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Coulomb drag as a signature of the paired quantum Hall state

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Motivated by the recent Coulomb drag experiment of M. P. Lilly *et al.* [Phys. Rev. Lett. **80**, 1714 (1998)], we study the Coulomb drag in a two-layer system with Landau-level filling factor $\nu=1/2$. We find that the drag conductivity in the incompressible paired quantum Hall state at zero temperature can be finite. The drag conductivity is also greatly enhanced above T_c, at which the transition between the weakly coupled compressible liquids and the paired quantum Hall liquid takes place. We discuss the implications of our results for the recent experiment. [S0163-1829(99)51712-8]

A double-layer system of two-dimensional electron gases (2DEG) allows an unusual measurement of scattering mechanism.[1,2] If there is no tunneling between the two layers, momentum can be transferred only via electron-electron scattering due to the interlayer Coulomb interaction. As a result, if a current is driven through one of the subsystems (active layer), then another current is induced in the other system (passive layer). The magnitude of the induced current is a measure of the interlayer scattering rate. In real experiments, an induced voltage is measured in the passive layer where no current flows. The ratio between the measured voltage in the passive layer and the driven current in the active layer is the so-called transresistivity or the drag resistivity. In the case of a double-layer 2DEG system in the absence of external magnetic field, only the quasiparticles within an energy band of width kT near the Fermi surface participate in scattering processes. This leads to a T^2 temperature dependence of the drag resistivity at low temperatures.[3,4] When the filling fraction becomes one-half in the presence of high magnetic fields, the 2DEG in each layer supports an unusual form of compressible liquid.[5] Namely, the quasiparticles of the half-filled Landau level are composite fermions, which are the electrons with Chern-Simons flux quanta attached to them. Chern-Simons field fluctuations due to the density fluctuations of electrons lead to a more singular low-energy interlayer scattering rate.[6–8] Theoretically it was found that the drag resistivity goes as $T^{4/3}$ for a pure system and $T^2\ln T$ for a diffusive system.

Recently, Coulomb drag measurement was done for double layers of half-filled Landau levels.[1] In the experiment, it was indeed found that the drag resistivity is much enhanced compared to that of 2DEG in the zero magnetic field. However, even though the temperature dependence can be fit to $T^{4/3}$ for a range of intermediate temperatures, the experiment revealed much richer physics at low temperatures. It was observed that (a) the drag resistivity has a minimum at a certain temperature below which the drag becomes very sensitive to disorder and the applied current; and (b) the drag resistivity *seems* to be finite at the zero temperature.

Motivated by this experiment, we study the Coulomb drag in the paired quantum Hall state limit. Incompressible paired quantum Hall states with two electron species were suggested some years ago, based on both numerical simulations and effective action approaches.[9–12] In particular, it was suggested that, in double layers of Landau-level filling factor $\nu=1/2$, composite fermions in one layer can establish the pairing correlation with composite fermions in the other layer below a certain temperature, T_c.[12] Though such a pairing correlation of composite fermions, which is responsible for the incompressibility of the paired quantum Hall state, *does not* lead to conventional long-rang order of electrons, it does introduce short-range pairing correlation of electrons, i.e., quantum fluctuations of electron pairs. The following question arises: How does the short-range pairing correlation developed by electrons in the incompressible phase affect the Coulomb drag?

In this paper, we study the transport properties of this incompressible phase and the temperature dependence of various transport coefficients. We find the following results. (i) At $T=0$, the drag conductivity can be finite in the incompressible paired quantum Hall state. Its temperature dependence for $T<T_c$ strongly depends on disorder. (ii) Above T_c, the drag conductivity is enhanced by $\sigma_{12}^{xx}\propto(e^2/\hbar)(k_F l)^{-2}T/(T-T_c)$. Here $k_F^{-1}=l_B$ is the Fermi wavelength and much shorter than the mean free path of the electrons l. The Hall drag conductivity exhibits a similar enhancement near T_c. We also obtain the drag resistivities below and above T_c. We discuss the implications of these results to the experiment and suggest that the observed anomaly could be interpreted as a signature of the formation of an incompressible double-layer paired quantum Hall state at low temperatures.

