[1]D. A. Cusano, Solid State Commun. 2, 353 (1964).

[2]C. J. Hwang, J. Appl. Phys. 40, 4591 (1969).

[3]Vilms and Spicer (Ref. 7) have shown that the internal recombination efficiency and radiative lifetime can also be obtained from measurements of external efficiency, optical-absorption spectra, surface photovoltaic effect, and an approximate calculation of the minority carrier mobility. Yu. A. Moma and V. P. Sushkov [Sov. Phys. Semicond. 1, 1278 (1968)] also obtained the same quantities from measurements of minority carrier lifetime, the external efficiency, and a rough allowance for the surface recombination.

[4]G. Lasher and F. Stern, Phys. Rev. 133, A553 (1964).

[5]The band edges of the conduction and valence bands will shift relative to the position of the edges in a pure crystal because of carrier-carrier and carrier-impurity interaction [see, for example, P. A. Wolff, Phys. Rev. 126, 405 (1962)]. The nominal band edges here refer to these shifted edges.

[6]C. J. Hwang, J. Appl. Phys. 40, 3731 (1969).

[7]J. Vilms and W. E. Spicer, J. Appl. Phys. 36, 2815 (1965).

[8]M. D. Sturge, Phys. Rev. 127, 768 (1962).

[9]C. J. Hwang, J. Appl. Phys. 38, 4811 (1967); 39, 1654 (1968).

[10]R. J. Carbone and P. Langaker, Appl. Phys. Letters, 4, 32 (1964); C. J. Hwang, Rev. Sci. Instr. 42, 1084 (1971).

[11]C. J. Hwang, J. Appl. Phys. 42, 4408 (1971).

[12]K. L. Ashley and J. R. Biard, Proc. IEEE Trans. Elec. Devices, ED-14, 429 (1967); D. B. Wittry and D. F. Kyser, J. Appl. Phys. 38, 375 (1967); and L. W. Aukerman, M. F. Miller, and M. McColl, J. Appl. Phys. 38, 685 (1967).

[13]N. M. Kolchanova and D. N. Nasledov, Fiz. Tverd. Tela 8, 1097 (1966) [Sov. Phys. Solid State 8, 876 (1966)]; D. B. Wittry and D. F. Kyser, Proceedings of the International Conference on the Physics of Semiconductors, Kyoto, 1966 (unpublished); J. Phys. Soc. Japan Suppl. 21, 312 (1966).

[14]J. I. Pankove, Phys. Rev. 140, A2059 (1965).

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Dynamics of Electrons and Phonons in Semiconductors

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We present a theoretical study of the evolution of the coupled electron-phonon system in the presence of a strong electric field. A nondegenerate semiconductor of constant carrier density is considered and a numerical solution of the coupled transport equations is performed, the electrons being represented by a Maxwell-Boltzmann distribution with a time-dependent electron temperature $T_e(t)$. It is shown that both $T_e(t)$ and the phonon distribution $N_q(t)$ undergo large deviations even for times of the order of 1 μsec after the application of the electric field. The electron temperature is shown to be generally a nonmonotonic function of time and results are presented for a variety of carrier concentrations and electric field strengths. The calculated phonon distribution generally differs considerably from a Bose-Einstein distribution, so that a phonon temperature cannot be defined.

I. INTRODUCTION

The application of a strong electric field to a semiconductor generally results in significant deviations of both the electron and phonon distributions from their equilibrium values. The phonon deviations are particularly large at low temperatures, where the phonon lifetimes are long,[1-3] being usually determined by boundary scattering.

In such cases one must deal with the coupled electron-phonon system, thus abandoning the usual Bloch assumption of an unperturbed phonon distribution. The effect of phonon heating on the steady-state electrical conductivity has been studied by several authors,[2-4] while it is only recently that studies of the time-dependent case have been undertaken.[5-7] Here one is explicitly interested in the evolution of the electron and phonon distributions when an electric field is applied to the system. This is often of interest experimentally since the electric field pulses employed are often short or of comparable duration to the time necessary to establish the steady state τ_h. The latter is commonly of the order of the phonon transit time L/c, L and c being an approximate sample dimension and sound velocity, respectively, multiplied by a factor η which characterizes the acoustic mismatch between the solid and the heat bath. For the helium-semiconductor contact, $\eta \approx 100$,[4] and the time necessary to reach the steady state is of the order of 10 to 100 μsec.

