RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 59, NUMBER 20 15 MAY 1999-II

Resistance of quasi-one-dimensional wires

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(Received 14 October 1998)

The problem of resistance of thin wires is revisited. A formalism is developed for the localization regime to include inelastic scatterings of a given frequency of occurrence decided by the amount and nature of disorder capable of producing a certain number of tunneling states in the system. Questions about dependence on the bulk resistivity of the localization correction to resistance and the magnitude of this correction are addressed. The reason for the long standing discrepancies on these issues is found in the fact that Thouless' [Phys. Rev. Lett. **39**, 1167 (1977); Solid State Commun. **34**, 683 (1980)] original results pertained to very high frequency of inelastic events while in the contemporary experiments this frequency should have been much lower. [S0163-1829(99)50820-5]

I. INTRODUCTION

Effects of localization and electron-electron (e-e) interaction on the electrical transport in thin metal wires once was a hotly pursued subject.[1–9,12–21,23] A fresh look at the results shows that the questions as to which of these effects dominates in the wires, and what is the magnitude of its influence on resistance, remained unresolved. We have investigated these problems which are relevent to basic aspects of localization in quasi-one dimension.

Classically a thin wire of cross section A, length L, and of a material of bulk impurity resistivity ρ_B has a resistance $R_0=\rho_B L/A$ with its electronic states extended across length L. Thouless[1] calculated the localization correction ΔR to the metallic resistance R_0 believing that localization effects become increasingly more dominant with decreasing temperature since this should rapidly increase the time τ_i between inelastic collisions with phonons and enable more and more electrons to diffuse over distances of the order of localization length. This contention was however not borne out by experiments[4–9] which showed considerably less effect of localization.

The importance of inelastic scattering from tunneling states or two-level systems[10,11] was realized at this stage when Black *et al.*[12] found that it reduces the scattering time τ_i and therefore also the effect of localization considerably. Altshuler *et al.*[13] on the other hand, studied *e-e* interactions as inelastic processes and the experiments[14] on ultrathin wires of Cu, Ni, and AuPd alloy showed that the interaction effects could be comparable to, if not dominate over, the localization effects in metallic samples. Other experiments[15,16] demonstrated the simultaneous presence of localization and interaction effects. Yet another experiment[17] on ultrathin Pt wires showed a very large increase in resistance—about 10 to 100 times the previous values—which was too large to be explained by interaction effects. This could be attributed[17] to one-dimensional (1D) localization. Later, magnetoresistance studies on Al wires[18] also supported Thouless' ideas on 1D localization. The question of relative roles of localization and interactions in metallic wire resistance thus remained ambiguous.

The experiments[4–9,12] showed that $\Delta R/R_0$ depends on A and T as A^{-1} and $T^{-1/2}$, respectively. Interestingly, both the interaction theory of Altshuler *et al.*[13] with $\tau_i=\tau_{ee}$ and the localization theory of Thouless[2] with $\tau_i=\tau_{TS}$ yield the A and T dependence of $\Delta R/R_0$ as A^{-1} and $T^{-1/2}$, respectively. Clearly it is hard to decide from the A and T dependences whether the existing experimental results favor the interaction or the localization effects. From the point of view of theory one faces the following difficulty.

A wire although thin on atomic length scale can behave as a bulk material in case it is not long enough to satisfy the Thouless criterion of one dimensionality.[1] Such a metallic wire with disorder would possess a mobility edge separating the metallic and the insulating regimes. Then the e-e interaction effects would be meaningful if the Fermi level (E_F) lay on the metallic side while the localization effects would dominate if the E_F were on the insulating side (i.e., in the midst of localized states) for the electrons trapped in localized states would normally not be able to interact with each other. Therefore in a given quasi-1D sample it must be ascertained first whether it satisfies Thouless criterion of one dimensionality, and if it does not, then whether E_F lies on the metallic or the insulating side of the mobility edge. Since this information is not available regarding the samples in question, one cannot decide whether localization theory or interaction theory ought to be applicable to the experiments of Refs. 4–9 and 12.

