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New Proposition of the Mechanism of Self-Focusing of Laser Beams in Semiconductors*

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We wish to propose in this paper a new mechanism of the self-focusing of laser beams in semiconductors which rests on the energy-dependent scattering of carriers. While this is the only important mechanism of self-focusing in elemental semiconductors, our elementary analysis of the effect reveals that it can be of comparable significance with the one recently proposed by Tzoar and Gersten in nonparabolic semiconductors.

In recent papers, Tzoar and Gersten[1,2] have proposed a mechanism of self-focusing of laser beams in semiconductors arising from the nonparabolicity of the conduction-band structure, quite different from the conventionally known mechanisms.[3] They have shown that the nonparabolicity mechanism gives rise to an enormously high value of the nonlinear dielectric constant when compared with the other mechanisms, e.g., electrostriction,[4] Kerr effect,[5] nonlinear electronic polarization,[6] thermal effects,[7] etc., and thus is much more important in nonparabolic semiconductors.

In this paper, we wish to draw attention towards a new mechanism which has not been considered by Tzoar and Gersten[1,2] and previous workers,[3-7] and which turns out to be of considerable significance in semiconductors. This arises from the energy dependence of the relaxation time of the carriers in analogy with the energy dependence of their mass considered in Refs. 1 and 2.

To demonstrate the essential features of our mechanism, we make the following simplifying assumptions to make the mathematical model simpler.

(i) The electric vector of the laser beams is not so high as to significantly disturb the isotropic Maxwellian-Fermi-Dirac distribution at the lattice temperature. We shall assume further that it is possible to speak of an "average" energy of the carriers, which in case of Maxwellian distribution is $\frac{3}{2}k_0T$ (k_0 is the Boltzmann constant, T is the lattice temperature, and heating effects are neglected—which is presumably justified in view of high frequencies of laser beams) and in case of Fermi-Dirac is $\frac{3}{5}\epsilon_F$.

(ii) Only fundamental harmonic motion of electrons under laser excitation is of relevance to us; therefore, the higher harmonics would be neglected wherever they occur.

We shall be calculating the nonlinear contribution to the dielectric constant arising from the process of energy-dependent scattering (EDS) treating the semiconductor as parabolic. The self-focusing length may then be calculated by the standard eikonal technique of Akhmanov *et al.*[3] or the variational treatment of Tzoar and Gersten.[2] We shall not give these calculations since they are not of primary importance to this paper.

The equation of motion for electrons in an electric field $\vec{E}=\vec{E}_0e^{i(\omega t-\vec{k}\cdot\vec{r})}$ and dissipating momentum in collisions with the vibrating lattice and/or donor ions is

$$\hbar\frac{d\vec{K}}{dt}=-e\vec{E}_0e^{i(\omega t-\vec{k}\cdot\vec{r})}-\frac{\hbar\vec{K}}{\tau}, \tag{1}$$

where $\vec{K}$ is the average electron wave vector due to field $\vec{E}$, τ is the relaxation time, and other symbols have their usual meanings (here $\vec{k}$ is complex in general). To proceed further, we need to substitute the explicit dependence of relaxation time on energy; we assume here that there exists for the relaxation time a power law of the type

$$\tau=\tau_0(\bar{\epsilon}_E/\bar{\epsilon}_0)^{n/2}, \tag{2}$$

where $\bar{\epsilon}_E$ is the total average energy in the presence of the field (consisting of the thermal and drift parts) and $\bar{\epsilon}_0$ is the average energy in absence of the field. We may mention that for predominant momentum loss by ionized impurity scattering (which is quite likely in most of the semiconductors with electron concentration of the order of 10^{16} cm^{-3} at temperature ~80 °K) $n=3$ in parabolic semiconductors.[8] For other (single) scattering processes, $n=-1$ for dominant acoustic phonon scattering and +1 for dominant elastic optical-

phonon scattering. These details would be found necessary when we arrive at final expressions.

