Energy-Conservation Considerations in the Characterization of Impact Ionization in Semiconductors*

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A simple expression for the ionization coefficient of charged carriers in a semiconductor as a function of electric field and lattice temperature has been developed by simultaneously fitting three physical asymptotic cases to Baraff's result. These cases are for low field (Shockley), high field (Wolff), and limitations imposed by energy conservation at high electric field or when the energy loss by phonon scattering is negligible. Given the threshold energy for ionization and the optical-phonon energy, our expression requires a single additional parameter to predict experimental results. Although the final expression is thus essentially a one-point fitting, it reproduces experimental data over as much as four decades of ionization coefficient with better accuracy than frequently used empirical two-parameter expressions. Excellent fits with much of the existing electric field dependence of the ionization coefficients for electrons and holes in Ge, Si, GaAs, and GaP were obtained. The temperature dependence was examined in the cases of GaAs and Si and excellent agreement was obtained in the case of GaAs. Some data on Si were found to be in considerable error in the sense that the data do not appear to be consistent with energy conservation.

I. INTRODUCTION

In the following paper we describe attempts to find a physically meaningful analytic function which describes the ionization coefficient of charged carriers in semiconductors over a wide range of electric fields. In Sec. II a trial analytic expression is deduced and shown to satisfy a number of conditions consistent with physical reality. In Sec. III this expression is fitted to existing theoretical predictions. In Sec. IV the physical significance of experimentally measurable parameters is discussed. In Sec. V the results of curve fitting to experimental results for Ge, Si, GaAs, and GaP at 300 °K and temperature-dependent results for GaAs are presented.

The phenomenon of avalanche multiplication in semiconductors is important in the operation of a variety of semiconductor devices. This phenomenon can be described in the most basic manner by specifying the electric field dependence of the ionization coefficient $\alpha(\mathcal{E})$ of a charge carrier as a function of electric field $\mathcal{E}$. Although much work has been done on this phenomenon, there still has not been presented a reasonable analytical expression for $\alpha(\mathcal{E})$ which covers a wide range of electric field.

There exist two electric field regions where simple physical models should be valid. In the so-called high-field range, charged-carrier energy loss to the lattice system is small in comparison with the energy gained from the field. Then lattice scattering tends to randomize the energy distribution of the carriers and Wolff's[1] expression has been assumed to apply:

$$\alpha(\mathcal{E}) \propto \exp[-3E_r E_i/(q\lambda\mathcal{E})^2] , \tag{1}$$

where E_r is the optical-phonon energy, E_i is the "threshold" energy for ionization (production of electron-hole pairs), and λ is the mean free path for optical-phonon scattering of energetic carriers. In the so-called low-field region, only those carriers which do not experience any phonon scattering reach the threshold energy[2] and contribute to the production of electron-hole pairs. Then the following expression due to Shockley[2] should apply:

$$\alpha(\mathcal{E}) \propto \frac{q\mathcal{E}}{E_r} e^{-E_i/q\mathcal{E}\lambda} . \tag{2}$$

Many experimental results have been reported for several semiconductors, but unfortunately, as pointed out by Baraff, experimental data lie between the two regions. Baraff has combined the two models and has obtained a result which is widely accepted.[3,4] The Baraff result is shown in Fig. 1 in his original universal form. This result is a computer calculation. Crowell and Sze[5] have obtained an expression which represents Baraff's results fairly well using a power series with a nine-point fitting. This expression cannot be extrapolated beyond the range of the Baraff calculation and does not have an obvious connection with the physical model.

