Magnetic Field Dependence and Q of the Josephson Plasma Resonance*

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(Received 15 May 1972)

The results of an experimental study of the magnetic field dependence of the Josephson-plasma-resonance frequency and linewidth in Pb–Pb oxide–Pb tunnel junctions are reported. In the presence of an external magnetic field, the plasma mode is found to be sensitive to an antisymmetric component of supercurrent density which is not observed in conventional measurements of the field-dependent critical current. The frequency and field dependence of the plasma-resonance linewidth are interpreted as evidence that the previously unobserved quasiparticle-pair-interference tunnel current predicted by Josephson exists and has the expected magnitude but the opposite sign.

I. INTRODUCTION

One of the characteristic features of the electrodynamic behavior of Josephson tunnel junctions is a plasmalike mode of oscillation in which electron pairs tunnel back and forth at a characteristic plasma frequency. Anderson was the first to note that the existence of this mode follows from the theory of the Josephson effects,[1] and Josephson has discussed it in some detail.[2,3]

The Josephson plasma mode was first observed experimentally by Dahm *et al.*[4] These authors found that the dependence of the plasma frequency on the dc Josephson supercurrent and the temperature agreed quantitatively with predictions of a simple theory and reported "qualitative agreement" with the expected magnetic field dependence.

We report here the results of an extended experimental study of the magnetic field dependence of the Josephson plasma resonance. Significant deviations from the predictions of the simple theory were found. We have also made a detailed study of the plasma-resonance linewidth and its dependence on frequency and magnetic field. The results suggest that a quasiparticle-pair-interference component of the tunnel current, predicted by Josephson[2] but previously unobserved, does in fact exist and has the expected magnitude but an unexpected sign.

II. THEORY

The origin of the plasma mode can be understood easily in terms of the basic Josephson equations applicable to a small tunnel junction,

$$I = I_1 \sin\varphi \ , \tag{1}$$

$$\frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar} \ , \tag{2}$$

where I is the supercurrent in the junction barrier, I_1 is the maximum supercurrent, φ is the pair phase difference across the barrier, and V is the voltage across the barrier. Although a supercurrent can flow at $V=0$ (the dc Josephson current) any change in the supercurrent will, in general, be accompanied by a nonzero voltage. The junction thus exhibits the characteristic of an ideal inductance. The effective inductance can be obtained by differentiating Eq. (1) and inserting Eq. (2),

$$L^{-1} = V^{-1}\frac{\partial I}{\partial t} = \frac{2eI_1}{\hbar}\cos\varphi \ . \tag{3}$$

Together with the junction capacitance C, this effective inductance leads to a mode of oscillation of the superconducting pairs with a characteristic frequency

$$\omega_p = (LC)^{-1/2} = \omega_J(\cos\varphi)^{1/2} \ , \tag{4}$$

where $\omega_J = (2eI_1/\hbar C)^{1/2}$. For typical tunnel-junction parameters, $f_J = \omega_J/2\pi$ is of order 10 GHz. The electric field in this mode is longitudinal, i.e., perpendicular to the barrier plane and parallel to the supercurrent.

In writing Eqs. (3) and (4), we have not made explicit the fact that the plasma oscillation corresponds to an oscillation of φ; i.e., the inductance is time dependent (parametric). Furthermore, if a magnetic field is present in the barrier, either an externally applied field or a field generated by the supercurrent itself, φ will vary spatially in the plane of the barrier, and so will the supercurrent density. We now consider these factors further.

Consider the "one-dimensional" in-line tunnel-junction geometry shown in Fig. 1. We assume that the magnetic field is in the y direction, so that φ depends only on z, with the z dependence determined by[2]

$$\frac{\partial\varphi}{\partial z}=\frac{2\mu_0 edH_y}{\hbar}\quad . \tag{5}$$

Here $d=2\lambda+l$, λ is the superconducting penetration depth, l is the barrier thickness, and we have used SI units. Following Dahm *et al.*,[4] we write the relative pair phase φ as

$$\varphi(z,\,t)=\varphi_0(z)+\delta\varphi(t)\ , \tag{6}$$

where $\delta\varphi(t)$ is assumed small; i.e., we restrict ourselves to the small-oscillation limit. The interaction of the plasma mode with external fields and currents can be taken into account by combining the Josephson and Maxwell equations to obtain a nonlinear differential equation for $\delta\varphi$,[4]

$$\frac{\partial^2\delta\varphi}{\partial t^2}+\frac{1}{RC}\frac{\partial\delta\varphi}{\partial t}+\omega_J^2\langle\sin[\varphi_0(z)+\delta\varphi]\rangle=\frac{2e}{\hbar C}(I_{dc}+I_{rf})\ . \tag{7}$$

The R which appears in the second term is an effective shunt resistance across the junction barrier and is intended to represent all of the possible sources of dissipation, e.g., quasiparticle tunnel currents, rf losses in the superconducting films, etc. I_{dc} and I_{rf} are dc and rf currents supplied by external sources, and the angular brackets denote a spatial average over the junction area.

