have fitted their results to the negative component of expression (1). The excellent quality of the fits to CdS,[1] Si, and InAs[9] data, and presumably other degenerate materials, provides further evidence for the general validity of our empirical expression and the model it represents.

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Brillouin Scattering from Transverse Phonons in Rare-Gas Crystals

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A mechanism is investigated for Brillouin scattering from transverse modes in rare-gas crystals, which is related to that which produces Raman scattering. Consideration is given to the intensity to be expected for the anomalous T_2 mode in the (110) direction in bcc ^{3}He; observation of this mode is concluded to be experimentally feasible. However, this mechanism is not compatible with the observed elasto-optic coefficients of xenon.

In a previous paper[1] we have analyzed the scattering of light from solid helium, giving attention to the spectral distribution and intensity of one- and two-phonon Raman scattering. We subsequently have produced[2] numerical computation of these quantities in the fcc, bcc, and hcp crystal structures, for comparison with current experiments.[3] The discussion given[1] of Brillouin scattering was not complete, however, in that an account was given of scattering from acoustic modes only of longitudinal polarization. It is nevertheless true that scattering from pure transverse acoustic modes has already been observed[4] in certain rare-gas crystals. Furthermore, the extension of these results to the bcc phase of helium would do much to clarify the rather unusual properties[5] inferred[6] for the slow transverse (T_2) branch, particularly near the (110) direction. We here investigate a mechanism for transverse-mode Brillouin scattering, and elaborate on the intensity to be expected for the (110) T_2 mode in bcc ^{3}He. However, this model fails to predict the observed[4] elasto-optic constants of xenon, which may indicate the inapplicability of the Lorentz–Lorenz dielectric model to this material.

Our starting point in this analysis is the one-phonon scattering rate for a model crystal composed of neutral atoms with dipole polarizability α,

$$(\tau^{-1})_1 = \left(\frac{2\pi N}{V}\right)^2 \sum_{\vec{k}_f} \omega_i \omega_f n_i (n_f+1) N^{-1} \sum_{\vec{k}\lambda} \delta(\vec{k}_{if} - \vec{k}) \frac{\pi\hbar}{M\omega_{\vec{k}\lambda}} \left(\frac{\delta(\omega_{if} + \omega_{\vec{k}\lambda})}{e^{\beta\omega_{\vec{k}\lambda}} - 1} + \frac{\delta(\omega_{if} - \omega_{\vec{k}\lambda})}{1 - e^{-\beta\omega_{\vec{k}\lambda}}} \right)$$
$$\times \left| \alpha \hat{\epsilon}_f \cdot \hat{\epsilon}_i \sum_{\sigma} i\vec{k} \cdot \vec{e}^{\sigma}_{\vec{k}\lambda} - \alpha^2 \sum_{\vec{\tau}} \sum_{\sigma,\sigma'} \langle (\vec{e}^{\sigma}_{\vec{k}\lambda} - \vec{e}^{\sigma'}_{\vec{k}\lambda} e^{-i\vec{k}\cdot\vec{r}_{\sigma\sigma'}}) \cdot \nabla [\hat{\epsilon}_f \cdot \overleftrightarrow{T}(\vec{r}_{\sigma\sigma'}) \cdot \hat{\epsilon}_i] \rangle \right|^2 . \quad (1)$$

This equation and its notation are taken directly from Eq. (18) of Ref. 1, except that the factor $e^{-i\vec{k}\cdot\vec{r}_{\sigma\sigma'}}$ in the last line here has not been neglected. The α^2 term in the matrix element accounts for Raman scattering from optical modes in the non-Bravais crystals. Confining attention here exclusively to acoustic modes, the wave vectors $\vec{k}$ are sufficiently small that the phonons are thermally populated, $\beta\omega_{\vec{k}\lambda} \ll 1$; and the polarization vectors of the n atoms in the unit cell are all in phase, $\vec{e}^{\sigma}_{\vec{k}\lambda} \cong n^{-1/2} \hat{e}_{\vec{k}a}$, where $a = 1, 2, 3$ indexes the acoustic modes. Then the Brillouin scattering rate is

$$(\tau^{-1})_B = \frac{2\pi nN}{V} \sum_{\vec{k}_f} \omega_i \omega_f n_i (n_f+1) V^{-1} \sum_{\vec{k}} \delta(\vec{k}_{if} - \vec{k}) \sum_a \frac{\pi k_B T}{M\omega_{\vec{k}a}^2} [\delta(\omega_{if} - \omega_{\vec{k}a}) + \delta(\omega_{if} + \omega_{\vec{k}a})]$$

