Low-Temperature Magnetoresistance in Degenerate *n*-Type Si

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Transverse magnetoresistance of degenerate *n*-type Si has been measured in the liquid-helium temperature range. A characteristic negative magnetoresistance is observed. The data are analyzed by a semiempirical expression originally used for CdS by us, thus providing further evidence for the general validity of our expression and for the localized-magnetic-moment model.

We have shown in another paper,[1] hereafter referred to as I, that the observed magnetoresistance in degenerate CdS doped with indium can be analyzed as the sum of an anomalous negative component and a positive contribution due to magnetic freeze out. It was shown that, although the qualitative features of the negative component at low fields are consistent with the localized-magnetic-moment model of Toyozawa,[2] quantitative disagreements exist with the calculations based on the second-order perturbation expansion of the exchange Hamiltonian. Based on experimental evidence from the resistivity data following Kondo,[3] we suggested that third-order terms in the expansion of the exchange Hamiltonian be used.

A semiempirical expression

$$\Delta\rho/\rho_0 = -B_1^2 \ln(1+B_2^2H^2) + B_3^2H^2/(1+B_4^2H^2) \qquad (1)$$

was derived. The parameters B_1 and B_2 are related to various physical quantities associated with exchange interaction. B_3 and B_4 are related to the mobility and the density of states (see I).

It was our contention in I that this expression should fit the data for other degenerate semicon-

TABLE I. Values of the parameters to fit Eq. (1) in the text. Sample numbers give the values of carrier concentration at room temperature, e.g., $4.0-18 = 4.0\times 10^{18}$ cm^{-3}, etc.

Sample	$\rho_{300\,°K}$ (Ω cm)	Temp. (°K)	B_1 (10^{-1})	B_2 $(10^{-3}\ G^{-1})$	B_3 $(10^{-5}\ G^{-1})$	B_4 $(10^{-5}\ G^{-1})$
4.0–18	0.0104	4.2	0.593	0.119	0.257	0.271
		1.3	0.0387	0.142	0.424	0.583
7.0–18	0.008	4.2	0.436	0.179	0.160	0.167
		1.3	0.326	1.298	0.197	0.424
7.5–18	0.0066	4.3	0.433	0.165	0.169	0.193
		2.8	0.408	0.287	0.179	0.250
		1.33	0.349	0.654	0.195	0.275
1.0–19	0.0055	4.3	0.374	0.285	0.169	0.164
		2.8	0.413	0.291	0.166	0.167
		1.29	0.379	0.356	0.180	0.249
1.6–19	0.00431	4.2	0.339	0.218	0.132	0.330
		1.2	0.317	0.141	0.119	0.323
1.3–19[a]		4.2	0.177	0.744	0.104	0.280

[a]Sample of Ref. 5.

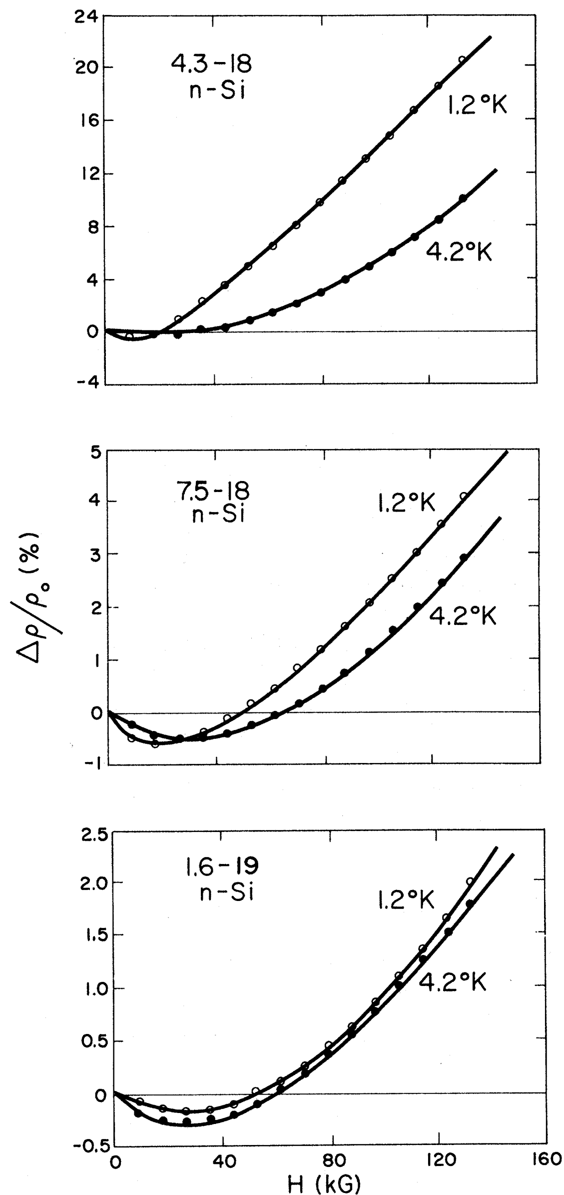

FIG. 1. Transverse magnetoresistance vs magnetic field for *n*-Si samples. The lines through the data points show the fits to Eq. (1) in the text.

