RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 59, NUMBER 6 1 FEBRUARY 1999-II

Electronic vortex structure and quasiparticle scattering in the cuprate superconductor $Nd_{2-x}Ce_xCuO_y$

R. P. Huebener, O. M. Stoll, and S. Kaiser
Physikalisches Institut, Lehrstuhl Experimentalphysik II, Universität Tübingen, Morgenstelle 14, D-72076 Tübingen, Germany
(Received 2 November 1998)

In a superconductor with *s*-wave symmetry of the order parameter, in the superclean limit, the density of states $N(\varepsilon)$ of the quasiparticles in the vortex core remains zero up to the minigap ε_0 where $N(\varepsilon)$ shows a sharp upturn. Another strong increase of $N(\varepsilon)$ occurs near the superconducting energy gap Δ. These features of $N(\varepsilon)$ have important consequences for the electric-field dependence of the flux-flow resistivity. A phenomenological discussion of the resulting effects is presented and related to the two intrinsic steps in the flux-flow resistance of the cuprate superconductor $Nd_{2-x}Ce_xCuO_y$ observed recently. [S0163-1829(99)50106-9]

The accepted picture for describing the electronic structure of the vortex cores in the mixed state of the classical superconductors is a cylinder residing in the normal state. The radius of this normal cylinder is given by the coherence length ξ. Here the underlying assumption is the validity of the dirty limit: the energy smearing $\delta\varepsilon=\hbar/\tau_p$ due to the mean electronic scattering time τ_p is larger than the energy gap Δ. Here $\hbar$ is Planck's constant divided by 2π. Apparently, in the classical superconductors the condition $\hbar/\tau_p>\Delta$ is well satisfied for the quasiparticles in the vortex cores. As an important consequence the phase space for quasiparticle scattering in the vortex cores is identical to that in the normal state. In combination with the concept of vortex motion due to an applied electric transport current (Lorentz force) these ideas yield the Bardeen-Stephen model of the flux-flow resistivity.[1]

In the cuprate superconductors the situation is much different, since the cuprates reside in the clean or even superclean limit because of their small coherence length ξ. Now we have the condition $\hbar/\tau_p\ll\Delta$, and the energy spectrum of the quasiparticles in the vortex core can be obtained only from a detailed quantum-mechanical calculation. Here Andreev reflection[2] of the quasiparticles at the boundary of the vortex cores plays a central role. The energy spectrum of the Andreev bound states in an isolated vortex line has been calculated by Caroli, De Gennes, and Matricon,[3] and also by Bardeen *et al.*[4] For the energy levels ε_i (measured from the Fermi energy ε_F) they obtained

$$\varepsilon_i=(n+\tfrac{1}{2})\frac{\Delta^2}{\varepsilon_F}, \tag{1}$$

where n is an integer. The ratio Δ^2/ε_F can also be cast in the form $\Delta^2/\varepsilon_F=2\hbar^2/(m\xi^2)$ (m=quasiparticle mass). Since in the cuprate superconductors the coherence length is nearly 100 times smaller than in the classical superconductors, the level distance Δ^2/ε_F in the former is up to 10^4 times larger than in the latter. The lowest bound state lies at the energy $\varepsilon_0=\frac{1}{2}\Delta^2/\varepsilon_F$ above the Fermi energy and is referred to as the minigap. In the case $\hbar/\tau_p<\Delta^2/\varepsilon_F$ we deal with the superclean limit. Kramer and Pesch[5] and subsequently Bardeen and Sherman[6] have shown that in the limit $T\ll T_c$ the vortex core shrinks with decreasing temperature, leading to an increased level spacing, such that Eq. (1) must be replaced by

$$\varepsilon_i=\left(n+\frac{1}{2}\right)\frac{\Delta^2}{\varepsilon_F}\ln\left(\frac{T_c}{T}\right). \tag{2}$$

Hence, the minigap is increased to the value $\varepsilon_0=\frac{1}{2}(\Delta^2/\varepsilon_F)\ln(T_c/T)$.

