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# Spectral Distribution of the Photomagnetoelectric Circulating Current in Semiconductors

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The investigation of the spectral distribution of the photomagnetoelectric (PME) circulating current is made possible by the new method of measurement based on the magnetic flux created by this current. The formula for PME-circulating-current magnetic flux is derived and the results of calculations of the magnetic flux as a function of absorption coefficient are presented in graphical form. The measured spectral distributions are in good qualitative agreement with theoretical distributions.

## INTRODUCTION

The idea has been introduced that the so-called circulating current flows in a semiconductor sample when the photomagnetoelectric (PME) effect takes place.[1,2] The photomagnetomechanical effect gives us some information about PME circulating current.[3] This effect is not suitable for measurement of spectral distribution because of a spurious effect caused by the diamagnetism of the sample. The new method enabling measurement of spectral distribution is based on the following idea: Circulating current can be understood as a current winding. The PME circulating current is created and disappears in a sample placed in a magnetic field when the sample is illuminated by a chopped radiation; the magnetic flux created by this current induces a measurable voltage in coils.

Results presented here are the first direct proof of the existence of the PME circulating current.

## MAGNETIC INDUCTION FLUX OF PME CIRCULATING CURRENT

The PME circulating current has been shown[2] to flow in a sample as illustrated in Fig. 1(a). If a long enough sample or a sample with end electrodes is considered the current flow approaches the case illustrated in Fig. 1(b).

The current flow illustrated in Fig. 1(b) is considered in our derivation. The resulting magnetic flux is a sum of the magnetic flux of elementary current windings as in Fig. 2. The magnetic flux of the elementary winding is divided into two parts, a part for $y < y_0$ and a part for $y > y_0$, and each part is expressed separately. We have

$$d\phi = \frac{J_x^{(oc)}(y)\, v}{R + R_0}\, dy , \tag{1}$$

where $J_x^{(oc)}(y)$ is the circulating current density,

$$R = v/|\,y_0 - y\,|\,l\,\mu$$

is the magnetic resistance of the part of the sample around which the elementary current $J_x^{(oc)}(y)\,dy$ flows, $y_0$ is the coordinate of the point where $J_x^{(oc)}(y_0) = 0$, $v$ is the width of the sample, $l$ is the length of the sample, $\mu$ is the magnetic permeability of the sample, and $R_0$ is the magnetic resistance through which the magnetic flux flows outside the sample.

The total magnetic flux equals

$$\phi = \int_0^{y_0} \frac{v J_x^{(oc)}(y)}{[v/(y_0 - y)l\mu] + R_0}\,dy - \int_{y_0}^{w} \frac{v J_x^{(oc)}(y)}{[v/(y - y_0)l\mu] + R_0}\,dy\,, \qquad (2)$$

where $w$ is the thickness of the sample. The circulating current density can be expressed as[2]

$$J_x^{(oc)}(y) = -\frac{\sigma}{G}\,J^{(sc)} - q\theta D\frac{dP}{dy}\,, \qquad (3)$$

where $\sigma$ is the conductivity of the illuminated sample, $G = \int_0^d \sigma dy$, $\theta$ is the Hall angle, $q$ is the elementary charge, $D$ is the ambipolar diffusivity, $P$ is the minority-excess-carrier density, and

$$J^{(sc)} = -\,q\theta \int_{P(0)}^{P(w)} D\,dP$$

is the PME short-circuit current.

The magnetic flux $\phi$ can be calculated if $J_x^{(oc)}(y)$ is expressed according to expression (3) and if excess-carrier density is expressed according to Refs. 2 and 4. Calculations performed in such a way are lengthy and the final expression is very complicated and not easy to understand. To get applicable expressions, $R_0 \ll R$ is assumed, a condition easy to fulfill in experiment, and expression (2) is simplified. We have then

$$\phi = l\mu \int_0^w J_x^{(oc)}(y)\,(y_0 - y)\,dy\,. \qquad (4)$$

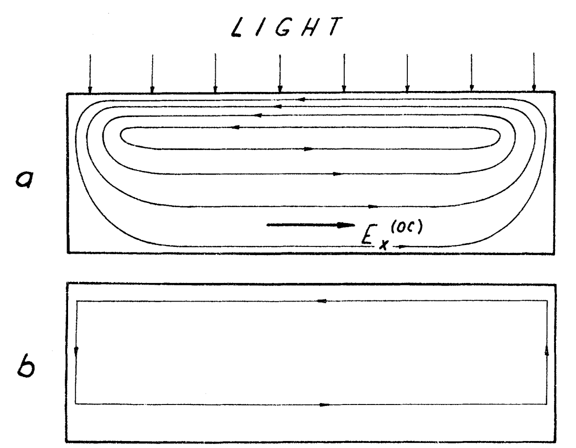


FIG. 1. PME circulating current in a semiconducting sample: (a) without end electrodes, (b) with end electrodes.

