Transport Properties of a Dirty Two-Band Superconductor in the Mixed State near H_{c2}

I-Ming Tang
Department of Physics, American University of Beirut, Beirut, Lebanon
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The mixed-state ultrasonic-attenuation and thermal-conductivity coefficients are obtained for a transition-metal superconductor containing a high concentration of nonmagnetic impurities. Expressing the transport coefficients as the sums of correlation functions of the individual bands, the attenuation and thermal-conductivity coefficients are obtained by the use of an equivalence theorem due to Maki. The resulting two-band expressions exhibit the linear field dependences observed in dirty transition-metal superconductors in the gapless state near H_{c2}. The two-band mixed-state thermal-conductivity coefficient for a dirty superconductor is then used qualitatively to show why the slope of the normalized thermal conductivity minus 1 versus the applied field for two dirty transition-metal superconductors ($Nb_{80}Mo_{20}$ and $Nb_{85}Mo_{15}$) having only slightly different l/ξ_0 ratios agrees with the one-band mixed-state thermal-conductivity expression of Caroli and Cyrot for the case of the dirtier $Nb_{80}Mo_{20}$ specimen, while for the case of the $Nb_{85}Mo_{15}$ specimen the slope is much greater than that predicted by the one-band expression.

I. INTRODUCTION

The recent discoveries of a second energy gap[1,2] and a second transition temperature[3] in pure niobium superconductors point to the need of using a model other than the one-band BCS model[4] to describe the superconducting states in niobium and other transition metals (TM).[5] A two-band model which predicted the existence of a second energy gap and a second transition temperature was proposed by Suhl, Matthias, and Walker[6] (SMW) for describing the pure TM superconductors. Shortly after this two-band model was introduced, Garland[7] discussed the effects of impurities on the two-band superconductors. He showed that the two-band model was appropriate to those TM superconductors satisfying the clean-limit condition $l/\xi_0 \gg 1$ (l being the electronic mean free path and ξ_0 being the coherence length) and that the usual one-band model[4] was appropriate for dirty (in the Anderson sense[8]) TM superconductors. Gusman[9] has shown that Garland's conditions regarding the appropriateness of the two-band description of the TM superconductors were too stringent. Treating the effects of the nonmagnetic impurity scattering by the Born approximation, Gusman showed that the two-band description of the TM superconductors could be extended to those superconductors satisfying the intermediate-limit condition $l \sim \xi_0$. In these superconductors, the electronic states in the two bands are still distinguishable from each other. However, the energy gaps $\Omega_{s(d)}$ appearing in the excitation spectrum of the two bands are the same.

Further studies on the effects of nonmagnetic impurities (in the region of high impurity concentration) on two-band superconductors have been carried out by several Russian authors.[10-14] Moskalenko and co-workers[10-12] have studied the effects of nonmagnetic impurities on the specific heats, the critical thermodynamic magnetic field, and the optical absorption by a two-band superconductor containing a high concentration of nonmagnetic impurities. Kolpagiu and co-workers[13,14] have studied the effects of a persistent current on various properties of a dirty two-band superconductor. Since the dirty two-band superconductor considered in all of these studies satisfied the dirty-limit condition, the energy gaps for the two bands were treated as being equal.

The categorizing of a two-band superconductor as being either a clean or a dirty superconductor is especially important if we are to consider the transport properties of the two-band superconductors near the upper critical field. In a series of papers,[15-18] the present author has obtained the two-band expressions for the various transport properties of a clean TM superconductor near H_{c2}. The expressions for both the ultrasonic attenuation[15] and the thermal conductivity[16] coefficients exhibited the experimentally observed $(H_{c2}-H)^{1/2}$ field dependences. With these expressions, the author was able to explain the observed purity dependences in the ultrasonic attenuation[19] and thermal conductivity[20] in niobium superconductors near H_{c2}. To obtain these expressions, it was necessary to make a conjecture regarding the densities of states in the two bands. This was necessary because the usual procedure for evaluating the various correlation functions appearing in the definitions of the transport properties yields unphysical results.[21] The unphysical results arise only when we attempt to evaluate the correlation functions for a *clean* two-band superconductor near H_{c2} by an iteration of the Gor'kov equations for the superconductor in high fields.[22] In the case of a *dirty* two-band superconductor near the upper critical field, the iteration of the two-band Gor'kov equations will not produce any divergent terms.[23] Therefore, it will be possible to evaluate the various correlation functions near the upper critical field by expanding them in powers of the energy gaps.

