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Suppression of Zeeman splitting in a GaAs quantum dot

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The Zeeman splitting of the first excited level of a two-dimensional parabolic GaAs quantum dot is studied perturbatively in the presence of the polaronic interaction. It is shown that the Zeeman effect is suppressed considerably in a GaAs quantum dot because of the polaronic effect and becomes strongly size dependent if the dot size is reduced below a few nanometers. [S0163-1829(99)51312-X]

Recent years have witnessed a flurry of investigations[1] in the field of quantum dots which are ultra-low-dimensional structures in which the motion of the charge carriers is confined in all the spatial directions. The confinement length or the effective size of a quantum dot is typically of the order of a few nanometers. At this length scale, quantum effects become extremely important and therefore these objects may be regarded as ideal quantum systems. Indeed, these systems show many novel physical properties[2] that are quite different from those of their bulk counterparts and are extremely interesting from the point of view of fundamental physics and for their potential applications in microelectronic device technology, for example, in ultrafast computers.

One of the recent interests in the area of quantum dots has been to investigate the role of electron-phonon interactions[3] on the electronic properties. In this connection, polaronic effects in quantum dots have been studied quite extensively.[4] However, to our knowledge, no attempt has so far been made to obtain direct experimental evidence of the polaronic effects in quantum dots. These effects, if present, will influence the transport and optical properties of a quantum dot device quite significantly. It is therefore very important, in this context, to calculate polaronic effects that can be measured experimentally and thus the existence or otherwise of these effects can be substantiated unambiguously. In the present paper we make an attempt in this direction. We calculate the Zeeman splitting of the first excited level of a two-dimensional (2D) polar semiconductor quantum dot with parabolic confinement in the presence of an external magnetic field applied normal to the plane of the dot and apply our results to a GaAs quantum dot. For the sake of mathematical simplicity we shall neglect the size quantization of phonons and treat the relevant phonon modes within the framework of the Fröhlich model. This model is certainly not very rigorous for very small confinement lengths but may still serve as a good enough approximation to capture some of the most important electron-phonon interaction effects in polar quantum dots.

The Hamiltonian for a magnetopolaron in a 2D quantum dot with parabolic confinement can be written in the symmetric gauge as

$$H'=-\frac{\hbar^2}{2m^*}\nabla^2_{\rho'}-i\hbar\frac{\omega_c'}{2}\left(x'\frac{\partial}{\partial y'}-y'\frac{\partial}{\partial x'}\right)+\frac{1}{2}m^*\omega'^2\rho'^2+\hbar\omega_{LO}\sum_{\mathbf{q}'}b^\dagger_{\mathbf{q}'}b_{\mathbf{q}'}+\sum_{\mathbf{q}'}(\xi'_{\mathbf{q}'}e^{-i\mathbf{q}'\cdot\boldsymbol{\rho}'}b^\dagger_{\mathbf{q}'}+\mathrm{H.c.}),\quad(1)$$

where all vectors are two dimensional, $\boldsymbol{\rho}(x',y')$ is the position vector of the electron and m^* is its Bloch effective mass, $\omega'=[\omega_h'^2+(\omega_c'^2/4)]^{1/2}$, ω_h' being the frequency of the confining harmonic potential and ω_c' the bare cyclotron frequency, ω_{LO} is the dispersionless longitudinal optical (LO) phonon frequency, $b^\dagger_{\mathbf{q}'}(b_{\mathbf{q}'})$ is the creation (annihilation) operator for an LO phonon of wave vector $\mathbf{q}'$, and $\xi'_{q'}$ is the electron-phonon interaction coefficient. We shall use the Feynman units in which the energy is scaled by $\hbar\omega_{LO}$, length by r_o where $r_o=q_o^{-1}$, q_o being an inverse length defined by $\hbar^2q_o^2/m^*=\hbar\omega_{LO}$, volume by r_o^2 and wave vectors by q_o. In these units the Hamiltonian (1) reads