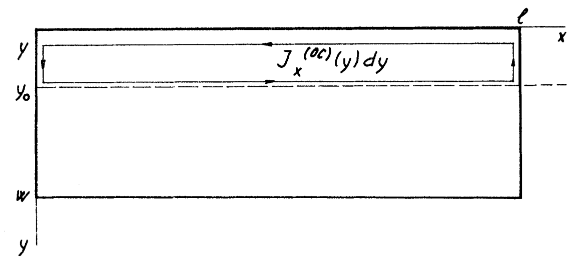


FIG. 2. Elementary current winding considered in calculation.

It is advantageous to express the circulating current density as a sum of an odd and an even component. The odd component represents one current winding in a sample, and the even one represents two current windings, the resultant magnetic flux of which equals zero. As shown below it is satisfactory to consider the odd component of the magnetic flux only for total-magnetic-flux calculation, when the solution for the density $P$ in the form derived[2,4] is employed. We have

$$\begin{aligned}\phi &= l\mu \int_0^w J_x^{(oc)}(y)(y_0 - y)\,dy \\ &= l\mu \int_{-w/2}^{w/2} J_x^{(oc)}(\tfrac{1}{2}w + \xi)(\xi_0 - \xi)\,d\xi \\ &= l\mu \int_{-w/2}^{w/2} \bar{J}_x^{(oc)}(\xi)(\xi_0 - \xi)\,d\xi \\ &= l\mu \int_{-w/2}^{w/2} \bar{J}_{x\,\mathrm{odd}}^{(oc)}(\xi)\,\xi\,d\xi\,,\end{aligned}$$

where $y = \frac{1}{2}w + \xi$. The integral

$$\int_{-w/2}^{w/2} \bar{J}_x^{(oc)}(\xi)\,\xi_0\,d\xi = 0\,,$$

because[2]

$$\int_{-w/2}^{w/2} \bar{J}_x^{(oc)}(\xi)d\xi = 0.$$

It is easy to see that the integral of the even component equals zero:

$$\int_{-w/2}^{w/2} \bar{J}_{x\,\mathrm{ev}}^{(oc)}(\xi)\,\xi\,d\xi = 0\,.$$

If the density $P$ in expression (3) previously derived[4] is substituted in (4) we have for $W \neq K$ and $W = w/L$, $K = kw$, $S_1 = s_1 w/D$, $S_2 = s_2 w/D$ ($L$ is the diffusion length; $k$ is the absorption coefficient of incident light; $s_1$, $s_2$ are surface recombination velocities; $\theta$ is the Hall angle; and $I$ is the photon current density)

$$\phi=-2\mu lq\theta I\,\frac{w^2}{W}\,\frac{K}{W^2-K^2}\left[\left(\frac{(K-S_2)(W-S_1)\,e^{-K}-(K+S_1)(W+S_2)\,e^{W}}{(W+S_1)(W+S_2)\,e^{W}-(W-S_1)(W-S_2)\,e^{-W}}\,e^{-W/2}\right.\right.$$

$$\left.\left.+\frac{(K-S_2)(W+S_1)\,e^{-K}-(K+S_1)(W-S_2)\,e^{-W}}{(W+S_1)(W+S_2)\,e^{W}-(W-S_1)(W-S_2)e^{-W}}\,e^{W/2}\right)\left[\tfrac{1}{2}W\cosh(\tfrac{1}{2}W)-\sinh(\tfrac{1}{2}W)\right]+\frac{W}{K}e^{-K/2}\left[\tfrac{1}{2}K\cosh(\tfrac{1}{2}K)-\sinh(\tfrac{1}{2}K)\right]\right]. \tag{5}$$

