E}}_\perp)\right], \tag{20}$$

where

$$\sigma' = q n_0 \mu / (1+\eta^2) .$$

For the transverse circularly polarized wave, $\vec{\mathrm{E}}_\perp$ may take the form of either $\vec{\mathrm{a}}_+ E_+$ or $\vec{\mathrm{a}}_- E_-$, and using Eqs. (9) and (10) it is easily shown that

$$\vec{\eta} \cdot (\vec{\mathrm{E}}_\perp^* \times \vec{\mathrm{E}}_\perp) = \mp i\eta |E_\pm|^2 . \tag{21}$$

Recalling that $\tilde{\xi} = \tilde{k} v_0/\omega$, $P_r = \mathrm{Re}(\frac{1}{2}\tilde{P})$ is obtained with the aid of Eqs. (20) and (21) as follows:

$$P_r = \tfrac{1}{2}|E_\pm|^2 \left[(\sigma_e' + \sigma_h') - (\beta/\omega)(v_{0e}\sigma_e' + v_{0h}\sigma_h') \mp (\alpha/\omega)(v_{0e}\sigma_e'\eta_e + v_{0h}\sigma_e'\eta_h)\right], \tag{22}$$

where the phase constant β and the amplitude constant α are, respectively, the real and imaginary parts of the complex propagation constant $\tilde{k} = (\beta + i\alpha)$, which is given by the dispersion relation, either Eq. (13) or Eq. (14).

It should be noted that under the condition $\omega \ll \omega_{ch} < \omega_{ce}$, the coefficients A_1, A_2, D_1, and D_2 are simplified to the following form:

$$\begin{aligned}
A_1 &= (\pm 1/\omega\epsilon_0 v_{0e})(v_{0e}\sigma_e'\eta_e + v_{0h}\sigma_h'\eta_h) , \\
A_2 &= (1/\omega\epsilon_0 v_{0e})(v_{0e}\sigma_e' + v_{0h}\sigma_h') , \\
D_1 &= \epsilon_l \mp (1/\omega\epsilon_0)(\sigma_e'\eta_e + \sigma_h'\eta_h) , \\
D_2 &= (1/\omega\epsilon_0)(\sigma_e' + \sigma_h') , \\
\bar{\beta} &= c\beta/\omega , \quad \bar{\alpha} = c\alpha/\omega , \quad \delta = v_{0e}/c .
\end{aligned} \tag{23}$$

The upper and the lower signs are taken for the right- and left-hand waves, respectively.

Using Eq. (23), Eq. (22) can be expressed as

$$P_r = \tfrac{1}{2}\omega\epsilon_0 |\tilde{E}_\pm|^2 \left[D_2 - \delta(\bar{\beta}A_2 + \bar{\alpha}A_1)\right] . \tag{24}$$

However, the quantity within the square bracket is equal to $-2\bar{\beta}\bar{\alpha}$, which can be seen when the imaginary part of the left-hand side is equated to that of the right-hand side of Eq. (16). Consequently, Eq. (24) is simplified and becomes

$$P_r = -\omega\epsilon_0 |\tilde{E}_\pm|^2 \bar{\beta}\bar{\alpha} . \tag{25}$$

It is of interest to note that $P_r < 0$ when $\bar{\alpha}\bar{\beta} > 0$, i.e., when $\beta\alpha > 0$, which requires the complex propagation constant $\tilde{k} = \beta + i\alpha$ to lie in the first or the third quadrant of the complex $\tilde{k}$ plane. For example, when $\beta > 0$ and $\alpha > 0$ the transverse electromagnetic wave grows spatially in the positive z direction. This is reasonable since the power is being extracted from the medium by the wave. On the other hand, if $\beta > 0$ but $\alpha < 0$, $\beta\alpha < 0$ so that $P_r > 0$ and the power is being transferred to the medium, i.e., the power is being absorbed by the medium so that the wave should suffer an attenuation.

In the absence of the static electric field, i.e., $E_0 = 0$, δ vanishes so that Eq. (18) yields

$$\tilde{K} = \pm \tilde{K}_0 = \pm(p_0 - ia_0) , \tag{26}$$

where the upper sign and the lower sign are taken for the positively and the negatively traveling waves, respectively, and p_0 and a_0 are given by

$$\begin{aligned}
p_0 &= \left\{\tfrac{1}{2}\left[(D_1^2 + D_2^2)^{1/2} + D_1\right]\right\}^{1/2} , \\
a_0 &= \left\{\tfrac{1}{2}\left[(D_1^2 + D_2^2)^{1/2} - D_1\right]\right\}^{1/2} .
\end{aligned} \tag{27}$$

In this case $\bar{\beta} = p_0$ and $\bar{\alpha} = -a_0$ so that Eq. (25) becomes

$$P_r = \omega\epsilon_0 |E_\pm|^2 p_0 a_0 . \tag{28}$$

Since p_0 and a_0 are both real and positive quantities,

P_r is real and positive also. Thus the power is transferred to the medium, and an instability of system does not occur. This suggests that for the wave to grow spatially it is necessary for the electron and the hole to drift and thus requires the presence of longitudinal static electric field in the system.

