Hysteresis properties at zero temperature in the dipolar random-field Ising model

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We present a modified two-dimensional random-field Ising model, where a dipolar interaction term is added to the classic random-field Hamiltonian. In a similar model it was already verified that the system state can exhibit domains in the form of stripe patterns, typical of thin materials with strong perpendicular anisotropy. In this work we show that the hysteresis loops obtained at zero temperature can display a strict similarity with the loops obtained in thin magnetic materials such as garnet films. In our model the processes of domain nucleation and domain-wall motion are well separated in time as the system evolves. This remarkable fact allowed us to better understand the nucleation process in this family of spin systems. [S0163-1829(99)14101-8]

I. INTRODUCTION

Many attempts have been made in the past to introduce some modifications in the Ising Hamiltonian. An interesting attempt is the random-field Ising model[1–4] in which a random local disorder term, constant in time, is added to the exchange interaction. This term permits one to describe the presence of disorder in a medium in which the exchange energy is the only spin-spin interaction. Another possible change is the addition of the dipolar energy, recently used[5–8] as a way to describe the magnetostatic energy of the system. In this work we will describe an Ising spin system whose Hamiltonian includes both the terms described. The behavior thus obtained is most interesting, as the exchange and dipolar interaction are very different in their properties: the first is ferromagneticlike and short range, the second is antiferromagneticlike and long range. The random-field term is local to the single spin, and describes the local disorder.

The interest of this approach lies in the possibility of describing the hysteretic properties of magnetic materials such as garnet films. These are thin magnetic films with a high uniaxial anisotropy perpendicular to the film plane, and, up to now, no satisfying model has been developed to explain their characteristic hysteresis loops.[9,10] As we will show, our model proves apt to describe these kinds of materials. In fact, the terms included in the Hamiltonian are the dominant ones in these films. The exchange interaction provides the basic ferromagnetic coupling, while the dipolar interaction term represents a good approximation to the strong magnetostatic energy present, due to the high surface-to-thickness ratio of the film. Last, the random-field term is used to describe the possible imperfections present in the lattice.

The system behavior will be analyzed in the limit of zero temperature. We believe it to be of some interest, as in many magnetic systems thermal activated phenomena are negligible in a broad temperature range. Moreover, we will assume that the evolution of our system occurs at time scales much shorter than the external field rate of change: a behavior known as rate-independent hysteresis. The consequence of this assumption is that, during an irreversible state change (avalanche) the external field can be considered constant.
Our interest will be focused on two key aspects of the system behavior: the domain structure and the hysteresis loop.

The domain structure in the films described is the stripe or labyrinth structure,[11,12] where the magnetization is oriented in the direction perpendicular to the film plane, and the domains appear as convoluted stripes with a high degree of branching to decrease the magnetostatic energy. In Refs. 5, 6, and 7 it has been shown that, in the framework of a dipolar/Ising model, the tuning of the relative strengths of dipolar and exchange interactions lets the system exhibit a stripe domain pattern. We verified in our work that the presence of disorder changes the structure of the domains that evolve from the stripe structure to a state characterized by isolated domains as the disorder is increased.

Another characteristic of thin magnetic films, particularly in the case of garnet films, is that the hysteresis loops show a well-defined nucleation jump, followed by a wide region where only domain-wall motion is present and the losses are very low. We will show that, in our model, for the same parameter values that let the system exhibit the stripe domain structure, the hysteresis loop shows the same key features of the garnet film loop. We find in fact a very small coercive field and a characteristic nucleation jump (NJ) as the system is magnetized. We verified also that an increase of the disorder strength can quickly destroy the NJ, changing the resulting loop shape in the typical random-field Ising model (RFIM) hysteresis loop.[1–4]

To better understand this model we also studied the energy variation of the system as the system state changes along the hysteresis loop. In particular we focused on the free energy change ΔF during the first nucleation event. Keeping the disorder strength ν fixed and varying the dipolar to exchange strength ratio D, we observed the presence of a maximum in ΔF for a critical $D_c(\nu)$ value. When the disorder strength ν increases, the $D_c(\nu)$ value shifts towards lower D values, and ΔF becomes smaller as the NJ disappears.