One of the first analyses of the time-dependent case is that of Paranjape and Paranjape.[5] In their paper an approximate analytic solution was presented, which was based on a simplified treatment of the phonon distribution, where several time-de-

pendent terms were approximated as constant. Subsequently, Baumann, Kocevar, and Kriechbaum[6] attempted a numerical solution of this problem, but instead of calculating the simultaneous evolution of both electrons and phonons, they took the electron temperature to be constant. This approximation does not appear to be appropriate and was reconsidered in a further paper by Baumann, Kriechbaum, and Kahlert.[7]

In this paper, we report a detailed theoretical study of the coupled dynamics of carriers and phonons in semiconductors. The time dependences of both the electron and phonon distribution functions are considered simultaneously, and a numerical solution of the relevant coupled equations allows us to deduce the evolution of both systems. Results will be presented for a typical nondegenerate semiconductor, and an application is made to n-type germanium.

The carrier concentration is assumed constant in order to avoid an explicit treatment of the impact-ionization process. This in itself is an extremely interesting problem, especially in the light of the present work, where the nonequilibrium nature of the phonon distribution is important. In addition to the effect of the phonons on the free-electron distribution, and therefore on the ionization coefficient, there is obviously also a direct effect on the recombination and "thermal excitation" of the bound states. Even a numerical solution of the electron-phonon-impurity system is a formidable task and would undoubtedly contribute to an understanding of the unexplained phenomena observed during impact ionization.[8,9]

Since we are interested in the low-temperature problem, we only consider electron scattering due to impurities and intravalley acoustic phonons. The neglect of optical-phonon scattering is justified as long as the heating of the electron system is not too significant. The inclusion of optical modes in the problem would require a detailed knowledge of just how these modes decay into acoustic phonons.

Since in the absence of optical modes the electrons are scattered quasielastically, the anisotropic part of the electron distribution function is small for weak electric fields. The phonon distribution will then also be approximately isotropic, and we are therefore essentially studying the energy exchange between the electron-phonon system. For simplicity, the electron distribution is represented by a Maxwell-Boltzmann distribution with a time-dependent electron temperature, the latter being determined by an energy-balance equation. The phonons are considered to interact only with the electrons; boundary scattering is neglected, as are phonon-phonon processes. The neglect of the latter limits the applicability of the present analysis to low "temperatures," while the neglect of boundary scattering is only valid for times short compared to $\tau_h \approx \eta L/c$.

In Sec. II we set up the coupled equations for the electron and phonon distributions under the assumptions of a simple band model for the crystal and an elastically isotropic crystal. In Sec. III the numerical method used to solve these equations is given, and the results are presented and discussed in Sec. IV.

II. COUPLED EQUATIONS FOR CARRIER AND PHONON DISTRIBUTIONS

The carrier distribution $f(\vec{k})$ is taken to be Maxwellian at a temperature T_e, such that

$$f(\vec{k}) = [n/N_c(T_e)]\, e^{-\epsilon_k/k_0T_e}\,, \qquad (2.1)$$

where n is the carrier concentration and $N_c(T_e)$ the effective density of states of the carriers:

$$N_c(T_e) = 2(2\pi m k_0 T_e/\hbar^2)^{3/2}\,,$$

where m is the effective mass of the carriers, ϵ_k their energy

$$\epsilon_k = \hbar^2 k^2/2m\,,$$

and k_0 is Boltzmann's constant.

The effect of the electrons on the evolution of the phonon distribution N_q is given by

$$\frac{\partial N_q}{\partial t} = \left(\frac{\partial N_q}{\partial t}\right)_e = -\frac{N_q - \overline{N}_q(T_e)}{\tau_{p\text{-}e}}\,, \qquad (2.2)$$

with[10]

$$\frac{1}{\tau_{p\text{-}e}(q,T_e)} = \frac{n\pi^{1/2} m^{1/2} E_1^2}{2^{1/2}\hbar\rho u (k_0T_e)^{1/2}\overline{N}_q(T_e)} \times \exp\left[-\frac{\hbar^2}{2mk_0T_e}\left(\frac{q}{2}+\frac{mu}{\hbar}\right)^2\right]\,. \qquad (2.3)$$

In Eqs. (2.2) and (2.3), $\overline{N}_q(T_e)$ is the Planck distribution at temperature T_e,

$$\overline{N}_q(T_e) = \frac{1}{\exp(\hbar\omega_q/k_0T_e) - 1}\,,$$

where ω_q is the frequency of the lattice vibration of wave vector $\vec{q}$. In Eq. (2.3), ρ is the mass density of the crystal, u is the longitudinal sound velocity in the medium, and E_1 is the acoustic-deformation-potential constant. Equation (2.2) simply expresses the tendency of the phonon distribution to evolve towards a Bose distribution at temperature T_e, and $\tau_{p\text{-}e}$ represents the mean time for the phonons to reach equilibrium with the electrons. Any additional energy-loss term for phonons could be easily added to Eq. (2.2) without providing any further computational difficulty.

