# Suppression of the tunneling current in a bilayer electron system

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In this work, we have numerically integrated the effective-mass nonlinear Schrödinger equation for an electron wave packet in a bilayer electron system in space and time. Due to the nonlinear effective-mass equation, it is found that the charge dynamically trapped in both wells produces a reaction field which modifies the system resonant condition. Considering an electron-electron interaction, we have calculated the tunneling times between the two quantum wells when a change of carrier density is induced in one quantum well. At large electronic sheet densities, we have also shown the possibility of having a suppression of the tunneling current due to nonlinear effects. [S0163-1829(99)07111-8]

In the last years, the physics of strongly coupled two-dimensional electron gases have attracted considerable interest for possible 2D-2D (two dimensional) tunneling devices.[1] These quantum systems constitute double layers of 2D electron gases separated by a thin barrier. With the application of parallel and perpendicular magnetic fields to such bilayer heterostructures, a variety of phenomena has been observed at several electron sheet densities. In the experiments, most magnetotransport measurements on double quantum wells have been for high magnetic fields perpendicular to the growth plane, where the electronic kinetic energy is quenched and Coulomb interactions dominate.[2,3] In such a case, interesting phenomena arise from the combined effect of tunneling and electron-electron interactions.[4] In addition, and when the magnetic field is applied in the growth plane, recent work has also investigated the magnetotransport properties in which case single-particle tunneling dynamics dominate the interactions between both electron semiconductor layers.[5]

At zero magnetic field, many-body interactions in such low-dimensional electronic systems have also been the object of a great attention.[6] The tunneling between two parallel 2D electron gases has been recently investigated as a function of the carrier density and temperature.[7] In such an experiment, the carrier densities in both semiconductor layers were independently controlled by two different applied biases. Thus the relationship between the applied interlayer voltage and the carrier densities $n_{1,2}$ in both quantum wells was determined. In this way, Turner *et al.*[7] investigated the zero-field differential tunneling conductance as a function of the applied interlayer voltage. From the width of the obtained tunneling resonance, the lifetime of electrons within a 2D electron gas was measured as a function of electron density and temperature. As a result, they obtained electron-impurity, electron-phonon, and electron-electron contributions to the overall scattering time.[7]

In a double-quantum-well system, we know that the electron dynamics is determined by the different competing processes of carrier tunneling and carrier scattering. The carriers tunnel from one well to the other as long as the tunneling time is shorter than other scattering times.[8] Using time-resolved optical techniques, Emiliani *et al.*[8] studied the different competing processes of carrier tunneling, exciton formation, and exciton recombination inside a double-quantum-well system. A very good agreement between the predictions of the model and the experimental data was reached, considering the tunneling time and the different scattering time values.

In a bilayer electron system, the lifetime of two-dimensional electrons in the semiconductor layer plane has been measured by tunneling spectroscopy,[9] and different scattering rates have been obtained. In view of such recent tunneling spectroscopy experiments,[9] we note that one remaining key question is the theoretical analysis of the tunneling times in a double-layer electron system. In the presence of an interacting 2D electron gas, the theoretically calculated tunneling times could be used to investigate the different competing processes in the quantum heterostructure. Then the carrier dynamics in a bilayer electron system could be also determined as the same manner as in the photoluminescence experiments.[8]

However, we note that, at present, the electron dynamics between parallel two-dimensional electron gases has not been theoretically studied from a time-dependent point of view. In principle, and with the using of a time-dependent wave function for the charge density in the semiconductor growth direction, the tunneling rate values can be easily obtained.

In view of the above comments and from a theoretical point of view, in this work we shall study the time-dependent evolution of an electron wave packet considering electron-electron interactions in a double-quantum-well system. The method of calculation will be based on the discretization of space and time for the carrier wave functions. We shall show that the tunneling time values and the system resonant condition can be strongly modified due to many-body interactions.

In order to study the charge-density dynamics in the structure growth direction, we need to solve the time-dependent Schrödinger equation associated with an spinless electron in a double-quantum-well potential. The wave function $\psi$ in the $z$ axis will be given by the nonlinear Schrödinger equation[10]

$$i\hbar \frac{\partial}{\partial t}\psi(z,t)=\left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2}+V_{e\text{-}e}(|\psi|^2)+V_{qw}(z)\right]\psi(z,t) \tag{1}$$

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where $m^*$ is the standard GaAs effective mass, $V_{qw}$ is the quantum well potential and $V_{e\text{-}e}$ is the potential given by the electron-electron interaction in the heterostructure region. Such a many-body potential is given by Poisson's equation

$$\frac{\partial^2}{\partial z^2} V_{e\text{-}e}(z,t) = \frac{en}{\varepsilon} |\psi(z,t)|^2, \tag{2}$$

where $n$ is the initial electronic sheet density in both quantum wells. We should point out that in Eq. (1) the $V_{e\text{-}e}$ many-body potential is a wave-dependent quantity. Such a result is given by Eq. (1), where $V_{e\text{-}e}$ depends on the wave function form.