II. THE MODEL

The model is defined using a two-dimensional random-field Ising Hamiltonian, with the addition of a dipolar interaction term between the single spins s_i:

$$\aleph = -\frac{J}{2}\sum_{\langle ij\rangle} s_i s_j + \frac{P}{2}\sum_{\substack{i,j\\ i\neq j}} \frac{s_i s_j}{r_{ij}^3} - V\sum_i h_i^c s_i - H\sum_i s_i . \tag{1}$$

The four terms are, in sequence: the exchange interaction term; the dipolar interaction term; the random-field term; the external field interaction term.

The J and P parameters express the strength of the exchange and dipolar interactions, while the V constant represents the strength of the local random fields h_i^c. The random fields are obtained from a Gaussian distribution with zero mean and unit variance. The exchange interaction acts at a nearest-neighbor level only (indicated by the subscript $\langle ij\rangle$), while the dipolar interaction is long range, with $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ indicating the relative distance in lattice units. The applied field H acts as the external driving force, and is used to magnetize the system.

The number of parameters can be reduced, normalizing the terms to the exchange interaction strength J; we will consider as relevant parameters the ratios $D = P/J$ and $\nu = V/J$. The external field will be normalized too as $H = \mathsf{H}/J$. The Hamiltonian is then rewritten as

$$\aleph' = -\frac{1}{2}\sum_{\langle ij\rangle} s_i s_j + \frac{D}{2}\sum_{\substack{i,j\\ i\neq j}} \frac{s_i s_j}{r_{ij}^3} - \nu\sum_i h_i^c s_i - H\sum_i s_i . \tag{2}$$

The Hamiltonian is dependent on the spin configuration (system state) $\{s_i\}$ and on the external field H. Then, the variation of the Hamiltonian can be written as

$$d\aleph' = -\sum_i \left(\sum_{\langle j\rangle_i} s_j - D\sum_{j\neq i} \frac{s_j}{r_{ij}^3} + \nu h_i^c + H \right) ds_i - \left(\sum_i s_i \right) dH = -\sum_i h_i ds_i - NM dH, \tag{3}$$

where the magnetization is defined as $M = (1/N)\Sigma_i s_i$, and we have defined the local field h_i experienced by s_i. Equation (3) shows that the system energy can change as a consequence of two possible factors: the inversion of a spin, or a change in the external field.

The study we will present is at zero temperature: the system state is considered to be stable when each spin is directed according to the sign of the local field h_i, and no temperature fluctuations are considered that can invert a single spin in the opposite direction, as long as the applied field H does not change. Then the stability condition can be written as

$$s_i = \mathrm{sgn}(h_i). \tag{4}$$

The algorithm used in the simulations is the following. The external field H is varied according to a predefined field history. When the local field of a spin changes its sign, the corresponding spin is flipped, an event that can trigger an avalanche of many spins, due to the coupling of the spin with the lattice. The unstable spins are randomly flipped one at the time, and the local fields are recalculated at each step, until all the spins satisfy Eq. (4). During the avalanche the external field does not change, so that we are able to affirm that the simulation is performed under quasistatic conditions.

III. SIMULATIONS

A. Parameter description and boundary conditions

Once the parameter set $\{D, \nu\}$ is known, the relative strengths of the interactions are defined. The simulations we performed explored the possible hysteresis loop shapes and the domain structures as the parameters of the model, $\{D, \nu\}$, are modified.

A great part of the results was obtained with a spin lattice having $N^2 = 2500$ spins (side $N = 50$). We observed that this number of spins is sufficient so that the results do not depend on N. The first check was performed on the loop shapes: apart from a greater discretization of the Barkhausen jumps at low N, the loops are invariant at N values higher than 50. Moreover, the domain structure was inspected with particular care after nucleation and at the coercive field. The typical domain width seems to depend on the $\{D, \nu\}$ parameters only, not on N.[6] These checks were performed until $N = 200$.