When $W=K$ we have

$$\phi=-\mu lq\theta I\,\frac{w^2}{W}\left\{\left[\tfrac{1}{2}W\cosh(\tfrac{1}{2}W)-\sinh(\tfrac{1}{2}W)\right]\left[\left(\frac{(K+S_2)+(K-1-S_2)(K-S_1)}{(K+S_1)(K+S_2)\,e^{K}-(K-S_1)(K-S_2)e^{-K}}\,e^{-K}-\frac{2+W}{2K}+2\right)e^{-W/2}\right.\right.$$

$$\left.\left.+\frac{(K-S_2)+(K-1-S_2)(K-S_1)}{(K+S_1)(K+S_2)\,e^{K}-(K-S_1)(K-S_2)\,e^{-K}}\,e^{-K}\,e^{W/2}\right]-e^{-W/2}(\tfrac{1}{2}W)^2\sinh(\tfrac{1}{2}W)\right\}. \tag{6}$$

The values of the magnetic flux $\phi$ as a function of normalized absorption coefficient for different values of parameters of the sample $S_1$, $S_2$, and $W$ were calculated according to (2) for $R_0=0$ by means of the computer Minsk II. Results of these calculations are presented in Fig. 3. The sign of the magnetic flux is changed for low values of surface-recombination velocity $S_1$ as in curves 1–6 of Fig. 3. For greater values of $S_1$ the sign of $\phi$ remains unchanged as in curves 7–9 of Fig. 3. The region of sign change is shifted towards the greater values of absorption coefficient for greater values of diffusion length. The values of magnetic flux are greater when the bulk recombination of material is greater (greater values of $W$).

The general expression for the magnetic flux (5) can be substantially reduced in a few cases: (i) For thick sample $W \gg 1$, strong absorption $K \gg W$, we have

$$\phi=\mu lq\theta I w^2/2(S_1+W). \tag{7}$$

(ii) When, in addition, the surface recombination of the illuminated surface is negligible ($S_1 \ll W$), we have

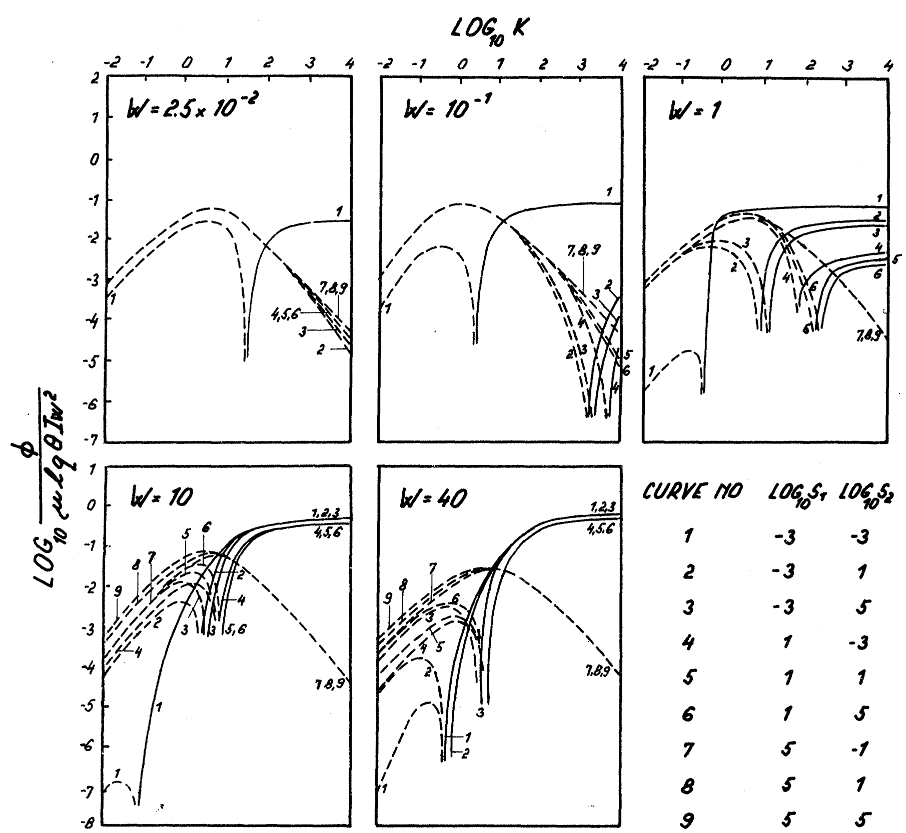


FIG. 3. Magnetic flux of the PME circulating current as a function of normalized absorption coefficient for various values of lifetime and surface recombination velocity (calculated values). Dashed curves are negative flux, solid curves are positive flux.