Since the electron-momentum relaxation time τ is extremely short compared to the relatively

slow variations considered here, the current density is given by $j = ne\mu E$, where μ is the electron mobility corresponding to the instantaneous electron temperature and phonon distribution. The power input per electron is then given by μeE^2, where E is the electric field and where the mobility is given by the usual expression

$$\mu = e\langle\tau\rangle/m \ . \qquad (2.4)$$

Therefore, the energy balance for electrons gives

$$\left\langle \frac{d\epsilon}{dt} \right\rangle = \mu(t)\, eE^2 - \frac{1}{n} \sum_{\vec{q}} \hbar\omega_q \left(\frac{\partial N_q}{\partial t}\right)_e \ . \qquad (2.5)$$

In Eq. (2.4), the relaxation time τ is given by

$$\tau^{-1} = \tau_a^{-1} + \tau_I^{-1} \ , \qquad (2.6)$$

where τ_I and τ_a are the electron-momentum relaxation times due to impurities and phonons, respectively. We do not make use of the Brooks-Herring formulation of τ_I based on the Born approximation, which would give an incorrect value of the mobility at very low temperatures.[11] Nevertheless, we make the usual assumptions on the energy dependence of τ_I and on the temperature dependence of μ_I, where

$$\tau_I = C\epsilon^{3/2} \ , \qquad (2.7)$$

$$\mu_I = \alpha T_e^{3/2} \ . \qquad (2.8)$$

The electron relaxation time due to acoustic phonons τ_a is determined by the usual expression

$$\frac{1}{\tau_a} = \frac{mE_1^{\,2}}{4\pi\hbar^2\rho u}\, \frac{1}{k^3} \int_0^{2k} (N_q + \tfrac{1}{2})\, q^4 dq \ . \qquad (2.9)$$

From Eqs. (2.4), (2.6), (2.7), and (2.9), we see that μ is a function of N_q and T_e.

The evolution in time of the electron temperature $T_e(t)$ and of the phonon distribution $N_q(t)$ can be determined from Eqs. (2.2)–(2.9). It is readily verified that the electrons "follow" the phonons almost adiabatically; i.e., the phonon distribution may be considered instantaneously fixed when calculating the electron distribution. The ratio of the characteristic response times is essentially given by the ratio of the specific heats. This needs some modification in the present context, since only a small part of the phonon distribution effectively interacts with the electrons, namely, the long-wavelength one. Even with this reduction of the effective phonon specific heat, the latter remains quite large compared to the electronic specific heat. This allows us to neglect the $\langle d\epsilon/dt\rangle$ term in Eq. (2.5), which introduces a relative error of the order of 10^{-3}, as we have verified *a posteriori*. Equation (2.5) therefore becomes

$$\mu(N_q, T_e)\, eE^2 = \frac{1}{n} \sum_{\vec{q}} \hbar\omega_q \left(\frac{\partial N_q}{\partial t}\right)_e \equiv P \ . \qquad (2.10)$$

The calculation is carried out for the case of a typical n-type semiconductor with a simple band model.

III. NUMERICAL METHOD

According to our adiabatic approximation, immediately after the electric field is applied to the equilibrium system of electrons and phonons at helium temperature T_0, the electron temperature becomes T_e^0 while the phonon distribution is still the thermal equilibrium one $\overline{N}_q(T_0)$ at $T_0 = 4.2\,^\circ\mathrm{K}$. Therefore, we first perform a numerical evaluation of T_e^0 by substituting $\overline{N}_q(T_0)$ for N_q in Eqs. (2.2) and (2.9), and then determining T_e such that Eq. (2.10) is satisfied. Then a small time interval Δt is considered. During this time, N_q undergoes a small variation δN_q given by Eqs. (2.2) and (2.3). The energy balance is no longer satisfied so that a variation of T_e, δT_e must be introduced to restore the energy balance. Having thus determined the value of δN_q and δT_e, we then continue to the next time interval, etc. We have verified that the calculated electron temperatures and phonon distributions satisfy Eq. (2.10) to within about a few percent throughout the entire calculation.

The choice of Δt is not an arbitrary one. Let us consider the variation of $1/\tau_{p\text{-}e}$ with q at given T_e and n. This is shown in Fig. 1 for two values of T_e ($T_e = 53\,^\circ\mathrm{K}$ and $T_e = 32\,^\circ\mathrm{K}$, $n = 5\times10^{14}\ \mathrm{cm}^{-3}$). It is seen that the electron-phonon energy relaxation rate $\tau_{p\text{-}e}^{-1}$ has a maximum value $[(\tau_{p\text{-}e})^{-1}]_{\max}$, which depends on T_e. The exponential decrease of $(\tau_{p\text{-}e})^{-1}$ at high energies is simply due to the exponential tail of the electron distribution function, which determines the number of electrons capable of interacting with the high-frequency phonons. The time interval Δt is chosen such that

$$\Delta t (1/\tau_{p\text{-}e})_{\max} \ll 1 \ . \qquad (3.1)$$

Two programs were set up; the first one was used to calculate T_e^0; the second one was used to study the evolution of $T_e(t)$ and $N_q(t)$ with time. Both calculations were performed on an IBM 360-75.