Two types of boundary conditions (b.c.'s) are possible. In the first case (*open b.c.*'s) the spins on the lattice boundary have a smaller number of neighbors (2 or 3). This fact has two effects: these spins are less coupled to the lattice, the absolute value of the exchange interaction being no more than $4J$ or $6J$, so it is more difficult to make them flip; moreover, since beyond them there are no more spins, these spins create a barrier, and the avalanches created near the boundaries have fewer directions along which they are able to expand. In the second case (*periodic b.c.*'s) we eliminate the effect of the boundary by closing the lattice on itself, using a toroidal topology. Using the periodic b.c.'s, no barrier at all is put along the propagation of the avalanche. Of course this has no consequences at all on the spurious effects caused by finite dimension of the system.

The b.c.'s choice has small visible effects on the loop shape. However, a study made on the distribution of the size of the avalanches showed some differences. The main reason is that big avalanches are hindered in the case of open b.c.'s, as we just said, having a smaller number of directions along which to evolve. The effect of the b.c.'s choice is even visible on the shape that the domains take on, and on the domain evolution. As we observed, in the case of open b.c.'s the boundary spins are less coupled to the lattice. In this case, a reversed spin frame remains present until the system is almost saturated, then the frame spins reverse too. Apart from this effect, the domain topology is however unaffected by the b.c.'s choice. In the following, unless where specified, we will consider just the case of periodic b.c.'s, to be able to neglect boundary effects.

B. Hysteresis loops

The hysteresis loops show a great variety of behaviors, when the $\{D, \nu\}$ parameters are changed. In Fig. 1 is shown a case with low disorder ν. When the dipolar interaction strength is lower than the exchange coupling ($D < 0.5$), the hysteresis loop shows two big avalanches, spanning a great part of the lattice. If $D = 0$, just two system states are pos-

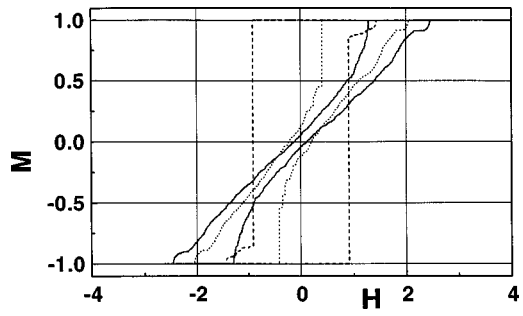

FIG. 1. Hysteresis loops, $\nu=10^{-6}$; $D=0.6$ (straight line), $D=0.5$ (dotted line), $D=0.35$ (dashed line).

sible, with $M=\pm 1$. Instead, an increase in the value of D let us observe a smaller nucleation jump. In garnet films[9,10] the hysteresis loops behave in exactly the same way, with a nucleation jump followed by a region characterized by smaller losses, typical of domain-wall motion.

The dependence on the parameter ν is shown in Fig. 2: as ν is increased, the NJ magnitude decreases to zero. In fact, a high enough value of the disorder ν causes the spins to flip independently, and, as a consequence, the height of the irreversible jumps decreases. In our simulations we verified that it is a sufficient disorder-to-exchange strengths ratio $\nu \approx 10^{-2}$ to destroy almost completely the NJ. It is shown in Figs. 3, 4 the influence of the D and ν variation on the NJ, i.e., on the total magnetization variation at the first spin reversal after saturation.

The NJ originates by the exchange interaction. A complete magnetization reversal in a field interval $\Delta H \rightarrow 0$ is contrasted by the two other terms in the Hamiltonian: the dipolar interaction and the disorder term. In the region $0.5 < D < 2$, where the NJ is still well defined, the dipolar field contribution is strong enough that a disorder-to-exchange ratio $\nu \approx 10^{-2}$ is sufficient to destroy the NJ. As the model approaches the RFIM description instead ($D<0.5$), the dipolar term becomes negligible, and a greater ν value is necessary to hinder the infinite avalanche.