$$H=H_o+H_{ep}=H_e+H_p+H_{ep},\quad(2)$$

with

$$H_e=-\frac{1}{2}\nabla^2_{\boldsymbol{\rho}}+\frac{\omega_c}{2}L_z+\frac{1}{2}\omega^2\rho^2,\quad(3)$$

$$H_p=\sum_{\mathrm{q}}b^\dagger_{\mathrm{q}}b_{\mathrm{q}},\quad(4)$$

$$H_{ep}=\sum_{\boldsymbol{\rho}}(\xi_q e^{-i\mathbf{q}\cdot\boldsymbol{\rho}}b^\dagger_{\mathbf{q}}+\mathrm{H.c}),\quad(5)$$

where $\boldsymbol{\rho}(x,y)=\boldsymbol{\rho}'/r_o$, $\mathbf{q}=\mathbf{q}'/q_o$, $\omega_h=\omega_h'/\omega_{LO}$, $\omega_c=\omega_c'/\omega_{LO}$, $\omega=\omega'/\omega_{LO}=(\omega_h^2+\frac{1}{4}\omega_c^2)^{1/2}$, $L_z=-i[x(\partial/\partial y)-y(\partial/\partial x)]$ and $|\xi_q|^2=(\sqrt{2}\pi\alpha/vq)$, where v is the dimensionless area of the 2D dot and α is the electron-phonon coupling constant.

Since most polar semiconductor quantum dots available today are weak electron-phonon systems, we employ the second-order Rayleigh-Schrödinger perturbation theory (RSPT) to obtain the ground- and the first excited-state magnetopolaron self-energy corrections. The second-order RSPT correction to the electron self-energy due to the polaronic interaction is given by

$$\Delta E_{nm}=-\sum_{n'm'}\sum_{\mathrm{q}}\frac{|\langle\psi^{(o)}_{n'm'}(\boldsymbol{\rho})|\xi_q e^{-i\mathbf{q}\cdot\boldsymbol{\rho}}|\psi^{(o)}_{nm}(\boldsymbol{\rho})\rangle|^2}{(E^{(o)}_{n'm'}-E^{(o)}_{nm}+1)},\quad(6)$$

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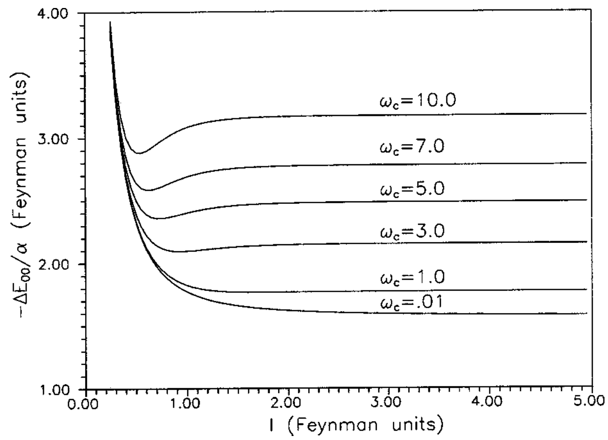

FIG. 1. Polaronic corrections ΔE_{oo} (in Feynman units) to the GS energy of an electron in a 2D parabolic quantum dot as a function of the confinement length l (in Feynman units) for several values of the magnetic field.

where

$$H_e \psi_{nm}^{(o)}(\boldsymbol{\rho}) = E_{nm}^{(o)} \psi_{nm}^{(o)}(\boldsymbol{\rho}), \tag{7}$$

$$\psi_{nm}^{(o)}(\boldsymbol{\rho}) = \frac{1}{\sqrt{2\pi}} e^{im\theta} \left(\frac{2\omega n!}{(n+|m|!)} \right)^{1/2} \times (\sqrt{\omega}\rho)^{|m|} L_n^{|m|}(\omega\rho^2) e^{-\omega\rho^2/2}, \tag{8}$$