Work along similar lines has already started. Chow has attempted to calculate the effects of nonmagnetic impurity scattering on the upper critical field of a dirty two-band superconductor[24,25] by an iteration of the Gor'kov equations. The effects of the nonmagnetic impurities were handled using a technique developed by Maki.[26] However, in his studies, Chow treats the two energy gaps as being unequal.

Therefore, in Sec. II we will briefly review the two-band model used in this paper. Our discussion of the effects of the nonmagnetic impurity scattering on the two-band superconductor will follow somewhat the discussion given by Moskalenko *et al.*[10–12] Using the same assumptions employed by Gusman[9] and Moskalenko *et al.*, we will find that in the limit of high impurity concentrations the two energy gaps will have the same value. Then by using the Green's functions obtained in Sec. II, the ultrasonic-attenuation coefficient for longitudinal waves in a dirty two-band superconductor near H_{c2} will be obtained in Sec. III. We will find that the attenuation coefficient is just the sum of the coefficients for the attenuation in the individual bands and that the two-band attenuation coefficient exhibits the linear field dependence which is characteristic of the transport properties of dirty superconductors in the mixed state near H_{c2}. We will obtain the thermal-conductivity coefficient for a dirty two-band superconductor in the gapless state near the upper critical field in Sec. IV. We will find again that the two-band thermal-conductivity coefficient is just the sum of the coefficients for the individual bands.

Finally, in Sec. V we will discuss the possible use of the two-band transport expressions to expand some discrepancies between experimental results and the theoretical predictions. In particular, we will use the two-band thermal-conductivity coefficient to explain qualitatively the results of the thermal-conductivity measurements on some impure niobium superconductors in the mixed state.

II. TWO-BAND MODEL

The Hamiltonian which describes the two-band system considered in this study is given by[6] (in the absence of the applied field)

$$H = H_0 + H_{\text{pair}} + H_{\text{imp}} , \tag{1}$$

where

$$H_0 = \sum_{k,\sigma} \epsilon_s(k)\, c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,\sigma} \epsilon_d(k)\, d^\dagger_{k\sigma} d_{k\sigma} , \tag{2}$$

$$\begin{aligned} H_{\text{pair}} = &- \sum_{kk'} V_{ss}\, c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} \\ &- \sum_{kk'} V_{dd}\, d^\dagger_{k\uparrow} d^\dagger_{-k\downarrow} d_{-k'\downarrow} d_{k'\uparrow} \\ &- \sum_{kk'} V_{sd}\,(c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} d_{-k'\downarrow} d_{k'\uparrow} + d^\dagger_{k\uparrow} d^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}) , \end{aligned} \tag{3}$$

$$\begin{aligned} H_{\text{imp}} = \sum_{i,k,k,\sigma} & [v_{ss}(k-k')\, c^\dagger_{k\sigma} c_{k'\sigma} e^{i(k-k')R_i} \\ & + v_{dd}(k-k')\, d^\dagger_{k\sigma} d_{k'\sigma} e^{i(k-k')R_i} \\ & + v_{sd}(k-k')\,(c^\dagger_{k\sigma} d_{k'\sigma} + d^\dagger_{k\sigma} c_{k'\sigma})] . \end{aligned} \tag{4}$$

