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# Track-Effect Account of Scintillation Efficiency for Random and Channeled Heavy Ions of Intermediate Velocities

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The high-velocity formulation of the track-effect theory of scintillation efficiency is generalized for applicability at intermediate velocities (velocities spanning the peak in $dE/dx$) and applied to room-temperature data for $O^{16}$ in NaI(Tl) both for random and channel trajectories. A core-correction term is incorporated into the representation of $dE/dx$. Electron velocities are taken into account in the calculation of track width $R_{max}(v)$. The contribution to $dL/dE$ from the region of high energy-deposit density is included. Quantitative agreement within experimental error is achieved for the $O^{16}$ random-response data over the range $0.9 \leq v \leq 3.6$, where $v$ is in units of $10^9$ cm/sec, including the region of intermediate velocities characterized by a reversal of curvature and leveling off of $dL/dE$. The channel response is treated only so far as to indicate applicability of the theory. Values for the ratio $[dL/dE(\text{channel})]/[dL/dE(\text{random})]$ are accounted for both in approximate magnitude and qualitative dependence on particle velocity.

## I. INTRODUCTION

In a recent work,[1,2] a track-effect theory was proposed to account for the general characteristics of the heavy-ion pulse-height data of a wide range of scintillating crystals. The model was applied to the high-velocity ($v \geq 2.2\times10^9$ cm/sec) data of Newman and Steigert[3] for room-temperature bombardment of NaI(Tl) with ions in the range $5 \leq Z \leq 10$. Subsequent data of Altman *et al.*[4,5] for $O^{16}$ bombardment of NaI(Tl) extend the pulse-height characteristics to intermediate velocities, i.e., velocities which span the peak in $dE/dx$ and demonstrate that channeling enhances scintillation efficiency $dL/dE$ in NaI(Tl). The latter is particularly apparent when particle velocities are reduced to the intermediate range. Indeed, this phenomenon was predicted in an earlier work,[6,7] and quantitative estimates of relative pulse heights, $L(\text{channel})/L(\text{random})$, were made.

In the present paper, the track-effect theory[2] is generalized for applicability at intermediate as well as high velocity, and a quantitative account is given for the channeling effect and for the behavior of the new intermediate-velocity data for randomly incident ions. A core-correction term is incorporated into the representation of specific energy loss $dE/dx$. Electron velocities are taken into account in the calculation of the track width $R_{max}(v)$. The contribution to $dL/dE$ from the region of high energy-deposit density is included. A quantitative account is provided for $dL/dE$ data[3-5] corresponding to room-temperature bombardment of NaI(Tl) by $O^{16}$ ions in the approximate range $0.9 \leq v \leq 3.6$, where $v$ is in units of $10^9$ cm/sec. Estimates are made of the ratio of $dL/dE$ (channel) to $dL/dE$ (random) which agree with observation both in magnitude and qualitative behavior. Insights which result from this treatment regarding the track effect and its role in luminescence are discussed.

## II. GENERAL FORMULATION OF THEORY

Theoretical analysis of heavy-ion pulse-height characteristics of scintillators demands consideration of a two-step process in which energy is transferred by the particle to the crystal lattice and then transported from the track and subsequently absorbed at luminescence centers. For purposes of analysis, pulse-height data are generally reduced to $dL/dE$, the so-called "scintillation efficiency," from which the total pulse may be generated through integration over the particle trajectory. Hence, all theoretical considerations refer to processes which occur in a differentially thin section of crystal perpendicular to the particle track. The present treatment concerns velocities $v \gtrsim v_{TF}$, where $v_{TF}$ is the characteristic Thomas-Fermi velocity of the penetrating ion. In this domain, elastic-collision losses constitute a

