Third Side of the Lampert Triangle: Evidence of Traps-Filled-Limit Single-Carrier Injection*

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(Received 9 December 1971)

Rosental and Kalda have pointed out that the full theoretical solution of the Lampert triangle in the traps-filled-limit region predicts a more gentle slope than the experimentally observed results reported by Henderson and Ashley in neutron-irradiated silicon. While this is true, the experimental results are consistent with the simple space-charge-limited model. Although Rosental and Kalda suggest that the experimental results are due to high-field or double-injection effects, further experimental data tend to confirm the original assertion of traps-filled-limit behavior.

Rosental and Kalda have completed an "exact" theoretical analysis of the third side of the Lampert triangle, presumably based upon the implicit formulation discussed in the Appendix of Lampert's original paper,[1] using the Henderson–Ashley[2] experimental data for neutron-irradiated silicon.

On the basis of the simple space-charge-limited-current (SCLC) concept of conduction, as noted by Lampert,[1,3] one expects an essentially vertical rise in the log-current–vs–log-voltage curve at the traps-filled-limit (TFL) threshold voltage,

$$V_{\mathrm{TFL}} = n_{t0}\, e L^2 / 2\epsilon \,, \qquad (1)$$

where n_{t0} is the unfilled equilibrium trap density, e is the specific electron charge, L is the diode length, and ϵ is the dielectric permittivity. Our original results, shown in Fig. 1, fit this concept very well.

It is true, however, that if the seldom-used "exact" model of Lampert is applied, the TFL region shows a somewhat more gentle slope, which increases in verticality as the ratio $A = n_{t0}/n_0$ of the equilibrium trap to equilibrium free-electron density increases. In our original results, $A > 2 \times 10^4$. Rounding this to an even decade, $A \simeq 10^4$, the "exact" analysis gives the universal curves shown in the recent and well-known publications by Lampert and co-workers.[4,5] A regional approximation method (p. 16 of Ref. 4 or p. 70 of Ref. 5) gives a sensibly vertical TFL region, whereas the simplest SCLC concept gives a perfectly vertical line. Viewing the complete Lampert triangle in perspective, the varied detailed assumptions which lead to slightly different calculated slopes in the TFL region fail to give convincing evidence to dispute the notion that the experimental curve shows a true TFL region based upon the essential correctness of Eq. (1). It should further be pointed out that the determination of n_{t0}, as used in these calculations, was itself based upon the experimentally determined vertical region, using Eq. (1). Therefore only the original SCLC results are entirely self-consistent.

Figure 2, taken from a log-log X-Y plotter at the time of the original work, is based upon similarly irradiated silicon. The various temperatures $T_1 < T_2 < T_3 < T_4$ were achieved by elevating the diode to increasingly warmer positions in the Dewar, above the surface of the liquid nitrogen. Note that the TFL region is preceded by a square-law region, because the temperatures render the thermodynamic Fermi level below the trap, thus causing the trap to appear shallow rather than

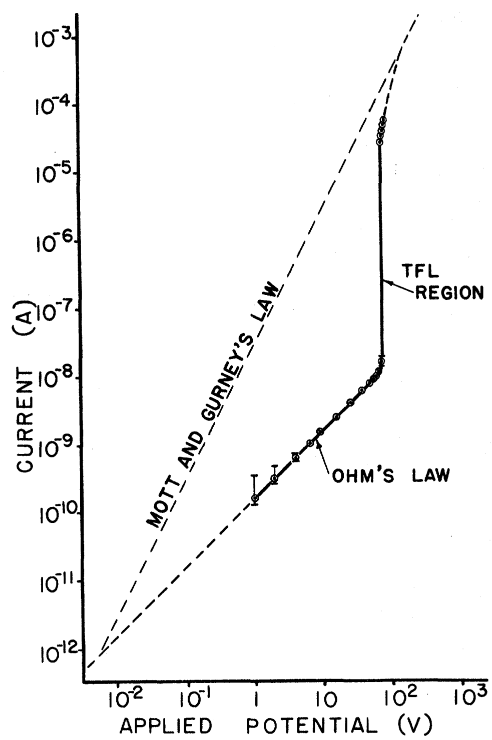

FIG. 1. Experimentally determined Lampert triangle in silicon irradiated to 1.1×10^{16} neutrons cm^{-2} (>0.1 MeV).

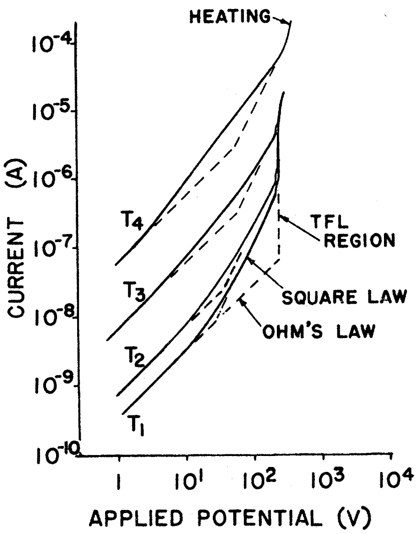

FIG. 2. Volt-ampere plots for neutron-irradiated silicon at increasingly higher temperatures.

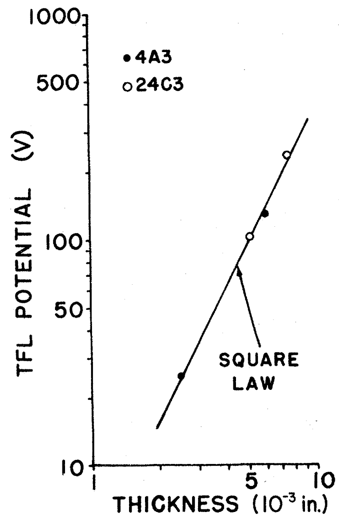

FIG. 3. Threshold voltage at onset of TFL region vs device thickness.

deep. This general behavior, including the square-law region, is included in the Lampert theory. (The small anomaly near the TFL region is believed to be due to early threshold at a point near a reduced thickness at the edge of the sample.)

Rosental and Kalda proceed to conclude that our experimental results were due to either traps emptying by field or double injection. As pointed out earlier by Bube,[6] one must be on the alert for such possibilities. However, for high-field emission one would anticipate a linear dependence of threshold voltage vs thickness, whereas Fig. 3 clearly shows a square-law dependence (for material irradiated to approximately the same level as in the original paper). This is in precise agreement with Eq. (1).

The field in the original results was approximately 6×10^3 V cm^{-1}. Although this is sufficiently high to edge into the warm-electron region in unirradiated material,[7] the unity slope in the Ohm's-law region in Fig. 1 up to threshold indicates essentially thermal equilibrium of the free carriers. Therefore, trap avalanche is not anticipated. Although trap avalanche has been reported in germanium[8] at liquid-helium temperatures, as well as in some intermetallic compounds noted in Rosental and Kalda's references, the authors know of no such observations in silicon at liquid-nitrogen temperatures, even in unirradiated silicon.

With proper electroding and preparation, the authors have observed double injection in neutron-irradiated[9] (lower irradiation level than that in the present material) and thallium-doped[10] silicon, with associated negative differential resistance at threshold. The authors know of no verified double-injection experiment where a vertical rise in current does not either just precede[11] or follow[12] such a negative-resistance region. Further, in surface-band-bending studies conducted in the laboratory over the past three years, the process used to fabricate the subject single-injection devices has shown to give distinct downbending, thus creating a blocking condition for hole injection.

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*Work supported in part by the National Science Foundation and the National Aeronautics and Space Administration.

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