Here $c_{k\sigma}$ ($c^\dagger_{k\sigma}$) is an annihilation (creation) operator for an electron in the s band having a spin σ. $\epsilon_s(k)$ is the energy of the s electrons measured from the Fermi level. V_{ss} is the electron-phonon coupling constant for BCS-like pair formation within the s band. V_{sd} is the coupling constant for pair formation between the s and d electrons in the region where the two bands overlap. $v_{ss}(k-k')$ and $v_{sd} \times (k-k')$ are the intraband and interband scattering potential of the nonmagnetic impurities for the s electrons, respectively. $d_{k\sigma}$, $d^\dagger_{k\sigma}$, $\epsilon_d(k)$, etc., are similarly defined for the d electrons. The effects of an applied magnetic field can be incorporated easily by going over to the spatial transformation of Eq. (1) and making the substitution

$$\vec{\nabla}^2 \to (\vec{\nabla} - ie\,\mathrm{A})^2 . \tag{5}$$

Disregarding the effects of the magnetic field for a moment, we now define the energy gaps of our two-band superconductor by

$$\begin{aligned} \overline{\Delta}_s = N_s V_{ss} k_B T \sum_n \frac{\tilde{\Delta}_s(\tilde{\omega}_{ns})}{[\tilde{\omega}^2_{ns} - \tilde{\Delta}^2_s(\tilde{\omega}_{ns})]^{1/2}} \\ + N_d V_{sd} k_B T \sum_n \frac{\tilde{\Delta}_d(\tilde{\omega}_{nd})}{[\tilde{\omega}^2_{nd} - \tilde{\Delta}^2_d(\tilde{\omega}_{nd})]^{1/2}} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \tilde{\Delta}_d = N_d V_{dd} k_B T \sum_n \frac{\tilde{\Delta}_d(\tilde{\omega}_{nd})}{[\tilde{\omega}^2_{nd} - \tilde{\Delta}^2_d(\tilde{\omega}_{nd})]^{1/2}} \\ + N_s V_{sd} k_B T \sum_n \frac{\tilde{\Delta}_s(\tilde{\omega}_{ns})}{[\tilde{\omega}^2_{ns} - \tilde{\Delta}^2_s(\tilde{\omega}_{ns})]^{1/2}} . \end{aligned} \tag{7}$$

The impurity-dependent quantities [$\tilde{\Delta}_d(\tilde{\omega}_{nd})$, $\tilde{\Delta}_s(\tilde{\omega}_{ns})$, $\tilde{\omega}_{ns}$, and $\tilde{\omega}_{nd}$] can be obtained in the usual manner[27] and are defined as

$$\begin{aligned} \tilde{\Delta}_s(\tilde{\omega}_{ns}) = \overline{\Delta}_s + \frac{1}{2\tau_s} \frac{\tilde{\Delta}_s(\tilde{\omega}_{ns})}{[\tilde{\omega}^2_{ns} - \tilde{\Delta}^2_s(\tilde{\omega}_{ns})]^{1/2}} \\ + \frac{1}{2\tau_{sd}} \frac{\tilde{\Delta}_d(\tilde{\omega}_{nd})}{[\tilde{\omega}^2_{nd} - \tilde{\Delta}^2_d(\tilde{\omega}_{nd})]^{1/2}} , \end{aligned} \tag{8}$$

$$\tilde{\Delta}_d(\tilde{\omega}_{nd}) = \bar{\Delta}_d + \frac{1}{2\tau_d}\frac{\tilde{\Delta}_d(\tilde{\omega}_{nd})}{[\tilde{\omega}_{nd}^2 - \tilde{\Delta}_d^2(\tilde{\omega}_{nd})]^{1/2}} + \frac{1}{2\tau_{ds}}\frac{\tilde{\Delta}_s(\tilde{\omega}_{ns})}{[\tilde{\omega}_{ns}^2 - \tilde{\Delta}_s^2(\tilde{\omega}_{ns})]^{1/2}} , \tag{9}$$

$$\tilde{\omega}_{ns} = \omega_n + \frac{1}{2\tau_s}\frac{\tilde{\omega}_{ns}}{[\tilde{\omega}_{ns}^2 - \tilde{\Delta}_s^2(\tilde{\omega}_{ns})]^{1/2}} + \frac{1}{2\tau_{sd}}\frac{\tilde{\omega}_{nd}}{[\tilde{\omega}_{nd}^2 - \tilde{\Delta}_d^2(\tilde{\omega}_{nd})]^{1/2}} , \tag{10}$$

$$\tilde{\omega}_{nd} = \omega_n + \frac{1}{2\tau_d}\frac{\tilde{\omega}_{nd}}{[\tilde{\omega}_{nd}^2 - \tilde{\Delta}_d^2(\tilde{\omega}_{nd})]^{1/2}} + \frac{1}{2\tau_{ds}}\frac{\tilde{\omega}_{ns}}{[\tilde{\omega}_{ns}^2 - \tilde{\Delta}_s^2(\tilde{\omega}_{ns})]^{1/2}} . \tag{11}$$