The dc component of Eq. (7) is simply the well-known dc Josephson supercurrent

$$I_{dc}=I_1\langle\sin\varphi_0(z)\rangle\ . \tag{8}$$

If we assume a harmonic variation of I_{rf} at frequency ω and look for a solution for $\delta\varphi$ [and hence, by Eq. (2), for V_{rf}] at frequency ω, we can define a complex junction impedance[5]

$$Z_J(\omega)=\frac{V_{rf}}{I_{rf}}=\frac{\omega/C}{j(\omega^2-\omega_p^2)+\omega\omega_p/Q}\ , \tag{9}$$

where

$$\omega_p^2=\omega_J^2\langle\cos\varphi_0(z)\rangle \tag{10}$$

and

$$Q=\omega_p RC\ . \tag{11}$$

The resonance in $Z_J(\omega)$ at the plasma frequency provides a means for experimental observation of the plasma mode. However, for reasons discussed by Dahm *et al.*,[4] it is advantageous to detect

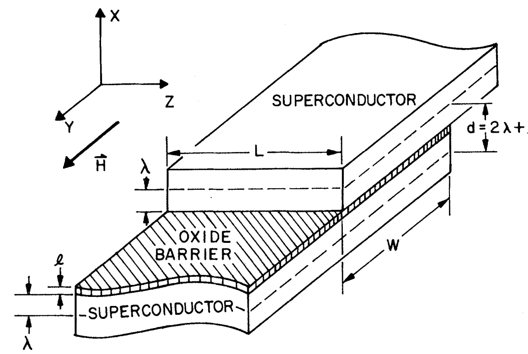

FIG. 1. Junction geometry.

the junction response at frequency 2ω rather than ω. An analysis of the second-harmonic terms in Eq. (7) yields a second-harmonic signal $V_{rf}(2\omega)=Z_{12}I_{rf}(\omega)$, where

$$Z_{12}=\frac{\omega_J^4}{2I_1^2}\times\frac{I_{rf}I_{dc}\omega/C}{[j(\omega^2-\omega_p^2)+\omega\omega_p/Q]^2[j(4\omega^2-\omega_p^2)+2\omega\omega_p/Q]}\ . \tag{12}$$

Note that resonant responses occur at both $\omega_p=\omega$ and $\omega_p=2\omega$.

If the magnetic field H_y is uniform and self-fields (those generated by the supercurrent itself) are neglected, Eq. (5) leads to a space-dependent relative pair phase

$$\varphi_0(z)=\varphi_0+(2\mu_0 edH_y/\hbar)z\ . \tag{13}$$

[The self-fields can be neglected only if the junction dimension L is small compared with the Josephson penetration depth $\lambda_J=(\hbar/2\mu_0 edJ_1)^{1/2}$, where J_1 is the Josephson supercurrent density amplitude.] The averages appearing in Eqs. (8) and (10) can then be evaluated,

$$\langle\sin\varphi_0(z)\rangle=\sin\varphi_0\left|\frac{\sin\pi\Phi_y}{\pi\Phi_y}\right| \tag{14}$$

and

$$\langle\cos\varphi_0(z)\rangle=\cos\varphi_0\left|\frac{\sin\pi\Phi_y}{\pi\Phi_y}\right|\quad , \tag{15}$$

where Φ_y is the flux contained in the junction, measured in units of the flux quantum $\Phi_0=h/2e$. From Eqs. (8) and (14), we obtain the well-known result[6] that the maximum dc current which can flow through the junction is

$$I_M=I_1\left|\frac{\sin\pi\Phi_y}{\pi\Phi_y}\right|\quad . \tag{16}$$

At $I_{dc}=I_M$, $\varphi_0=\pm\frac{1}{2}\pi$, depending on the direction of current flow.

The plasma frequency [Eqs. (10) and (15)] can be expressed as a function of I_{dc} and Φ_y,

$$\omega_p^2 = \omega_J^2 \left[\left(\frac{\sin \pi\Phi_y}{\pi\Phi_y} \right)^2 - \left(\frac{I_{dc}}{I_1} \right)^2 \right]^{1/2} . \tag{17}$$

If we identify Φ_y with Φ_M, the flux through the junction when $I_{dc} = I_M$, Eq. (17) takes the form

$$\omega_p^2 = \omega_J^2 \left| \frac{\sin \pi\Phi_M}{\pi\Phi_M} \right| \left(1 - \frac{I_{dc}^2}{I_M^2} \right)^{1/2} , \tag{18}$$

which is the result derived in Ref. 4.

III. EXPERIMENTAL DETAILS

The microwave detection system used in these experiments was essentially the same system used by Dahm *et al.*[4] A block diagram is shown in Fig. 2. A complete discussion of the operation of the system has been given by Denenstein.[5] Briefly, a small microwave signal at frequency ω is applied to the junction and any signal at 2ω generated by junction nonlinearity is detected. Separation of the input and output signals in frequency makes possible high detection sensitivity. The input frequency is 4–6 GHz and the superheterodyne receiver operates over the range 8–12 GHz. The system is sensitive to both the amplitude and the phase of the second-harmonic signal. The output of the detection system is recorded on an X-Y recorder as a function of the dc junction current.

To ensure that data were obtained in the small-oscillation regime in which the theory of Sec. II is applicable, the input signal was attenuated until the position and shape of the observed plasma resonance ceased to vary with input power, then a further 3–5-dB attenuation was added before data were taken. The resulting power levels were typically less than 10^{-6}-W input power and less than 10^{-18}-W detected second-harmonic power. All experiments were performed in a shielded room to minimize noise.

The junctions used were Pb–Pb oxide–Pb tunnel junctions fabricated by standard methods in the geometry of Fig. 1. Junction dimensions were typically $L \times W = 0.37 \times 0.15$ mm. The junction critical currents were such that $0.5\lambda_J < L < 4\lambda_J$ for the various junctions used. Most of the experiments were performed at 4.2 K.