Substituting Eq. (2) in (1), and assuming that the drift energy is much smaller than the thermal energy, we obtain

$$\frac{d\vec{K}}{dt} = -\frac{e\vec{E}_0}{\hbar}\exp[i(\omega t - \vec{k}\cdot\vec{r})] - \frac{\vec{K}}{\tau_0}\left(1 - \frac{n\hbar^2K^2}{4m\bar{\epsilon}_0}\right) . \tag{3}$$

Rather than attempting a general solution of nonlinear differential equation (3), we solve it by the iterative process realizing that the term making it nonlinear is small in our considerations. Thus, the first-order iterative solution is

$$\vec{K} = -\frac{e\vec{E}_0\tau_0}{\hbar}\left(1 - \frac{ne^2E_0^2\tau_0^2}{4m(1+\omega^2\tau_0^2)\bar{\epsilon}_0} - i\omega\tau_0\right) \times\left(\omega^2\tau_0^2 + 1 - \frac{ne^2E_0^2\tau_0^2}{2m(1+\omega^2\tau_0^2)\bar{\epsilon}_0}\right)^{-1}\exp[i(\omega t - \vec{k}\cdot\vec{r})] . \tag{4}$$

From Eq. (4), it is straightforward to obtain the nonlinear dielectric constant. Making the assumption that $\omega^2\tau_0^2 \gg 1$ (as is the case for laser frequencies), and using the relation $\epsilon_{eff} = \epsilon_l + 4\pi\sigma_i/\omega = \epsilon_0 + \epsilon_2E_0^2$, where the effective dielectric constant is the sum of the contributions from the lattice (ϵ_l) and the electronic part, it is possible to obtain the nonlinear dielectric constant $\epsilon_2E_0^2$ as (here σ_i is the imaginary part of the electronic conductivity)

$$\epsilon_2E_0^2 = -\frac{4\pi Ne^2}{m\omega^2}\left(\frac{ne^2}{4m\bar{\epsilon}_0\omega^2}\right)\frac{E_0^2}{1+\omega^2\tau_0^2} . \tag{5}$$

We note here that the coefficient ϵ_2 (in the notation of Tzoar and Gersten[2]) becomes zero if $\tau_0 \to \infty$ and/or $n \to 0$, as it should be (the latter corresponds to a constant relaxation time). In the same notation of Ref. 2, we can also obtain

$$\epsilon_0 = \epsilon_l - \frac{4\pi Ne^2}{m\omega^2}\left(1 - \frac{1}{1+\omega^2\tau_0^2}\right) \simeq \epsilon_l - \frac{4\pi Ne^2}{m\omega^2} . \tag{6}$$

At this stage, we may compare our result with the one obtained by Tzoar and Gersten. As expected, Eq. (6) is the same as the corresponding expression of Ref. 2. Further, denoting our result [Eq. (5)] as $(\epsilon_2)_{EDS}$ (i.e., due to energy-dependent scattering) and Tzoar and Gersten's result as $(\epsilon_2)_{NP}$ [i.e., due to nonparabilicity], we may obtain

$$(\epsilon_2)_{EDS}/(\epsilon_2)_{NP} = -\tfrac{4}{3}n(c^*/\bar{v})^2(\omega\tau_0)^{-2} . \tag{7}$$

Thus, although the product $(\omega\tau_0)^{-2}$ is much less than unity, the ratio in Eq. (7) is not necessarily small, since in general, c^*, which has been termed as "relativistic" speed by Tzoar and Gersten, is much greater than $\bar{v}$, the average speed of carriers in thermal equilibrium. If the crystal under consideration is nondegenerate, $\bar{v}$ can be replaced by $(3k_0T/m)^{1/2}$ and $(6\epsilon_F/5m)^{1/2}$ for degenerate semiconductors. For InSb, if we use the same parameters as in Ref. 2, the ratio in Eq. (7) turns out to be of the order of 10^{-2}, and indeed our mechanism is relatively less significant in InSb than the nonparabolicity mechanism. Nevertheless, in other III-V semiconductors, this ratio is of the order of unity primarily since in semiconductors other than InSb, the bottom-of-the-conduction-band mass is not unusually small. Also, we may recall that for ionized impurity scattering our mechanism becomes more and more important as τ_0 decreases with increase in donor concentration.