IV. RESULTS AND DISCUSSION

Different carrier concentrations from 10^{13} to $5\times10^{16}\ \mathrm{cm}^{-3}$ and different field intensities from 5 to 20 V cm^{-1} were considered. We assume an acoustic-deformation-potential constant $E_1 = 10$ eV, which is typical for an n-type semiconductor. The electron effective mass m is taken as $0.25m_0$ (m_0 is the free-electron mass). Under these assumptions, the results obtained are shown in Table I and Figs. 2, 3, 5–9.

Table I shows the variations of T_e^0, the initial electron temperature, with the carrier concentration n and with the electric field intensity E. It is

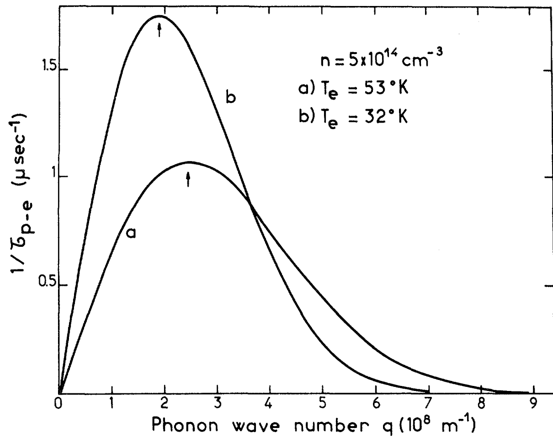

FIG. 1. Electron-phonon relaxation frequency $1/\tau_{p-e}$ vs phonon wave number q. The acoustic-deformation-potential constant is taken as $E_1 = 10$ eV.

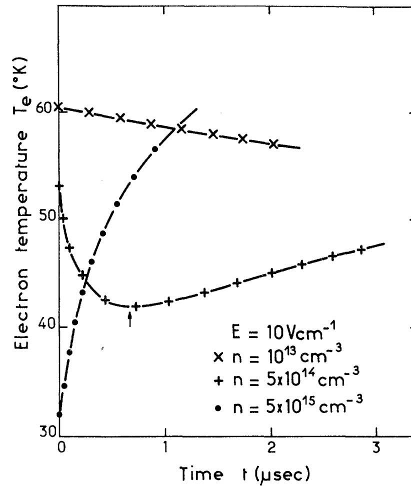

FIG. 3. Electron temperature $T_e(t)$ vs time t for three electron concentrations.

seen that T_e^0 is an increasing function of E and a decreasing function of the carrier concentration n. This latter result is expected, since the electron mobility $\mu(N_q, T_e)$ depends on the ionized impurity concentration (i.e., the carrier concentration) through the electron-impurity mobility: It is thus a decreasing function of n and, therefore, the rate at which energy is supplied to the carriers by the electric field also decreases with n at fixed T_e and E. The phonon-limited mobility and the rate of energy dissipation per electron are, on the other hand, independent of n.

From Figs. 2 and 3 it is seen that $T_e(t)$ exhibits two types of time dependence; it may first decrease and then increase with time, the minimum occurring at a time t_m, which is a function of the carrier concentration n and of the electric field E (type A), or it may increase monotonically with time (type B). The curves in Fig. 2 are of type A, and we see that t_m increases with the electric field. The dependence on n is shown in Fig. 3, where the evolution of T_e is presented for three different carrier concentrations. The upper curve corresponds to the lowest doping (10^{13} cm^{-3}), and the corresponding t_m occurs at a time longer than the scale of the figure. As the doping is increased to 5×10^{14} cm^{-3} in the second curve, the minimum moves in towards the origin. Finally, for $n = 10^{15}$ cm^{-3}, there is no longer a minimum, type-B behavior, and T_e increases monotonically with time.