Now we discretize time by a superscript $\vartheta$ and spatial position by a subscript $j$. Thus $\psi \rightarrow \psi_j^{\vartheta}$. The various $z$ values become $j\,\delta z$ in the conduction band, where $\delta z$ is the mesh width. Similarly, the time variable takes the values $\vartheta\,\delta t$, where $\delta t$ is the time step. In this way, and to treat the time development, we have used an unitary propagation scheme for the evolution operator obtaining a tridiagonal linear system that can be solved by standard numerical methods.[11] In addition, we have also solved Poisson's equation associated with $V_{e\text{-}e}$ using another standard tridiagonal numerical method for each $t$ value.[10]

In the experiments,[7] the carrier densities in both semiconductor layers were independently controlled by two different applied biases. In such conductance measurements, the carrier density in the lower semiconductor layer was fixed. Meanwhile, the carrier density in the upper layer was controlled by a gate voltage. Taking this into account, we have taken two Gaussian wave packets centred in both quantum wells as our initial wave function. In principle, the height of both wave packets are given by two different coefficients $c_1$ (left quantum well) and $c_2$ (right quantum well). If the applied interlayer voltage is zero, we have a symmetric wave function and both Gaussian coefficients are identical, i.e., $c_1 = c_2$. However, if a change of carrier density $\delta n$ is induced in the first quantum well, the ratio between both gaussian coefficients can be written as $c_1/c_2 = 1 + 2\,\delta n/n$, and the total electron sheet density is given by $n + \delta n$. As a result, an asymmetric wave function at $t = 0$ is obtained.

In Fig. 1, we show the amplitude of the wave function $|\psi|^2$ and the conduction-band potential at several times. We have numerically integrated Eqs. (1), and (2) using an initial 2D electron sheet density equal to $n \sim 6.0 \times 10^{11} \mathrm{cm}^{-2}$ for the two quantum wells. In Fig. 1, we take a change of carrier density $\delta n = 0.6 \times 10^{11}$ cm$^{-2}$ in the left quantum well. Then the equations are numerically solved using a spatial mesh size of 0.5 Å and a time mesh size of 0.2 a.u. and a finite box (4000 Å) large enough so as to neglect border effects. We have considered a GaAs/$Ga_{0.7}Al_{0.3}As$ double-quantum-well system which consists of two 100-Å -wide quantum wells separated by a barrier of thickness equal to 16 Å . At $t = 0$, we have assumed a symmetric wave function that is created in the center of both quantum wells.

The numerical integration in time allows us to obtain the probability of finding the charge density $Q_{a,b}$ inside a quantum-well region $[a,\ b]$ at any time $t$,

$$Q_{a,b}(t) = \int_a^b dz\ \ |\psi(z,t)|^2 \tag{3}$$

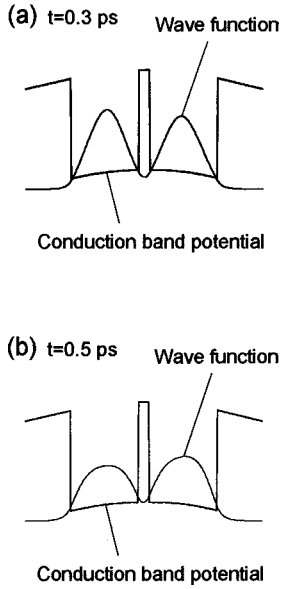


FIG. 1. (a) Conduction-band potential and wave function at $t = 0.3$ ps. (b) Conduction-band potential and wave function at $t = 0.5$ ps. We have taken an initial 2D electron sheet density equal to $n \sim 6.0 \times 10^{11}$ cm$^{-2}$, and a change of carrier density $\delta n = 0.6 \times 10^{11}$ cm$^{-2}$ in the left quantum well.

where $[a,b]$ are the quantum-well limits. In Fig. 2, we plot the charge density $Q_{a,b}$ in the left quantum well versus time. The total charge density in both quantum wells has been taken equal to 1. The change of carrier density in the left quantum well has been taken equal to $\delta n = 0.6 \times 10^{11}$ cm$^{-2}$. Such a $\delta n$ value is small. However, the existence of the tunneling oscillations between the two quantum wells is clearly shown in Fig. 2. It is found that the amplitude of the tunneling oscillations is increased as we increase $\delta n$. We can also notice that smooth curves have not been obtained from our numerical integration. The different lines shown in Fig. 2 display small oscillations (see the inset of Fig. 2). These small oscillations are independent of the time-mesh size. Such a result is given by Eq. (1) where $V_{e\text{-}e}$ depends on the wave function form in our nonlinear Schrödinger equation. Due to the $V_{e\text{-}e}$ potential being a wave-