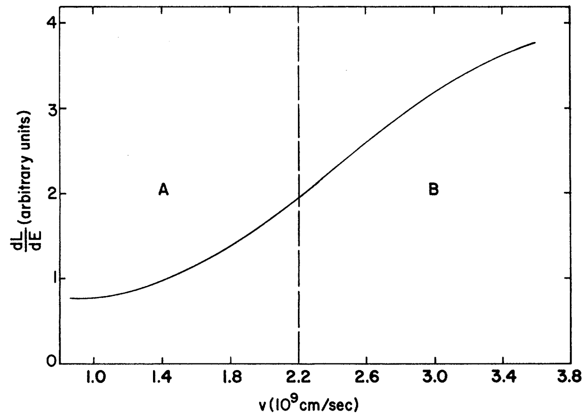


FIG. 1. Data for scintillation efficiency versus velocity for randomly incident $O^{16}$ ions in NaI(Tl) at room temperature (Refs. 3 and 5). Regions $A$ and $B$ correspond to intermediate and high velocities, respectively.

fraction of the total energy not exceeding approximately 5%[8,9] and hence shall be neglected.

A general formulation of the problem is embodied in the following equation:

$$\frac{dL}{dE}=\frac{1}{dE/dx}\int_{R=0}^{R=R_{\max}(v)}\left(\frac{dL}{dE}\right)_{\rm local}\rho(Z,v,R)\,2\pi R dR\ , \quad (1)$$

where $(dL/dE)_{\rm local}$ is the differential light output per unit of energy deposit at a distance $R$ from the particle track, $\rho(Z,v,R)$ is the corresponding energy density as a function of the atomic number and velocity of the particle, $R_{\max}(v)$ is the maximum distance from the track at which energy is deposited, and $dE/dx$ is the specific energy loss. Implicit in this relation is the assumption that luminescence processes result primarily from thermalized electronic energy deposition, e.g., recombination of initially free electrons and holes at luminescence centers. There is much experimental evidence to indicate that this assumption is particularly valid for the alkali iodides (see Ref. 2 for a survey of pertinent findings), and most probably valid for a wide range of other types of scintillating media.[10]

The two major ingredients required for an evaluation of $dL/dE$ are the energy-deposition functions, $\rho(Z,v,R)$ and $(dL/dE)_{\rm local}$. The former involves pure energy-deposit considerations, while the latter requires, for a first-principles treatment, detailed identification of the various luminescence mechanisms operative in a given phosphor, and a measure of their relative importance, as well as corresponding knowledge concerning the various competitive processes. Generalizing and making tractable the problem of calculating $(dL/dE)_{\rm local}$ is a major feature of the track-effect theory. It is noted that some of the basic ideas of the track-effect theory presented in Ref. 2 were stated earlier in the work of Luntz and Bartram,[7] and derive from features of the δ-ray model of Meyer and Murray.[11]

## III. DATA

In Fig. 1, $dL/dE$ vs $v$ (in units of $10^9$ cm/sec) is displayed for randomly incident $O^{16}$ ions in NaI(Tl) at room temperature (Tl concentration $\cong 0.1$ mole%). The curve is generated by reducing and normalizing the pulse-height data of Altman, Dietrich, and Murray[5] ($0.86 \le v \le 1.77$) to the $O^{16}$ data of Newman and Steigert[3] ($1.47 \le v \le 4.40$), with nearly perfect agreement in the overlap region. The vertical dotted line at $v = 2.20$ serves to delineate, approximately, the regions of high and intermediate velocity. This separation is discussed below. In addition, the dotted line serves to locate an inflection in the curvature of $dL/dE$. The high-velocity formulation of the track-effect theory[2] provides a reasonable account of the data in region $B$, but can never predict a flattening out of $dL/dE$ versus velocity at lower velocities, no matter what choice of parameters. In the following sections, the model is generalized and applied to the data of Fig. 1, as well as corresponding data for channeled ions.

## IV. GENERALIZATION OF MODEL

The representations of both $\rho(Z,v,R)$ and $(dL/dE)_{\rm local}$ as presented in Ref. 2 are inadequate for application at intermediate velocities. The necessary modifications are discussed below.