If we replot Baraff's results as shown by the dashed lines in Fig. 2, the vertical axis is

$$\alpha(\mathcal{E})E_i/q\mathcal{E} = [(\alpha\lambda)(E_i/q\mathcal{E}\lambda)] ,$$

i.e., the ratio of the energy lost via ionization to the energy gained from the electric field. When the electric field is increased, if we consider that

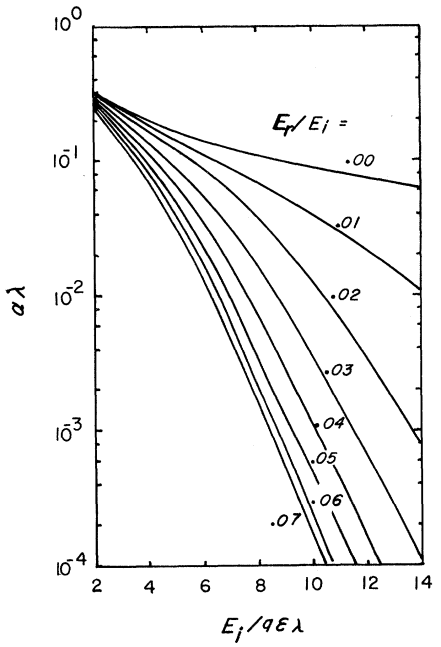

FIG. 1. Baraff's calculation of normalized ionization coefficient vs normalized inverse electric field with r (the ratio of the characteristic phonon energy loss to the ionization energy) as a parameter.

the cross section for ionization rises rapidly [as $(E-E_i)^2$] above E_i^3, the chance to lose energy to the lattice system should become smaller, and at infinite field, $\alpha E_i/q\mathcal{E}$ should tend to unity:

$$\alpha E_i/q\mathcal{E} \to 1 \quad \text{as} \quad \mathcal{E} \to \infty . \tag{3}$$

Also when $E_r \to 0$ this ratio should become unity for any $\mathcal{E}$ since there is then no mechanism to lose energy except by ionization:

$$\alpha E_i/q\mathcal{E} \to 1 \quad \text{as} \quad E_r \to 0 . \tag{4}$$

The Baraff universal result, however, is calculated assuming that only ionization occurs for energies above E_i and that the mean free path is the same as that for optical-phonon scattering for energies below E_i. Since Baraff assumes that all energy is lost subsequent to ionization, his results should not satisfy Eqs. (3) and (4).

In this paper we take the Baraff results at low field where the specific mean free path for energies above E_i is not important, and consider the implication of fitting the data with a function which satisfies Eqs. (1)–(4).

II. ANALYTICAL APPROXIMATION

An appropriate analytical expression for ionization rates must satisfy the four asymptotic relationships discussed in Sec. I. Also, $\alpha(\mathcal{E})$ must be an even, monotonic-increasing function of $\mathcal{E}$ and (E_i/E_r).

Initially it will be helpful to define a few normalized parameters as follows:

$$x \equiv E_i/q\mathcal{E}\lambda \tag{5}$$

is the ratio of the ionization threshold energy to the energy gained from the electric field in one mean free path for optical-phonon scattering;

$$r \equiv E_r/E_i \tag{6}$$

is the ratio of the optical-phonon energy to the ionization scattering energy, and

$$y \equiv \ln(\alpha E_i/q\mathcal{E}) = y(x, r) \tag{7}$$

is the logarithm of the efficiency with which energy from the electric field is used to produce additional carriers.

Consider a series expansion which can satisfy the above general requirements, i.e.,

$$y = \sum_{n=-\infty}^{\infty} \alpha_n (x^2 + a^2)^{n+\frac{1}{6}} + C , \tag{8}$$