The electronic structure of the vortex cores in the clean or superclean limit is expected to have important consequences for the quasiparticle scattering in the core region and, hence, for the flux-flow resistivity. In the following we examine these consequences using only phenomenological concepts. This discussion is strongly motivated by our recent observation of an intrinsic step structure in the flux-flow resistance of the cuprate superconductor $Nd_{2-x}Ce_xCuO_y$ (NCCO).[7]

We restrict our discussion to a cuprate superconductor with *s*-wave symmetry of the order parameter, such as NCCO. We assume negligible coupling between the CuO_2 planes such that we deal with a nearly two-dimensional electronic system. The vortex axis is oriented parallel to the crystallographic *c* axis. Since the density of states (DOS) $N(\varepsilon)$ remains zero up to the minigap ε_0 above and below the Fermi energy, in the limit $k_BT\ll\varepsilon_0$ quasiparticle scattering in the vortex cores is strongly reduced (k_B=Boltzmann's constant). In our discussion we assume that the DOS $N(\varepsilon)$ in the vortex cores is approximately unchanged due to the electric field generated by the vortex motion. In the complete absence of scattering, under the influence of the Lorentz force of an applied electric current the vortices will move along with the current. Only an electric Hall field will be generated and dissipation remains zero. However, any residual quasiparticle scattering causes a component of the vortex motion perpendicular to the applied current and a resistive electric field F.[8–12] Due to this field the energy of the quasiparticles is shifted to the value $\varepsilon=eFv_F\tau_p$ (Ref. 13) (e=elementary charge, v_F=Fermi velocity). If this energy shift reaches the value of the minigap we have

$$\varepsilon=\varepsilon_0=\frac{1}{2}\frac{\Delta^2}{\varepsilon_F}\ln\left(\frac{T_c}{T}\right)=eF_1v_F\tau_p \tag{3}$$

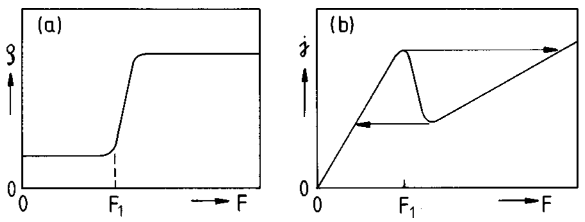

FIG. 1. (a) Electric resistivity ρ versus electric field F with a step in resistivity at the field F_1. (b) Current density j versus field F resulting from $\rho(F)$ of part (a). Negative differential resistivity starts at F_1. Current bias yields the jumps and hysteresis indicated by the arrows.

and the DOS $N(\varepsilon)$ available for quasiparticle scattering strongly increases. We denote the field at which this happens by F_1. As a result the flux-flow resistivity strongly increases with electric field and we encounter negative differential resistivity. This is shown schematically in Fig. 1. For simplicity, in our discussion we ignore flux pinning. For current biased operation we obtain a step in the resistive voltage and hysteresis. We claim that the first of the two intrinsic resistive flux-flow voltage steps that we have observed recently in NCCO films near 1.9 K,[7] is caused by exactly this effect. Depending upon the magnetic field B this first step appeared at 100–200 μV, corresponding for the samples used to the field value $F_1=(3-5)\times 10^{-3}$ V/cm in the range $B=0.8-1.6$ T. Taking $F_1=4\times 10^{-3}$ V/cm and the following values for NCCO: $\Delta=4$ meV,[14] $v_F=10^7$ cm/s,[15] $\varepsilon_F=30$ meV, $T_c=21.3$ K, and $T=1.9$ K, we obtain from Eq. (3) $\varepsilon_0=0.63$ meV and $\tau_p=1.6\times 10^{-8}$ s. Such a long scattering time is just the signature of the strongly reduced phase space for quasiparticle scattering for energies below the minigap. Multiplying this value of τ_p with the Fermi velocity v_F yields the ballistic path length $l=v_F\tau_p=1.6$ mm. This length is much longer than the typical dimensions of the NCCO samples we have studied experimentally:[7] thickness $d=90-100$ nm, width $w=40$ μm, length $L=360$ μm. However, we must keep in mind that we are dealing with the quasiparticles trapped in the vortex core and undergoing many Andreev reflections during their energy shift when exposed to the electric field F. This is shown schematically in Fig. 2. This concept of multiple Andreev reflections requires that the quantity $k_F\xi_{ab}$ is sufficiently large compared to 1 (k_F=Fermi wave vector). From the parameter values for NCCO given above and taking $\xi_{ab}=8$ nm,[14] we have $k_F\xi_{ab}=6-7$. From this the idea of multiple Andreev reflections appears reasonable. The ballistic path length $l=1.6$ mm corresponds to a total of $l/2\xi_{ab}=10^5$ Andreev reflections. It is only in this way that the quasiparticles in the vortex core can ''climb up'' to the energy ε_0 of the minigap. For the value $\tau_p=1.6\times 10^{-8}$ s the energy smearing is $\hbar/\tau_p=4.1\times 10^{-8}$ eV and can be neglected compared to the minigap ε_0.