IV. RESULTS AND DISCUSSION

A. ω_J

The constant $\omega_J = (2eI_1/\hbar C)^{1/2}$ can be obtained experimentally in two ways: (i) The maximum (temperature-dependent) dc Josephson current I_1 and the junction capacitance C can be measured directly, and ω_J calculated from the above formula; (ii) the observed plasma frequency ω_p can be plotted vs $(\cos\varphi_0)^{1/2}$, using the known dependence of I_{dc} on $\sin\varphi_0$, and hence $\cos\varphi_0$, and ω_J determined by extrapolation to the limit $\cos\varphi_0 \to 1$. We have used both methods. The junction capacitance was mea-

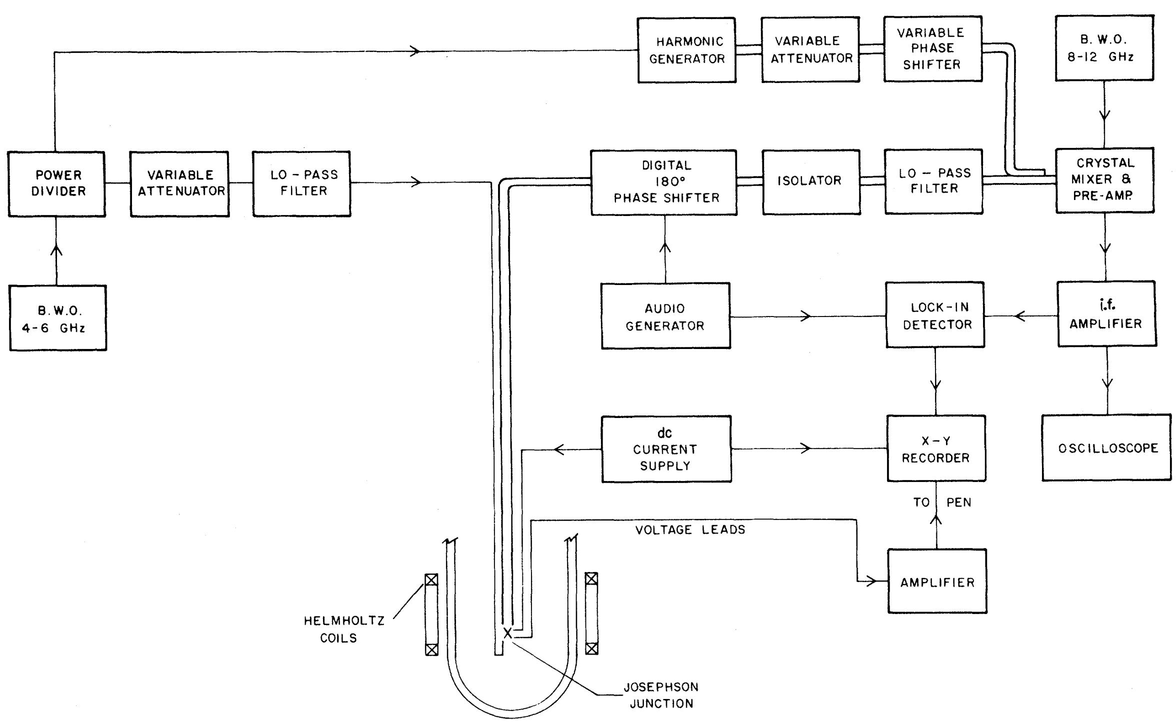

FIG. 2. Block diagram of the second-harmonic detection system (from Ref. 5). B. W. O. stands for backward wave oscillator.

sured using a standard capacitance bridge. It was necessary to reduce the junction temperature below 1.8 K in order to reduce the quasiparticle conductance sufficiently to permit accurate capacitance determinations. The capacitance is not expected to vary significantly with temperature in the range 1.8–4.2 K, so that these measurements should yield good values for the capacitance at 4.2 K where the plasma-resonance experiments were done. The values of ω_J obtained by these two methods agreed within 2%!

B. Magnetic Field Dependence

The magnetic field dependence of the plasma resonance was studied by applying an external magnetic field (using the Helmholtz coils shown in Fig. 2) and recording the output signal of the detection system as a function of I_{dc}. This yielded data on the value of I_{dc} at resonance I_R and the linewidth ΔI_R. The maximum dc current I_M for each magnetic field was also recorded (with no microwave fields applied to the junction). Since the magnetic field could be varied much more readily than the second-harmonic detection frequency, data were generally taken at a given frequency for a series of magnetic fields. Most of the data were taken on the first lobe of the diffraction pattern because in most of our junctions the plasma frequency on the higher lobes fell below the frequency range of the detection system. However, data were obtained on the third, fourth, and fifth lobes in one low-resistance junction in which the zero-field plasma frequency was very high.

In Fig. 3, we have plotted the experimentally observed I_M vs $H_y = H_e$ (the externally applied magnetic field) for one of the junctions (normal-state resistance $R_N = 0.42\ \Omega$, $f_J = 15.6$ GHz, $L/\lambda_J = 1.5$). Using the experimentally observed I_R (for a fixed value of ω_p) and I_M we have also plotted $F(I_R, I_M) = I_1(\omega_p/\omega_J)^2\,(1 - I_R^2/I_M^2)^{-1/2}$ vs H_e. If Eq. (18) is valid, $F(I_R, I_M)$ should be identical with I_M. Clearly, the two curves do not coincide. $F(I_R, I_M)$ differs significantly from I_M over most of the range. Similar deviations have been observed in all the junctions we have studied. (For the first lobe of the diffraction pattern, we have made measurements for ω_p between 0.3 and $0.9\omega_J$ and junction dimensions between 0.5 and $1.5\lambda_J$.) Although L is comparable to λ_J, the calculations of Ferrell and Prange[7] and the experiments of Schwidtal and Finnegan[8] indicate deviations of less than 2% for $L = \lambda_J$. The use of a symmetric in-line geometry should minimize deviations due to the asymmetric self-fields inherent in cross-type geometries. However, the validity of Eq. (18) does depend on the assumption that Φ_y is independent of I_{dc} for a given applied magnetic field. We therefore consider the possibility of self-field effects which