$$\times \left| \alpha(\hat{\epsilon}_f \cdot \hat{\epsilon}_i)(i\vec{k} \cdot \hat{e}_{\vec{k}a}) - \alpha^2 \sum_{\vec{\tau}} n^{-1} \sum_{\sigma,\sigma'} i\vec{k} \cdot \vec{\tau}_{\sigma\sigma'} \langle \hat{e}_{\vec{k}a} \cdot \nabla [\epsilon_f \cdot \overleftrightarrow{T}(\vec{r}_{\sigma\sigma'}) \cdot \hat{\epsilon}_i] \rangle \right|^2 , \quad (2)$$

and the Brillouin scattering efficiency is

$$S_B = \frac{nN}{V} \left(\frac{\omega_i}{c}\right)^4 (n_f+1) \sum_a \frac{\pi k_B T}{M s_{\vec{k}a}^2} [\delta(\omega_{if} - \omega_{\vec{k}a}) + \delta(\omega_{if} + \omega_{\vec{k}a})] \left| \alpha(\hat{\epsilon}_f \cdot \hat{\epsilon}_i)(\hat{k} \cdot \hat{e}_{\vec{k}a}) - \alpha^2 \sum_{\vec{\tau}} n^{-1} \sum_{\sigma,\sigma'} \hat{k} \cdot \vec{\tau}_{\sigma\sigma'} \langle \hat{e}_{\vec{k}a} \cdot \nabla (\hat{\epsilon}_f \cdot \overleftrightarrow{T}(\vec{r}_{\sigma\sigma'}) \cdot \hat{\epsilon}_i) \rangle \right|^2 . \quad (3)$$

By comparison with the standard formulas for Brillouin scattering in terms of Pockels's elasto-optic coefficients p_{ijkl}, we obtain the following expression for this model:

$$p_{ijkl} = \alpha \delta_{ij} \delta_{kl} - \alpha^2 \sum_{\vec{\tau}} n^{-1} \sum_{\sigma\sigma'} (\vec{\tau}_{\sigma\sigma'})_l \langle \nabla_k \overleftrightarrow{T}_{ij}(\vec{r}_{\sigma\sigma'}) \rangle . \quad (4)$$

Although the first term in this expression was quoted previously[1] and leads to scattering from longitudinal modes only, the second term will contribute to scattering from pure transverse modes.

The α^2 term in Eq. (4) is fully symmetric on the three indices i, j, k, and has zero trace on any two of them, because of the assumption that the dielectric properties of the material are due exclusively to dipole interactions. Furthermore, since the elasto-optic tensor can be shown[7] to be symmetric on the two indices k, l if the crystal is not birefringent, hence the α^2 contribution has complete symmetry on all indices. These facts immediately lead to the sum rule

$$\sum_l (p_{ijll} - 3p_{illjl}) = 0 \quad \text{for all } i, j . \quad (5)$$

Let us now specialize to the case of cubic Bravais lattices. The three independent elements of the elasto-optic tensor can be taken as usual to be p_{11}, p_{12}, and p_{44}, which are further interrelated because of Eq. (5):

$$p_{11} - p_{12} + 3p_{44} = 0 . \quad (5')$$

We can thus write the elements in the form

$$p_{11} = \alpha + 2c(\alpha^2/a^3) ,$$
$$p_{12} = \alpha - c(\alpha^2/a^3) , \quad (6)$$
$$p_{44} = -c(\alpha^2/a^3) ,$$

in terms of a single dimensionless parameter c, with a being the nearest-neighbor distance. Numerical evaluation of the lattice sum defined by Eq. (4), in the limit of negligible zero-point averaging, gives the estimates

$$c \sim 1.2 \text{ (bcc)}, \quad c \sim 1.6 \text{ (fcc)} . \quad (7)$$

Previous experience[2] with the effects of zero-point averaging in helium indicates that c will be reduced from the values (7) by a factor of roughly two in this material.

Equations (6) permit a simple order-of-magnitude estimate for the intensity of Brillouin scattering from pure transverse modes in helium. We adopt the values $\alpha = 0.2 \times 10^{-24}$ cm^3 and $a \sim 3.6 \times 10^{-8}$ cm. The velocity of the (110) T_2 mode is taken to be[6] $s = 0.8 \times 10^4$ cm/sec, compared with a typical longitudinal velocity[6] of $s = 6.5 \times 10^4$ cm/sec. The (110) T_2 mode scatters with integrated intensity

$$S_B = \frac{N}{V} \left(\frac{\omega_i}{c}\right)^4 \frac{\pi k_B T}{Ms^2} [\tfrac{1}{2}(p_{11} - p_{12})]^2 \cos^2 \tfrac{1}{2}\theta \quad (8)$$

for unpolarized source and detector, where θ is the scattering angle. (The $\cos^2 \frac{1}{2}\theta$ factor indicates that backward scattering is suppressed and that the observation is effectively restricted to angles $\theta \lesssim 90°$.) We then estimate that the ratio of scattering intensities from the (110) T_2 mode at temperature $T = 1$ K to the Raman-active optical mode[3] in the hcp phase is ~ 5. It should thus be entirely feasible to observe experimentally this rather unusual transverse mode in bcc ^{3}He, and perhaps to compare its behavior in bcc ^{4}He.