where $a = a(r)$ is a monotonic function of r and the

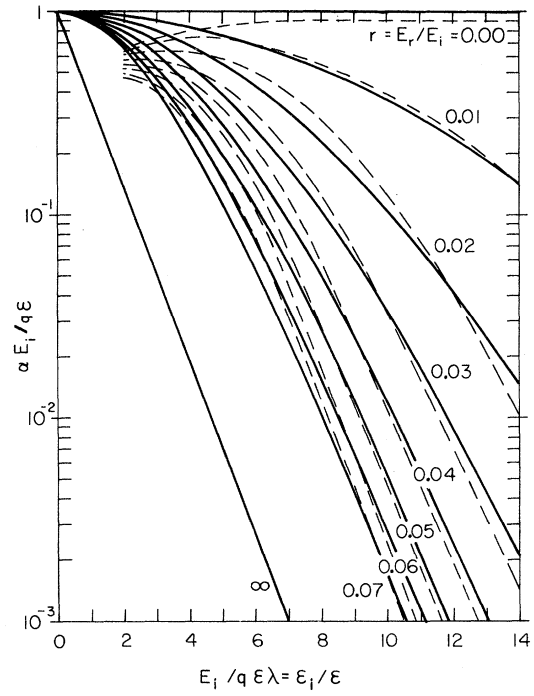

FIG. 2. Replotted Baraff result (broken line) and our analytic "one-point fitting" (solid line) with r as a parameter. The ordinate is the ratio of the carrier energy lost via ionization to the energy gained from the electric field. The abscissa is the normalized inverse electric field (the ratio of a characteristic electric field $\mathcal{E}_i$ to the electric field). The straight line for $r = \infty$ is the Shockley asymptote.

coefficients α_n are independent of r and x. In addition $0 \leq \delta < 1$.

The Shockley model [Eq. (2)] requires that $dy/dx \to -1$ as $x \to \infty$. This condition yields the following restrictions:

$$\delta = \tfrac{1}{2}, \tag{9}$$

$$n \leq 0, \tag{10}$$

and

$$\alpha_0 = -1 . \tag{11}$$

If we wish our results to approach those of the Wolff model in the high-field limit, we must place the additional restriction on y that

$$\frac{d^2y}{dx^2} \to -6r \quad \text{as } x \to 0 . \tag{12}$$

Here, the preexponential term in Eq. (1) has been assumed to be $q\mathcal{E}/E_i$ to satisfy the energy conservation relation in the high-field limit. This requirement gives rise to the condition

$$\sum_{n=-\infty}^{-1} \alpha_n (2n+1)\, a^{2n-1} = a^{-1} - 6r . \tag{13}$$

Energy conservation at high field [cf. Eq. (3)] requires that

$$\sum_{n=-\infty}^{0} \alpha_n a^{2n+1} = -C . \tag{14}$$

All these restrictions can be satisfied by Eq. (13) and the following equation:

$$y \equiv -(x^2+a^2)^{1/2} + a + \sum_{n=-\infty}^{-1} \alpha_n [(x^2+a^2)^{n+1/2} - a^{2n+1}] . \tag{15}$$

We have not yet imposed the condition $y \to 0$ as $r \to 0$, which is also required by conservation of energy [cf. Eq. (4)].

III. CURVE FITTING TO THE BARAFF RESULT

In Sec. II, we presented a general functional form for α. Formally we could obtain an exact expression if we used an infinite number of points from the Baraff results. Since this is impractical and the high-field asymptotic region of the Baraff result is obviously not applicable for our model, we have tried to fit the expression to the lower field portions of the Baraff results.

First, we tried a two-point fitting for each value of r. We must retain the leading term in our expression [Eq. (15)] as one of the two terms, because this term is the only one which survives in the Shockley asymptotic region. The other term was chosen from the higher-order terms to subsequently satisfy the Wolff regime via an appropriate choice of order and weighting. The expression for the two-point fitting can then be written as

$$y = a' - (x^2+a^2)^{1/2} + (a-a')\,[(x^2/a^2)+1]^{a\,(1-6ar)/2\,(a-a')} . \tag{16}$$

Note that when a approaches a' the higher-order term drops out quickly when x increases beyond a.