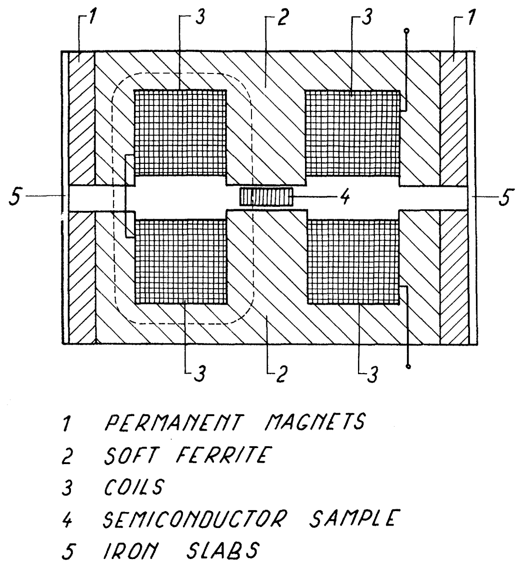


FIG. 4. Cross section of the measuring apparatus.

$$\phi = \tfrac{1}{2}\mu l q \theta I L w . \tag{8}$$

These expressions may be used to determine the diffusion length or the lifetime [they do not depend on $S_2$ or even on $S_1$ (Fig. 3), $W=10$, $W=40$]. (iii) For negligible volume recombination ($W \ll 1$, $K \gg 1$)

$$\phi \to 0 , \tag{9}$$

the magnetic flux disappears (Fig. 3, $W = 2.5 \times 10^{-2}$). (iv) When $S_1 \ll W$, $K \ll 1$, we have

$$\phi = \mu l q \theta I L^2 (K/W^2) . \tag{10}$$

(v) When $S_1 \gg W$, $S_2 \ll W$, $K \ll 1$, we have

$$\phi = -\mu l q \theta I L^2 (K/W^2) . \tag{11}$$

## EXPERIMENTAL ARRANGEMENT AND RESULTS

The cross section of the measuring apparatus is illustrated in Fig. 4. The constant magnetic field in which the sample (4) is placed is created by means of permanent magnets (1). Monochromatic illumination is chopped by a chopper (not illustrated) and illuminates the sample perpendicularly to the plane of the cross section. The ac magnetic flux of PME circulating current flows through the circuit made of soft ferrite (2) and by means of this flux the ac voltage is induced in the coils (3). The chopper was constructed in such a way that the illumination increased and decreased linearly with time. The chopping frequency was chosen low enough, $f = 214$ Hz, to avoid relaxation effects.[5] The two coils were connected in series with each other and the capacitance was connected to them to reach resonance at the frequency measured. The coil voltage was measured by the homodyne rectifier voltmeter Unipan model No. 202B with selective nanovoltmeter Unipan model No. 208. Thus the amplitude of the first harmonic component of the voltage induced in the coils was measured. It was necessary to place the apparatus proper in a threefold magnetic screening. The number of coil turns was 12 000. The optical system was composed of a projection lamp KP-GT 8-V 50-W Kondo and a mirror monochromator Zeiss model SPM-1. The selectivity was about $10^{-15}$ Wb. The resistivity of germanium samples was in the region 2–25 $\Omega$ cm, the dimension of samples was about $0.4 \times 0.4 \times 2.0$ cm$^3$.

The measured PME-circulating-current magnetic flux as a function of normalized absorption coefficient is presented in Fig. 5. Three dependences are always plotted for a certain value of parameter $W$. The first curve corresponds to the case when both illuminated and unilluminated surfaces were etched to give $S_1 = S_2$ (curve 1), in the second case the illuminated surface was etched and the unilluminated surface sandblasted to give $S_1 \ll S_2$ (curve 2), and in the third case the illuminated surface was sandblasted and the unilluminated was etched to give $S_1 \gg S_2$ (curve 3). The measured samples were etched in boiling $H_2O_2$ in the same way.

The sign of the measured magnetic flux is changed for lower values of surface recombination $S_1$ (surface etched). For greater values of surface recombination (surface sandblasted) the sign remains unchanged in accord with the theoretical conclusion. Further, it can be stated that the region of sign reversal is shifted towards greater values of absorption coefficient for greater values of the diffusion length ($W$ smaller). The first maximum of curve 1 is always lower than that of curve 2, and the values of magnetic flux are greater for samples of stronger bulk recombination ($W$ larger) which is in accord with the theory. From the facts mentioned above it may be concluded that there is a good qualitative agreement of our measurement with theoretical predictions, which verifies the correctness of our idea concerning the PME circulating current.