$$E_{nm}^{(o)} = (2n+|m|+1)\omega + \frac{m}{2}\omega_c, \tag{9}$$

where $L_n^{|m|}(\omega\rho^2)$ is the associated Laguerre polynomial. Zhu and Gu[5] have also considered the same weak-coupling magnetopolaron problem but have restricted their study to the strong magnetic-field limit. They have evaluated Eq. (6) by taking $n'=n$ and $m'=m$. Even in the strong field limit, this approximation is not very attractive for large confinement lengths and for small confinement lengths also it introduces sizable errors that can make a qualitative difference in the behavior of the energies. One may notice that the infinite sum over n', m' occurring in Eq. (6) is the Green's function for the unperturbed problem. For the magnetic field alone the corresponding Green's function was first derived by Sondheimer and Wilson[6] and for a magnetic field with a three-dimensional harmonic oscillator potential it was first obtained by Lepine and Matz.[7] In the present case the unperturbed problem involves a two-dimensional harmonic oscillator in a magnetic field. This Green's function can be obtained exactly for all values of the magnetic field for the ground state and for small values of the magnetic field ($\omega + \omega_c/2 < 1$) for the first two excited states. We obtain

$$G = \sum_{n'm'} \frac{\psi_{n'm'}^{*}(\boldsymbol{\rho})\psi_{n'm'}(\boldsymbol{\rho}')}{(E_{n'm'}^{(o)} - E_{nm}^{(o)} + 1)} = \int dt e^{-(1-E_{nm}^{(o)})t} \frac{\omega}{2\pi \sinh(\omega t)} \times \exp\left[-\frac{\omega}{2}\{\rho^2+\rho'^2\}\coth\omega t - 2\boldsymbol{\rho}\cdot\boldsymbol{\rho}' \frac{\cosh(\omega_c t/2)}{\sinh(\omega t)} - 2i(x'y - y'x)\frac{\sinh(\omega_c t/2)}{\sinh(\omega t)} \right]. \tag{10}$$

We finally get

$$E_{nm} = E_{nm}^{(o)} + \Delta E_{nm},$$

where E_{nm} is the perturbed energy and

$$\Delta E_{oo} = -\frac{\alpha\sqrt{\pi}\sqrt{\omega}}{2} \int_o^\infty dt \frac{e^{-t}}{[1 - e^{-\omega t}\cosh(\omega_c t/2)]^{1/2}}, \tag{11}$$

$$\Delta E_{o,\pm 1} = \frac{-\alpha\sqrt{\pi}\sqrt{\omega}}{\sqrt{2}} \int_o^\infty dt \times \frac{e^{-[1-\omega_\mp(\omega_c/2)]t}[2f(g\mp h) + h^2 - f^2]}{(1-e^{-2\omega t})[f(gf+h^2)]^{3/2}}, \tag{12}$$

where $f = 1 + \coth(\omega t) - \cosh(\omega_c t/2)/\sinh(\omega t)$, $g = 1 + \coth(\omega t) + \cosh(\omega_c t/2)/\sinh(\omega t)$, and $h = \sinh(\omega_c t/2)/\sinh(\omega t)$. It may be noted that results (11) and (12) are exact to order α.

We define the renormalized cyclotron frequencies as $\omega_\pm^* = (E_{o,\pm 1} - E_{oo})/\hbar$ and the corresponding cyclotron masses as $m_\pm^* = m^*(\omega_c/\omega_{c\pm}^*)$. It is possible to obtain simple analytical expressions for the magnetopolaron self-energy corrections in different limiting cases. However, we shall present below our numerical results for a wider range of the parameter values.