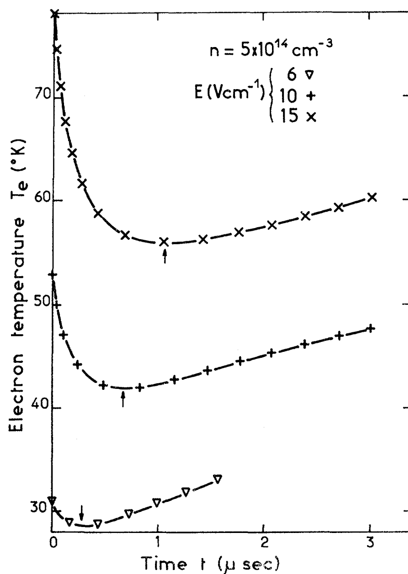

FIG. 2. Electron temperature $T_e(t)$ vs time t for different electric field intensities.

The existence of type-A behavior depends on the decrease in mobility due to an increase in the phonon population. When the variation of the mobility with phonon number is sufficiently weak, one observes type-B behavior. This is clearly seen in Fig. 3, where in the lower curve the mobility depends only weakly on N_q, because of the strong impurity scattering. This conclusion is readily demonstrated from Eqs. (2.10) and (2.2) in the case where the mobility is independent of N_q. We shall first show that dT_e/dt is initially positve. This follows from differentiating Eq. (2.10) at $t=0$:

$$eE^2 \frac{\partial\mu}{\partial T_e}\frac{dT_e}{dt} = \frac{\partial P}{\partial T_e}\frac{dT_e}{dt} - \frac{1}{n}\sum_{\vec{q}}\frac{\hbar\omega_q}{\tau_{p-e}}\frac{\partial N_q}{\partial t} . \tag{4.1}$$

Since $\partial N_q/\partial t$ is everywhere positive at $t=0$, dT_e/dt is positive if $\partial P/\partial T_e > eE^2\partial\mu/\partial T_e$. To show this, we consider the variation of T_e induced by an increment in the electric field δE for the case where *the phonons are assumed to remain in equilibrium*:

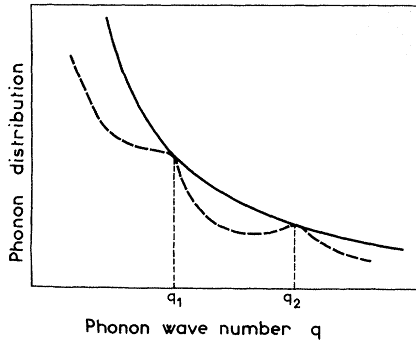

FIG. 4. The solid line gives the Planck phonon distribution at $T_e(t_2)$. The dashed line gives an arbitrary phonon distribution at the same time t_2 with $\partial N_q/\partial t = 0$ for the phonon wave numbers q_1 and q_2.

$$eE^2 \frac{\partial \mu}{\partial T_e} \delta T_e + 2e\mu E \delta E = \frac{\partial P}{\partial T_e} \delta T_e . \qquad (4.2)$$

The sign of δT_e in this case is determined by the same condition as above. For a normal system, we expect δT_e to be positive for an increase in the electric field, which is readily demonstrated explicitly for the impurity-scattering case. Therefore, dT_e/dt is initially positive.

We proceed to show that dT_e/dt remains nonnegative for a phonon-independent mobility by means of a proof by contradiction. If dT_e/dt is negative at some time $t > 0$, T_e obviously has a maximum at some time t_1. In this case, Eq. (4.1) at time t_1 yields

$$\left(\sum_{\vec{q}} \frac{\hbar \omega_q}{\tau_{p-e}} \frac{\partial N_q}{\partial t} \right)_{t=t_1} = 0 . \qquad (4.3)$$

This is clearly only possible if $\partial N_q/\partial t$ assumes negative values. By Eq. (2.2), on the other hand, we see that this requires $N_q > \bar{N}_q(T_e)$. Since $\partial N_q/\partial t$ is initially positive, there must be a time $t_2 \leq t_1$ such that for some q, $(\partial N_q/\partial t) = 0$, while $(\partial N_q/\partial t) > 0$ for all other q (Fig. 4). Since at the points of contact in Fig. 4, $N_q = \bar{N}_q(T_e)$ and $(\partial N_q/\partial t) = 0$, $\partial N_q/\partial t$ would therefore not become negative in the following interval Δt if $\bar{N}_q(T_e)$ was stationary. In fact, the only way that these curves can cross in this infinitesimal time interval is for $\bar{N}_q(T_e)$ to decrease; i.e., T_e must decrease. But this is impossible, since by hypothesis dT_e/dt is positive for $t < t_1$. We may therefore conclude that the nonmonotonic time dependence of T_e is due to the effect of the phonons on the mobility.

In Fig. 5 is shown a typical evolution of the phonon distribution; as expected, N_q increases monotonically with time and the heating of the phonon population is most significant for the long-wavelength phonons. It is readily seen that the number of phonons increases considerably even for times of the order of 1 μsec.