### A. Representation of $\rho(Z,v,R)$

The $Z$ and $v$ dependence in the energy-deposition function $\rho(Z,v,R)$ is assumed separable from the dependence on $R$ over almost the entire region of energy deposit, as discussed in Ref. 2, and is contained in a factor $f(Z,v)$. In Fig. 2, $\rho/f(Z,v)$ is plotted as a function of $R$ to indicate the form of the assumed energy-deposit profile for arbitrary $Z$ and $v$. The factor $f(Z,v)$ employed in Ref. 2 is expressed as $Z^{*2}/v^2$, where $Z^*$ is an effective atomic number for the incident particle,

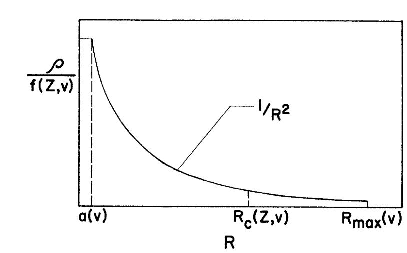


FIG. 2. Model for energy-deposit density as a function of distance from the track of a penetrating ion.

and where the ratio $Z^{*2}/v^2$ constitutes the leading term in high velocity $dE/dx$.[2] In the present work, a more general approach is taken, where $f(Z, v)$ is obtained from the normalization condition

$$\int_0^{R_{\max}(v)} \rho(Z, v, R) 2\pi R \, dR = \frac{dE}{dx}. \quad (2)$$

In order to perform the normalization, expressions for $dE/dx$, $R_{\max}(v)$, and $a(v)$ must be provided. The extension to intermediate velocities necessitates modification of both $dE/dx$ and $R_{\max}(v)$. The parameter $R_c(Z, v)$ serves as a sharp cutoff between regions of high and low energy-deposit density, and is utilized in the representation of $(dL/dE)_{\text{local}}$ as discussed below. The assumed $1/R^2$ falloff, employed and discussed in Ref. 2, is critically examined in a later section of the paper.

### 1. $dE/dx$

An adequate representation of total stopping power is given by the expression[12]

$$\frac{dE}{dx} = \frac{4\pi Z^{*2} e^4}{mv^2} n \left[ \ln\left(\frac{2mv^2}{I}\right) - \frac{C}{Z} \right] , \quad (3)$$

which is equivalent to Bethe's formulation[13] for the stopping of high-velocity protons, modified by an effective charge $Z^*e$ (defined in Ref. 2) so as to be made applicable to heavy ions, and further modified, for purposes of the present application, by the inclusion of an average core-correction term $C/Z$ ($Z$ being an effective atomic number for the target medium) to compensate for the inability of the particle to excite tightly bound inner-shell electrons as its velocity decreases. The parameter $n$ is the average electron density of the medium, and $I$ is the mean excitation energy, evaluated by Sternheimer for NaI to be 0.427 keV.[14] The core-correction term, which was not included in the high-velocity treatment of Ref. 2, is evaluated by fitting $dE/dx$ of Eq. (3) to the corresponding semiempirical curve of Newman and Steigert[3] for $O^{16}$ in NaI.

In Fig. 3, calculated values for $dE/dx$ resulting from use of a core correction $C/Z$ equal to 0.125 are superimposed on the Newman-Steigert curve. In addition, values employed in Ref. 2, obtained without the core correction, are shown for comparison. The vertical dotted line at $2.2 \times 10^9$ cm/sec serves to delineate regions of high and intermediate velocities. Region $B$ is characterized by an approximate $v^{-2}$ falloff with increasing velocity. In this region, each of the theoretical representations is in agreement with the semiempirical curve to within experimental error. Hence, as might be expected, the core correction does not play a very important role at high velocity. In contrast, it is apparent that the values for $dE/dx$ calculated without the use of a core correction increase too rapidly in region $A$. Hence, for present purposes, the core correction is employed, and the representation of $dE/dx$ in Eq. (3) is regarded as adequate. It is noted that the falloff of the calculated values at the low-velocity end is at least qualitatively consistent with observation.[12] However, stopping-power theory is relatively incomplete in the region surrounding the peak. Therefore, the precise quantitative character of the falloff is uncertain.