Results from the two-point fitting have suggested that $a' \approx a$ (with $a - a'$ positive). Then if we neglect the higher-order term, we have a fairly simple expression, i.e.,

$$y = a - (a^2+x^2)^{1/2} . \tag{17}$$

Here, a is a function of r and can be obtained for a single value of r from a single-point fitting. The resultant a is plotted in Fig. 3 as a function of r. The vertical lines for each r represent the range of a which corresponds to curve fitting at different values of x. The open dots represent the case where our expression crosses the Baraff result at $x = 10$. In general, curves from Eq. (17) cross the Baraff results twice for a single value of r. The dotted line corresponds to the case where our expression touches the Baraff result at only a single point. The solid line corresponds to the value we have chosen as most representative of the low-field Baraff results. For this line,

$$a(r) = 0.217 r^{-1.14} . \tag{18}$$

The results of this fitting procedure can be compared with the Baraff results by an examination of Fig. 2. The curve-fitting scheme represents the Baraff result fairly well for large x and appears reasonable in the high-field asymptotic region. The single-point fitting of the curve is not significantly poorer than the two-point fitting as far as the Baraff results are concerned. Equations (17) and (18) do not exactly fit the Wolff result [Eq. (12)] but provide a reasonable approximation: Instead of an exponent of $-3rx^2$, as $x \to 0$, our result gives an asymptotic exponent of $-2.38 r^{1.14} x^2$.

IV. PHYSICAL SIGNIFICANCE OF EMPIRICAL PARAMETERS

Our final expression, Eq. (17), exhibits the Shockley asymptotic region and satisfies the energy-conservation relation, but only qualitatively shows the Wolff asymptote. From Eqs. (17) and (18) it is not immediately obvious how unambiguously one can deduce the empirical parameters E_i, E_r, and λ from experimental data. We can get more insight into the answer to this problem by introducing the following two characteristic fields:

$$\mathcal{E}_i \equiv E_i/q\lambda , \tag{19}$$

the field at which the threshold energy is reached in one mean free path, and

$$\mathcal{E}_r \equiv E_r/q\lambda , \tag{20}$$

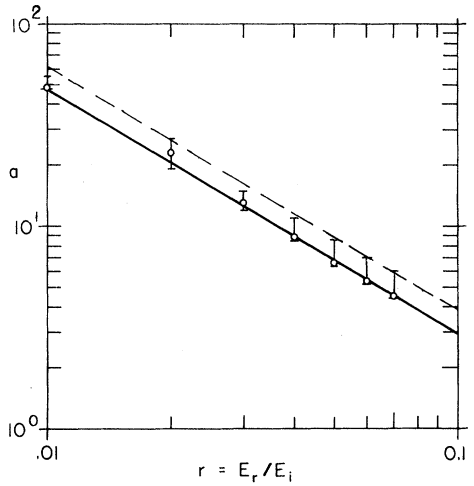

FIG. 3. a vs r from the one-point fitting. The vertical bars represent the range of a from all possible fittings to Baraff result; the dashed line represents the fitting which touches the Baraff results at a single point; the open circle is the result of a fitting at $x=10$; the solid line is the selection as most representative fitting.

the field at which the phonon energy is reached in one mean free path.

Then from Eq. (17), we have

$$y = \ln(\alpha E_i/q\mathcal{E}) = 0.217(\mathcal{E}_i/\mathcal{E}_r)^{1.14} - \{[0.217(\mathcal{E}_i/\mathcal{E}_r)^{1.14}]^2 + (\mathcal{E}_i/\mathcal{E})^2\}^{1/2} . \quad (21)$$

From this result we see that $\mathcal{E}_i$ determines the slope of the y-vs-$\mathcal{E}^{-1}$ relationship at low fields, $\mathcal{E}_r$ determines the field above which appreciable curvature occurs in the y-vs-$\mathcal{E}^{-1}$ relationship, and E_i determines the asymptotic form of α vs $\mathcal{E}^{-1}$ when $\mathcal{E} \gg \mathcal{E}_i$. When a given set of experimental data are analyzed it becomes more difficult to obtain meaningful values of $\mathcal{E}_i$, $\mathcal{E}_r$, and E_i in that order. Note that if Chynoweth's law

$$\alpha = \alpha_\infty e^{-b/\mathcal{E}} \quad (22)$$

appears to apply (where α_∞ and b are empirical constants), effects of $\mathcal{E}_r$ and E_i cannot be separated empirically. A reasonable value of $\mathcal{E}_i$ can only be deduced from Chynoweth's law if $\mathcal{E} \approx \mathcal{E}_r$. Then the ionization rate must be extremely small because, on the average, the charge carrier must lose almost all its energy via phonon generation. Note also that then neither E_i nor λ are determined, but only their ratio is established.