In the above expressions, $1/2\tau_s$, $1/2\tau_d$, $1/2\tau_{sd}$, and $1/2\tau_{ds}$ are the scattering amplitudes for s-s, d-d, s-d, and d-s band transitions, respectively. They are defined as

$$1/2\tau_{s(d)} = \pi n_i N_{s(d)}(0) \langle |v_{s(d)}|^2 \rangle_\Omega \tag{12}$$

and

$$1/2\tau_{sd(ds)} = \pi n_i N_{d(s)}(0) \langle |v_{sd}|^2 \rangle_\Omega , \tag{13}$$

where n_i is the impurity concentration and $N_{s(d)}(0)$ is the density of states of the $s(d)$ band at the Fermi surface.

The changes in the densities of states due to the nonmagnetic impurity scattering can be obtained from the definition of the density of states:

$$N_{s(d)}(\omega) = N_{s(d)}(0)\,\mathrm{Re}[u_{s(d)}(u_{s(d)}^2 - 1)^{-1/2}] , \tag{14}$$

where the u's are defined as

$$u_{s(d)} = \tilde{\omega}_{ns(d)} / \tilde{\Delta}_{s(d)}(\tilde{\omega}_{ns(d)}) . \tag{15}$$

If we substitute expressions (8)–(11) into the above definitions of the u's, we obtain the following coupled equations:

$$u_s = \frac{\omega_n}{\bar{\Delta}_s} + \frac{1}{2\tau_{sd}\bar{\Delta}_s}\frac{u_d - u_s}{(u_d^2 - 1)^{1/2}} , \qquad u_d = \frac{\omega_n}{\bar{\Delta}_d} + \frac{1}{2\tau_{ds}\bar{\Delta}_d}\frac{u_s - u_d}{(u_s^2 - 1)^{1/2}} . \tag{16}$$

In the case of high impurity concentrations, the second terms on the right-hand side of Eqs. (16) become the dominant terms.[9,10–12] The solutions to Eqs. (16) are readily obtainable and are

$$u_s \approx u_d \approx \omega_n / \Omega_G , \tag{17}$$

with

$$\Omega_G = \frac{N_s(0)\bar{\Delta}_s + N_d(0)\bar{\Delta}_d}{N_s(0) + N_d(0)} . \tag{18}$$

Substitution of the u's given by (17) into the densities of state (14) shows that Ω_G acts as the energy gap in the excitation spectrum of both bands.

To determine the effects of the magnetic field on the two-band superconductor, it is necessary to retrace our way back to Eq. (1) in its spatial representation and then to construct the Gor'kov equations for the superconductor in the absence of the impurities. The effects of the impurity scattering would be incorporated through the technique developed by Maki[26] and which was used by Chow[24,25] in his attempt to calculate the effects of nonmagnetic impurities on a dirty two-band superconductor. However, we may also use the following equivalence theorem of Maki[28]:

Theorem. The correlation functions in the type-II superconductors in the mixed state have expressions equivalent to those for a current-carrying state, as long as the contribution from the fluctuations of the order parameter is negligible.

Then it will not be necessary for us to use the Gor'kov equations to calculate the various correlation functions appearing in the two-band definitions of the ultrasonic-attenuation coefficient and the thermal-conductivity coefficient. A careful analysis by Maki and Fulde[29] showed that the expressions for the transport coefficients, except for the electromagnetic conductivity, for a dirty one-band superconductor in the mixed state near H_{c2} are equivalent to those found in the current-carrying case.