### 2. $R_{max}(v)$

The track width $R_{\max}$, or maximum transverse distance from the particle track at which energy is deposited, is obtained in Ref. 2 by maximizing the transverse component of the practical range of secondary electrons. The resulting expression takes the form

$$R_{\max}(v) \propto T_{\max}^{1.35} , \quad (4)$$

where $T_{\max}$ is the maximum possible energy transfer to an electron. The latter is represented by the simple classical result

$$T_{\max} = 2mv^2 , \quad (5)$$

where $m$ is the mass of the electron. In the high-velocity formulation of Ref. 2, $v$ is taken as the velocity of the incident particle. In so doing, the electrons are regarded as initially at rest. This is not a valid approximation for present purposes. In Eq. (5), a relative velocity, $v_{\text{rel}}$, for particle-electron collisions must be used. Representing the collision velocity by its maximum value

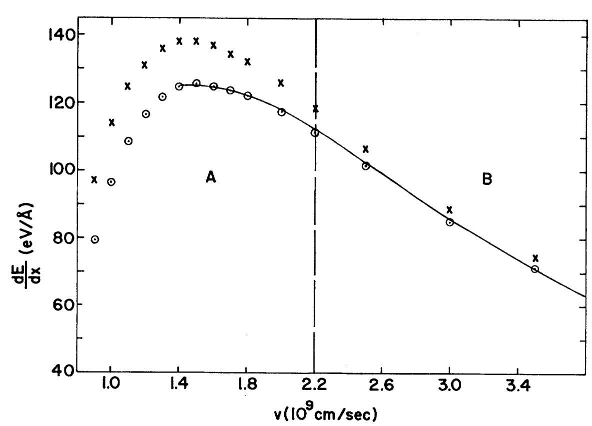


FIG. 3. Specific energy loss versus velocity for $O^{16}$ ions in NaI: Crosses represent calculated values from Ref. 2, without use of a core correction. Open circles represent values calculated with a core correction. The solid curve represents the semiempirical data of Ref. 3. Regions $A$ and $B$ correspond to intermediate and high velocities, respectively.

$$v_{rel} = v + v_e , \quad (6)$$

where $v$ is the particle velocity and $v_e$ is the orbital velocity of the target electron, the problem of calculating $R_{max}$ becomes one of providing an adequate expression for $v_e$, for those electrons which travel furthest from the particle track.

It is easily shown that the maximum value for the excess energy of secondary electrons, $T_{max}$ less the binding energy, is greatest for electrons with highest $v_e$. Thus, in the limit of high velocity, when the incident particle is capable of deep enough atomic penetration so as to excite all electrons, $v_e$ should be taken as the $K$-shell velocity. However, with decreasing particle velocity, inner shells become relatively incapable of excitation, and the value of $v_e$ decreases. Thus, $v_e$ is a function of the velocity of the incident particle.

For present purposes, a characteristic average velocity is employed, namely, the velocity corresponding to the mean excitation energy in Eq. (3). In particular,

$$v_e = (2I/m)^{1/2} , \quad (7)$$

which for NaI yields the approximate magnitude $1.33 \times 10^9$ cm/sec. The resulting expression for $R_{max}(v)$ employed in the present calculation is

$$R_{max}(v) = 132\,[v + (2I/m)^{1/2}]^{2.70}\ \text{Å} , \quad (8)$$

where $v$ and $(2I/m)^{1/2}$ are in units of $10^9$ cm/sec. Values range from approximately $10^3$–$10^4$ Å, decreasing according to Eq. (8) with decreasing particle velocity.

### 3. $a(v)$

The primary-excitation region, i.e., the region of direct particle-electron interaction, extending out to the plasma-screening distance $a(v)$ from the particle track, is assumed to be characterized by a constant density of energy deposit.[2] The parameter $a(v)$ is given by $v/\omega_0$, where the natural frequency $\omega_0$ is calculated assuming a plasma comprised of the $I^-(5s)$ and $I^-$ $(5p)$ electrons.[7] Values for $a(v)$ range from approximately 1 to 3 lattice constants.