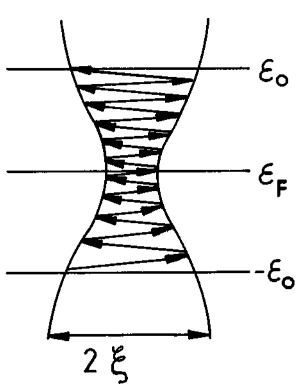

FIG. 2. In the presence of the electric field F generated by flux flow, in the limit $T\rightarrow 0$, the quasiparticles occupying the bound states in the vortex core below the Fermi energy ε_F are raised energetically. They undergo many Andreev reflections in bridging the energy gap to the next higher and unoccupied bound-state level.

It is important to note that the electronic vortex structure expressed in Eqs. (1)–(3) has been calculated in the limit $B\rightarrow 0$ where the interaction between the vortices remains negligible. On the other hand, the experiments of Ref. 7 have been performed at intermediate magnetic fields. At 1.9 K the first resistive flux-flow voltage step has been observed up to about $B=1.6$ T. Because of the vortex-vortex interaction the value of the minigap in Eqs. (1)–(3) may have to be modified. However, calculations regarding this question have not been reported and are highly interesting.

We still have to compare the time $\tau_p=1.6\times 10^{-8}$ s with the life time τ_L of the vortex while traversing the sample. Denoting the vortex velocity by v_φ, we have $\tau_L=w/v_\varphi$. Using the relation $F=v_\varphi B$ and Eq. (3) we obtain

$$\frac{\tau_L}{\tau_p}=\frac{ewBv_F}{\varepsilon_0} \tag{4}$$

yielding $\tau_L/\tau_p=6.3\times 10^3$ for the values given above and for $B=1$ T. We see that the lifetime τ_L of the vortex is sufficiently long compared to τ_p and does not restrict the quasiparticle energy shift up to ε_0. Furthermore, Eq. (4) indicates that the ratio τ_L/τ_p is independent of the electric field, as one would expect.

We have seen that the energy shift of the quasiparticles from their exposure to the electric field, as expressed in Eq. (3), results in a strong increase of the flux-flow resistivity at the field value F_1. Next we examine this field-dependent resistivity $\rho(F)$ in more detail. The observed voltage jump at fields in the range $F_1=(3-5)\times 10^{-3}$ V/cm combined with hysteresis[7] indicates the appearance of negative differential resistivity [see Fig. 1(b)]. As we have discussed in more detail elsewhere,[16] this requires that the flux-flow resistivity $\rho(F)$ increases stronger than linearly with the electric field. A phenomenological model for this behavior must go beyond the concepts we have used above and which have lead to Eq. (3). In our case it is the energy dependence $\tau_p(\varepsilon)$ of the scattering time which must be included. From a Taylor expansion we have

$$\tau_p(\delta\varepsilon)=\tau_p(0)+\frac{\partial\tau_p}{\partial\varepsilon}\,\delta\varepsilon. \tag{5}$$

$\delta\varepsilon$ is the energy increment of the quasiparticles resulting from their drift velocity $\delta\mathbf{v}$ in the electric field $\mathbf{F}$:

$$\delta\varepsilon=e\cdot\mathbf{F}\cdot\delta\mathbf{v}\cdot\tau_\varepsilon. \tag{6}$$

