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Ultrasonic Attenuation in Copper, Silver, and Gold

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Second- and third-order elastic-constant data have been used to determine the Grüneisen mode γ $\langle\gamma\rangle$, average-square Grüneisen constant $\langle\gamma^2\rangle$, and nonlinear constant D for Cu, Ag, and Au. The attenuation suffered by longitudinal ultrasonic waves propagating in the $\langle 100\rangle$ and $\langle 110\rangle$ directions and transverse waves polarized along $\langle 100\rangle$ and $\langle 1\bar{1}0\rangle$, owing to the phonon viscosity and thermoelastic phenomena, have been evaluated for the three metals at 293 °K. The phonon viscosity and dislocation drag along the $\langle 100\rangle$ and $\langle 1\bar{1}0\rangle$ directions are also discussed.

INTRODUCTION

Thermal attenuation of ultrasonic waves, particularly the part arising because of the interaction of waves with phonon gas, i.e., the phenomenon of phonon viscosity and the dislocation drag, provide a good probe[1] for the study of dislocations in solids. Ultrasonic attenuation in some dielectric and other crystals has been extensively studied in the recent past. The interaction of acoustic-wave phonons with thermal phonons accounts for a dominant portion of this attenuation. Thermal-phonon relaxation time τ_{th} decreases with an increase in temperature and generally at room temperature $\tau_{th} \ll 1/\omega$, where ω is the angular frequency of the acoustic wave.[2] Hence the interaction between various phonon modes becomes insignificant, and a statistical model of phonon gas having macroscopic parameters, which may be varied by sound energy, is described. The two well-known absorptions in this region are (i) phonon viscosity loss (Akhieser loss)[3] occurring because of the relaxational flow of thermal energy among various phonon branches at different temperatures and (ii) thermoelastic attenuation arising from the thermal conduction between the compressed and expanded parts of the acoustic waves. For shear-wave propagation, the volume remains intact and there is no heating effect. Hence the thermoelastic loss is absent in this case. The phonon viscosity in solids, which is the analog of shear viscosity in liquids, damps the motion of both the types of dislocations (screw and edge dislocations) in a crystal.[1] This damping is represented by the drag coefficient B. The acoustic attenuation, phonon viscosity, and drag coefficient are theoretically predicted for Cu, Ag, and Au at 293 °K.

THEORY

The expression for the acoustic attenuation produced because of the phonon-viscosity effect for longitudinal and shear ultrasonic waves are, respectively,

$$\alpha_l = \frac{E_0\omega^2(D_l/3)\tau_l}{2\rho V_l^3}, \qquad (1)$$

$$\alpha_s = \frac{E_0\omega^2(D_s/3)\tau_s}{2\rho V_s^3}, \qquad (2)$$

where the condition $\omega\tau \ll 1$ has already been assumed. Here E_0 is thermal energy density, ω is angular frequency, ρ is density, and V is the velocity of ultrasonic wave. The subscripts l and s represent longitudinal and shear. The two relaxa-

TABLE I. Primary physical constants calculated for three Debye solids.

Metal	V_l (m/sec)	V_s (m/sec)	K (cal/sec cm °K)	C_v (10^7 erg/cm^3 °K)	E_0 (10^9 erg/cm^3)	τ_{th} (10^{-10} sec)
Cu	4322	2916	1.0057	3.307	6.673	0.345
Ag	3411	2079	0.7069	2.358	5.705	1.013
Au	3161	1467	0.9177	2.401	5.742	1.354

TABLE II. Grüneisen number and nonlinearity constant along the ⟨100⟩ and ⟨110⟩ directions for Cu, Ag, and Au.

Direction	Metal	$\langle\gamma\rangle$	$\langle\gamma\rangle^2$ Longitudinal	$\langle\gamma\rangle^2$ Shear	D Longitudinal	D Shear
	Cu	1.99	7.10	1.35	46.74	12.16
⟨100⟩	Ag	2.46	12.86	2.74	93.69	24.62
	Au	2.55	10.43	2.13	69.90	19.17
	Cu	2.34	8.09	7.71	49.45	69.42
⟨110⟩[a]	Ag	2.55	10.21	17.00	68.22	153.00
	Au	2.74	9.67	10.03	59.31	90.27

[a]For shear waves, $\langle\gamma\rangle^2$ and D are along $\langle 1\bar{1}0\rangle$.

tion times τ_l and τ_s are related as

$$\tfrac{1}{2}\tau_l = \tau_s = \tau_{\rm th} = 3K/C_v\bar{V}^2 , \quad (3)$$

where $\tau_{\rm th}$ is thermal relaxation time for the exchange of acoustic and thermal energies, K is thermal conductivity, C_v is specific heat, and $\bar{V}$ is the Debye average velocity. The nonlinearity constant D in Eqs. (1) and (2) is obtained from the second- and third-order elastic-constant data using theoretical formulas for γ_i^1, γ_i^5, and

$$D = 9\langle(\gamma_i^j)^2\rangle - 3\langle\gamma_i^j\rangle^2(\rho C_v T/E_0) , \quad (4)$$

where γ_i^j are the Grüneisen numbers corresponding to a particular direction of propagation and polarization. The thermoelastic attenuation is obtained from

$$\alpha_l = \omega^2\langle\gamma_i^j\rangle^2 KT/2\rho V_l^5 . \quad (5)$$

A similar relation holds for shear waves. The attenuations in Eqs. (1), (2), and (5) are in units of Np/cm[2]. The phonon viscosities associated with the two types of waves are

$$\eta_l = 2D(E_0K/C_v\bar{V}^2) , \quad \eta_s = D_s(E_0K/C_v\bar{V}^2) . \quad (6)$$