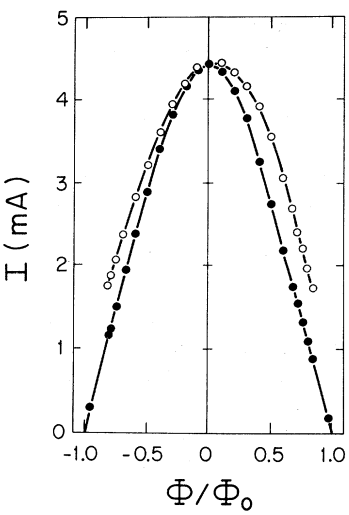

FIG. 3. Maximum dc current I_M (solid circles) and $I_1(f_p/f_J)^2[1-(I_R/I_M)^2]^{-1/2}$, $f_p = 9.0$ GHz (open circles) vs the applied flux $\Phi_y = \mu_0 H_y\, Ld$. Smooth curves have been fitted to the data.

might occur for $I_{dc} < I_M$ but not for $I_{dc} = I_M$. Such effects would not play a role in conventional experiments where I_M is measured as a function of H_e, but could affect a plasma resonance observed at $I_R < I_M$.

As Anderson[1,9] and others have pointed out, a Josephson tunnel junction may be regarded as a weak superconductor. Thus, in response to an external magnetic field, shielding supercurrents flow in the junction. The spatial distribution of these shielding currents is characterized by the Josephson penetration depth λ_J, which is analogous to the London penetration depth in a bulk superconductor. For large junctions ($L \gg \lambda_J$) the shielding currents lead to drastic modifications of the I_M-vs-H_e diffraction pattern. These have been investigated in detail both theoretically[9–11] and experimentally.[12–15] For small junctions ($L \ll \lambda_J$) the results for I_M vs H_e are consistent with the linear spatial variation of φ proportional to H_e described by Eq. (13). Measurements of I_M vs H_e, however, correspond to a special case in which the supercurrent density J is perfectly symmetric.[11] For $I_{dc} < I_M$, J will have both a symmetric and an antisymmetric component, as illustrated in Fig. 4.[11] While the symmetric component will contribute no additional magnetic flux through the junction, the antisymmetric component will result in a circulating current

$$I_c = I_1\,[(1-\cos\pi\Phi_y)/\pi\Phi_y]\cos\varphi_0\ , \tag{19}$$

and hence a net magnetic flux proportional to I_c. This flux can affect the plasma frequency significantly even for $L \leq \lambda_J$. For $I_{dc} = I_M$, $I_c = 0$.

In Fig. 5(a) we have plotted I_M and a second quantity, $I_M' = I_1(\sin\pi\Phi_y'/\pi\Phi_y)$, as a function of H_e. I_M' was calculated from the observed I_R using Eq. (17) (ω_p = const). Φ_y' is the flux in the junction when $I_{dc} = I_R$. Although I_M is symmetric in the field, I_M' is slightly asymmetric, and the average value has been used here. From these data the difference $\Delta\Phi = \Phi_M - \Phi_y'$ can be obtained. The results for $\Delta\Phi$ are plotted in Fig. 5(b). The solid line corresponds to $\Delta\Phi = \beta I_c$, where β is a geometry-dependent constant determined by fitting the data. A value of $\Delta\Phi$ for $\Phi_y = 0.4\Phi_0$ obtained by applying Ampere's law to the current density of Eq. (19) is indicated in Fig. 5(b) as the solid triangle. Experimental uncertainties associated with determining I_M and I_M', the junction geometry, and the externally applied flux can account for the difference between this value and the fitted one. The expected dependence of I_c (and hence $\Delta\Phi$) on ω_p^2 and the expected scaling of β with the junction dimension perpendicular to the applied field were observed. Taken together, these results imply that the effective flux through a tunnel junction in an external magnetic field is a function of the current, *independent* of L/λ_J (for $L/\lambda_J \leq 1$). Experiments involving the measurement of the maximum Josephson current I_M as a function of the external magnetic field are not sensitive to this additional flux, but the present plasma-resonance experiment is.

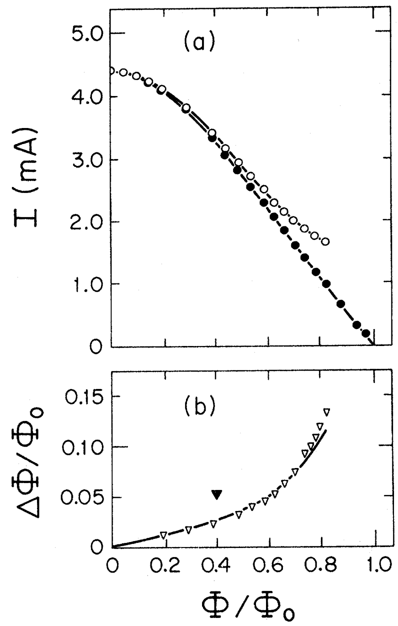

FIG. 5. (a) Maximum current I_M (solid circles) and $I_M' = I_1(f_p^4/f_J^4 + I_R^2/I_1^2)^{1/2}$, $f_p = 9.0$ GHz (open circles), as a function of the applied flux. Smooth curves have been fitted to the data. (b) The difference $\Delta\Phi$ (open triangles) as a function of the applied flux obtained from (a). The line through the data is the function βI_c ($\beta = 0.0229$ mA^{-1}). The direct numerical result for $\Delta\Phi$ is shown as a solid triangle.