τ_ε is the quasiparticle energy relaxation time (which can be different from and is generally longer than the scattering time τ_p). Writing

$$\delta \mathbf{v} = \delta \mathbf{p}/m = e \cdot \mathbf{F} \cdot \tau_p / m \tag{7}$$

(p = momentum) and inserting this into Eq. (6), we obtain

$$\delta\varepsilon = \frac{e^2F^2\tau_\varepsilon\tau_p}{m}, \tag{8}$$

and using Eq. (5)

$$\tau_p(\delta\varepsilon) = \tau_p(0)\left[1 + \frac{\partial\tau_p}{\partial\varepsilon}\frac{e^2F^2\tau_\varepsilon}{m}\right]. \tag{9}$$

At the energy near the minigap ε_0, where the DOS $N(\varepsilon)$ strongly increases with ε, the derivative $\partial\tau_p/\partial\varepsilon$ is negative. The field-dependent flux-flow resistivity is then given by

$$\rho(F) - \rho(0) = \frac{\rho(0)\alpha F^2}{1-\alpha F^2} \tag{10}$$

with

$$\alpha = \frac{e^2\tau_\varepsilon}{m}\left|\frac{\partial\tau_p}{\partial\varepsilon}\right| \tag{11}$$

being a positive quantity. Equations (8)–(10) explain the step in the flux-flow resistivity at the field F_1 observed for current-biased operation. In particular, they yield the stronger than linear increase of $\rho(F)$ with the electric field, needed for the appearance of negative differential resistivity.[16] We emphasize that this discussion only provides a qualitative understanding of the onset of negative differential resistivity at the field value F_1. A more complete treatment of the field-dependent resistivity $\rho(F)$ requires more theoretical work and must go beyond the first term of the Taylor expansion of Eq. (5). [Because of this, the singularity in Eq. (10), at which $\alpha F^2 = 1$, is also beyond the validity range of this equation.]

In Ref. 16 we have discussed in detail the instability and hysteresis of the voltage-current characteristic resulting from the field dependence $\rho(F)$ displayed in Fig. 1(a). Here, following the resistivity step at field F_1, at higher fields not too far above F_1 the resistivity $\rho(F)$ is assumed to remain approximately constant at its higher level. The actual behavior of $\rho(F)$ above field F_1 can only be determined from quasi-voltage-biased measurements which have not yet been performed. However, for the current-biased measurements of Ref. 7 the field regime above but close to F_1 remains inaccessible since the field immediately jumps to values of about 1000 times F_1.

In addition to the resistive flux-flow voltage step in the range 100–200 μV we have observed a second step in the range 200–300 mV corresponding to electric fields of 5–8 V/cm.[7] We identify this second step with the strong upturn of the DOS $N(\varepsilon)$ near the energy $\varepsilon = \Delta$. We denote the field at which this happens by F_2. Equation (3) is then replaced by

$$eF_2 v_F \tau_p = \Delta. \tag{12}$$

Based on the values given above, from Eq. (12) we obtain $\tau_p = (5-8)\times 10^{-11}$ s. A sharp upturn of the DOS $N(\varepsilon)$ near $\varepsilon = \Delta$ has been found in theoretical calculations of the electronic vortex structure at intermediate magnetic fields, based on the quasiclassical approximation.[17–19] Therefore, our interpretation of the second resistive flux-flow voltage step in terms of this upturn of $N(\varepsilon)$ and Eq. (12) appears reasonable.

In this paper we have restricted our discussion to the case of a superconducting order parameter with *s*-wave symmetry. In the case of *d*-wave symmetry the node lines with zero energy gap are expected to strongly affect the phase space for quasiparticle scattering and, hence, the flux-flow resistivity. A discussion of this subject is beyond the scope of this paper. However, the experimental search for a possible step structure in the field-dependent flux-flow resistivity $\rho(F)$ in the cuprates with *d*-wave symmetry such as $YBa_2Cu_3O_{7-\delta}$ is expected to be difficult because of flux pinning.

Financial support of the Deutsche Forschungsgemeinschaft and useful discussions with N. Schopohl are gratefully acknowledged.

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