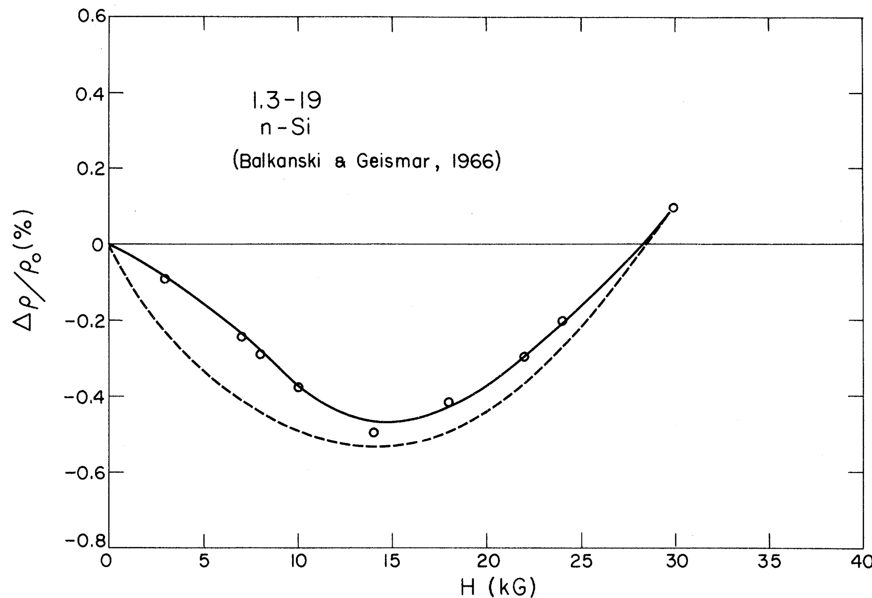

FIG. 2. Fits to expression (1) in the text (solid line) and to that of Hedgcock and Raudorf (Ref. 8) to the data on n-type Si by Balkanski and Geismar (Ref. 5).

ductors that show negative magnetoresistance. The literature is abundant with publications on the observance of negative magnetoresistance in many degenerate semiconductors (Refs. 3–15 in I).

In this paper, we wish to report our measurements of transverse magnetoresistance in degenerate n-type Si and the validity of expression (1). Single crystals of Si doped with phosphorus were obtained from Monsanto Chemical Co. They were cut ultrasonically into spider shapes. Both oriented and unoriented samples were used. The size of the samples was $0.1\times 0.2\times 0.6$ cm^3. Ohmic electrical contacts were formed by electroless nickel plating. The measurements were made in the temperature range 1.2–4.2 °K. The data were taken at the Frances Bitter National Magnet Laboratory, Cambridge, Mass., where the fields could be varied up to 150 kG.

Table I gives the room-temperature carrier concentration and resistivity of the samples used. In Fig. 1 are plotted representative data for three of these samples. These data are consistent with the observations of magnetoresistance in other degenerate semiconductors, i.e., the magnetoresistance is negative at low fields, then passes through a minimum, and becomes positive at higher fields. The lower the carrier concentration or temperature, the lower is the field required for this change of sign in magnetoresistance. These data are also consistent with the magnetoresistance studies in Si reported by other workers.[4–6]

As has been discussed in detail in I, the present data and the earlier data on Si by other workers,[4–6] as well as data on other materials, do not show an H^2 dependence at low fields as predicted by Yosida's[7] model. In fact, Roth *et al.*[4] fitted their data on Si, in particular, to an empirical expression of the form $\Delta\rho/\rho_0 = -aH^c + bH^2$. They showed that the value of c was generally less than one, approaching 0.5 for high-concentration samples.

We have fitted the data on Si samples to the empirical expression (1). The fits are shown by the solid lines through the data points in Fig. 1. The values of the parameters $B_1 - B_4$ for these three samples and the other samples (data not shown) are also given in Table I. The values of these parameters follow a consistent concentration and temperature dependence, similar to that observed for CdS samples in I.