This problem can be resolved if one can identify another parameter on which the dependence of $\Delta R/R_0$ is different in the above two regimes. The bulk resistivity ρ_B can be a candidate. The experiments show that $\Delta R/R_0 \sim \rho_B$ whereas both Thouless and Altshuler *et al.* suggest $\sqrt{\rho_B}$ dependence. At least in the localization-based theory of Thouless there seems to be a discrepancy that might account for this disagreement: The inelastic diffusion length $\sqrt{D\tau_i}$ is assumed to be a constant, independent of ρ_B.[2] With $D\sim\rho_B^{-1}$, this implies $\tau_i\sim\rho_B$ which leads to an anomalous result[19] that τ_i should become shorter as the wire is made cleaner.

II. THE ISSUE OF BULK RESISTIVITY

We take up the issue of the ρ_B dependence of $\Delta R/R_0$ and investigate it within the framework of localization theory. We treat the inelastic scattering probabilistically and thereby introduce the element of disorder (which is the source of

ρ_B)—the larger the disorder the greater the probability of finding a tunneling state (TS) responsible for the inelastic scattering.

As in Thouless[1] we describe the electron by a wave packet made up of localized wave functions much larger than the size of the wave packet. The wave packet diffuses until either an inelastic event causes the electron to scatter into a new wave packet, or it reaches the boundary of one of its constituent localized states. The wave packet can diffuse a range of distances, involving a range of ''step lengths,'' from a_0, the interatomic distance, to l, the localization length—depending on the temperature and the number of TS's present in the system. We find that the result of Thouless[2] is retrieved in our approach in the limit of high temperature and/or large numbers of TS's, both of which make the step length small. For relatively lower concentration of TS's and/or lower temperature that ensure larger step lengths—of the order of l—we obtain the ρ_B dependence of $\Delta R/R_0$ as in the experiments besides the correct A and T dependences. Another significant discrepancy removed here is that of the magnitude of ΔR—the estimate of Thouless[2] is an order of magnitude higher than the experimental values.

According to the scaling ideas[21] states in a quasi-1D wire of cross-section area, A should be localized with decay length $l=(\pi\hbar/e^2)A/\rho_B$. For thin enough wires this makes the resistance R depend exponentially on length L, as $(e^{L/l}-1)$.[22] Thouless[2] calculated the leading localization correction ΔR to the metallic resistance R_0 as

$$\frac{\Delta R}{R_0}=\frac{\rho_B\sqrt{D\tau_i}/A}{\pi\hbar/e^2}. \tag{1}$$

Here D is the diffusion constant $\sim\rho_B^{-1}$, and $1/\tau_i$ is the decay rate due to inelastic collisions[1] which goes as $\sim T^2$ and $\sim T$ for phonons and TS's,[9] respectively. So, if the TS scattering is dominant, $\Delta R\sim\rho_B^{1/2}A^{-1}T^{-1/2}$ as compared to the experimental $\Delta R\sim\rho_B A^{-1}T^{-1/2}$.

Using e-e interaction as the inelastic process, Altshuler *et al.*[13] found

$$\frac{\Delta R}{R_0}=\frac{(2-F)}{(\pi\hbar/e^2)A}\,2\rho_B\left(\frac{4D\hbar}{k_BT}\right)^{1/2}; \tag{2}$$

F is a screening parameter. Abrahams *et al.*[19] also found the same result which, like Thouless,[2] also shows $\Delta R\sim\rho_B^{1/2}A^{-1}T^{-1/2}$. Note that Abrahams *et al.*[22] showed that $\tau_i\equiv\tau_{ee}\sim T^{-d/2}$ (d being the Euclidian dimensionality of the system) while $\tau_{ee}\sim T^{-2}$ for clean systems of any d. Contrast this with $\tau_i\equiv\tau_{TS}\sim T^{-1}$ which holds in all dimensions[23] and $\tau_{e-ph}\sim T^{-d_{ph}}$ where d_{ph}, the phonon dimensionality, is 1 and 2, respectively for clean and dirty 1D systems.[1] It should be pointed out that the τ_i in $\sqrt{D\tau_i}$ (which is inelastic diffusion length) in Eq. (1) cannot be replaced by τ_{ee} because as suggested above Eq. (1) is valid in the insulating regime.