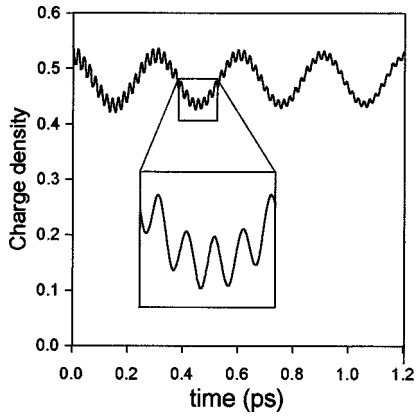


FIG. 2. Charge density in the left quantum well ($Q_{ab}$) vs time. We have taken an initial 2D electron sheet density equal to $n \sim 6.0 \times 10^{11}$ cm$^{-2}$, and a change of carrier density $\delta n = 0.6 \times 10^{11}$ cm$^{-2}$ in the left quantum well.

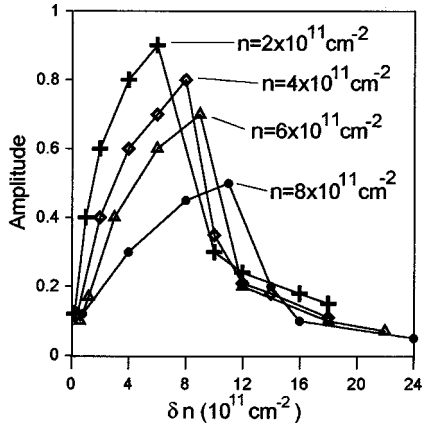


FIG. 3. Amplitude of the tunneling oscillations vs $\delta n$. We have taken four different electron sheet density values.

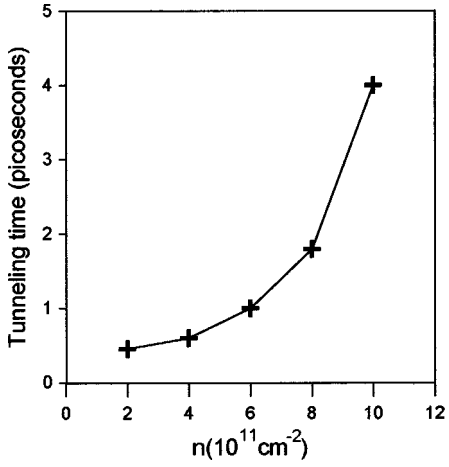


FIG. 4. Tunneling time at resonant condition vs electron sheet density.

dependent quantity, the charge dynamically trapped in the two wells produces a reaction field which modifies the time evolution of the system. As a consequence, the probability of finding an electron in the left quantum well displays small oscillations as time progresses.

In Fig. 3, we have plotted the amplitude of the tunneling oscillations versus the change of carrier density $\delta n$ in the left quantum well. The charge density in both quantum wells have been taken equal to 1. In principle, we have taken an initial sheet density equal to $n=2.0\times10^{11}$ cm$^{-2}$. In Fig. 3 it is clearly shown the existence of resonant tunneling peaks. Such a result can be easily explained as follows.

Up to a certain value of the change of carrier density ($\delta n=6.0\times10^{11}$ cm$^{-2}$), we have found that the amplitude of the tunneling oscillations is increased as we increase $\delta n$. The electron energy levels of both wells are exactly aligned at $\delta n=0$. Therefore, if $\delta n\sim0$ the charge density will oscillate between both wells with a certain tunneling period. If $\delta n$ is increased, the amplitude of the oscillations will also be increased due to the field-induced tunneling process. However, we know that if the potential difference between both wells is higher than the level splitting, the resonant condition is not obtained, and, then, the tunneling process is not allowed. Such a result is clearly shown in the numerical data plotted in Fig. 3. In Fig. 3, we have found the existence of oscillations up to a certain value of $\delta n$.

Figure 4 shows the different resonant tunneling peaks at different $n$ values. Taking into account the relation between the oscillation amplitude and the tunneling current, we can see that our results are consistent with the experimental data obtained by Eisenstein, Pfeiffen and West[3] and Turner *et al.*[7] In the experiments, the authors found a peaked tunneling conductance at $\delta n\sim0$ in the absence of an applied magnetic field. If $n$ is increased, we also note the existence of two new effects in Fig. 3. First, the amplitude peak position is shifted to a higher $\delta n$ value. Second, the amplitude peak value is decreased.

This shift in the amplitude peak can be easily explained as follows. Up to a certain resonant value of $\delta n$, we have field-induced tunneling oscillations due to the existence of a potential difference between both wells. Such a potential value is given by the ratio between $\delta n$ and $n$. Consequently, the resonant $\delta n$ value is increased as we increase $n$.