The strong dependence of the NJ height on the disorder ν can possibly explain why this typology of hysteresis loops has not been found in previous works. While it is certainly true that a frozen disorder in the lattice is different from a simulated temperature effect in a Monte Carlo simulation, still there exist some common characteristics. In the case studied, the local field acting on a spin s_i is given from Eq. (3). From the stability condition in Eq. (4), we know that the spin s_i will flip each time the external field crosses the value

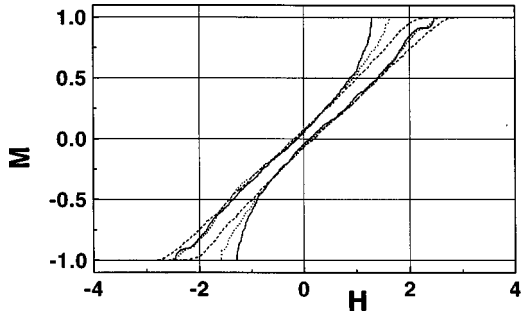

FIG. 2. Hysteresis loops, $D=0.6$; $\nu=10^{-6}$ (straight line), $\nu=10^{-2}$ (dotted line), $\nu=10^{-1}$ (dashed line).

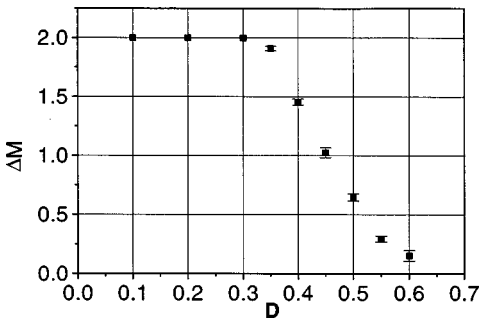

FIG. 3. Nucleation jump height: $\nu=10^{-6}$, as a function of D.

$$H_i = -\sum_{\langle j \rangle_i} s_j + D \sum_j \frac{s_j}{r_{ij}^3} - \nu h_i^c . \tag{5}$$

As the ν value grows higher, the H_i values are more widespread, and the single spins flip independently.

In a Monte Carlo simulation we do not have the $-\nu h_i^c$ term. Instead a spin will flip when the external field reaches the value $H_i = -\Sigma_{\langle j \rangle_i} s_j + D\Sigma_j (s_j / r_{ij}^3)$, or, if this condition is not satisfied, will flip anyway with a probability $e^{-\Delta E/kT}$, where ΔE is the energy difference between the state in which the spin is flipped, and the current state. Therefore, if the temperature T is high enough, the spins will tend to flip independently.

Then we have shown that a high enough frozen disorder ν, or a high enough temperature, cause the spins to flip independently, and, as a consequence, the height of the irreversible jump decreases. In our simulations we verified that it is a sufficient disorder-to-exchange strengths ratio $\nu \approx 10^{-2}$ to destroy almost completely the NJ.

Concerning the nucleation field, we observed that it changes in a linear way with the parameter D: when $\nu = 10^{-6}$ we have (for the nucleation field from negative saturation): $H_n \cong H_n^0 - 9D$, with $H_n^0 \cong 4$. This law is followed fairly well, although a greater error is expected at higher ν values. The law derives from the increased instability of the saturated state as the dipolar interaction increases. The exact value of H_n^0 is explained once the model is investigated at $D=0$. From Eq. (3), and from the stability condition Eq. (4), we know that the spin s_i will flip when the external field crosses the value given in Eq. (5). If the system is negatively saturated, the disorder is low, and $D=0$, we see that all the spins will be unstable at $H \cong 4$. This value will decrease once D start increasing.