III. ULTRASONIC ATTENUATION

The expression for the ultrasonic attenuation of longitudinal waves in a clean two-band superconductor has been obtained by the present author in Ref. 15. Since the ultrasonic-attenuation coefficient was expressed in terms of various correlation functions and did not depend on a specific type of Green's functions, it may be used for the case of a dirty two-band (type-II) superconductor in a high magnetic field. The defining expression for the attenuation in the dirty two-band superconductor is then

$$\alpha_L^S = \mathrm{Re}\,\frac{q^2}{i\omega\rho_{\mathrm{ion}}}V_s \Big[\langle [\tau_{szz}, \tau_{szz}] \rangle_{q\omega} - \frac{2p_F^2}{3m_s}\langle [\tau_{szz}, n_s] \rangle_{q\omega} + \left(\frac{p_F^2}{3m_s}\right)^2 \langle [n_s, n_s] \rangle_{q\omega} + \langle [\tau_{dzz}, \tau_{dzz}] \rangle_{q\omega} - \frac{2p_F^2}{3m_d}\langle [\tau_{dzz}, n_d] \rangle_{q\omega} + \left(\frac{p_F^2}{3m_d}\right)^2 \langle [n_d, n_d] \rangle_{q\omega} \Big] , \tag{19}$$

where the various terms have the same definitions as in the reference cited. Again, the retarded products $\langle [A, B] \rangle_{q\omega}$ are obtained by analytical continuation of the thermal products.

As mentioned in Sec. II, the expression for the ultrasonic attenuation in the mixed state will be obtained by finding the attenuation coefficient for a dirty two-band superconductor in the current-carrying case. This is done just because of the mathematical convenience involved. In both cases, use has been made of the fact that in the gapless region the ratio between the impurity-dependent frequency $\tilde{\omega}_{ns(d)}$ and the energy gap $\tilde{\Delta}_{s(d)}(\tilde{\omega}_{ns(d)})$ can be expanded in inverse powers of the u's.

By considering longitudinal waves in the limit $ql \gg 1$ (q being the wave vector), only the third and sixth terms in (19) contribute to the attenuation coefficient, i.e.,

$$\alpha_L^S = \mathrm{Re}\left(\frac{q^2}{i\omega\rho_{\mathrm{ion}}} V_s\right)\left[\left(\frac{p_F^2}{3m_s}\right)^2 \langle [n_s, n_s] \rangle_{q\omega} + \left(\frac{p_F}{3m_d}\right)^2 \langle [n_d, n_d] \rangle_{q\omega}\right] , \quad (20)$$

where

$$n_{s(d)} = \sum_\sigma \psi^\dagger_{s(d)\sigma} \psi_{s(d)\sigma} . \quad (21)$$

The thermal products from which the retarded products in (20) can be obtained are[30]

$$\langle [n_j, n_j] \rangle_{q\nu} = T\sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\,[\rho_3 \mathcal{G}_j(\vec{p}, \omega_n)\rho_3 \mathcal{G}_j(\vec{p}-\vec{q}, \omega_n')] , \quad (22)$$

where $\mathcal{G}_j(\vec{p}, \omega_n)$ is the matrix representation of the Green's function, i.e.,

$$\mathcal{G}_j(\vec{p}, \omega_n) = \begin{bmatrix} G_j(\vec{p}, \omega_n) & F_j(\vec{p}, \omega_n) \\ -F_j^*(-\vec{p}, -\omega_n) & -G_j(-\vec{p}, -\omega_n) \end{bmatrix} \quad (23)$$

and $\omega_n' = \omega_n - \omega_2$. p_3 is the third Pauli matrix. Taking the trace and performing the indicated integrations, the density-density correlation functions (22) for the individual bands become

$$\langle [n_j, n_j] \rangle_{q\nu} = G_j + \frac{\pi N_j}{v_{jF}\vec{q}} \pi T \sum_{\omega_n}\left(1 - \frac{u u' + 1}{(u^2-1)^{1/2}(u'^2-1)^{1/2}}\right), \quad (24)$$

where the u's are given by (17) in the limit of high impurity concentrations. The first term in (24) is normalized so that (24) reduces to the correct normal-state expression in the limit $\Omega_G \to 0$.[30] For small frequencies, the analytic continuation of the thermal density-density correlation function (24) gives

$$\langle [n_j, n_j] \rangle_{q\omega} = c_j + i\omega \frac{\pi N_j(0)}{v_{jF}\vec{q}} \times \int_0^\infty \frac{d\omega'}{2T} \cosh^{-2}\left(\frac{\omega'}{2T}\right) \frac{1}{2}\left(1 + \frac{|u'|^2 - 1}{|u'^2 - 1|}\right) . \quad (25)$$