## CONCLUSION

In the present paper the idea of the PME circulating current was treated. The dependences of magnetic flux of PME circulating current as a function of normalized absorption coefficient were calculated and compared with dependences measured. This comparison is made on the basis of a new contactless method of measuring the PME effect. The obtained results verify the correctness of our idea of the PME circulating current.

It is necessary to mention one very important fact here. The changes of magnetic susceptibility

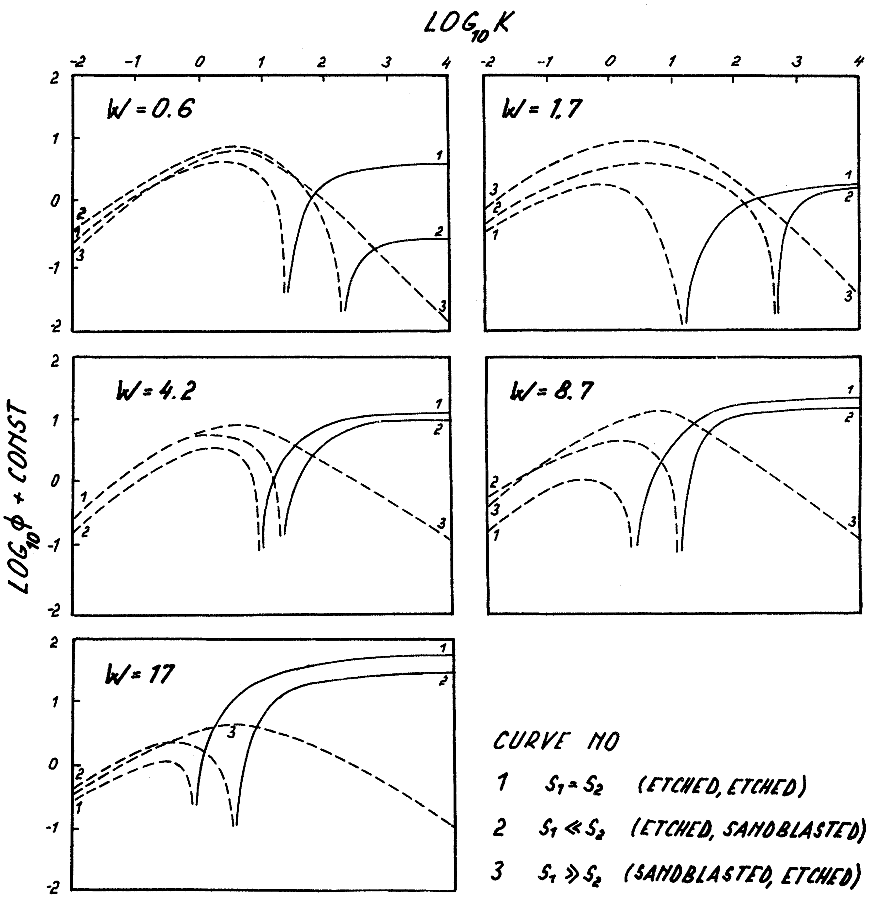


FIG. 5. Magnetic flux of the PME circulating current as a function of normalized absorption coefficient for various values of lifetime and surface recombination velocity (measured values). Dashed curves are negative measured flux, solid curves are positive measured flux.

with illumination were not considered in our paper. Provided the changes of magnetic susceptibility are caused by changes of excess-carrier density with illumination, then on the basis of simple estimation it is evident that these changes, if any, are not measurable under the condition of the given sensitivity and illumination. Simultaneous reorientation of magnetic dipoles of all excess carriers (about $10^{12}$ cm$^{-3}$) causes the total change of magnetic flux of the order of $10^{-17}$ Wb. This is beyond the sensitivity of our apparatus. It is clear that by increasing either the sensitivity or the illumination such a change might be measurable.

In our opinion, the presented information on the PME circulating current might be useful for two reasons: On the one hand, results presented may be a basis for a contactless method of measuring the diffusion length in semiconductors, and on the other hand, our findings may be useful for investigation of changes of magnetic susceptibility with illumination.

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