III. LOCALIZATION CORRECTION TO RESISTANCE

To tackle the problem of ρ_B dependence of ΔR in the insulating regime considering diffusion of wave packets constructed out of broad localized states[1,2] we distinguish between the two regimes defined in terms of the step lengths traveled by the wave packets on the scale of localization length l: the *short-step regime* where the distance traveled between two consecutive inelastic events or the step length l_r is of the order of interatomic distance, i.e., a_0 or a few multiples of it such that l is sufficiently larger than $l_r=ra_0$, $r=1,2,3,\ldots$; and the *long-step regime* where the step lengths L_r can be almost of size l, $L_r=rl$ with $r=0,1,2,\ldots$. Here $r=0$ corresponds to two inelastic events occurring within a span L_0 such that $l>L_0>l_r$; $r>1$ indicates step lengths protracted beyond l by Mott's variable-range hopping[11]—if the packet diffuses full localization length l without encountering an inelastic event, it can tunnel into a nearby degenerate or nearly degenerate localized state. The inelastic lifetime in the two regimes shall be represented by $\tau_<$ and $\tau_>$, respectively—while $\tau_<$ is dominated by direct inelastic collisions, $\tau_>$ is dominated by incoherence time or the typical time taken in reaching the edge of a localized state.

l_r and L_r have number distributions $s(l_r)$ and $S(L_r)$ in a wire of length L. In general they will peak at different values of segment length with the constraint

$$\sum_{r=1}^{N}s(l_r)l_r+\sum_{r=0}^{n}S(L_r)L_r=L, \tag{3}$$

where $N=L/a_0$ and $n=L/l$. The dimensionless resistance ρ, scaled in $\pi\hbar/e^2$, is just an incoherent sum with segments contributing as resistances in series:

$$R=\sum_{r=1}^{N}s(l_r)(e^{l_r/l}-1)+\sum_{r=0}^{n}S(L_r)(e^{L_r/l}-1). \tag{4}$$

The incremental resistance ratio will be

$$\frac{\Delta R}{R_0}=\sum_{r=1}^{N}p_r\left[\left(\frac{e^{l_r/l}-1}{l_r/l}\right)-1\right]+\sum_{r=0}^{n}P_r\left[\left(\frac{e^{L_r/l}-1}{L_r/l}\right)-1\right], \tag{5}$$

where we have defined fractional contributions to the total length,

$$p_r\equiv s(l_r)l_r/L,\quad P_r\equiv S(L_r)L_r/L;$$
$$\sum_{r=1}^{N}p_r+\sum_{r=0}^{n}P_r\equiv(1-\alpha)+\alpha=1, \tag{6}$$

and have assumed metallic behavior *over the span of each stretch of diffusion* of size l_r or L_r, so that for $l_r,L_r<l$ from Eq. (4) we have $R\sim R_0=L/l$. In general the packet will make relatively fewer excursions over distances larger than l without meeting an inelastic event since the probability for undergoing variable-range hopping is exponentially smaller [$\sim\exp(-T^{-1/2})$] than that for finding an inelastic scatterer ($\sim T^{-1/2}$, see later). So, *on average* we will always take $L_r<l$. In these conditions Torodov's scaling[22] will also give the same result as (5). Each term in (5) dominates separately in one of the regimes described above.

IV. SHORT STEPS

For frequent inelastic scatterings from TS's or phonons p_r is sharply peaked to order $O(N^{-1/2})$ about $\langle l_r\rangle<l$. Then the leading localization correction from the packets able to dif-

fuse over short distances in the range of localization length l, is

$$\frac{\Delta R_<}{R_0} \simeq (1-\alpha)\frac{\langle l_r\rangle}{2l} \simeq (1-\alpha)\frac{\rho_B}{A}\frac{\sqrt{D\tau_<}}{\sqrt{2}(\pi\hbar/e^2)}. \quad (7)$$

The result is essentially the same as that of Thouless [Eq. (1)] but is derived here more directly from the exponential length dependence of R given by scaling theory of localization. The insight we gain is that the result of Thouless belongs to the short-step regime. If $\tau_<$ is small due to frequent scatterings from phonons then this will be the weak localization regime.[24] However, $\tau_<$ can also be small due to scatterings from TS even at low temperatures if the disorder is sufficiently high. In this situation also, in spite of the disorder being rather high, the effect of localization would be reduced to some extent. Thus at high disorders (and low temperatures) a competition between strong localization and inelastic scattering from TS—both produced by strong disorder—decides the value of $\tau_<$ or the extent to which the effect of localization is felt.