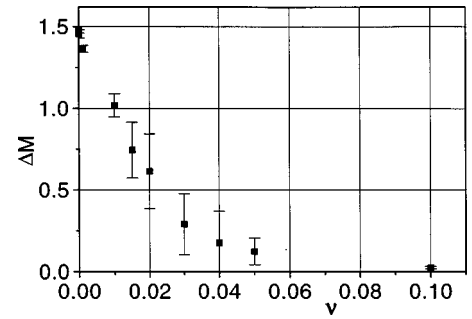

FIG. 4. Nucleation jump height: $D=0.4$, as a function of ν.

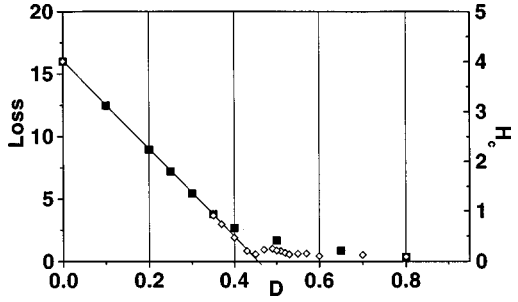

FIG. 5. Hysteresis loss (squares) and coercive field (diamonds) as a function of the dipolar strength D, for $\nu=10^{-6}$.

When $D\neq 0$, we can observe that the dipolar part h_i^D of the local field is the source of the demagnetizing field that helps the system to nucleate the first reversed domains. Then, the real nucleation field will be obtained by subtracting this demagnetizing field from H_n^0:

$$H_n=H_n^0-H_D=H_n^0-D\sum_{j\neq i}\frac{1}{r_{ij}^3}, \tag{6}$$

where the calculation is performed at negative saturation, and s_i is the first flipping spin. The value of the second term gives the result searched, approximately equal to $(-9D)$.

An error is expected when comparing with the simulation results, because the nucleation field depends even on the parameter ν, not only on D. In fact, increasing ν, the first nucleation avalanches start at fields lower than the fields predicted by Eq. (6), due to the coda in the random-field Gaussian distribution of the random fields.

The total hysteresis loss and the coercive field are shown in Fig. 5, as a function of the parameter D. We observe that two regimes are clearly visible. For $D<0.35$, hysteresis loss and coercive field decrease in an almost linear way. In this regime, the increasing dipolar interaction lets the system become unstable at smaller fields, but the exchange interaction is nevertheless strong enough to completely reverse the system magnetization in just one jump, as the first unstable spin reverses. As $D>0.35$, the dipolar interaction prevents the system from saturating after the first spin reversal, letting it set in some barely stable states after a first NJ.

C. Energy study

To investigate more deeply the nucleation phenomena, we studied the energy behavior as the parameters of our problem were changed. Figure 6 shows the value of the free energy

$$F=\aleph'/N+HM$$

$$=\frac{1}{N}\left(-\frac{1}{2}\sum_{\langle ij\rangle}s_is_j+\frac{D}{2}\sum_{\substack{i,j\\ i\neq j}}\frac{s_is_j}{r_{ij}^3}-\nu\sum_i h_i^c s_i\right)$$

as a function of the magnetization. These values were acquired during magnetization from negative saturation to positive saturation. A clearly recognizable parabolic shape is evident at high D values, while as D is lowered the NJ does not allow us to record the intermediate energy values (in the figure, starting from $D=0.5$). The parabolic shape at high D is an evident feature of the dipolar interaction, whose contribution to the system energy is $E\approx DM^2$. As the dipolar interaction strength decreases, we observe that the curvature of the energy curve becomes lower.