VI. DISCUSSION

Since the wave function with a phase factor of the form $e^{i(\omega t - kz)}$ is being considered, the wave may be spatially growing in the direction of wave propagation when either $\beta > 0$ and $\alpha > 0$ (i.e., $\tilde{k}$ is in the first quadrant of $\tilde{k}$ plane), or $\beta < 0$ and $\alpha < 0$ (i.e., $\tilde{k}$ is in the third quadrant). On the other hand, the wave under consideration may be spatially decaying in the direction of propagation, when either $\beta < 0$ and $\alpha > 0$ (i.e., $\tilde{k}$ is in the second quadrant) or $\beta > 0$ and $\alpha < 0$ (i.e., $\tilde{k}$ is in the fourth quadrant). For each type of polarization, the complex plot of the propagation constant, either $\tilde{k}_l$ or $\tilde{k}_r$, has two branches. Branches 1 and 2 refer to the plots in which the minus and the plus algebraic signs associated with the square-root sign in Eq. (18) are taken.

The complex plot of propagation constant k_l of the left-hand wave as a function of the wave frequency f for different values of static electric field E_0, with $B_0 = 13$ kG is illustrated in Fig. 1(a) for an intrinsic indium antimonide at room temperature [case (i)]. This plot shows that when $E_0 = 0$, i.e., in the absence of carrier drift, branch 1 is located in the second quadrant while branch 2 is in the fourth quadrant. Branch 1 represents the negatively traveling wave, and branch 2 represents the positively traveling wave; both decay in the direction of propagation. The wave frequency f is assumed to be a real and positive quantity in this paper.

It is of interest to note that branch 2 remains in the fourth quadrant while branch 1 may be extended into the first quadrant as E_0 is increased from zero. For example, at $f = 0.04$ GHz, branch 1 is extended into the first quadrant at about $E_0 \simeq 80$ V/cm, while at $f = 0.1$ GHz this occurs at about $E_0 \simeq 120$ V/cm. The growth rate $\alpha_l > 0$ increases only slightly with E_0 (see the plot in the first quadrant of Fig. 1). This type of instability is referred to as "anomalous instability."[11]

The complex plot if $\tilde{k}_l$ for n-type indium antimonide at liquid-nitrogen temperature [case (ii)] is illustrated in Fig. 2(a). Once again, when $E_0 = 0$, branches 1 and 2 are located in the second and fourth quadrants, respectively, so that no spatial growth of wave is possible. As E_0 is increased, branch 2 remains in the fourth quadrant, while branch 1 is extended into the first quadrant so that a spatial growth of the wave is possible (this involves an anomalous instability). This type of instability, associated with the left-hand polarized helicon wave in a material in which electrons are dominant, has been studied by various workers.[7-9]

For a given frequency, $|\alpha_l|$ remains practically unchanged with E_0, while β_l increases algebraically with $|E_0|$ in both branches. On the other hand, in branch 1, for a given $E_0 \neq 0$ and B_0, there is a critical frequency f_0 below which $\alpha_l > 0$ and $\beta_l > 0$. The growth rate α_l increases with $f < f_0$ in this case [see the first quadrant of Fig. 2(a)].

The attenuation rate $|\alpha_l|$ at $f = 0.1$ GHz can be considerably reduced by increasing B_0 [Fig. 2(b)]. This figure shows that an increase in E_0 leads to a reduction of $|\alpha_r|$ somewhat in the lower magnetic field range, while practically no effect on $|\alpha_l|$ in the range of $B_0 \leq 100$ kG is apparent. For the range of B_0 considered, where $|\alpha_l| > |\alpha_r|$, $|\alpha_r|$ decreases with B_0 at a much faster rate than $|\alpha_l|$ does. Furthermore, $|\alpha_r|$ is reduced to a sufficiently small value for the range $B_0 > 3$ kG, so that the propagation of the right-hand wave is possible.