In terms of $dE/dx$, $R_{max}$, and $a$, the energy deposition function of Fig. 2 is given by

$$\rho(Z, v, R) = \frac{dE/dx}{2\pi R^2 \ln(\sqrt{e}\, R_{max}/a)} \quad \text{for } R_{max} > R > a , \quad (9)$$

where $e$ is the base of natural logs.

## B. Representation of $(dL/dE)_{local}$

The functional form of $(dL/dE)_{local}$ vs $R$ (distance from the track) anticipated on the basis of the track-effect theory is displayed in Fig. 4. Three regions are delineated. In region I, corresponding to the immediate vicinity of the track, it is assumed that light-production efficiency is negligibly small as a consequence of nearly complete dominance by high-density competitive effects. Significant although lesser competition characterizes region II, whereas the role of competitive effects is regarded negligible over region III.

In the high-velocity treatment of Ref. 2, the contributions to $dL/dE$ from regions of high density are disregarded. This is justified by the correspondingly large fraction of the energy loss deposited far from the track. However, as the particle slows down, the track width decreases and, simultaneously, $dE/dx$ increases. An increasing fraction of the energy loss is deposited close to the track, and the corresponding contribution to $dL/dE$ can no longer be disregarded. Further, as mentioned previously, the resulting functional form of $dL/dE$ in Ref. 2 is incapable of accounting for the behavior of the data at intermediate velocities.

For present purposes, $(dL/dE)_{local}$ is represented by the linear-falloff model shown in Fig. 4, where the boundary between regions II and III is defined by $R_c(Z, v)$, corresponding to a critical value of energy-deposit density $\rho_c$ treated as an adjustable parameter of the model. The one other adjustable parameter is the value $c_0$ of the maximum light-production efficiency.

## C. $dL/dE$

Employing the linear-falloff representation of $(dL/dE)_{local}$, and the model for $\rho(Z, v, R)$ shown in Fig. 2, the resulting expressions for $dL/dE$

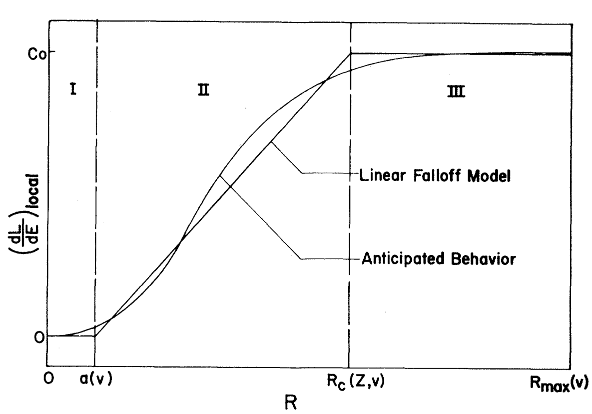


FIG. 4. Model for the local light-production efficiency as a function of distance from the track, superimposed on a curve representing its anticipated behavior. Regions I, II, and III correspond to the immediate vicinity of the track, and the high- and low-density domains, respectively.

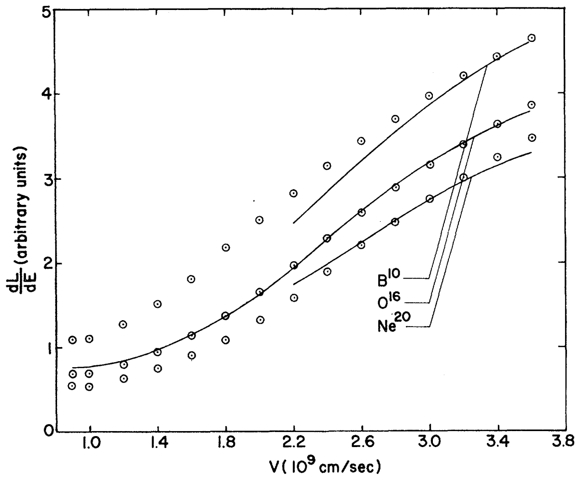


FIG. 5. Calculated values (represented by open circles) of scintillation efficiency versus velocity for a set of heavy ions in NaI(Tl) superimposed on data from Refs. 3 and 5.