Substitution of (25) into the definition of the ultrasonic-attenuation coefficient (20) gives

$$\frac{\alpha_L^S}{\alpha_L^n} = \frac{\alpha_L^n(s)}{\alpha_L^n} \int_0^\infty \frac{d\omega}{2T} \cosh^{-2}\left(\frac{\omega}{2T}\right) \frac{1}{2}\left(1 + \frac{|\omega_{sn}|^2 - \Omega_G^2}{|\omega_{sn}^2 - \Omega_G^2|}\right) + \frac{\alpha_L^n(d)}{\alpha_L^n} \int_0^\infty \frac{d\omega}{2T} \cosh^{-2}\left(\frac{\omega}{2T}\right) \frac{1}{2}\left(1 + \frac{|\omega_{dn}|^2 - \Omega_G^2}{|\omega_{dn}^2 - \Omega_G^2|}\right) , \quad (26)$$

where $\alpha_L^n(s(d))$ is the normal-state attenuation coefficient for a system comprised of $s(d)$ electrons only. The above expression remains valid for arbitrary $\vec{q}l$ if terms of order l/ξ_0 are neglected.[30] For fields close to H_{c2}, the ultrasonic-attenuation coefficient in a dirty two-band superconductor in the mixed state becomes

$$\frac{\alpha_L^s}{\alpha_L^n} = 1 - \sum_{j=s,d} \frac{\alpha_L^n(j)}{\alpha_L^n} \frac{1}{2(2\pi T)^2}\left[\rho_j^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho_j) - \psi^{(2)}(\tfrac{1}{2}+\rho_j)\right]\Omega_G^2 , \quad (27)$$

where the function $\psi^{(n)}$ denotes the higher derivatives of the digamma functions and $\rho_j = \tau_{tr,j}^2 v_{jF} q^2/12\pi T$.

To see that the above attenuation coefficient will predict a linear field dependence, we need only to look at Eqs. (18) and (7) for a typical two-band superconductor [one in which $N_d(0) \gg N_s(0)$].[31] When $N_d(0)$ is much greater than $N_s(0)$, the energy gap Ω_G will be approximately equal to $\bar{\Delta}_d$. If we now look at (7), we see that for $N_d(0) \gg N_s(0)$, the energy gap $\bar{\Delta}_d$ is defined by a self-consistent equation similar to that of the order parameter in the one-band BCS model.[4]

IV. THERMAL CONDUCTIVITY

As was done in Ref. 16, the thermal conductivity can be obtained by using the Kubo formula[32]

$$K_S = \mathrm{Im}[(\omega T)^{-1} P(i\omega)]_{\omega=0} , \quad (28)$$

where $P(i\omega)$ is obtained from the heat-current correlation function $P(\omega_\nu = 2\pi i\nu T)$ by analytic continuation. It can easily be verified that the heat current in the two-band superconductor containing impurities is of the form

$$j_i^h(\vec{r}) = -\tfrac{1}{2}\sum_j \sum_\sigma \frac{1}{2m_j}(\dot{\psi}^\dagger_{j\sigma}\vec{\nabla}_i\psi_{j\sigma} + \vec{\nabla}_i\psi^\dagger_{j\sigma}\cdot\dot{\psi}_{j\sigma}) , \quad (29)$$

where the summation is over the two bands. Within the Hartree-Fock approximation, the two-band correlation function takes the form

$$P_i(\omega_\nu) = \langle [j_{si}^h, j_{si}^h] \rangle_{0\omega_\nu} + \langle [j_{di}^h, j_{di}^h] \rangle_{0\omega_\nu} , \quad (30)$$

with $j_{s(d)i}^h$ being the heat current carried by the $s(d)$ electrons in the i direction.