An alternative approach for analysis of the observed negative magnetoresistance was provided by Hedgcock and Raudorf.[8] They presented a two-band model with a sharp mobility edge which produces a negative magnetoresistance. A linear field dependence of the negative magnetoresistance is deduced. Hedgcock and Raudorf fitted the data for one of the Si samples of Balkanski and Geismar,[5] as shown by the dashed line of Fig. 2. The poor fit at low fields indicates that a simple linear field dependence is not sufficient, nor do the data of Balkanski and Geismar show H^2 dependence. The fit to our expression (1) is shown by the solid line in Fig. 2. The fit is excellent and the values of the parameters used to fit this sample are also given in Table I. The values are comparable to those of our samples.

Recently, Bauer and Kahlert[9] have reported the negative magnetoresistance of n-type InAs and

have fitted their results to the negative component of expression (1). The excellent quality of the fits to CdS,[1] Si, and InAs[9] data, and presumably other degenerate materials, provides further evidence for the general validity of our empirical expression and the model it represents.

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Brillouin Scattering from Transverse Phonons in Rare-Gas Crystals

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A mechanism is investigated for Brillouin scattering from transverse modes in rare-gas crystals, which is related to that which produces Raman scattering. Consideration is given to the intensity to be expected for the anomalous T_2 mode in the (110) direction in bcc ^{3}He; observation of this mode is concluded to be experimentally feasible. However, this mechanism is not compatible with the observed elasto-optic coefficients of xenon.

In a previous paper[1] we have analyzed the scattering of light from solid helium, giving attention to the spectral distribution and intensity of one- and two-phonon Raman scattering. We subsequently have produced[2] numerical computation of these quantities in the fcc, bcc, and hcp crystal structures, for comparison with current experiments.[3] The discussion given[1] of Brillouin scattering was not complete, however, in that an account was given of scattering from acoustic modes only of longitudinal polarization. It is nevertheless true that scattering from pure transverse acoustic modes has already been observed[4] in certain rare-gas crystals. Furthermore, the extension of these results to the bcc phase of helium would do much to clarify the rather unusual properties[5] inferred[6] for the slow transverse (T_2) branch, particularly near the (110) direction. We here investigate a mechanism for transverse-mode Brillouin scattering, and elaborate on the intensity to be expected for the (110) T_2 mode in bcc ^{3}He. However, this model fails to predict the observed[4] elasto-optic constants of xenon, which may indicate the inapplicability of the Lorentz–Lorenz dielectric model to this material.

Our starting point in this analysis is the one-phonon scattering rate for a model crystal composed of neutral atoms with dipole polarizability α,

$$(\tau^{-1})_1 = \left(\frac{2\pi N}{V}\right)^2 \sum_{\vec{k}_f} \omega_i \omega_f n_i (n_f+1) N^{-1} \sum_{\vec{k}\lambda} \delta(\vec{k}_{if}-\vec{k}) \frac{\pi\hbar}{M\omega_{\vec{k}\lambda}} \left(\frac{\delta(\omega_{if}+\omega_{\vec{k}\lambda})}{e^{\beta\omega_{\vec{k}\lambda}}-1} + \frac{\delta(\omega_{if}-\omega_{\vec{k}\lambda})}{1-e^{-\beta\omega_{\vec{k}\lambda}}}\right)$$
$$\times \left| \alpha \hat{\epsilon}_f \cdot \hat{\epsilon}_i \sum_{\sigma} i\vec{k}\cdot\vec{e}^{\,\sigma}_{\vec{k}\lambda} - \alpha^2 \sum_{\vec{\tau}} \sum_{\sigma,\sigma'} \langle (\vec{e}^{\,\sigma}_{\vec{k}\lambda} - \vec{e}^{\,\sigma'}_{\vec{k}\lambda} e^{-i\vec{k}\cdot\vec{r}_{\sigma\sigma'}}) \cdot \nabla[\hat{\epsilon}_f \cdot \overleftrightarrow{T}(\vec{r}_{\sigma\sigma'}) \cdot \hat{\epsilon}_i] \rangle \right|^2 . \quad (1)$$

This equation and its notation are taken directly from Eq. (18) of Ref. 1, except that the factor $e^{-i\vec{k}\cdot\vec{r}_{\sigma\sigma'}}$ in the last line here has not been neglected. The α^2 term in the matrix element accounts for Raman scattering from optical modes in the non-Bravais crystals. Confining attention here exclusively to acoustic modes, the wave vectors $\vec{k}$ are sufficiently small that the phonons are thermally populated, $\beta\omega_{\vec{k}\lambda} \ll 1$; and the polarization vectors of the n atoms in the unit cell are all in phase, $\vec{e}^{\,\sigma}_{\vec{k}\lambda} \cong n^{-1/2}\hat{e}_{\vec{k}a}$, where $a = 1, 2, 3$ indexes the acoustic modes. Then the Brillouin scattering rate is