In the framework of composite fermion theory,[3] the response functions of electrons can be expressed in terms of those of composite fermions; as a consequence, the in-plane conductivity, Hall conductivity, and drag conductivity, as well as Hall drag conductivity can be expressed in terms of the composite fermion polarizabilities:

$$\sigma_{11,12}^{xx}=\frac{\text{Im}}{2\Omega}\lim_{\Omega,\mathbf{Q}\to 0}\left(\frac{\pi_-^{cc}}{1+c^2\pi_-^{cc}\pi_-^{dd}}\pm\frac{\pi_+^{cc}}{1+c^2\pi_+^{cc}\pi_+^{dd}}\right)$$

$$\sigma_{11,12}^{xy}=\frac{e^2}{8\pi\hbar}\lim_{\Omega,\mathbf{Q}\to 0}\left(\frac{c^2\pi_-^{dd}\pi_-^{cc}}{1+c^2\pi_-^{cc}\pi_-^{dd}}\pm\frac{c^2\pi_+^{dd}\pi_+^{cc}}{1+c^2\pi_+^{cc}\pi_+^{dd}}\right), \tag{1}$$

where $\pi_\pm^{cc}=\pi_{11}^{cc}(i\Omega,\mathbf{Q})\pm\pi_{12}^{cc}(i\Omega,\mathbf{Q})$, $\pi_\pm^{dd}=\pi_{11}^{dd}(i\Omega,\mathbf{Q})\pm\pi_{12}^{dd}(i\Omega,\mathbf{Q})$. $\pi_{\alpha\beta}^{dd}$ and $\pi_{\alpha\beta}^{cc}$ denote the density-density and current-current polarization matrices of composite fermions, respectively, defined in the space of the layer index $\alpha,\beta=1,2$. $c=i4\pi/Q$ comes from the Chern-Simons transformation. In the incompressible double-layer quantum Hall liquid limit, we introduce the Green's functions of composite fermions defined in a generalized Nambu space in Matsubara representation[12]

$$\hat{G}=\begin{pmatrix}\tilde{G}, & \tilde{F}\\ \tilde{F}^+, & -\tilde{G}\end{pmatrix},\quad \tilde{G}=\begin{pmatrix}G_{11}, & 0\\ 0, & G_{22}\end{pmatrix},$$

$$\tilde{F}=\begin{pmatrix}0, & F_{12}\\ F_{21}, & 0\end{pmatrix}. \tag{2}$$

Here $\tilde{G}$ and $\tilde{F}$ are defined in layer-index space,

$$G_{11,22}=\frac{i\omega+\xi_{\mathbf{p}}}{\omega^2+\Delta^2+\xi_{\mathbf{p}}^2},\quad F_{12,21}=\frac{\Delta}{\omega^2+\Delta^2+\xi_{\mathbf{p}}^2}, \tag{3}$$

where $\xi_{\mathbf{p}}=\mathbf{p}^2/2m-\epsilon_F$ and $\omega=(2n+1)\pi T$. We first consider the clean limit $\tau\Delta\gg 1$, where τ is the elastic mean free time. Δ is determined by the self-consistent equation $\Delta(\omega)=T\Sigma_\Omega g(\Omega)F_{12}(\omega-\Omega)$, where $g(\Omega)$ is the interaction constant in the interlayer particle-particle channel. The external field vertices are renormalized accordingly,[13,14] as shown in Fig. 1(a). To simplify the calculation we neglect the energy dependence of $g_{12}(\Omega)$ and Δ. We also ignore intralayer Fermi liquid renormalization effects. To the leading order in the small parameter Δ/ϵ_F, when $v_FQ,\Omega\ll\Delta$, the diagrams in Fig. 1(a) yield

$$\hat{\Gamma}_0=\hat{\tau}_3+\hat{\tau}_2\frac{\Omega\Delta}{\Omega^2+v_s^2Q^2},\quad \hat{\mathbf{\Gamma}}=\mathbf{v}_F, \tag{4}$$