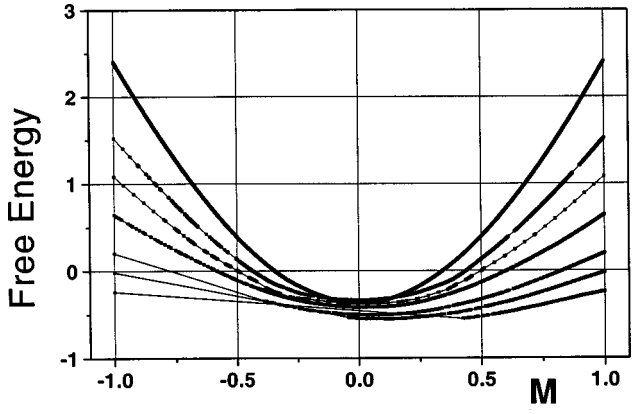

FIG. 6. Free energy as a function of magnetization, for $\nu=10^{-6}$. Top to bottom: $D=0.8$, $D=0.7$, $D=0.6$, $D=0.5$, $D=0.45$, $D=0.4$, $D=0.35$.

The energy value at remanence decreases with D. This trend is visible, until no stable remanence states are present (in the figure, for $D=0.4$, $D=0.35$). In fact, as D decreases the exchange interaction becomes dominant, increasing the dimension ΔM of the NJ. For $D<0.5$ its value is $\Delta M>1$, and no remanence state is defined. In the $D<0.35$ limit, we reach the Ising cycle, in which the saturation states are the only stable states.

If we study the free-energy variation ΔF as the system nucleates the first domain (Fig. 7), we can observe a clearly defined minimum at a value D_c. In fact, at high D values the loop shows a very small NJ, because only small domains can nucleate, due to the dipolar interaction. At low D instead, the low curvature of the free energy (Fig. 6) causes this jump to be very small in energy, notwithstanding the high ΔM value. The D_c value is dependent on the disorder present, in fact, a strong enough ν value can destroy the NJ for any given D value.

D. Domain structure

The same variety of behaviors that we observed in the hysteresis loops is present when the domain structure and

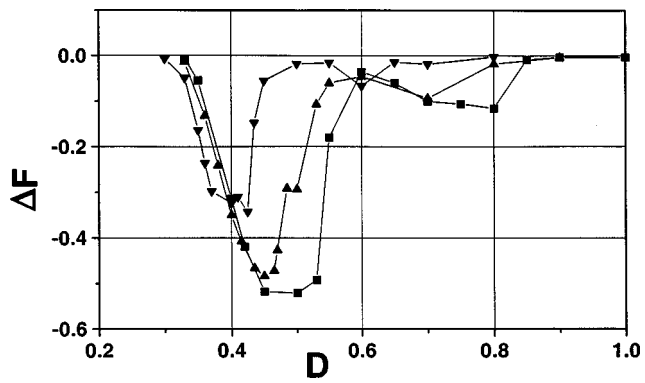

FIG. 7. Change in free energy at nucleation, as a function of D, for $\nu=10^{-6}$ (squares), $\nu=10^{-3}$ (up triangles), $\nu=10^{-2}$ (down triangles).

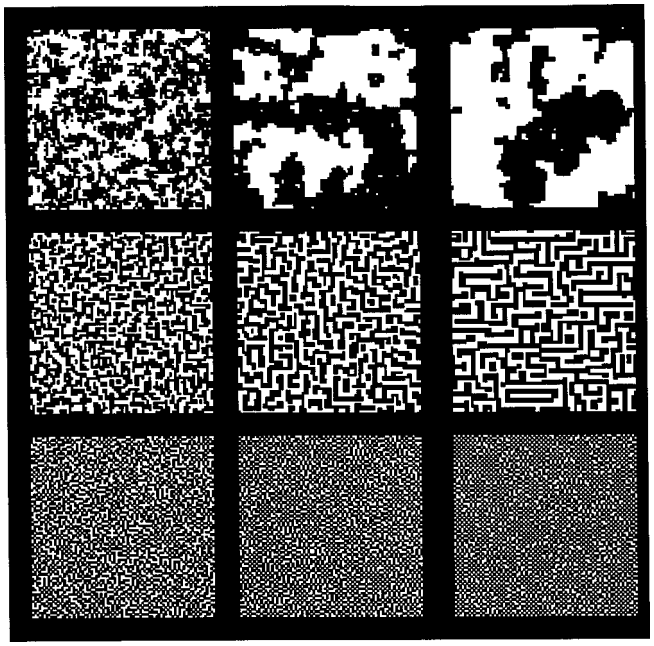

FIG. 8. System states at the coercive field. Top row: $D=0$, $\nu=10$, $\nu=2$, $\nu=1.2$. Middle row: $D=0.6$, $\nu=1$, $\nu=0.1$, $\nu=10^{-6}$. Bottom row: $D=3$, $\nu=10$, $\nu=1$, $\nu=10^{-6}$.