C. Plasma Resonance Q

We have made a study of the linewidth of the resonance in Z_{12} [Eq. (12)] at $\omega_p = 2\omega$. For this case the signal $V_{rf}(2\omega)$ is half-maximum when

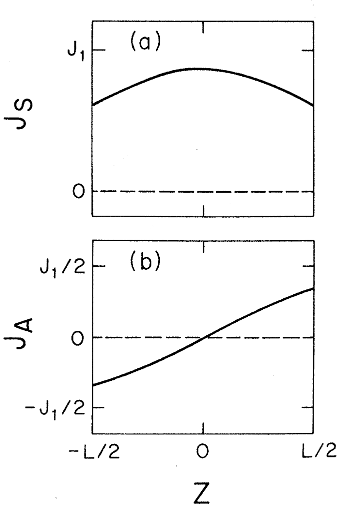

FIG. 4. Current density as a function of position in the junction for $I_{dc} < I_M$: (a) the symmetric part $J_S = J_1 \sin\varphi_0 \times \cos(2\pi z\Phi_y/L\Phi_0)$; (b) the antisymmetric part $J_A = J_1 \cos\varphi_0 \times \sin(2\pi z\Phi_y/L\Phi_0)$. $\sin\varphi_0 = 0.86$; $\Phi_y = 0.25\,\Phi_0$.

$\Delta\omega_p^2 \equiv \omega_p^2 - (2\omega)^2 = \pm 2\omega\omega_p/Q$. Now $\Delta\omega_p^2$ can be related to ΔI_{dc}, the observed variable, by expressing Eq. (17) in differential form. We find

$$|\Delta\omega_p^2| = \left(\frac{2e}{\hbar\omega_p}\right)^2 \frac{I_{dc}|\Delta I_{dc}|}{C^2} \tag{20}$$

and

$$Q^{-1} = \left(\frac{2e}{\hbar\omega_p^2}\right)^2 \frac{I_R\,\Delta I_R}{2C^2}\ , \tag{21}$$

where ΔI_R is defined as the full width of the output signal at half-maximum. Most of our measurements were made at 4.2 K. At this temperature the observed Q's were typically about 50. For one junction, the temperature was decreased to 1.3 K, and the Q was observed to increase by about a factor of 2. In general, the Q displayed a frequency and magnetic field dependence. Before discussing these dependences we consider possible contributions to the observed Q.

There are several sources of dissipation which might be expected to contribute to the linewidth of the plasma resonance. These include (a) dissipation associated with the quasiparticle tunnel current, (b) rf absorption in the superconducting films, (c) dielectric loss in the oxide barrier, and (d) radiation of rf power by the junction. The total Q will be given by $Q^{-1} = \sum_i Q_i^{-1}$, where Q_i is a Q associated with the ith dissipation mechanism.

The quasiparticle current Q, Q_{qp}, is determined by the quasiparticle tunnel current which flows at voltages within a few microvolts of $V=0$. (This is the order of magnitude of the amplitude of the rf voltages across the junction in the present experiments.) Now the quasiparticle tunnel current is a highly nonlinear function of voltage, so that, in general, it is not possible to represent quasiparticle-current effects by a simple constant shunt resistance. Theoretical calculations based on the BCS theory yield a quasiparticle current proportional to $V\ln(kT/eV)$ in the small-voltage limit; the quasiparticle conductivity diverges logarithmically as $V\to 0$, according to this theory. However, gap anisotropy and quasiparticle damping effects not included within the simple BCS theory would be expected to remove the logarithmic singularity at $V=0$, just as they are known to remove the step singularity in the quasiparticle current at $V=2\Delta/e$ predicted by the BCS theory (Δ is the superconducting energy-gap parameter). We therefore expect that the quasiparticle current will in reality be a linear function of voltage within a region about $V=0$ of width comparable with the observed voltage width of the quasiparticle current jump at $V=2\Delta/e$, i.e., at least some tens of microvolts. Consequently, we assume that it is appropriate to use a constant effective quasiparticle shunt resistance R_{qp} in the low-voltage region of interest here. Because Q_{qp} dominates the total Q (as we shall see below), the validity of this assumption is confirmed by the experimental fact that the properties of the observed plasma resonance are independent of amplitude under the conditions of our experiments (see Sec. III).

Various experimental data suggest that R_{qp} is of the same order of magnitude as R_N, the normal-state junction resistance, at the reduced temperature used in most of the present plasma-resonance experiments. If we take $R_{qp}=R_N$, and use the theoretical relation $R_N=\pi\Delta/2eI_1$, Q_{qp} can be estimated from Eq. (11) using typical junction parameters. The result is $Q_{qp}\sim 100$, roughly what we observe for the total Q. Whatever the precise relation between R_{qp} and R_N, we would expect R_{qp} to be proportional to R_N, and hence Q_{qp}^{-1} should be proportional to I_1 for experiments performed under similar circumstances on essentially ideal tunnel junctions with different I_1's.[16] Q_{qp} should be essentially frequency independent over the frequency range of our experiments.