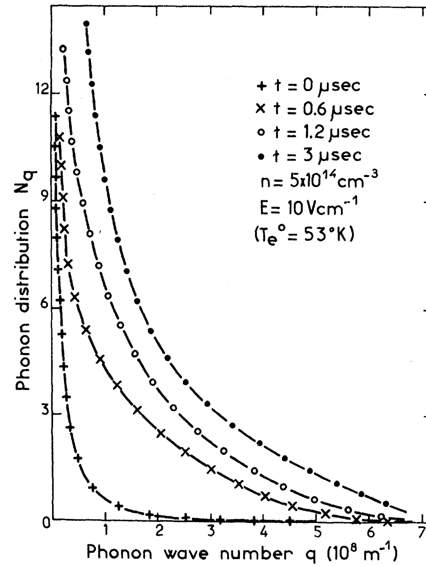

FIG. 5. Phonon distribution N_q vs phonon wave number q at different times. The curve at $t = 0$ μsec is the thermal-equilibrium phonon distribution $\bar{N}_q(T_0)$ at $T_0 = 4.2$ °K.

In Fig. 6 is shown the field dependence of the phonon distribution, calculated 1.2 μsec after the application of the electric field. We note that these curves cross one another, so that large electric fields (high electron temperatures) are most effective in heating the high-energy phonons at the expense of the low-energy phonons. This can be understood from Fig. 1, where we note a crossover in the $1/\tau_{p-e}$ curves at different electron temperatures.

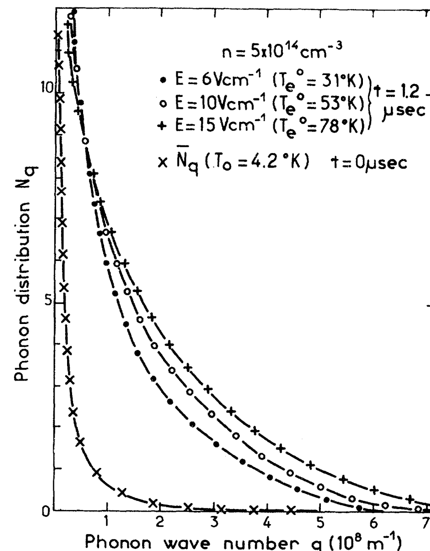

FIG. 6. Phonon distribution N_q vs wave number q for three electric field intensities at time $t = 1.2$ μsec.

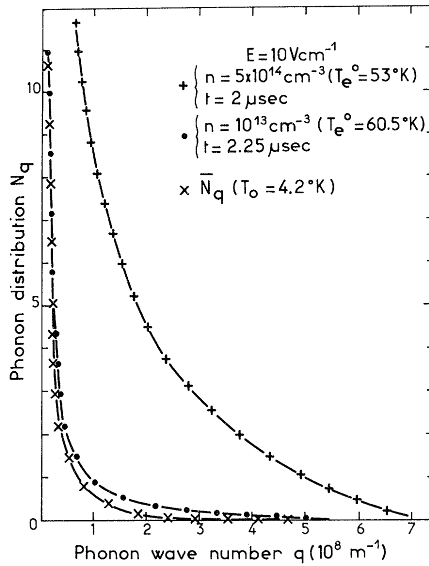

FIG. 7. Phonon distribution N_q vs phonon wave number q for two electron concentrations at about the same time.

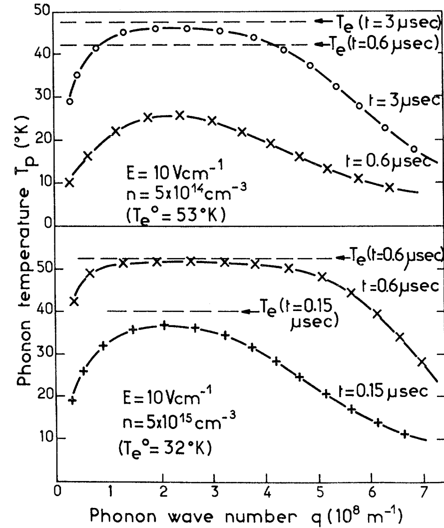

FIG. 9. Phonon temperature T_p vs wave number q at different times and different electron concentrations. The dashed lines represent the electron temperature.

The electron concentration dependence of N_q is shown in Fig. 7. As expected from the linear variation of $1/\tau_{p-e}$ with n, the increase in the phonon population depends strongly on the electron concentration. If in fact there were no impurity scattering, one could calculate a universal curve for the phonon distribution (at a given field), the time scale being simply linear in n^{-1}. This is readily seen from Eq. (2.2).