V. LONG STEPS

If the inelastic events from TS's and/or phonons are of such frequency that the wave packets can manage to diffuse over distances of the order of l with some nonzero probability, then on scale l we will have

$$\frac{\Delta R_>}{R_0} = \frac{\alpha\sum_{r=0}^{n} P_r\left[\frac{e^r-1}{r}-1\right]}{\sum_{r=0}^{n} P_r} \approx \alpha\left[\frac{(e^{\langle r\rangle}-1)}{\langle r\rangle}-1\right] \simeq \frac{\alpha}{2}\langle r\rangle. \quad (8)$$

For peaked distribution P_r, r has been replaced by $\langle r\rangle$, the coarse grained average of L_r/l on the scale l. The last result in Eq. (8) is obtained for $\langle r\rangle < 1$, neglecting $\langle r\rangle^3$ and higher powers of $\langle r\rangle$. In this regime, although $\langle L_r\rangle < l$, there will be some excursions made by the packets over distances of order l or a few multiples of it.

In a general sense we can include the frequency of inelastic events in our formulation by introducing a probability $P_>$ for a packet to take a step of size l (i.e., move to another localized state after having diffused through one of size l) and a probability $P_<$ of not diffusing (i.e., encountering an inelastic scattering *well* within a distance of l); $P_> + P_< = 1$. We can represent such a trajectory as a sequence of randomly arranged vertical partitions and horizontal bars—the former representing steps of zero size on a scale l and the latter, steps of an average size of order l. The partitions, $n_<$ in number, represent inelastic events, and the bars, $n_>$ in number, represent diffusion steps with, $n_> + n_< = n$. This will enable us to estimate the coarse grained average $\langle r\rangle$ which will be the average occupation fraction in a box bounded by vertical partitions, i.e.,

$$\langle r\rangle \simeq \overline{n_>}/(\overline{n_<-1}) \approx \overline{n_>/n_<}. \quad (9)$$

The most probable values $\overline{n_<}$ and $\overline{n_>}$ are found by maximizing the binomial distribution $W(n_>) = n!P_>^{n_>}P_<^{n_<}/n_>!n_<!$. Using Stirling's approximation and maximizing, we get

$$\langle r\rangle \simeq P_>/P_<. \quad (10)$$

The probabilities $P_>$ and $P_<$ defined in the strictly 1D model must be related to the physical decay or diffusion rates in the quasi-1D wire. In a thin wire, such that $A^{1/2}/l \ll 1$ (Ref. 25) the minimum energy states will be uniform across the wire, with roughly A/a_0^2 chains of states exponentially decaying along the wire. For time intervals greater than A/D, the diffusing packets can be taken as uniform in the transverse direction while diffusing ''one dimensionally'' at a rate D/l^2 along the wire. The 1D diffusion probability $P_>$ then depends directly on D/l^2.[26]

As outlined later the incoherence time $\tau_>$ is related to the decay time τ_l of any one of the localized states that make up the packet. Since there are A/a_0^2 chains of states across the wire, the packet decay rate, $1/\tau_>$, must be related to the 1D decay probability summed across the wire, $\Sigma P_< = (A/a_0^2)P_<$. That is, the decay in any one of the chains should bring about incoherence and decay in the overall combination. Since the incoherence time of the packet comes from the decay of any of the localized states, the rate per chain $(1/\tau_>)/(A/a_0^2)$ is to be related to the 1D probability $P_<$.

Equating the ratio of probabilities to the ratio of relevant rates, i.e.,

$$P_>/P_< = (2D/l^2)/(a_0^2/A\tau_>), \quad (11)$$

we finally get

$$\Delta R_>/R_0 = \alpha\rho_B^2 D\tau_>/\{Aa_0^2(\pi\hbar/e^2)^2\}. \quad (12)$$

We should determine $\tau_>$, the packet incoherence time. Each localized state used in the construction of a packet has an energy width $\hbar/\tau_l$ which should affect the intrinsic energy width of the packet and consequently its intrinsic incoherence time. To relate $\tau_>(\tau_l)$ to $\hbar/\tau_l$, since we do not know the details of how a particular packet is constructed, we must consider typical diffusion path lengths and work backwards.