V. CURVE FITTING TO EXPERIMENTAL DATA

Equations (17) and (18) have been used to study ten sets of experimental data for four different materials (Ge,[6] Si,[7-9] GaAs,[10] and GaP[11]). Rather than attempting a three-point fitting to determine E_r, E_i, and λ, we have chosen to show how the general form of Eq. (21) compares with the experimental data for a reasonable choice of E_r and E_i. We have used theoretical values of E_r and E_i and chosen λ as the only adjustable parameter. We have selected the values of λ to fit the experimental data at a single point on the α-vs-$\mathcal{E}$ relationship for each material. These values are tabulated in Table I. Here E_r values have been taken from Crowell and Sze[5] and E_i from Anderson and Crowell.[12] In Fig. 4, the experimental data are plotted using solid curves, and values from Eq. (21) are plotted by dashed curves. The points used to determine the λ values are indicated by heavy dots. We have also plotted two straight lines in the upper portion of each diagram. These lines are limits imposed by conservation of energy when $E_i = E_g$ (solid lines) and $E_i = \frac{3}{2}E_g$ (dotted lines). Thus, in any case α can never exceed the $E_i = E_g$ lines. Note that our expression and choice of parameters represents the experimental data fairly well for Ge, GaAs, GaP, and some data for Si,[7] but not for all data on Si. In Si some data[8,9] seem to saturate at values of α which would require $E_i \approx 10E_g$. This appears to be much too large since to a first approximation $E_i \approx \frac{3}{2}E_g$.[13] It is probable that there is an appreciable error in interpretation of such experimental results.

Our expression can also be used to predict the temperature dependence of $\alpha(\mathcal{E})$ if the temperature dependence of λ, $\mathcal{E}_r$, and E_i are assumed. Crowell and Sze[5] assumed that the appropriate E_r was the average energy loss per collision with an optical phonon and that λ was the mean free path for a collision independent of whether a phonon was generated or absorbed. Under these circumstances $\mathcal{E}_r$ is independent of temperature and

$$\lambda = \lambda_0 \tanh(E_{r0}/2kT) , \quad (23)$$

where λ_0 and E_{r0} are λ and E_r at 0 °K. The threshold energy was assumed to be proportional to the band-gap energy.[14,15]

Experimental temperature-dependence measure-

TABLE I. Impact ionization parameters at 300 °K.

Material	electron or hole	E_i (eV)	E_r (meV)	λ (Å)	$\mathcal{E}_i$ (10^6 V/cm)	$\mathcal{E}_r$ (10^4 V/cm)
Ge	e	0.8	19	36	2.2	5.3
	h	0.9		47	1.9	4.0
Si	e	1.1	51	48	2.3	10.6
	h	1.8		44	4.1	11.5
GaAs	e/h	1.7	22	33	5.2	6.7
GaP	e/h	2.6	38	31	8.4	12.3

ments on Si[5] and GaAs [16] were examined. The results for the case of GaAs are shown in Fig. 5. The data are fitted with the parameters listed in Table I. Thus, all the theoretical curves in this diagram are based on knowledge of the band structure, the optical-phonon energy, and an independent measurement of α at a single temperature and electric field. Our curve fitting reproduces the experimental data fairly well except for the 300 °K data. The experimental data for 300 °K do not appear to be consistent with the data at the other temperatures, but the data at the other temperatures are consistent with earlier experimental data at 300 °K.[10]

The correlation is not as good in the case of Si as in that of GaAs. The published data of α as a function of temperature for Si do, however, yield values of α at 300 °K, which are considerably higher at any given field than any of the data presented in Fig. 4.