The individual correlation functions in (30) are expressed in terms of the Green's function as

$$P_{i\nu}^j(\vec{q}) = \frac{1}{8m_j} T \sum_{\omega_n} \times \int \frac{d^3p}{(2\pi)^3} \omega_n\omega' \mathrm{Tr}[\rho_3 p_i \mathcal{G}_j(\vec{p}, \omega_n)\rho_3 p_i \mathcal{G}_j(\vec{p}-\vec{q}, \omega')] , \quad (31)$$

where $\mathcal{G}_i(p, \omega_n)$ is the matrix representation (23) and $\omega' = \omega_n - \omega_\nu$. Taking the trace and performing the indicated integration, we obtain

$$P_{i\nu}^{j}(\vec{q}) = \frac{N_j(0)}{4m_j^2}\, 2\pi T\tau_j \sum_{\omega_n} \omega_n\omega'\left(1 - \frac{u_i u_j' + 1}{(u_j^2-1)^{1/2}(u_j'^2-1)^{1/2}}\right), \quad (32)$$

where again the u's are given by (17) in the limit of high impurity concentrations. The analytic continuation of (32) leads to

$$K_S = \sum_{j=s,d} \frac{\tau_i N_j(0)}{2m_j T^2}\int_0^\infty d\omega\, \omega^2 \cosh^{-2}\left(\frac{\omega}{2T}\right)\frac{1}{2}\left(1 + \frac{|u_j|^2 - 1}{|u_j^2 - 1|}\right). \quad (33)$$

For fields close to the upper critical field and in the limit of high impurity concentrations, the thermal conductivity in the mixed state of a dirty two-band superconductor is

$$\frac{K_S}{K_N} = 1 - \frac{3}{2(\pi T)^2}\sum_{j=s,d}\frac{K_N(j)}{K_N}\rho_j[\rho_j\psi^{(2)}(\tfrac{1}{2}+\rho_j) + \psi^{(1)}(\tfrac{1}{2}+\rho_j)]\,\Omega_G^2\,, \quad (34)$$

where $\psi^{(n)}$ was defined in Sec. III. $K_N(j)$ is the thermal conductivity in the normal state of a system comprised of j-type electrons only.

Again if we consider a typical two-band superconductor, we would see that (31) would predict a linear field dependence. Naturally, the two-band expression reduces to the one-band expression as $N_s(0) \to 0$.[30]

V. CONCLUSION

While it may appear that the two-band expressions for the ultrasonic attenuation (31) and the thermal conductivity (34) in the mixed states of a dirty TM superconductor do not predict anything new which is not already predicted by the one-band expressions[23,33] (both the one- and two-band expression predict the linear field dependence observed in the measurements of these properties in various impure superconductors in the mixed state[34-36]), closer inspection will show that the slopes of the normalized coefficients minus 1 versus the applied field are different from those obtained from the one-band expressions. This is important in light of some measurements of the mixed-state thermal conductivity of various impurity-doped niobium superconductors by Lowell and Sousa[34] (especially the thermal conductivities of $Nb_{80}Mo_{20}$ and $Nb_{85}Mo_{20}$). They found that even though l/ξ_0 for each of the superconductors are only slightly different from each other, the observed slopes had completely different characteristics, i.e., the observed slope for the dirtier $Nb_{80}Mo_{20}$ superconductor was in reasonable agreement with the one-band expression,[33] while the observed slope for $Nb_{85}Mo_{15}$ was much greater than that predicted by the one-band theory. They found that as the samples became purer, the discrepancies between the observed and the predicted slopes became greater. Their findings are in agreement with the results of Wasim and Zebouni,[34] who found the slope for a niobium superconductor in the intermediate-limit category to be much greater than that predicted by the dirty one-band thermal conductivity. We can see qualitatively that the dirty two-band mixed-state thermal conductivity (34) will predict a greater deviation from the one-band slope as the samples become purer. This follows from the fact that as more impurities are added to the system, the density of states of the less-populated band becomes less. Therefore, the contribution to the slope by the less populated band becomes less. In some ways, this purity dependence is similar to the purity dependence seen in the observed transport properties of clean TM superconductors in the mixed state.[19,20]

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