Let us study the second point. In Fig. 3, we have found that the oscillation amplitude at a resonant condition is decreased as we increase the 2D sheet density. We shall explain this result considering our nonlinear effective-mass Schrödinger equation. In absence of electron-electron interaction, we know that the amplitude of the oscillating charge density is approximately equal to 1 at resonant condition.[10] However, it has been also shown that such a result can be modified due to nonlinear effects.[10] In the case of Ref. 10, no charge is present in the right well and, in principle, the initial resonant condition is approximately obtained for small carrier sheet density values. In our case, two Gaussian wave packets centered in both quantum wells is our initial wave function. Up to a certain resonant $\delta n$ value, we have field-induced tunneling oscillations. It is clearly shown in Fig. 3 that the resonant $\delta n$ value is increased as we increase $n$. Then the total electron sheet density in the double-well system $n+\delta n$ is also inreased. At high $n+\delta n$ values, the time-dependent evolution of the wave packets is strongly modified due to nonlinear effects, i.e., due to the $V_{e\text{-}e}(|\psi|^2)$ potential in Eq. (1). The charge dynamically trapped in the double-well system produces a reaction field which modifies the amplitude of the tunneling oscillations. As a consequence, the amplitude of the oscillating charge density is never equal to 1.

When thicker barriers are introduced in our numerical evolution, we also note the existence of an important effect in the tunneling process. The amplitude peak position is shifted to a lower $\delta n$ value. If the barrier thickness is increased, the level splitting value between both wells is also decreased. As a result, the resonant condition is shifted to a lower $\delta n$ value.

To obtain the tunneling times, we note the existence of tunneling charge oscillations between both quantum wells in Fig. 2. From a semiclassical point of view, the tunneling time value is given by

$$\tau=\frac{1}{\nu A}, \tag{4}$$

where $\nu$ is the oscillation frequency and $A$ the oscillation amplitude (transmission).[12] The amplitude and the frequency of the tunneling oscillations can easily be obtained from Fig.

2. And taking into account Eq. (4), the semiclassical values for the tunneling times can be easily calculated.

In Fig. 4, we plot the tunneling times $\tau$ at resonant condition versus $n$. We note that $\tau$ is exponentially increased as we increase the initial carrier sheet density. Such an effect can be understood considering that the amplitude of the tunneling oscillations at resonant condition is decreased if we increase $n$ (Fig. 3). Then, and from Eq. (4), the obtained $\tau$ values are also exponentially increased.

From a semiclassical point of view, the dynamics of an electron that is localized in a quantum well is determined by the two different competing processes of carrier tunneling to the other well and carrier scattering in the semiconductor layer plane. An electron tunnels from one well to the other as long as the tunneling rate is higher than the scattering rate in the quantum-well layer plane. The different scattering rate values has been recently investigated from an experimental point of view.[7]

From the $n$ and T dependence of the linewidth $\Gamma$, Turner *et al.*[7] obtained the electron-impurity, electron-phonon, and electron-electron contributions to the overall scattering rate $\Gamma$. In the temperature range considered, they found that the mobility is dominated by impurity scattering. At a fixed carrier density, $\Gamma$ exhibited an approximate $T^2$ temperature dependence. From the experiments over a range of carrier densities, they investigated the $n$ dependence of the linewidth $\Gamma$. In the high-$n$, low-$T$ regime, they found a constant scattering time $\tau_{sc} \sim 1.2$ ps.

At this point, we should remark that our calculated tunneling time in a bilayer system increases exponentially with increasing $n$ due to nonlinear effects (Fig. 4). Such an effect is given by the amplitude of the tunneling oscillations, that is strongly decreased as we increase $n$ (Fig. 3). Then $\tau$ is always higher than $\tau_{sc}$ if the electron sheet density is large enough (the experiment shows a constant and low scattering time $\tau_{sc}$ value at large electronic sheet densities). At the high-$n$ regime, the scattering process in the quantum-well layer plane is more efficient than the tunneling process between the two quantum wells. Consequently, a suppression of the tunneling current at large electronic sheet densities can be expected.

In summary, in this work we have numerically integrated a nonlinear effective-mass Schrödinger equation in a double-quantum-well system in space and time. Electron-electron interaction effects have been considered in our model through a Hartree potential. It is found that charge accumulation effects can play an important role in the tunneling process. If the carrier sheet density is increased, the system resonant condition is shifted to a higher $\delta n$ value. In addition, the obtained tunneling time between the quantum wells is exponentially increased as we increase $n$ due to nonlinear effects. Finally, we showed the possibility of having a suppression of the tunneling current in the bilayer system at the high-$n$ regime.

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