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Third Side of the Lampert Triangle in Fitting Experimental Data

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It is shown that when the experimental data are analyzed according to the simplified theory of the space-charge-limited currents proposed by Lampert, the use of the full solution in the traps-filled-limit region is requisite. In this context Henderson and Ashley's results do not prove to be in agreement with Lampert's theory.

Lampert[1] pointed out that any current-voltage characteristic of the space-charge-limited current (SCLC) in a given n-type semi-insulator with a given density of shallow donors is confined to a triangular region in a log-log plot. The bounding sides of this triangle are formed by two straight lines corresponding to Ohm's law and Mott and Gurney's law,[2] and a curved line corresponding to the traps-filled-limit (TFL) law.[1] With the particular density and location of a trapping level within the energy gap, it may be possible to observe all the sides of the triangle in a single material. Recently, Henderson and Ashley[3] reported that they succeeded in carrying out such an experiment. The main feature of the data they obtained on neutron-irradiated silicon at $T = 77$ °K consists in the dramatic verticality of the third side of the triangle, the TFL characteristic. In Henderson and Ashley's view their results are in full conformity with Lampert's theory.

The purpose of this note is first to draw attention to the very shape of the third side of the Lampert triangle to guard against the implication that the TFL characteristics given by Lampert's simplified SCLC theory are always very steep and have short transition regions, and second to consider Henderson and Ashley's data in a new light.

Lampert's theory is built up assuming no diffusive flow. With this assumption the problem has an analytic solution, as illustrated in Fig. 1(a), which shows a computed current-voltage characteristic with the complete Lampert triangle *ABC*. Unfortunately, the full solution is relatively complicated and therefore rarely used. Also the curve fitting by means of the first and second sides of the triangle can be easily done with the help of good approximations—Ohm's law and Mott and Gurney's law. On the contrary, on the third side the full solution has to be used to carry out fitting correctly. Neglecting this requirement often leads to error. This can be seen in the example of Henderson and Ashley's results.

These authors have interpreted their current-voltage characteristic on the basis of the SCLC. The very steep rise of the current vs voltage at 72 V, as shown in Fig. 1(b), they have connected with the TFL phenomenon—strong filling of traps by one-carrier injection. The measured rise is indeed very steep—by recording the difference in voltage between the bottom and top of the vertical portion of the TFL line [Fig. 1(b), points C and B, respectively] the authors had to use a high-resolution digital voltmeter. It is impossible to resolve these two points in the given voltage scale.

We now wish to compare Henderson and Ashley's results with the corresponding theoretical current-voltage characteristic computed according to Lampert's equations. In the calculations [results are shown in Figs. 1(a) and 1(b)] we have adjusted the product μS, where μ is the electron mobility and S is the effective electrode area, to obtain the best fit to Henderson and Ashley's data, and we have put the dielectric permittivity ϵ equal to 10^{-10} F/m, the magnitudes of other parameters being taken from Henderson and Ashley's work as follows: the thermal-equilibrium free-electron density $n_0 = 2.8 \times 10^{14}$

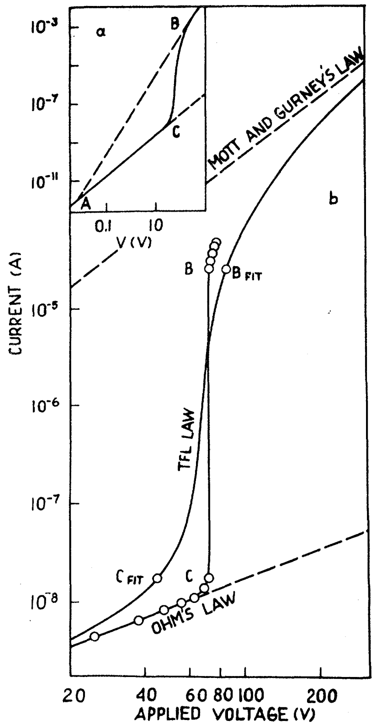

FIG. 1. Computed current-voltage characteristic to fit best Henderson and Ashley's results: (a) with the complete Lampert triangle; (b) with the third side of the triangle (TFL characteristic) only. Henderson and Ashley's data are likewise drawn in (b).

m^{-3}, the unfilled-trap density $n_{t0} = 6 \times 10^{18}\ \mathrm{m}^{-3}$, the trap energy $E_c - 0.17$ eV, and the thickness of the wafer $L = 121\ \mu\mathrm{m}$.

It is seen from Fig. 1(b) that the theoretical TFL characteristic has rather extended transition regions in the beginning and the end (within a decade of the voltage scale), which determine in a large measure the expected TFL behavior in many practically interesting cases. Figure 1(b) shows also that the theoretical curve in the TFL region differs considerably from Henderson and Ashley's experimental data. Fixing on the theoretical characteristic through the points B_{FIT} and C_{FIT} the current levels corresponding to the maximum and minimum of the vertical portion of the experimental curve, we see that the theory predicts a voltage difference 41 V between the "bottom" and "top" points on the nearly vertical section of the characteristic for the case considered. Obviously there must be no need to measure such a difference using a high-resolution digital voltmeter.

Hence Henderson and Ashley's results do not agree with Lampert's simplified theory. If diffusion were taken into account this conflict would increase even more.[4,5] Nevertheless, it should be observed that Lampert's simplified theory gives a far better fit to other experimental data, e.g., those obtained by Gregory and Jordan,[6] which after Henderson and Ashley are at variance with the theory. It appears therefore that Henderson and Ashley's results should rather have been interpreted on the basis of double injection or traps emptying by field (compare, for example, Refs. 7–10).

Elsewhere[11,12] the steady-state characteristics with the surprisingly narrow transition regions have been discussed following the simplified SCLC theory without reservations again.

The above figure illustrating some important features of the current-voltage characteristics in Lampert's simplified theory should forewarn about unsubstantiated reference to Lampert's theory in studying the currents in thin semi-insulator layers.

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