Josephson[2] has predicted a total tunnel current density of the form

$$J=J_1(V)\sin\varphi+[\sigma_0(V)+\sigma_1(V)\cos\varphi]\,V. \qquad (22)$$

Here the first term is the usual Josephson supercurrent, the second is the usual quasiparticle current, and the third is a dissipative but phase-dependent current which arises from interference effects between the quasiparticle and pair currents. So far as we know, no experimental evidence for or against the existence of this third current component has been reported. One of the motivations for our detailed study of the plasma resonance Q was to investigate this question. If this current component exists and is large enough, one would expect a *phase-dependent* contribution to the Q, with Q decreasing as $\cos\varphi_0$ increases. It has been estimated that $\sigma_1(V)$ is comparable with $\sigma_0(V)$ at low reduced temperatures,[4] so that if the quasiparticle resistance accounts for a major part of the total Q, as turns out to be the case, we would expect to observe this term.

The Q associated with rf losses in the superconducting films, Q_f, can be estimated using a variety of assumptions about the nature of the rf fields in the junction and the magnitude and behavior of the surface resistance in the frequency range of interest.[17] We have made such estimates and conclude that Q_f is of order 10^3 and that $Q_f \propto \omega^{-\alpha}$, where α lies between about 0.7 and 1, depending on how nearly the superconducting films approach the Pippard limit.

We cannot estimate the magnitude of the Q associated with dielectric loss in the barrier, Q_b, since we do not know the composition of the barrier (it might contain any or all of several oxides of Pb), and we have no information on the low-temperature conductivities of the Pb oxides. However, the Q of a capacitor completely filled with a dielectric with dielectric loss tangent $\tan\delta$ is given by $Q^{-1}=\tan\delta$. Therefore, $Q_b=\omega\epsilon\epsilon_0/\sigma$, where ϵ and σ are the dielectric constant and conductivity of the oxide. If ϵ and σ are frequency independent in the range of interest, we would expect $Q_b\propto\omega$ and independent of I_1. If, as often is the case for other dielectrics, $\tan\delta$ is nearly independent of frequency around 10 GHz, Q_b will be frequency independent.

The Q associated with radiation of power from the junction should be roughly the inverse of the radiation coupling efficiency typically found in tunnel junctions.[18] That is, $Q_r\sim 10^4$, so that this contribution to Q is completely negligible.

In Fig. 6 the dependence of the observed Q on I_1 is shown as a plot of Q^{-1} vs I_1 for four different junctions. The three points nearest the origin were all obtained at $f_p=9$ GHz, $H=0$. The fourth point corresponds to a low-resistance junction for which ω_J was so high that we could only observe the plasma resonance on the third and higher lobes of the magnetic field diffraction pattern. For this point, therefore, $H\neq 0$. Because of the magnetic field dependence of the Q (see below), the value of Q^{-1} at $H=0$ is probably somewhat higher than the value indicated. Nevertheless, this point supports the conclusion drawn from the other three, i.e., that Q^{-1} appears to be rather accurately proportional to I_1. This is the behavior expected if Q_{qp} dominates the total Q. If we assume that $Q = Q_{qp}$ and that $R_{qp} = \alpha R_N = \alpha\pi\Delta/2eI_1$, where α is a constant, the straight line shown in Fig. 6 corresponds to $\alpha = 1.2$. The smallness of the intercept at $I_1 = 0$ in Fig. 6 indicates that nonquasiparticle contributions to the Q are small. We estimate, neglecting Q_r^{-1}, $(Q_f^{-1} + Q_b^{-1}) \leq 10^{-3}$.

The frequency dependence of $f_p Q^{-1}$ for one junction is shown in Fig. 7. These data are typical of all the junctions we have studied. As the frequency of the detection system (and hence the value of f_p on resonance) is varied, the value of I_{dc} (and hence φ_0) on resonance varies accordingly. Values of $\cos\varphi_0$ corresponding to f_p are indicated at the top of Fig. 7. The dashed straight line is $(2\pi R_N C)^{-1}$, where R_N is calculated from the observed value of I_1 and the relation $R_N = \pi\Delta/2eI_1$, and C is the measured junction capacitance.[19]

A peak is evident in $f_p Q^{-1}$ near 10 GHz. The center of this peak occurs at half the lower fundamental geometrical frequency ($f_1 = 20$ GHz) of the junction. This suggests a coupling between the plasma mode and a geometrical mode of oscillation. The factor-of-2 decrease in the Q associated

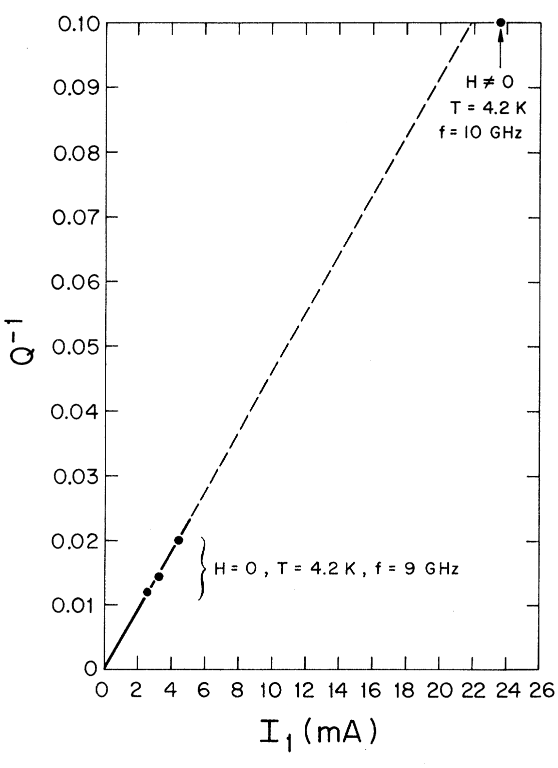

FIG. 6. Experimentally observed Q^{-1} vs I_1 for four different junctions.