The drag coefficients on the motion of screw and edge type of dislocations are as follows[4]:

$$B_{\rm screw} = 0.071\eta ,$$

$$B_{\rm edge} = \frac{0.0532\eta}{(1-\sigma)^2} + \frac{0.0079}{(1-\sigma)^2}\left(\frac{\mu}{\kappa}\right)^2\chi , \quad (7)$$

where σ is Poisson ratio, and

$$\mu = C_{44} , \quad \kappa = C_{11} + \tfrac{4}{3}C_{44} , \quad \chi = \eta_l - \tfrac{4}{3}\eta_s .$$

RESULTS AND DISCUSSIONS

Table I shows the values of primary physical constants calculated for the three Debye solids. The values of ρC_v and E_0 as a function of Θ/T are taken from literature,[5] and then $\tau_{\rm th}$ is calculated. From the elastic-constant data of Hiki and Granato,[6] $\langle\gamma_i^j\rangle$ and $\langle(\gamma_i^j)^2\rangle$ values were calculated using Mason's theoretical formulas[7,8] for cubic crystals. The $\langle\gamma_i^1\rangle$ and $\langle(\gamma_i^1)^2\rangle$ for longitudinal waves and $\langle(\gamma_i^5)^2\rangle$ for shear waves have been evaluated by averaging over 39 pure modes ($\langle\gamma_i^5\rangle = 0$). The ratio D_l/D_s along the ⟨100⟩ direction for the three metals lies in the range 5–17 (Table II) like other crystals. The maximum value of τ_l is 2.7×10^{-10} sec for gold, and hence we have chosen $f = 10$ MHz so as to maintain the condition $\omega\tau \ll 1$.

The drag coefficient for the motion of screw and edge type of dislocations and the values of attenuation at $f = 10$ MHz along the two directions ⟨100⟩ and ⟨110⟩ (⟨1$\bar{1}$0⟩ for shear waves) are presented in Table III. Like other fluorite-structure crystals,[9]

TABLE III. Attenuation of longitudinal and shear acoustic waves and drag coefficient for screw and edge dislocations at f=10 MHz along the ⟨100⟩ and ⟨110⟩ directions. Akh represents Akhieser and therm represents thermoelastic.

Direction	Metal	$\alpha_{\rm Akh}$ (dB/cm) Longitudinal	$\alpha_{\rm Akh}$ (dB/cm) Shear	$\alpha_{\rm therm}$ (dB/cm) Longitudinal	$B_{\rm screw}$ (poise) Longitudinal	$B_{\rm screw}$ (poise) Shear	$B_{\rm edge}$ (poise) Longitudinal	$B_{\rm edge}$ (poise) Shear
	Cu	0.171	0.072	0.006	0.507	0.066	0.808	0.205
⟨100⟩	Ag	1.486	0.862	0.026	2.563	0.337	5.775	0.955
	Au	1.020	1.397	0.016	2.572	0.353	7.638	1.119
	Cu	0.181	0.413	0.008	0.537	0.377		
⟨110⟩[a]	Ag	1.081	5.355	0.028	1.824	2.092		
	Au	0.865	6.580	0.017	2.183	1.662		

[a]For shear waves the polarization is along ⟨1$\bar{1}$0⟩.

TABLE IV. Thermoelastic attenuation calculated from K_p values obtained by extending the Leibfried-Schlömann formula. Akh represents Akhieser.

Metal	K_p (10^{-2} cal/sec cm °K)	$(\alpha/f^2)_{Akh}$ (10^{-16} sec^{2}cm^{-1}) Longitudinal ⟨100⟩	⟨110⟩	Shear ⟨100⟩	⟨1$\bar{1}$0⟩
Cu	6.89	1.28	1.36	0.54	3.10
Ag	1.76	2.60	1.89	1.51	9.38
Au	0.70	1.01	0.86	1.39	6.52

the α values along the two directions are unequal in the present case also. More than 90% of the total attenuation is caused by the phonon-viscosity effect. One infers from here that a major part of the absorbed acoustic energy is either transformed to thermal energy or is used up in equalizing the temperature difference of the phonon. The values of α and η (not given) are much higher than those for dielectric crystals[9] because of the high values of thermal conductivity for these metals. The ratio of longitudinal- to shear-phonon viscosity η_l/η_s follows the ratio D_l/D_s. Because

$$\eta_x = \tfrac{1}{3} D E_0 \tau_x \quad (x = l, s) .$$

For calculating the drag coefficient B, the condition $a = \frac{3}{4} b$, where a is the dislocation-core radius and b is Burger's vector, is assumed. While evaluating the damping constant B for edge dislocation, the compressional viscosity χ has also been considered. But, the contribution of this term is very small in the case of Ag and Au.

The above results will not be experimentally true, because in Eq. (3), we have used the values of total thermal conductivity (due both to the electrons and the phonons) instead of K_p, the thermal conductivity due to phonons only. For a metallic crystal at room temperature, most of the thermal energy is carried by electrons. But, for achieving an equilibrium among various phonon modes, only the phonons take part. Table IV shows the K_p values obtained by extending the Leibfried formula[10] of lattice thermal conductivity for monovalent metals to these metals also. From the comparison of Tables III and IV it is clear that now the thermoelastic process is more effective than the phonon-viscosity effect as it should be for a metal.[11] Moreover, the new α/f^2 values for Cu are much closer to the available experimental[12] data. The η and B values will also be modified accordingly. Further better results are expected from the exact knowledge of K_p values.

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