The time dependence of the mobility is shown in Fig. 8, where it is seen to decrease with time except in the lowest curve. The variation of the mobility is complicated since its dependence on T_e is nonmonotonic, being an increasing function of T_e when impurity scattering dominates, and a decreasing function of T_e for dominant phonon scattering. In addition there is obviously the N_q dependence of μ, which leads to a decrease in μ for an increase in N_q. These factors are all interdependent and, while a large carrier concentration tends to make the mobility only weakly N_q dependent, at least for small T_e, it also tends to make $\partial N_q/\partial t$ large. In general, a variety of behavior is possible for different carrier concentrations and electric fields.

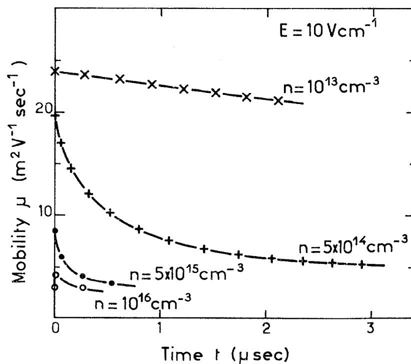

FIG. 8. Electron mobility μ vs time t for different impurity concentrations.

Let us now see to what extent a phonon temperature $T_p(q, t)$ may be usefully introduced by the definition

$$N_q(t) \equiv \frac{1}{\exp[\hbar\omega_q/k_0 T_p(q, t)] - 1} .$$

It is quite interesting to see how this temperature $T_p(q)$ depends on q at a given time. Where the electron-phonon interaction is strong, i.e., when $1/\tau_{p-e}$ is large, a phonon temperature $T_p(q)$ close to the electron temperature may be expected, since in the limit where $\tau_{p-e} \to 0$, $N_q \to \bar{N}_q(T_e)$, as can be seen in Eq. (2.2). The calculated $T_p(q)$ is shown in Fig. 9 for different times and carrier concentrations. It is seen that T_p depends strongly on q and has a maximum which corresponds to the maximum value of the $1/\tau_{p-e}$ curve (Fig. 1). $T_p(q)$ generally differs considerably from T_e, except in the neighborhood of the maximum, where it becomes nearly constant over a wide range of values of q, and close to T_e for long enough times. Therefore, a phonon temperature T_p could possibly be introduced in this case at least for intermediate-wavelength phonons.

We have so far considered the case of a typical nondegenerate n-type semiconductor. Let us now look at the particular case of n-type Ge for which

TABLE I. Calculated values of the initial electron temperature T_e^0 as a function of carrier concentration n and electric field intensity $E(E_1 = 10\ \text{eV})$.

	Electric field E (V/cm)									
	5	6	7	8	9	10	11	12	15	20
Carrier concentration n (1/cm^3)	Initial electron temperature (°K)									
10^{13}	33	39	44.5	50	55	60.5	65.5	70.5	85	108
5×10^{13}	30	35.5	41	47	52	57.5	62.5	67.5	82	105
10^{14}	29	34.5	40	46	51	56	61.5	66.5	81	104
5×10^{14}	25.5	31	37	42.5	48	53	58.5	63.5	78	101.5
10^{15}	23	29	34.5	40	45.5	51	56	61	76	99
5×10^{15}	7.5	10.5	15	20.5	26	32	37	43	58	82
10^{16}	5.5	6	7.5	10	13	18	23	29	45	70
5×10^{16}						5	5.5	6	7	15

two acoustic-deformation-potential constants Ξ_0 and Ξ_1 are, respectively, introduced in the rate of energy loss of electrons to acoustic modes and in the electron relaxation time. We take $\Xi_0 = 16.5$ eV and $\Xi_1 = 12.2$ eV.[12] Under these conditions the results are shown in Table II and Figs. 10 and 11 for a carrier concentration $n = 5\times10^{14}$ cm^{-3}. The curves in Fig. 10 are of type A. We note the influence of the acoustic-deformation-potential constant by comparing the curves a_1 and b in Fig. 10 and the two upper curves in Fig. 11: An increase of the acoustic-deformation-potential constant lowers the electron temperature. A comparison of our results for Ge with the work of Baumann *et al.*[7] is difficult since these authors only give the electron temperature and mobility at two times. Moreover, they use a value of the acoustic-deformation-potential constant considerably smaller than the usual one. In any case, their results do not appear to be even in qualitative agreement with our own.

V. CONCLUSION

The results presented above show that even a qualitative understanding of the response of a semiconductor to an intense electric field at low temperature requires a solution of the coupled dynamics of the electron-phonon system. Considerable variations of both the phonon and electron distribution occur even for time intervals as short as 1 μsec.

The phonon distribution is generally quite different from a Bose-like distribution and reflects the form of the electron-phonon relaxation time [Eq. (2.3)].