where $v_s=v_F\alpha_0(T)/\sqrt{2}$, v_F is the Fermi velocity, and $\alpha_0(T)$ is a temperature-dependent constant. $\alpha_0(T=0)=1$ and for $T\sim T_c$ (where $\Delta\sim 0$), $\alpha_0(T)=\sqrt{7\zeta(3)\Delta/2\pi^2T}\ll 1$. $\zeta(x)$ is the Riemann ζ function

$$\hat{\tau}_2=\begin{pmatrix}0, & \tau_1\\ -\tau_1, & 0\end{pmatrix},\quad \hat{\tau}_3=\begin{pmatrix}\tau_0, & 0\\ 0, & -\tau_0\end{pmatrix},$$

where τ_0, τ_1 are the unity matrix and x-component Pauli matrix in the layer space, respectively. We have chosen the Coulomb gauge $\nabla\cdot\mathbf{A}=0$ so that the vertex corrections to $\hat{\mathbf{\Gamma}}$ are zero. It is worth emphasizing that the vertex corrections in $\hat{\Gamma}_0$ are essential for preserving the gauge invariance of the theory.

The polarizability can be calculated in terms of the diagrams in Fig. 1(b). Taking into account $\hat{G},\hat{\Gamma},\hat{\mathbf{\Gamma}}$ given in Eqs. (2) and (4), we obtain the results in the incompressible paired quantum Hall liquid limit. At low temperatures $T\ll T_c$ in the limit $v_FQ\ll\Omega\ll\Delta$, we have

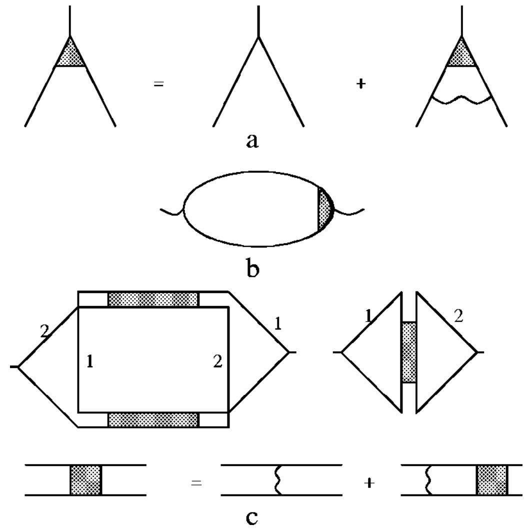

FIG. 1. (a) Diagrams for vertex corrections. Solid lines represent the composite fermion Green's functions $\hat{G}$ defined in Nambu space; shaded triangles represent the renormalized vertices. The wavy line stands for the irreducible interaction vertex. (b) Diagrams for the polarization of composite fermions. (c) Diagrams for the drag conductivity above the critical temperature T_c. Solid lines with index 1, 2 are composite fermion Green's functions in layers 1, 2, respectively.

$$\pi_+^{dd}(i\Omega,\mathbf{Q})=e^2\frac{\partial n}{\partial\mu}\left[\alpha_1(T)\frac{v_s^2Q^2}{-\Omega^2+v_s^2Q^2}+\beta_2(T)\Xi_1(\Omega,\mathbf{Q})\right],$$

$$\pi_-^{dd}(i\Omega,\mathbf{Q})=e^2\frac{\partial n}{\partial\mu}\beta_1(T)\Xi_1(\Omega,\mathbf{Q}),$$

$$\pi_+^{cc}(i\Omega,\mathbf{Q})=-\frac{N_se^2}{m}[1-\beta_1(T)\Xi_2(\Omega,\mathbf{Q})],$$

$$\pi_-^{cc}(i\Omega,\mathbf{Q})=-\frac{N_se^2}{m}[1-\alpha_1(T)-\beta_2(T)\Xi_2(\Omega,\mathbf{Q})]. \tag{5}$$

Here $\partial n/\partial\mu=m/2\pi$ is the thermodynamic density of states, m is the mass of composite fermions, and N_s is the superfluid density. $\beta_1(T)\approx\beta_2(T)$ and $\beta_2(T)=1-\alpha_1(T)$; $\alpha_1(T)$ is given by

$$\alpha_1(T)=\begin{cases}1-2\sqrt{\dfrac{\pi T}{\Delta}}\exp\left(-\dfrac{\Delta}{T}\right), & T\ll T_c,\\[2mm] \dfrac{\pi\Delta}{4T}, & T_c-T\ll T_c.\end{cases} \tag{6}$$