are as follows:

$$\frac{dL}{dE}=c_0\left(F_0+\frac{(R_c-a)-a\ln(R_c/a)}{(R_c-a)\ln(\sqrt{e}R_{max}/a)}\right) \quad \text{for } R_{max}>R_c, \qquad (10a)$$

$$\frac{dL}{dE}=c_0\left(\frac{(R_{max}-a)-a\ln(R_{max}/a)}{(R_c-a)\ln(\sqrt{e}R_{max}/a)}\right) \quad \text{for } R_{max}\leq R_c, \qquad (10b)$$

where $F_0$ is the fraction of energy loss per unit path length deposited outside the high-density regions (I and II of Fig. 4).

## V. COMPARISON WITH DATA

Theoretical $dL/dE$ contains two adjustable parameters; the cutoff density $\rho_c$ [incorporated into $R_c(Z, v)$] and the normalization constant $c_0$. With the aid of a nonlinear–least-squares-fit computer subroutine,[15] a "best" fit of the data of Fig. 1 was found to result from the choice of $\rho_c$ equal to $3.44\times10^5$ erg/cm$^3$. Corresponding values of $R_c(Z, v)$ for $O^{16}$ ions range from approximately 2700 to 3900 Å, increasing with decreasing velocity. It should be mentioned that the resulting value of $\rho_c$ is approximately two orders of magnitude smaller than the critical density obtained in the high-velocity treatment of Ref. 2, as well as the value obtained in the work of Katz and Kobetich.[16] However, in the present treatment, $\rho_c$ is interpreted as the density at which competitive effects just begin to be felt. This is a different interpretation than that employed in the other works. Further, in view of the sharp cutoff nature of the model, the precise value of $\rho_c$ is regarded as far less significant than details of track width, spatial distribution of energy deposit, and spatial variation of $(dL/dE)_{local}$.

In Fig. 5, calculated values of $dL/dE$ (represented by open circles) are compared with experiment. The comparison with $O^{16}$ data (same data as in Fig. 1) is the result of the "best" fit mentioned above. The $B^{10}$ and $Ne^{20}$ experimental curves are representative samples from the Newman-Steigert high-velocity data of Ref. 3, reduced to the form $dL/dE$ vs velocity.[2] The corresponding calculated points are evaluated from Eqs. (10a) and (10b), using the values of the two adjustable parameters obtained from the $O^{16}$ fit. Details of the comparison are discussed in Sec. VI.

## VI. DISCUSSION OF RANDOM-RESPONSE THEORY

The primary aim thus far has been to indicate the nature of the modifications required to make the track-effect theory of Ref. 2 applicable to intermediate velocities and, indeed, to demonstrate its applicability. The following are the major modifications: (a) A core-correction term is incorporated into the expression for the specific energy loss $dE/dx$. (b) Electron velocities are taken into account in the calculation of the track width $R_{max}(v)$. (c) The contribution to $dL/dE$ from the high-density region is included.

Applicability of the theory is apparent from the data fit displayed in Fig. 5 , in which the general features of $dL/dE$ are accounted for. In particular, the calculated scintillation efficiency falls off with decreasing velocity for fixed $Z$, and increases with decreasing $Z$ at fixed velocity. Further, quantitative agreement within experimental error is achieved down the entire range of $O^{16}$ data, including the region of intermediate velocities characterized by a reversal of curvature and leveling off of $dL/dE$.

It is regarded as unwarranted, for present purposes, to strive for better over-all quantitative agreement, in view of relatively large uncertainties in the high-velocity data. However, particular insight results from an examination of the sensitivity of the results to variations in several of the approximations made. This is done with regard to the $O^{16}$ data of Fig. 1.