In Fig. 1 we plot the ground-state (GS) polaronic correction to the electron self-energy, $-\Delta E_{oo}/\alpha$, as a function of the dimensionless confinement length l. We find that for small values of the magnetic field, the behavior is qualitatively similar to what one would observe in the absence of the magnetic field.[8] For instance, the polaronic effects can be extremely large if the dot is sufficiently small in size. Furthermore, as the confinement length increases, the polaronic correction decreases. This decrease is very rapid for small l but becomes very slow when l exceeds a certain value. At very large l the polaronic correction becomes essentially independent of l and gives the bulk limit. For large values of the magnetic field, however, we find that the behavior of the polaronic correction as a function of the confinement length is much more interesting. One can observe that in this case

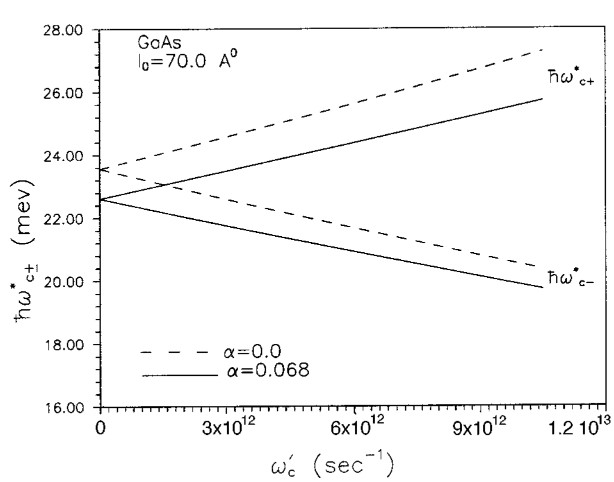

FIG. 2. Renormalized cyclotron resonance frequencies for a GaAs quantum dot of effective size 70 Å as a function of the magnetic field.

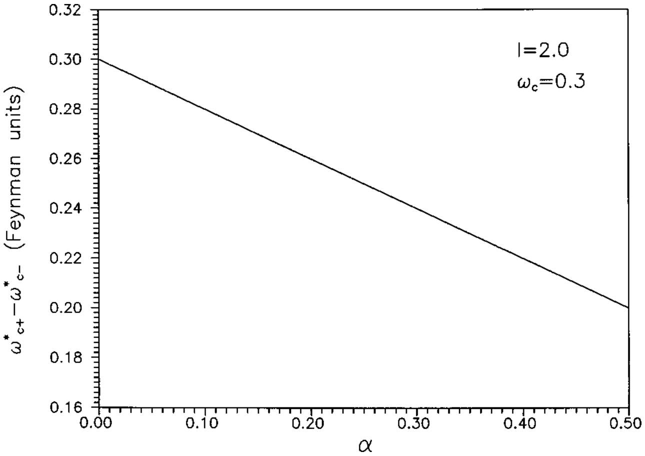

FIG. 3. Zeeman splitting (in Feynman units) as a function of α for a quantum dot with $l=2.0$ and for $\omega_c=0.3$.

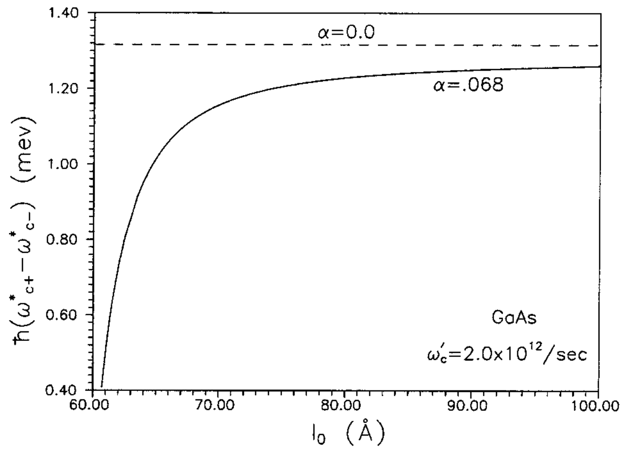

FIG. 4. Zeeman splitting (in meV) for a GaAs dot as a function of the confinement length (in Å) for a particular value of the magnetic field.

the polaronic correction develops a minimum at some value of l which is smaller than 1. As the magnetic field increases, this minimum becomes deeper and deeper and shifts towards smaller values of l.