evolution is considered. We give an example in Fig. 8 of the domain structures that can be obtained. The top row of the figure refers to the $D=0$ case (RFIM system), with varying degrees of disorder. At decreasing ν values we obtain at the coercive field larger domains that often originate from a single avalanche. The lowest ν value shown is $\nu=1.2$: at lower values the system exhibits a single irreversible jump from saturation to saturation, and no intermediate states are present.

The bottom row shows the effect obtained when the dipolar interaction dominates on the exchange. The state to the right is, apart from some imperfection due to the non-null disorder, the classic checkerboard pattern, in which each spin is surrounded by four reversed spins: this is the most stable state for a dipolar system. The increasing disorder (to the left) destroys this simple pattern. The most interesting system states, in a comparison with garnet films,[11,12] are obtained with an intermediate D range, approximately $0.5<D<2$ (stripe region), where stripe domains are present: in the middle row is shown the case $D=0.6$. It is worth noting that the range in which the stripe domains are present is exactly the same range in which the hysteresis loops show the NJ.

Concerning the nucleation phase in the stripe region, we observed the nucleation of stripes spanning the whole lattice. This is a reasonable behavior: after some neighboring spins have been reversed during the avalanche, all the spins edging the stripe are more unstable than the average, due to the exchange coupling with the spins belonging to the stripe. But the spins at the two extrema of the stripe are the most unstable, because it is in these two sites that the stabilizing dipolar interaction due to the reversed stripe is weaker. So the average disposition for the system is to nucleate thin stripes, extending themselves in both directions. This behavior is most interesting, since in garnet films the nucleation phase is not different: single stripes are nucleated one after the other, each one spanning the whole sample, starting at the edges near some imperfection.

The collective behavior of the stripes is again similar to the real world stripe domains: the stripes have a self-avoiding tendency, always arising from the dipolar interactions. So it has been often observed that the presence of stripes, elongating in a given direction during an avalanche, when coming in the proximity of another stripe suddenly change direction, try not to intersect it.

IV. CONCLUSIONS

The model presented has proven itself able to develop two key features present in many thin magnetic media: the labyrinth domain structure, and the characteristic hysteresis loop often found in garnet films. The stripe domain structure, already described in previous articles,[5–7] is found in a given range of the dipolar and disorder strengths. The labyrinth structure can be modified in many ways. If the disorder is too high the stripes become more fragmented. If the dipolar strength is too low, the stripes grow thicker and less elongated, while if it is too high the checkerboard pattern becomes dominant. The next logical step will be to obtain a comparison between the topological properties of the domains in this model, and the same properties as found in garnet stripe domains.

The study of the domain structure at nucleation showed a behavior that can be compared to the nucleation in thin magnetic films. Long stripes are nucleated, which span the whole lattice. Successive stripes nucleate in such a way that they are able to avoid other stripes already present. The nucleation process generates a sudden decrease in the total energy of the system. A study of the energy variation showed the presence of a clearly defined maximum of the energy dissipated for a given value of D.

The hysteresis loops described were not observed previously. A possible explanation is that the NJ can be easily destroyed when the simulation is nonstatic, or when the temperature is different from zero. An insight on this sensibility of the NJ is the described behavior as ν increases: if the disorder to exchange strength ratio is greater than $\nu=10^{-2}$, the NJ magnitude goes to zero. A parallel study of hysteresis properties and domain structure, both in this model and in real materials could lead to a deeper understanding of many phenomena, among which the nucleation process.

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