Firstly, the calculation is quite sensitive, particularly at the low-velocity end, to the manner in which $R_{max}(v)$ is represented. Various characteristic electron velocities were employed in Eq. (6). Values lower than that given by Eq. (7) result in a slower leveling off of $dL/dE$ than is exhibited by the data. Larger values result in a minimum occurring in the neighborhood of $1.2\times10^9$ cm/sec, followed by an upturn with decreasing velocity. Total disregard of $v_e$ results in a poor fit at all but the highest velocities. From continued analysis of pulse-height data in terms of the track-effect theory, it is hoped ultimately to develop a more accurate representation of $R_{\max}$, including an appropriate velocity dependence for $v_e$.

Secondly, a variation of the model was examined in which a $1/R^n$ falloff of energy-deposit density was assumed, with $n$ treated as an adjustable parameter. A "best" fit was found to result from a choice of $n$ equal to 2.01. This supports the assumed model of Fig. 2, and provides information regarding the spatial distribution of energy deposit about particle tracks.

Thirdly, a reasonable fit in the range of intermediate velocities can only be achieved when the contribution to $dL/dE$ from inside the high-density region is taken into account. One variation in the representation of $(dL/dE)_{\text{local}}$ was examined. In particular, a two-step function was assumed, in which constant values $c_i$ and $c_0$ were assigned in regions II and III, respectively, of Fig. 4. In this form, the model has three adjustable parameters instead of two. Nevertheless, the resulting fit over the range of intermediate velocities is poor. It is concluded that there is a significant spatial variation of $(dL/dE)_{\text{local}}$ in the high-density region. This supports the assumed model of Fig. 4 and, indeed, the major feature of the track-effect theory.

## VII. APPLICATION TO CHANNELING RESPONSE

An apparent anisotropy in the heavy-ion pulse-height response of CsI(Tl) scintillators was reported by Newman, Smith, and Steigert,[17] although no definitive evidence for the effect was provided. In a subsequent theoretical work,[7] it is argued that since $dL/dE$ falls off with increasing $dE/dx$, and since a channel trajectory is characterized by an anomalously low $dE/dx$, one might expect a channeled ion to produce light more efficiently than one randomly incident. Hence, it is postulated that the response characteristics of activated alkali-iodide scintillators are indeed anisotropic and, in particular, that the crystal channels are directions preferential for luminescence. Quantitative estimates were made of the ratio $L$(channel)/$L$(random) assuming—with expressed reservation as to validity—that the dependence of $dL/dE$ on $dE/dx$ is independent of particle trajectory. Values as high as four are obtained for $O^{16}$ in NaI(Tl) in the domain of intermediate velocities, and are found to exhibit a rapid falloff toward unity with increasing $v$.

Subsequent experiments at Harwell[18] provided definitive evidence that channeling is indeed operative in alkali halides. Very recent experiments at the University of Delaware[4,5] demonstrate a clear and unambiguous enhancement of luminescence by channeling. In the latter, the room-temperature pulse-height response of NaI(Tl) to ⟨110⟩ axially channeled $O^{16}$ ions of intermediate to high velocities is compared with the corresponding random response. In qualitative accord with prediction, $L$(channel)/$L$(random) is greatest in the region of intermediate velocities, and decreases with increasing $v$. However, the observed ratios are significantly lower than predicted, ranging only up to approximately 1.4. A major part of the discrepancy in magnitude can be understood in terms of the track-effect theory, and is attributed to the assumption that $dL/dE$ vs $dE/dx$ is independent of particle trajectory. In particular, consider two identical energetic particles, one channeled and one moving in a random direction, each undergoing the same $dE/dx$. According to channel-stopping-power theory,[7] the channeled ion is traveling with lower velocity, and hence deposits its energy closer to the track. Thus, the energy deposition of the channeled ion is characterized by higher density and, from the track-effect theory, results in lower light-production efficiency. Consequently, the assumption regarding the relation of $dL/dE$ to $dE/dx$ is invalid, and results in an overestimate of the ratio $L$(channel)/$L$(random). Additional factors contributing to the theoretical overestimate can be found in the representation of $dE/dx$(channel) employed in the calculation. The latter will not be discussed here.