VI. SUMMARY AND CONCLUSIONS

By superimposing three physical asymptotic cases on Baraff's theoretical calculation, we have obtained the following relationship between the ionization coefficient α and the electric field $\mathcal{E}$ in

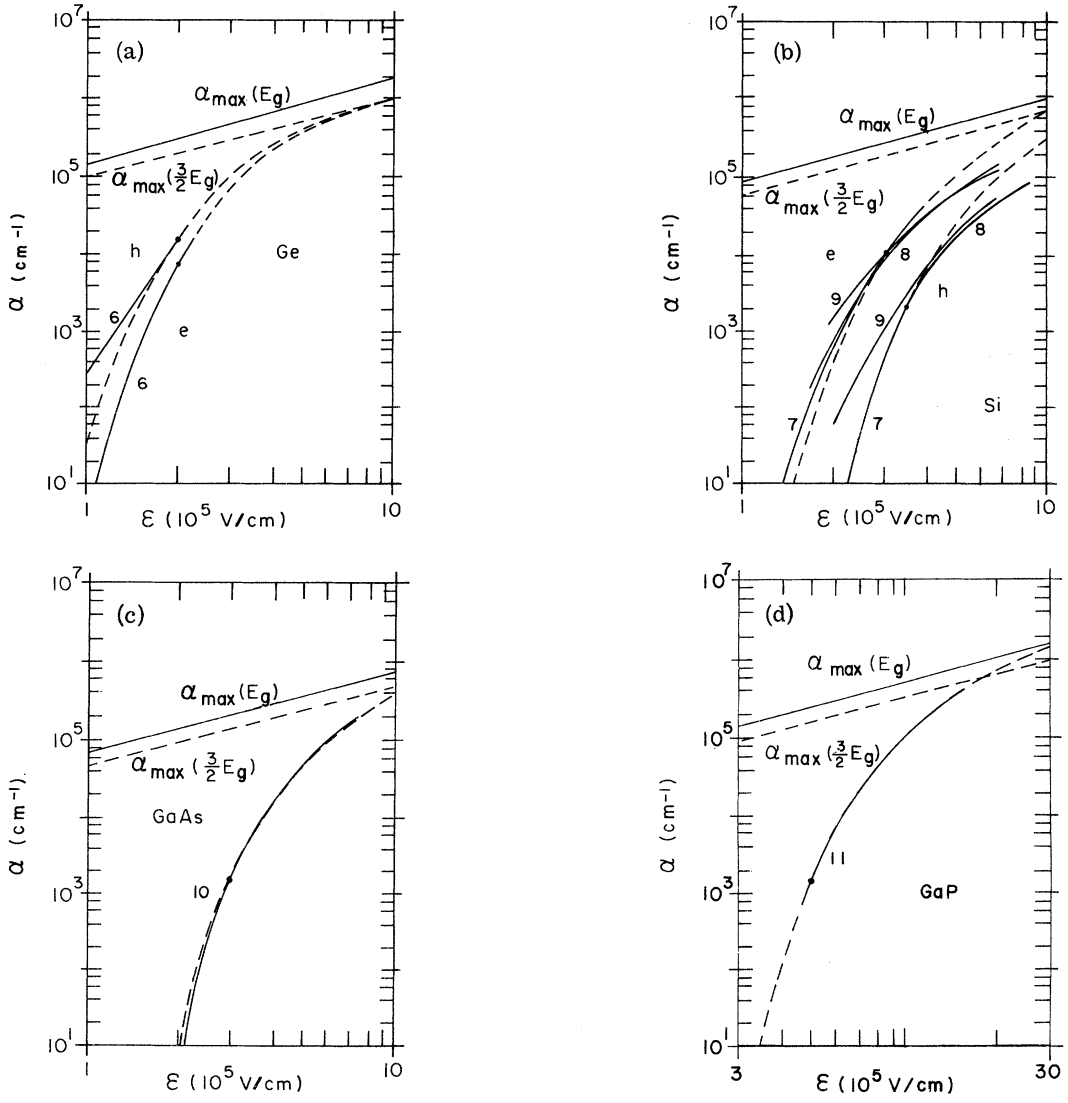

FIG. 4. Ionization rate α as a function of the electric field $\mathcal{E}$ for electrons (e) and holes (h) in selected semiconductors at 300 °K. (Solid line) experimental data (with reference numbers); (dotted line) theoretical fit; (dot) point used to determine mean-free-path fitting parameter. $\alpha_{max}(E_g)$: theoretical maximum α imposed by energy conservation. $\alpha_{max}(\frac{3}{2}E_g)$: theoretical maximum α for threshold ionization energy of $\frac{3}{2}E_g$.

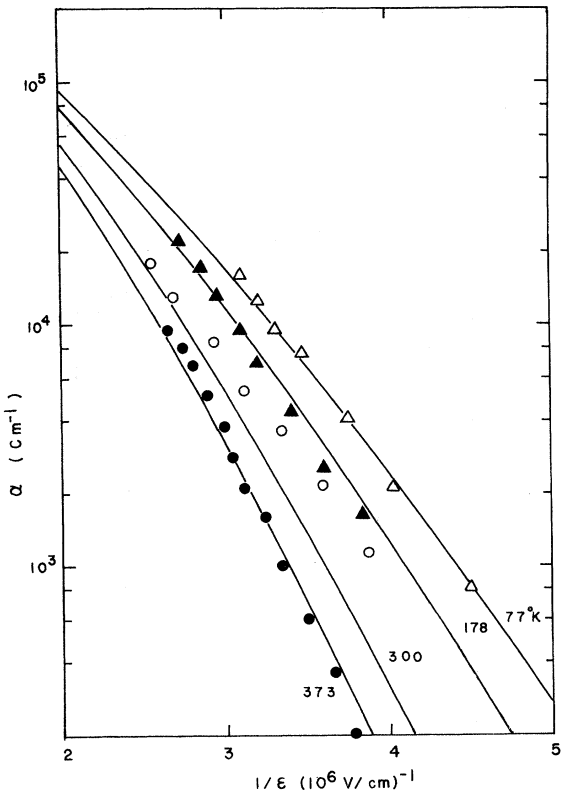

FIG. 5. Ionization coefficient as a function of the inverse electric field for electrons and holes in GaAs at selected temperatures. Experimental data (Ref. 16): (Δ) 77 °K; (▲) 178 °K; (○) 300 °K; (●) 373 °K. Predicted curves from parameters in Table I are given as solid lines.

terms of the threshold energy E_i, the optical-phonon energy E_r, and the mean free path for optical-phonon scattering λ:

$$\alpha = (q\mathcal{E}/E_i)\exp\{0.217\,(E_i/E_r)^{1.14} - [(0.217\,(E_i/E_r)^{1.14})^2 + (E_i/q\mathcal{E}\lambda)^2]^{1/2}\}. \quad (24)$$

Of these three parameters, λ is the least well known and is chosen as an adjustable parameter since the experimental data serve mainly to determine the parameter $\mathcal{E}_i = E_i/q\lambda$ [cf. Eq. (19)]. Thus, if either $\frac{3}{2}E_g$ or "exact" thresholds, which generally are closer to E_g,[12] are used for E_i, it does not greatly affect the result.

From our approach, once a single value of α is measured at some electric field and temperature, one can estimate α at any temperature and electric field. In this sense our expression is an empirical one-point fitting. Within this context it is desirable to contrast this result with those from two empirical expressions which have frequently been used in the past; namely, the Chynoweth law $\alpha = \alpha_\infty e^{-b/\mathcal{E}}$ and the power law $\alpha = \beta\mathcal{E}^p$, where $p \cong 6$. These expressions both are two-point fittings and applicable only at a single temperature for a given α_∞, b, β, and p. In spite of our expression having only one adjustable parameter, it reproduces most experimental data better than either of the above expressions.

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