We note from Eq. (5) that for positive values of n (i.e., for scattering mechanisms which become less and less effective as energy of the carrier increases) our mechanism predicts defocusing of beam rather than focusing (since ϵ_2 is negative[3]). Thus, for ionized impurity scattering and elastic optical-phonon scattering, defocusing of beam would occur if only our mechanism is prevalent; for acoustic-phonon scattering focusing would occur as usual. This can bear some interesting consequences. In III-V semiconductors, where it is generally felt that acoustic-phonon scattering plays a negligible role (except at very low temperatures), the two mechanisms of nonparabolicity and energy-dependent scattering would be in opposition to each other. Since the focusing-defocusing phenomenon is additive in character, it is possible in principle that there may not be any change in the undisturbed situation (i.e., beam radius uniform throughout) if the two mechanisms exactly annul each other. In normal situations, we may expect that the focusing length may be considerably increased beyond 2 mm (the focusing length calculated by Tzoar and Gersten for InSb taking normal parameters). In elemental semiconductors (Ge, Si), however, predominant momentum loss occurs by acoustic-phonon scattering for all practical range of temperatures, and therefore focusing should be observed. Thus, in these semiconductors, the observance of focusing or defocusing may itself lead to a possible knowledge of predominant scattering process (particularly the character whether net scattering increases or decreases with increase in energy). In III-V semiconductors, a comparison of theoretical and experimental self-focusing lengths can possibly lead to some knowledge of scattering processes.

We may point out, finally, some of the basic differences in the two mechanisms. The first and the most obvious is that the EDS mechanism is the only important mechanism in elemental semiconductors, where it is well known that the conduction-band structure is almost parabolic even near the edges of the Brillouin zone. The second is that consideration of EDS mechanism, in addition to the NP mechanism, takes into account the loss of the power of the beam due to absorption by the carriers. This part, which is contained in Eq. (4),

was omitted in arriving at Eq. (5). This was advertently neglected in view of the inequality $\omega^2\tau_0^2 \gg 1$, and assuming the sample length to be small. Moreover, it has been demonstrated earlier by Sodha *et al.*[9] that inclusion of absorption, unless very high, only slightly affects the self-focusing length (for a positive coefficient of absorption, it results in increase) and primarily changes the field pattern inside the sample, which is not relevant to us here. Thirdly, besides a linear absorption term there is a part contained in Eq. (4) which corresponds to nonlinear absorption, quadratic in field. This gives rise to interesting effects such as those discussed in Ref. 3. Essentially, inclusion of nonlinear absorption results in increase of self-focusing length. We had neglected this part also. However, if one wishes to take account of absorption for long samples, calculations can be made proceeding with Eq. (4) by evaluating the real part of the oscillatory current.

We finally conclude that the EDS mechanism outlined in this paper may contribute significantly to the self-focusing phenomenon of laser beams in semiconductors.

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Study of V_2O_3 by X-Ray Photoelectron Spectroscopy

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The band structure of V_2O_3 below the Fermi level has been studied by x-ray photoelectron spectroscopy both above and below the Néel temperature. Aside from normal bands associated with atomic states from 20 to 640 eV, a valence band with structure was observed in the range 0–10 eV. The results are compared to soft-x-ray-emission and -absorption results and to theoretical predictions concerning the band structure of V_2O_3.

I. INTRODUCTION

We report in this paper on the band structure of V_2O_3 below the Fermi level, as determined by x-ray photoelectron spectroscopy (commonly termed ESCA). Early applications of this technique have been reviewed by Siegbahn and co-workers[1]; additional studies have been reported for metals,[2,3] semiconductors,[4] and near insulators.[5–7] As far as we are aware, the technique has not been applied to materials which undergo a semiconductor-metal transition, such as V_2O_3. At 170 K this compound exhibits sharp changes in physical properties as a concomitant to alterations in magnetic and crystallographic properties. These matters are reviewed elsewhere.[8–11]

II. EXPERIMENTAL

The investigations were carried out with a Hewlett-Packard model No. 5950 A spectrometer with provision for dispersion compensation. Monochromatic Al$K\alpha_{1,2}$ radiation was obtained from a quartz-crystal disperser at high Bragg angle. Electrons, which are ejected by the interaction of