$\Xi_{1,2}$ have the following forms:

$$\Xi_1=\left(\frac{\partial n}{\partial \mu}\right)^{-1}\sum_{\mathbf{p}}\frac{n_F(\epsilon_{\mathbf{p}+\mathbf{Q}})-n_F(\epsilon_{\mathbf{p}})}{-\Omega+\xi_{\mathbf{p}+\mathbf{Q}}-\xi_{\mathbf{p}}+i\delta},$$

$$\Xi_2=\frac{2m}{N_s}\sum_{\mathbf{p}} v_F^2\frac{n_F(\epsilon_{\mathbf{p}+\mathbf{Q}})-n_F(\epsilon_{\mathbf{p}})}{-\Omega+\xi_{\mathbf{p}+\mathbf{Q}}-\xi_{\mathbf{p}}+i\delta}, \tag{7}$$

which were studied in detail in Ref. 3. When $\Omega \gg v_F Q$, $\Xi_{1,2}\propto Q^2/\Omega^2$.

The results in Eqs. (5), (6), and (7) can be interpreted in terms of two-fluid model. π_+^{dd} is the sum of the condensate contribution, which is proportional to α_1, and the quasiparticle contribution, which is proportional to $\beta_{1,2}$. π_+^{cc} is determined mainly by the condensate component. Asymmetrical polarizations π_-^{cc}, π_-^{dd} have contributions mainly from thermally excited quasiparticles. At $T \ll T_c$, the quasiparticle contributions are exponentially small because of the energy gap in the spectrum. At temperatures close to T_c, the condensate contribution becomes small. Following Eqs. (5), (6), and (7), we find that $\sigma_{11}^{yy}=0$, $\sigma_{11}^{xy}=e^2/4\pi\hbar$ for this incompressible paired quantum Hall state. The drag conductivity also vanishes in this limit, $\sigma_{12}^{yy}=\sigma_{12}^{xy}=0$ at $T<T_c$.

In the presence of random impurity potentials $V_{1,2}(\mathbf{r})$ in layer 1, 2, the composite fermions in different layers experience different random potentials. Composite fermions in layer 1 have to pair with those in layer 2 with a different spectrum. In this case the Hamiltonian acquires additional terms: $\frac{1}{2}[V_1(\mathbf{r})+V_2(\mathbf{r})](\psi_1^\dagger\psi_1+\psi_2^\dagger\psi_2)+\frac{1}{2}[V_1(\mathbf{r})-V_2(\mathbf{r})](\psi_1^\dagger\psi_1-\psi_2^\dagger\psi_2)$. Here ψ_1,ψ_2 are the composite fermion operators in layers 1 and 2, respectively. The second term acts like a random Zeeman magnetic field on composite fermions and effectively leads to the suppression of Δ. The impurity potentials pin the Chern-Simons flux in space[3] and break the time-reversal symmetry of the composite fermion system. This results in a further suppression Δ. Thus, in the strong disorder limit, the underlying composite fermion system becomes gapless.

In the weak disorder limit, $\tau\Delta \gg 1$, the energy gap of the quasiparticles remains open. The change of the superfluid density N_s and the sound velocity v_s is proportional to $1/\tau\Delta$ and is negligible. However, the quasiparticle contributions are dramatically changed. The longitudinal polarization $\Xi_1 = DQ^2/i\Omega$ takes the diffusion form, while the transverse one $\Xi_2=\tau^{-1}/(i\Omega+\tau^{-1})$ is Drude-like. Taking these into account, we find

$$\sigma_{12}^{xx}=\frac{\beta_1 k_F l}{1+\beta_1\beta_2(k_F l)^2}\frac{e^2}{\hbar},\quad \sigma_{12}^{xy}=\frac{-1}{1+\beta_1\beta_2(k_F l)^2}\frac{e^2}{8\pi\hbar} \tag{8}$$