A free wave packet made up of plane waves of momentum $\hbar\langle k\rangle$ broadens as it moves, with the coordinate space width $\Delta x \equiv \langle[x-\langle x(t)\rangle]^2\rangle^{1/2}$ proportional to the distance $\langle x(t)\rangle$ traveled. In the energy space its width narrows such that $\Delta E \geqslant \hbar K/\langle x(t)\rangle$ (with the constant $K \sim \hbar k/m$). We *assume* that for an elastically diffusing wave packet the same type of relation holds,

$$\Delta E \geqslant \hbar K/\sqrt{2Dt}. \quad (13)$$

Since the localized states decay in τ_l, the intrinsic energy width of a packet should be

$$\hbar/\tau_> \geqslant \hbar K/\sqrt{2D\tau_l}. \quad (14)$$

The K is estimated by considering a narrow packet of size $\sim a_0$ and initial intrinsic width $\hbar/\tau_0$ that does not narrow in its short lifetime τ_0. That is $\hbar/\tau_0 \geqslant \hbar K/\sqrt{2D\tau_0}$, or $\sqrt{2D/\tau_0} \geqslant K$. Using the equality sign for a rough estimate we obtain from Eq. (14)

$$\tau_{>} \approx \sqrt{\tau_0 \tau_l}. \tag{15}$$

Combining this with Eq. (12) we get

$$\frac{\Delta R_{>}}{R_0} = \alpha \left(\frac{\rho_B^2}{a_0^2 A} \right) D \sqrt{\tau_l \tau_0} / (\pi \hbar / e^2)^2. \tag{16}$$

An electron in a localized state can decay and hop to another localized state a distance $\sim l$ away by an inelastic process such as interaction with a TS. This rate differs from the free electron scattering rate by a factor $\sim l^{-1} |\int dx e^{-|x|/l} e^{-|x-l|/l}|^2 \sim O(1)$[12]. Setting τ_l^{-1} equal to the scattering rate off TS's, $\propto T$,[27] we obtain

$$\Delta R_{>} / R_0 \sim \rho_B A^{-1} T^{-1/2}, \tag{17}$$

in agreement with experimental behavior. Using the parameters of Ref. 9 one finds $\Delta R_{>} / R_0 = 4 \times 10^{-2}$ by setting $\alpha \approx 1$ as compared to the experimental value $\sim 2 \times 10^{-2}$ while Thouless' result of Eq. (1) is anomalously large.[9,19] Large τ_i (as opposed to the present small $\tau_{<}$), and the erroneous assumption of a ρ_B independent $\sqrt{D \tau_i}$ (which wrongly led to $\tau_i \sim \rho_B$) were apparently responsible for the high value of Thouless.

Combining the results (7) and (16) we will get

$$\frac{\Delta R_{>}}{R_0} = \frac{\rho_B}{A(\pi \hbar / e^2)} \left[(1-\alpha) \sqrt{\frac{D \tau_{<}}{2}} + \frac{\alpha \rho_B D \sqrt{\tau_l \tau_0}}{\pi \hbar / e^2} \right]. \tag{18}$$

This covers the physical conditions ranging from the extreme ones where an electron can barely diffuse a few atomic spacings, to the moderate one where it can make some excursions of the order of localization length. The conditions allowing many and longer excursions can be taken into account by including higher powers of $\langle r \rangle$ in Eq. (8).

VI. DISCUSSION

The electrical resistance of quasi-1D systems is seen to be sensitive to the frequency of inelastic scattering events. The identification of the physical conditions under which the results of Thouless[2] should be valid and those that would have been present in the experiments[6–9] done to verify his results, calls for new experiments under controlled conditions of disorder and temperature to study in detail the roles of inelastic scatterings from TS's and phonons separately in the electrical transport in quasi-1D systems. This should give more insight into the microscopic details of electron diffusion in the backdrop of localization. Experiments at very low temperatures should possibly reveal something new arising from the interaction between TS's which can change the nature of tunneling from coherent to incoherent.[28]

It should also be interesting to investigate the conditions under which the τ_{ee} takes over from τ_{TS}, and also the nature of this transition. This transition is expected to be different from the weak[24] to strong localization crossover. Even the latter requires more investigation following the finding of Herzog *et al.*[29] that granularity of wires is related with this transition being continuous or discontinuous. A uniform wire with the TS scatterings acting as short-ranged local events connected with other such events located in far off regions of the system by the nonlocal long-ranged variable-range hoppings, we suspect, may mimic the granular scenario.

ACKNOWLEDGMENT

Thanks are due to Professor S. R. Shenoy for extensive discussions and suggestions in the initial stages of this work.

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