It should be noted that for the parameters considered in Fig. 2(b) the frequency is low enough to make $\omega \ll \omega_{ce}$, ω_{pe}, ν_e and the magnetic field large enough to make $|\eta_e| = (\omega_{ce}/\nu_e) \gg 1$, the range where the transverse circularly polarized waves are commonly referred to as "helicon waves." Figure 2(b) clearly indicates that the right-hand wave is far less damped than the left-hand wave. This is one of the important properties of helicon wave since it permits the propagation of low frequency with relatively small collisional damping. The experimental observation of the propagation of helicons in indium antimonide at 77 °K made by Libchaber and Veilex, using circularly polarized microwave radiation at a frequency of 10 GHz, also indicates that only the correct sense of circularly polarized radiation propagated, while the opposite sense was highly damped. On the other hand, if B_0 is increased to sufficiently large value to make $\omega_{pe}^2/\omega\omega_{ce} \leq \epsilon_l$, the left-hand wave should also begin to propagate more easily. Furdyna[6] has confirmed this aspect of helicon transmission in experiment with indium antimonide at 35 GHz.

The variation of the amplitude constants α_l and α_r associated with branch 2 for a p-type indium antimonide at liquid-nitrogen temperature $T = 77$ °K is shown in Fig. 3(a) for $f = 0.1$ GHz. This figure shows that $\alpha_r < 0$ in the range of B_0 under consideration, so that no spatial growth of the right-hand wave is to be expected. However, there is a range of B_0 for which $\alpha_l > 0$, which is given by $2.5 \text{ kG} < B_0 < 55$ kG for $E_0 = 120$ V/cm, and $1.5 \text{ kG} < B_0 < 70$ kG for $E_0 = 150$ V/cm. Moreover, the value of $(\alpha_l)_{\max}$, the maximum growth rate, takes the value of 0.55 N_p/cm at $B_0 = 8.5$ kG for $E_0 = 120$ V/cm while it takes the value of 1.1 N_p/cm at $B_0 = 7.5$ kG for $E_0 = 150$ V/cm. For a given B_0 and f, there is also a threshold value $E_{0\,\mathrm{th}}$ for electrostatic field, above which α_l becomes positive so that the left-hand polarized wave may grow spatially, e. g., for $f = 0.1$ GHz, $B_0 = 70$ kG, $E_{0\mathrm{th}} = 150$ V/cm. It should be pointed out that in the range $B_0 < 100$ kG, the attenuation rate of the right-hand wave is much larger than that of the left-hand wave. However, in the high-magnetic-field region, say 130 kG $\leq B_0 \leq 150$ kG, both $\alpha_l < 0$ and $\alpha_r < 0$ are sufficiently small so that two types of circularly polarized waves can be expected to propagate with small attenuation. In this case, it is of interest to consider the effect of static electric field on Faraday rotation.

The variation of angle of rotation φ with E_0, as defined by Eq. (19), is shown in Fig. 3(b) for different values of B_0. Figure 3(b) shows that φ increases directly proportional to E_0 and it decreases inversely proportional to B_0. The microwave Faraday rotation in a n-type indium antimonide at liquid-nitrogen temperature has been studied by Furdyna in the same region of B_0 for the case $E_0 = 0$.[6] In the present investigation we also considered whether or not a change in E_0 from 0 to about 150 V/cm would cause the change of Faraday rotation in n-type indium antimonide at $f = 35$ GHz, but no significant change was observed.

In order to see how $\tilde{k} = \beta + i\alpha$ might move into the first quadrant from the second or the fourth quadrant by varying the parameters f, B_0, and E_0, it is helpful to consider the following approximate solution of Eq. (18):

$$\tilde{K} = \mp\tilde{K}_0 + \tfrac{1}{2}\delta\tilde{A} \;, \tag{29}$$

provided that

$$\left|\delta^2\tilde{A}^2/4\tilde{K}_0^2\right| \ll 1 \;. \tag{30}$$

The condition (30) is valid for most of the range of parameters considered, except for the range of low frequency, e. g., $f < 0.4$ GHz in case (i).

Equation (29) implies that

$$c\beta/\omega = \mp p_0 + \tfrac{1}{2}\delta\, A_1 \tag{31a}$$

and

$$c\alpha/\omega = \pm a_0 + \tfrac{1}{2}\delta A_2 \;, \tag{31b}$$

where the upper and the lower signs are taken for branches 1 and 2, respectively.