in the weak disorder limit at $T \ll T_c$. $\beta_{1,2}$ are given by Eq. (6), with Δ evaluated in the presence of disorder. Since Δ, which appears in the low-temperature asymptotic forms of $\beta_{1,2}$, is a function of the elastic scattering rate τ^{-1} itself, the drag conductivity as a function of temperature strongly depends on disorder. When T becomes close to T_c, $\sigma_{12}^{xx}=(1/k_F l)e^2/\hbar$ and $\sigma_{12}^{xy}=-(1/k_F l)^2 e^2/\hbar$. On the other hand, they become exponentially small when T goes to zero. A similar temperature dependence of the drag conductivity was also found in electron-hole double-layer system.[13] It is easy to confirm that, in both the pure and disordered limit, the following equalities hold:

$$\sigma_{11}^{xx}-\sigma_{12}^{xx}=0,\quad \sigma_{11}^{xy}-\sigma_{12}^{xy}=\frac{e^2}{4\pi\hbar}. \tag{9}$$

Equation (9) can be attributed to the incompressibility of the paired quantum Hall state and does not depend on disorder.

In the limit $\tau\Delta \ll 1$, the energy gap in the quasiparticle spectrum disappears. In this case, $\beta_{1,2}$ become of order unity even at $T=0$ and the exponential decay of the drag conductivity at low temperatures does not occur. As a result, for $T \ll T_c$, $\sigma_{12}^{yy}\approx(1/k_F l)e^2/\hbar$, $\sigma_{12}^{xy}\approx(1/k_F l)^2 e^2/\hbar$, remaining finite even at zero temperature. It is worth pointing out that in general Δ has energy dependence. However, note that π_+^{dd} in Eq. (5) manifests the existence of the Bogoliubov-Anderson mode in the spontaneously symmetry-broken state and π_+^{cc} reflects the off-diagonal long-range order in the composite fermion system. Thus, Eq. (8) follows as a consequence of the incompressibility of the paired quantum Hall state and does not depend on the detailed structure of Δ.

At high temperatures, the double-layer composite fermions are weakly coupled with each other. However, when the critical temperature T_c is approached, the current-current polarizability diverges due to the strong pairing fluctuations of composite fermions in the two layers. This is similar to the situations discussed in Ref 15. We find $\pi_{12}^{cc}(i\Omega,0)=i\Omega\sigma_{12}^{CF}$ in the $\Omega\to 0$ limit, where

$$\sigma_{12}^{CF}=\frac{e^2}{128\hbar}\frac{\eta^2 D^2}{T^3}\int d^2\mathbf{Q}\int_0^{+\infty} d\Omega\frac{Q^2 \mathrm{Im}L^R(\Omega,Q^2)}{\sinh^2\left(\frac{\Omega}{2T}\right)}\left[\mathrm{Im}L^R+\frac{64T^2\eta^{-2}D^{-2}}{\pi^3(l^{-2}+Q^2)Q^2}\mathrm{Im}\Psi\left(\frac{1}{2}+\frac{i\Omega+\eta DQ^2}{4\pi T}\right)\right]. \tag{10}$$

Here $\eta=7\xi(3)/2\pi^3 T\tau$ and Ψ is the digamma function. The effective interlayer interaction is calculated in terms of the diagrams in Fig. 1(c),

$$L^R(\Omega,Q^2)=\left(\frac{T-T_c}{T}+\frac{\pi}{8}\frac{\eta DQ^2+i\Omega}{T}\right)^{-1}. \tag{11}$$

We assume the temperature is close to T_c and $T-T_c \ll \tau^{-1}$. To leading order in $\tau(T-T_c)$, the contribution from the second term in Eq. (10) is negligible. Taking into account Eqs. (1) and (10) we obtain

$$\sigma_{12}^{xx}=\frac{\pi}{4(k_F l)^2}\frac{T}{T-T_c}\frac{e^2}{\hbar},\quad \sigma_{12}^{xy}=\frac{-\pi}{8(k_F l)^3}\frac{T}{T-T_c}\frac{e^2}{\hbar}. \tag{12}$$

Meanwhile $\sigma_{11}^{xx}=(2\pi/k_F l)e^2/\hbar$ and $\sigma_{11}^{xy}=e^2/4\hbar$ in zeroth-order perturbation theory with respect to the pairing fluctuations. When $T/(T-T_c)\sim k_F l$, $\sigma_{12}^{xx}\sim\sigma_{11}^{xx}$ and the perturbation method breaks down.