Finally, we examine the observed[4] elasto-optic coefficients of xenon, in order to test the predictions of our model. The ratios

$$p_{12}/p_{11} = 1.45, \qquad p_{44}/p_{11} = -0.10$$

have been found,[4] after allowing for a corrected definition[8] of p_{44}. Using the value $\alpha/a^3 = 4.5 \times 10^{-2}$ appropriate to xenon, we would predict

$$p_{12}/p_{11} \sim 0.81, \qquad p_{44}/p_{11} \sim -0.063,$$

which satisfy relation (5'). The experimental values not only violate Eq. (5') but are qualitatively inconsistent with the underlying dielectric model, because $p_{12}/p_{11} > 1$ and $p_{44}/p_{11} < 0$, as observed, cannot be reconciled with formulas (6). We can only ascribe these serious discrepancies to a breakdown[9] of the Lorentz-Lorenz approximation for the dielectric constant of xenon.

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Third Side of the Lampert Triangle in Fitting Experimental Data

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It is shown that when the experimental data are analyzed according to the simplified theory of the space-charge-limited currents proposed by Lampert, the use of the full solution in the traps-filled-limit region is requisite. In this context Henderson and Ashley's results do not prove to be in agreement with Lampert's theory.

Lampert[1] pointed out that any current-voltage characteristic of the space-charge-limited current (SCLC) in a given n-type semi-insulator with a given density of shallow donors is confined to a triangular region in a log-log plot. The bounding sides of this triangle are formed by two straight lines corresponding to Ohm's law and Mott and Gurney's law,[2] and a curved line corresponding to the traps-filled-limit (TFL) law.[1] With the particular density and location of a trapping level within the energy gap, it may be possible to observe all the sides of the triangle in a single material. Recently, Henderson and Ashley[3] reported that they succeeded in carrying out such an experiment. The main feature of the data they obtained on neutron-irradiated silicon at $T = 77$ °K consists in the dramatic verticality of the third side of the triangle, the TFL characteristic. In Henderson and Ashley's view their results are in full conformity with Lampert's theory.

The purpose of this note is first to draw attention to the very shape of the third side of the Lampert triangle to guard against the implication that the TFL characteristics given by Lampert's simplified SCLC theory are always very steep and have short transition regions, and second to consider Henderson and Ashley's data in a new light.

Lampert's theory is built up assuming no diffusive flow. With this assumption the problem has an analytic solution, as illustrated in Fig. 1(a), which shows a computed current-voltage characteristic with the complete Lampert triangle *ABC*. Unfortunately, the full solution is relatively complicated and therefore rarely used. Also the curve fitting by means of the first and second sides of the triangle can be easily done with the help of good approximations—Ohm's law and Mott and Gurney's law. On the contrary, on the third side the full solution has to be used to carry out fitting correctly. Neglecting this requirement often leads to error. This can be seen in the example of Henderson and Ashley's results.

These authors have interpreted their current-voltage characteristic on the basis of the SCLC. The very steep rise of the current vs voltage at 72 V, as shown in Fig. 1(b), they have connected with the TFL phenomenon—strong filling of traps by one-carrier injection. The measured rise is indeed very steep—by recording the difference in voltage between the bottom and top of the vertical portion of the TFL line [Fig. 1(b), points C and B, respectively] the authors had to use a high-resolution digital voltmeter. It is impossible to resolve these two points in the given voltage scale.

We now wish to compare Henderson and Ashley's results with the corresponding theoretical current-voltage characteristic computed according to Lampert's equations. In the calculations [results are shown in Figs. 1(a) and 1(b)] we have adjusted the product μS, where μ is the electron mobility and S is the effective electrode area, to obtain the best fit to Henderson and Ashley's data, and we have put the dielectric permittivity ϵ equal to 10^{-10} F/m, the magnitudes of other parameters being taken from Henderson and Ashley's work as follows: the thermal-equilibrium free-electron density $n_0 = 2.8 \times 10^{14}$