New theoretical estimates of the effect of channeling on the heavy-ion pulse-height response of NaI(Tl) scintillators are made, employing the track-effect theory. The concern is to compare the light-production efficiencies of channeled and random ions, by calculating their relative scintillation efficiency $[dL/dE\ (\text{channel})]/[dL/dE\ (\text{random})]$. Detailed quantitative comparison with experiment cannot meaningfully be made at this time, in view of uncertainties in the available data. In particular, with increasing velocity, not only does $dL/dE$ decrease, but the height and clarity of the channel peak as well. This is associated with a corresponding reduction of the channeling component of the incident beam, and results in uncertainty regarding the location of the channel peak, which is further amplified by the numerical differentiation involved in the data reduction. Hence, the present goal is limited to establishing whether or not the track-effect theory is capable of predicting at least the general behavior (i.e., velocity dependence) and approximate magnitude of the relative scintillation efficiency for $O^{16}$ in NaI(Tl). To this end, the simplifying assumption is made that the distinction between channel and random response is attributed only to the reduction of $dE/dx$ due to channeling and hence, from Eq. (9), to the scaling down of the density function $\rho(Z, v, R)$. In the calculation, $dE/dx$ (channel) is represented as

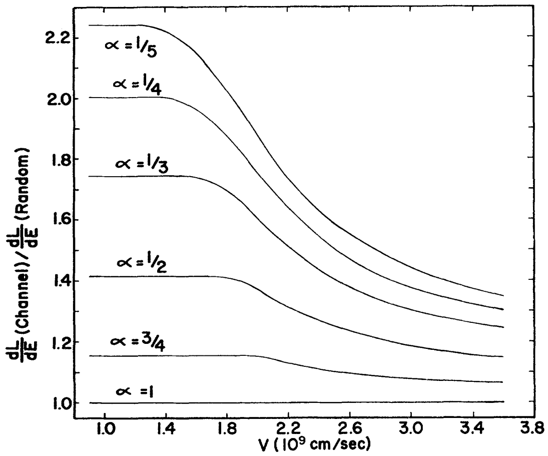


FIG. 6. Calculated values of the ratio of scintillation efficiency (channel-to-random) versus velocity for $O^{16}$ in NaI(Tl), for a set of values of the stopping-power scaling factor $\alpha$.

$$\frac{dE}{dx}\,(\text{channel}) = \alpha(v)\frac{dE}{dx}\,, \qquad (11)$$

where $\alpha(v)$ is a velocity-dependent scaling factor. It is further assumed that $\alpha(v)$ is approximately constant over the range of the data. For selected values of $\alpha$, the relative scintillation efficiency is calculated, employing Eqs. (10a) and (10b).

Results for several cases are displayed in Fig. 6. Experimental values are found to range from approximately 1.35 at the low-velocity end, and are observed to decrease with increasing $v$—although somewhat more rapidly than the calculated curves. The channeling calculation of Ref. 7 results in the approximate value of $\frac{1}{4}$ for the parameter $\alpha$. The corresponding curve in Fig. 6 indicates a maximum predicted ratio of 2.0. This is a considerable improvement over previous predictions.[7] Further, since the channeling calculation of Ref. 7 was performed for the special case of best channeled ions, i.e., ions which remain on the channel axis over their entire trajectory, it follows that the value of $\frac{1}{4}$ is extreme and should be viewed as a lower limit. The curve of Fig. 6 corresponding to $\alpha$ equal to $\frac{1}{2}$ provides the best agreement and, indeed, the value of $\frac{1}{2}$ is perhaps the most realistic estimate employed for the average stopping-power reduction due to channeling in thick crystals. In view of the rather rough nature of the approximations made in this calculation, the results are surprisingly good. In particular, it is demonstrated that the track-effect theory is indeed capable of accounting for the response to channeled ions in NaI(Tl), both in magnitude and qualitative behavior. Much work needs to be done in order to achieve a precise, detailed account of the effect. It is anticipated that application of the track-effect theory to this end may shed light on specific energy loss for channeled ions. The availability of additional data for a set of ions covering a range of $Z$ and extending down to lower velocities would be highly desirable.

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