It should be noted that $p_0\,(f, B_0)$ and $a_0(f, B_0)$ are both real and positive quantities. However, $A_1\,(f, B_0)$ and $A_2(f, B_0)$ may be either positive or negative depending on the sense of polarization as well as the type of material under consideration. For example, when $\omega < \omega_{ch}$, from Eq. (23), it is easily seen that $A_1 < 0$ for the right-hand wave, whereas $A_1 > 0$ for the left-hand wave in all three cases. On the other hand, $A_2 > 0$ for both the right- and left-hand wave in cases (i) and (ii) while $A_2 < 0$ for both the right- and left-hand wave in case (iii). The quantity $\delta = \mu_e E_0/c$ may be positive or negative according to whether $E_0 < 0$ or $E_0 > 0$, since $\mu_e < 0$.

For the left-hand wave in case (i), both A_1 and A_2 are positive. It is observed from Eqs. (31a) and (31b) that for branch 1, if $\delta > 0$ then δA_1 and δA_2 are both positive so that both β and α increase algebraically as δ increases. α remains positive but β may become positive when $\delta A_1 > 2p_0$, by sufficient increase in δA_1, so that branch 1 is extended into the first quadrant. The quantity δA_1 may be increased either by an increase of δ (i. e., by increasing $|E_0|$) or by an increase of A_1 by decreasing the wave frequency f. However, $|E_0|$ cannot be increased indefinitely, otherwise the validity of Ohm's law may be violated. As for branch 2, β remains positive and it increases with δ or with $|E_0|$, while α remains negative since $\delta A_2 < 2a_0$; but α becomes less negative as δ increases, so that the attenuation rate decreases with $|E_0|$ (see Fig. 1). However, when δ is reversed, (i. e., $E_0 > 0$), α becomes more negative so that the attenuation rate increases with $|E_0|$.

The above remarks on the behavior of the left-hand wave are also applicable to case (ii) [see Fig. 2(a)].

Finally, with regard to the Faraday rotation φ for a large magnetic field considered in Fig. 3(b), $\eta_h^2 \gg 1$. Since $A_{1l} = -A_{1r}$, where the subscripts l and r are introduced to refer to the left- and right-hand wave, respectively, and the contribution of the electron drift is negligible, with the aid of Eqs. (23) and (31a), Eq. (19) can be given as

$$\varphi = \varphi_0 - \tfrac{1}{2}\mu_0 v_{0h}(\sigma_h/\eta_h) \;, \tag{32}$$

where

$$\varphi_0 = \tfrac{1}{2}(\omega/c)(p_{0l} - p_{0r}) \;, \tag{33}$$

in which p_{0l} and p_{0r} are the values of p_0 (the refractive index) of the left- and right-hand waves, respectively, for the case where E_0 is absent. μ_0 denotes the permeability of vacuum. Recalling that $v_{0h} = \mu_h E_0$ and $\eta_h = \mu_h B_0$, with $\mu_h > 0$, Eq. (32) suggests that when $E_0 < 0$, $v_{0h} < 0$ so that φ increases proportionally with $|E_0|$ and φ is also inversely proportional to B_0, as shown in Fig. 3(b).

VII. CONCLUDING REMARKS

In the present study, the static electric field $\vec{E}_0$ is assumed to be in the negative z direction so that electrons are drifting in the positive z direction while the holes are drifting in the negative z direction. The static magnetic field $\vec{B}_0$ is taken in the positive z direction but the wave vector may be either in the positive or negative z direction.

The static electric field strengths under consideration are weak so that Ohm's law is not violated. It has been shown that when $|\vec{E}_0| \neq 0$, i.e., in the presence of carrier drift, the left-hand polarized wave traveling in the positive z direction may grow spatially in indium antimonide plasma for all three cases considered, provided that the system parameters f, B_0, and E_0 are in a proper combination.

It is of interest to note that the present study suggests the possibility of wave amplification for the left-hand polarized wave in case (i), i.e., in an intrinsic indium antimonide at room temperature. If this theoretical observation can be verified with the results of some laboratory experiments, then the possibility is suggested of making an amplifier out of indium antimonide at room temperature. In this case, indium antimonide does not have to be kept at low temperature, such as at a liquid-nitrogen temperature so that it would make construction of the device easier.

Although there have been a great number of experimental results reported on the microwave emission from indium antimonide at liquid-nitrogen temperature subjected to static electric and magnetic fields,[12–16] it appears that there is no experimental result available, reporting the detailed study of the propagation characteristic of a transverse circularly polarized wave in indium antimonide, which can be used for comparison. Moreover, the result on microwave emission cannot be used directly to check the results of the present study because in the microwave emission problem usually $|\vec{E}_0|$ must be sufficiently high, and thus the threshold value of E_0 for emission to occur may be outside of the range in which Ohm's law is valid.

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