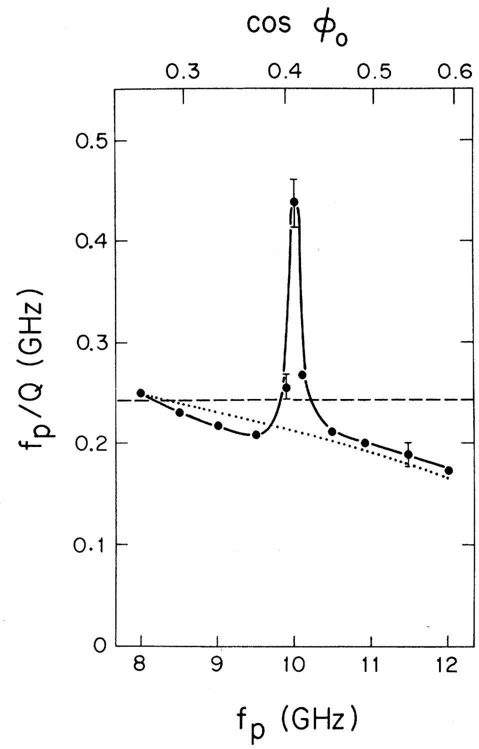

FIG. 7. $f_p Q^{-1}$ as a function of frequency. A smooth curve has been fitted to the data. The dashed line is $(2\pi R_N C)^{-1}$, $R_N = 0.46\Omega$, $C = 1.5$ nF. Typical error bars are shown. See text for discussion of dotted line.

with this peak is significant, since in general the Q of a cavity (resonance) will be reduced by a factor of 2 if the cavity (resonance) is critically coupled to a second cavity (resonance) with the same center frequency. This interpretation in terms of a coupling between modes is supported by detailed features of the observed line shape of the plasma-resonance line. The line shape had not only a relatively broad component but also a sharp spike which remained at a fixed value of I_{dc} as f was slowly varied near $\frac{1}{2} f_1$, while the broader component moved in the way expected for the plasma resonance.

Away from the peak shown in Fig. 7, $f_p Q^{-1}$ *decreases* monotonically with f_p. We recall that $f_p Q_f^{-1}(f_p)$ should increase roughly as f_p^2, and that we expect $f_p Q_b^{-1}(f_p)$ to be independent of f_p or perhaps to increase as f_p. If we postulate $f_p Q^{-1} = f_p Q_{qp}^{-1} = \beta(1 + \gamma\cos\varphi_0)$ as suggested by Eq. (22) (β and γ are constants), we find that we can fit the decrease rather well, *but only if γ has a negative sign*! The dotted line in Fig. 7 shows the fit for $\gamma = -0.8$. The limited accuracy of the present data and uncertainties due to the presence of the geometrical resonance and to possible contributions from Q_f and Q_b preclude an accurate determination of γ. If

Q_f^{-1} and Q_b^{-1} are completely negligible, we can estimate that $\gamma = -0.8 \pm 0.2$, where the uncertainty is intended to represent roughly one standard deviation. If we admit contributions from Q_f and/or Q_b of the maximum size permitted by our earlier conclusions derived from the data of Fig. 6, a negative value of γ with larger magnitude is required to offset the positive slope of these contributions. For example, if we assume that Q_b^{-1} is negligible, but Q_f^{-1} is 10^{-3} at 10 GHz and varies as f_p, then we would estimate $\gamma = -0.9 \pm 0.2$. In any case, we feel that our data clearly indicate the presence in Q_{qp} of a $\gamma \cos\varphi_0$ term with a *negative* γ of magnitude near (and probably slightly less than) one.

It might be possible to avoid the necessity of a negative γ by, say, supposing Q_b makes a larger contribution than the data of Fig. 6 would seem to allow, and in addition has an unusual frequency dependence. But, barring this or some other unforseen factor, we are led to the conclusion that the present experiments (i) provide the first experimental support for the reality of the quasiparticle-pair-interference current predicted by Josephson [Eq. (22)], (ii) indicate that the quasiparticle-pair conductivity and quasiparticle conductivity are of the same magnitude, as theoretically expected, but (iii) the quasiparticle-pair conductivity has *negative* sign![20]

The magnetic field dependence of the Q was studied in several junctions. Data for $f_p = 9$ GHz are shown in Fig. 8 as a function of the applied magnetic field. (These data are for the same junction as the data in Figs. 3–5 and 7.) The directly observed quantity $I_R \Delta I_R$ is plotted. This is directly proportional to Q^{-1} at fixed frequency [see Eq. (21)], provided Φ_y is taken to be independent of I_R.[21] The dashed line is a value of $I_R \Delta I_R$ calculated from the experimental values of R_N and C. The results are quite asymmetric for the two directions of magnetic field. In one direction a peak is evident, while in the opposite direction Q^{-1} monotonically decreases as the magnetic field increases. Essentially identical results were obtained for $f_p = 10.5$ GHz. Note that the Q actually increases by more than a factor of 3 as the field increases, and that it appears to be tending toward a very high value (∞ ?) in the vicinity of $\Phi = \Phi_0$.