In experiments, the drag resistivity is measured. The drag resistivity tensor can be obtained by inverting the conductivity tensor presented above. At $T > T_c$, taking into account Eq. (12), we get the corresponding drag resistivity

$$\rho_{12}^{xx} = \frac{\pi}{4(k_F l)^2} \frac{T}{T - T_c} \frac{\hbar}{e^2}, \tag{13}$$

which *increases* as the temperature is decreased toward T_c. The Hall drag resistivity is always zero in this model. The contribution discussed in the previous papers,[6,7] without taking into account the contribution from the pairing fluctuations, is a monotonically *decreasing* function of temperature, i.e., $(l_B T/d\epsilon_F)^{4/3}\hbar/e^2$. Here d is the interlayer spacing, assumed to be larger than the magnetic length. Since the contribution due to pairing fluctuations *diverge* as T_c is approached, we find that as far as

$$\frac{T}{T - T_c} \geqslant (k_F l)^2 \left(\frac{l_B T}{d\epsilon_F} \right)^{4/3}, \tag{14}$$

the result discussed here always overwhelms the contributions in Refs. 6 and 7. T_c is estimated as $(l_B/d)^2 \epsilon_F$ in Ref. 12. When $d \gg l_B$ and $T_c \ll \epsilon_F$, Eq. (14) can be easily satisfied. Thus the drag resistivity can develop a minimum as a function of temperature around T_c.

At $T < T_c$, following Eq. (8), we obtain

$$\rho_{11}^{xx} = \rho_{12}^{xx} = \frac{2}{\beta_1(T) k_F l} \frac{\hbar}{e^2}, \tag{15}$$

which indicates that the drag resistivity diverges at low temperatures in the weak disorder limit when a gap still exists. In the strong disorder limit, β_1 is of order unity even at $T = 0$ and ρ_{12}^{xx} remains finite at $T < T_c$. We therefore suggest that the transition between the incompressible paired quantum Hall state and the weakly coupled compressible double-layer state could be responsible for the anomalous temperature dependence of the drag resistivity observed in the experiment.[1]

In Ref. 1, no pronounced divergence was observed at low temperatures. Instead, drag resistivity was shown to be saturated at low temperatures. This at first seems to indicate that a gapless limit was reached in the experiment. In Ref. 1, $d \sim l_B \sim k_F^{-1} \sim 200$ Å; the in-plane longitudinal resistance is close to 3000 Ω and $l \sim k_F^{-1}$. Indeed, this yields $\tau\Delta \sim 1$, implying a gapless situation. However, to derive Eqs. (8) and (9), we have assumed that, in the low-temperature phase, thermal fluctuations are negligible. When $|T - T_c|/T_c \leqslant 1/k_F l$, fluctuations are strong and the results in Eqs. (8) and (13) are invalid. This sets the limit of the theory when compared with the experiment *quantitatively*. For the situation where $k_F l \sim 1$, the transition regime where thermal fluctuations are large could be of the same order as T_c. It is plausible that the lowest temperature in the experiment is still in the critical regime and the low-temperature incompressible phase discussed in this paper was smeared out in Ref. 1. To distinguish the gapless situation and the thermal fluctuation effects, we suggest studying double-layer systems with $d \gg l_B$, where the gapless limit can be reached ($\tau\Delta < 1$), while $\tau\epsilon_F$ is still greater than unity so that the critical regime is narrow.

Further complications arise when the pairing wave function also becomes inhomogeneous in space in the presence of macroscopic inhomogeneities in the sample. The drag current is then carried by electron pairs traveling along the percolating paths, which are strongly dependent on impurity configurations and the amplitude of applied currents. Finally, in the strong disorder limit, the mean-field approach is questionable due to strong quantum phase fluctuations present even at zero temperature. Solutions to these complications remain open.

Recently we became aware of a related work where the effect of the pairing fluctuation is also studied.[16] The discrepancy between some of the results in our initial manuscript and those of Ref. 16 was due to different boundary conditions.[17]

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