The time dependence of the electron temperature is generally nonmonotonic, the detailed form being a function of the impurity concentration and the applied electric field.

While we have assumed throughout that the electron concentration remains constant, it is interesting

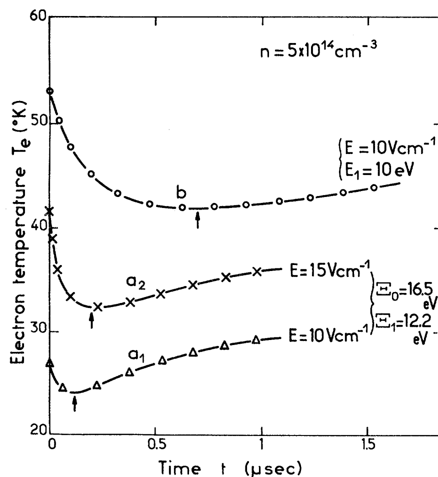

FIG. 10. Electron temperature vs time t : a_1, a_2 refer to n-type germanium; b is shown for comparison.

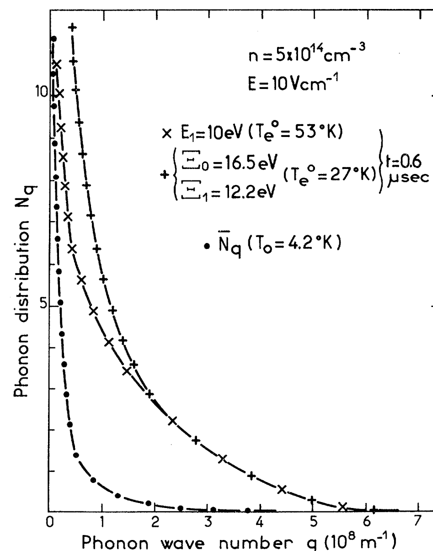

FIG. 11. Phonon distribution N_q vs wave number q.

TABLE II. Initial values of the electron temperature T_e^0 in n germanium at 4.2 °K for $n = 5\times 10^{14}$ cm^{-3}.

Electric field (V/cm)	5	6	7	8	9	10	11	12	15	20
T_e^0 (°K)	12	15	18	21	24	27	30	33	41.5	55

to consider the impact-ionization problem in the light of our results. If initially the electron temperature is sufficiently high to result in impact ionization, its subsequent decrease in time can result in deionization. At the same time, however, the phonon population is increasing and one has to reach some electron temperature below the critical temperature (corresponding to the initial breakdown field) for this to occur. As one loses electrons from the conduction band, τ_{p-e}^{-1} decreases and the phonons start to relax towards an equilibrium state at the temperature of the helium bath. This could be qualitatively represented by simply adding a relaxation term to Eq. (2.2):

$$\frac{\partial N_q}{\partial t} = \frac{\overline{N}_q(T_e) - N_q}{\tau_{p-e}} + \frac{\overline{N}_q(T_0) - N_q}{\tau_h} ,$$

where T_0 is the helium temperature and τ_h is the appropriate relaxation time discussed in the Introduction. This decrease in N_q then leads to an increase in T_e and a corresponding increase in n, etc. It seems quite plausible that under appropriate conditions this could lead to the type of instabilities associated with breakdown phenomena. This is rather difficult to treat quantitatively, since one requires a theory of impact ionization in the presence of time-dependent electrons and phonon distributions, the latter having a rather complicated q dependence.

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[1]V. V. Paranjape, Proc. Phys. Soc. (London) 80, 971 (1962).
[2]E. M. Conwell, Phys. Rev. 135, A814 (1964).
[3]H. Sato, J. Phys. Soc. Japan 18, 55 (1963).
[4]L. E. Gurevich and T. M. Gasymov, Fiz. Tverd. Tela 9, 106 (1967) [Sov. Phys. Solid State 9, 78 (1967)].
[5]V. V. Paranjape and B. V. Paranjape, Phys. Rev. 166, 757 (1968).
[6]K. Baumann, P. Kocevar, and M. Kriechbaum, J. Phys. Chem. Solids 31, 95 (1970).
[7]K. Baumann, M. Kriechbaum, and H. Kahlert, J. Phys. Chem. Solids 31, 1163 (1970).
[8]S. H. Koenig and R. D. Brown III, J. Phys. Chem. Solids 10, 201 (1959).
[9]J. F. Palmier, Phys. Rev. Letters 25, 13 (1970).
[10]J. M. Ziman, Phil. Mag. 1, 191 (1956).
[11]F. J. Blatt, Phys. Rev. 105, 1203 (1957).
[12]E. M. Conwell, *High Field Transport in Semiconductors*, Suppl. 9 of *Solid State Physics* (Academic, New York, 1957).