It is well known that the twofold degeneracy of the first excited level of a 2D parabolic dot potential is lifted in the presence of a magnetic field. This is the so-called Zeeman effect that splits the bare cyclotron frequency ω_c into two cyclotron frequencies ω_{c+}^* and ω_{c-}^*. With increasing magnetic field ω_{c+}^* increases, while ω_{c-}^* decreases. We have studied the behavior of the renormalized cyclotron resonance frequencies as a function of ω_c' in a GaAs quantum dot incorporating the electron-phonon interaction. The results are shown in Fig. 2 in which we also plot the case for $\alpha=0$ for the sake of comparison. One can notice that when the polaronic interaction is taken into account the cyclotron resonance frequencies decrease and furthermore their variation with the magnetic field also becomes slower, more so for larger magnetic fields. We have also studied the variation of the cyclotron resonance masses as a function of the confinement length l_o in a GaAs dot (not shown here). We find that the values of the cyclotron masses diminish with the reduction in the confinement length. Comparison with the $\alpha=0$ case shows that the polaronic interaction enhances the cyclotron masses.

It may be more useful from the point of view of experimental observation to study the behavior of the Zeeman splitting $[\hbar(\omega_{c+}^*-\omega_{c-}^*)]$ directly. Figure 2 shows that in a GaAs quantum dot the rate of increase of the Zeeman splitting with the magnetic field is slower than what would have been obtained if there were no electron-phonon interactions. This suggests that the polaronic interactions would suppress the Zeeman effect in a quantum dot. To confirm this finding we study the variation of the Zeeman splitting as a function of α in a quantum dot. The results are shown in Fig. 3. It is clear that the Zeeman splitting indeed decreases monotonically with increasing α in a linear way. The behavior can, however, become slightly nonlinear if higher-order perturbative corrections are included. Finally we show in Fig. 4 how the Zeeman effect would depend on the size of a polar quantum dot which is the most crucial issue in the present investigation. It is evident from the figure that in the absence of any polaronic interaction the Zeeman splitting is essentially independent of the confinement length, while for $\alpha=0.068$, i.e., in a GaAs quantum dot it is found to be strongly size dependent below a certain value of l_o. In fact, the Zeeman splitting decreases very rapidly with decreasing dot size below a few nanometers. We believe that this is a very interesting theoretical observation and should be experimentally measurable. Furthermore, this interesting property can be usefully exploited to obtain any desired resonance absorption behavior in a GaAs quantum dot by tuning the frequency of the confining potential or the effective dot size. The strong size dependence of the Zeeman splitting is not difficult to understand. In a parabolic quantum dot of polar semiconductors the excited unperturbed states $n=o$, $m=\pm1$ (plus zero phonon) strongly mix with the $n=0$, $m=0$ plus one phonon state resulting in the devaluation of the axial angular momentum of the pure first excited states of the quantum dot potential. This explains the suppression of the Zeeman splitting in the presence of the electron-phonon interaction. It is now well known that when the confinement length becomes comparable to the polaron size, the polaronic effects become extremely pronounced and increase sharply with the decrease in the confinement length. Thus it is expected that in the presence of the polaronic interaction the Zeeman effect will be strongly suppressed if the effective dot size is reduced below a few nanometers.

In conclusion, we have shown that for small magnetic fields the GS polaronic correction to the electron self-energy in a weak-coupling quantum dot increases monotonically as the dot size decreases and becomes considerably larger if the confinement length is made sufficiently small, while for large magnetic fields we observe that the polaronic correction develops a minimum that becomes deeper and deeper and shifts towards smaller values of l with increasing magnetic field. We have furthermore shown that the polaronic effect suppresses quite considerably the Zeeman splitting of the first excited level of a 2D parabolic GaAs quantum dot if the dot size is reduced below a few nanometers. This size-dependent Zeeman suppression is in our opinion a very interesting result which is a clear manifestation of the quantum size effect. The proposed size dependence of the Zeeman splitting should be observable through infrared magneto-optical experiments and would give an unambiguous experimental evidence of the existence of polarons in quantum dots.

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