In the lowest-resistance junction we studied ($R_N = 0.08\ \Omega$, $\lambda_J = 4L$), the Q of the plasma resonance measured on the third and fourth lobes of the diffraction pattern at $f_p = 9$ GHz was approximately 10, again in good agreement with a value calculated from the measured R_N and C. The dependence of $I_R \Delta I_R$ on magnetic field in this junction was also asymmetric in a manner similar to the data shown in Fig. 8. It is worth noting that for the third lobe the peak in $I_R \Delta I_R$ occurred for a smaller applied flux than that for the corresponding maximum in I_M, while for the fourth lobe the peak in $I_R \Delta I_R$ occurred at a slightly larger value of flux than that for I_M.

We have not been successful in extending the theory to account for this observed magnetic field dependence of the Q. We note, however, that the only obvious way in which a magnetic field might modify the Q as strongly as we observe is through the phase appearing in the quasiparticle-pair interference current term. The strong dependence of Q on magnetic field may thus be taken as further evidence for the existence of this current component.

V. CONCLUSIONS

We may summarize our conclusions as follows.

(i) In the presence of an external magnetic field, the Josephson plasma mode is sensitive to the antisymmetric component of the supercurrent density which exists at dc Josephson currents less than the (field-dependent) critical current, even in the limit $L < \lambda_J$. This antisymmetric component vanishes at the critical current and is not observed in conventional experiments which determine the field-dependent critical current.

(ii) The Josephson plasma resonance can couple to geometric resonances of the tunnel-junction structure. The effect of this coupling at zero dc voltage is to reduce the Q of the plasma mode at the characteristic frequencies of the geometrical resonances.

(iii) The linewidth or Q of the Josephson plasma mode is dominated by dissipation due to quasiparticle currents. The frequency and magnetic

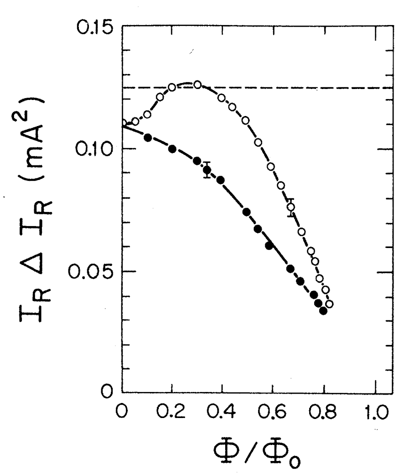

FIG. 8. $I_R \Delta I_R$ as a function of the applied magnetic flux, open circles for $\Phi/\Phi_0 > 0$, solid circles for $\Phi/\Phi_0 < 0$. The dashed line is the calculated value of $I_R \Delta I_R$ obtained using the experimental values of f_p, R_N, and C. Typical error bars are shown.

field dependence of the Q suggest that the quasiparticle-pair-interference current predicted by Josephson (but heretofore not experimentally observed) does exist and has the expected magnitude but an unexpected sign.

Note added in proof. The authors' surprise at the observation that the quasiparticle-pair-interference conductivity $\sigma_1(V)$ of Eq. (22) has negative sign stemmed from a common habit of regarding a "conductivity" as a positive quantity. Subsequent to the submission of this paper, a discussion with B.D. Josephson made it clear that the negative sign is in fact contained in the original report on the Josephson effects [B.D. Josephson, Phys. Letters 1, 251 (1962); see also Sang Boo Nam, Phys. Rev. 156, 470 (1966); and G. Rickayzen, *Theory of Superconductivity* (Interscience, New York, 1965)]. The observed sign is therefore in agreement with theory. U. K. Poulsen [Phys. Letters (to be published)] has carried out a numerical calculation of $\sigma_0(V)$ and $\sigma_1(V)$ using the BCS density of states and averaging over the logarithmic singularities in the conductivities at $V=0$. His results are in quantitative agreement with our experimental observations [see N. F. Pedersen, T. F. Finnegan, and D. N. Langenberg, in *Proceedings of the Thirteenth International Conference on Low Temperature Physics*, edited by R. H. Kropschot and K. D. Timmerhaus (University of Colorado Press, Boulder, Colo., 1972).

ACKNOWLEDGMENTS

We wish to thank A. Denenstein for valuable discussions. Also the help of Professor K. Saermark is gratefully acknowledged. One of us (N. F. P.) particularly wishes to thank Professor D. N. Langenberg and the University of Pennsylvania for their hospitality during his visit. Another one of us (T. F. F.) wishes to thank Professor K. Saermark and the Technical University of Denmark for their hospitality during the preparation of this manuscript.

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*Work supported in part by the U. S. Army Research Office (Durham), by the Advanced Research Projects Agency, and by the Statens Naturvidenskabelige Forskningsråd of Denmark.

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‡Work done in part while at the Technical University of Denmark.

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[16]Q_{qp} also depends on C, which depends on the barrier thickness, as does I_1. However, the dependence is linear for C, and exponential for I_1, so that C will vary very little as I_1 varies over several orders of magnitude.

[17]See, for example, H. Hahn, H. J. Halama, and E. H. Foster, J. Appl. Phys. 39, 2606 (1968).

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[19]The estimated 1σ uncertainty in this value of $R_N C$ is of order 5%.

[20]It is perhaps worth commenting that since the quasiparticle-pair conductivity represents an interference effect, a negative sign is not *a priori* unreasonable. Furthermore, it would seem to violate no hallowed laws of physics since $\cos\varphi_0 \leq 1$, so that if $\sigma_1(V) \leq \sigma_0(v)$ the total conductivity is always positive.

[21]If the dependence of Φ_y on I_R implied by Eq. (19) is included in deriving an expression for the Q, the result may be expressed in the form of a correction to Eq. (21). Although this correction term (which is a function of Φ_y) can be as large as 10% for the case considered here, the essential features of the magnetic field dependence are not altered by neglecting it.