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Quantum decoherence and weak localization at low temperatures

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With the aid of the Keldysh technique we develop a path-integral formalism which allows us to nonperturbatively study the quantum dynamics of electrons in a disordered metal in the presence of interactions. We derive a formally exact equation of motion for the electron density matrix and demonstrate that the effect of interaction of the electron with other electrons in a disordered metal is equivalent to that of an effective dissipative environment. We obtain nonperturbative results for the weak localization correction to the conductance $\delta\sigma$ and show that the effective decoherence length extracted from $\delta\sigma$ remains finite down to $T=0$ due to electron-electron interactions. [S0163-1829(98)06942-2]

I. INTRODUCTION

Recent experiments[1] attracted a great deal of attention to an old but fundamental question: how fast can a quantum particle lose information about its initial state in the presence of interaction? In other words, how fast can interaction destroy the quantum phase coherence? The answer to this question essentially depends on the type of interaction.

It follows from general principles of quantum mechanics that quantum coherence of the wave function cannot be destroyed due to *elastic* interaction with a static potential. Another physical situation may take place if the quantum particle interacts with other (quantum) degrees of freedom which play the role of an effective environment. In this case quantum dynamics of the particle cannot be described by the wave function but only by the density matrix. Various examples (to be discussed below) show that such interactions *may lead* to a destruction of quantum coherence.

A general approach to the problem was formulated by Feynman and Vernon[2,3] who demonstrated that the effect of environment can be taken into account by means of averaging over all its possible quantum states. As a result the environment variables are integrated out and quantum dynamics of the particle can be described only in terms of its own degrees of freedom. Within this approach, interaction with the external environment is taken into account by means of the so-called *influence functional* which appears in the (effective) action for the particle as a result of averaging over the bath variables. It is quite clear that specific properties of the environment are not important unless they explicitly enter the expression for the influence functional. In other words, the particle does not ''feel'' the difference between physically different bathes provided they are described by the same influence functional.

These ideas were developed further by Caldeira and Leggett[4] who showed that the above arguments can be used to describe quantum dynamics of dissipative systems and derived the effective action for the case of linear Ohmic dissipation. The same type of analysis was also developed by Schmid[5] who formulated a quasiclassical Langevin equation approach describing real time dynamics of a quantum particle in the presence of dissipation and quantum noise.

Although the above papers are dealing with the model of a bosonic environment it is obvious that the ideas[2,3] can be applied to a fermionic bath as well. This was done, e.g., by Ambegaokar, Eckern, and Schön[6] in the case of superconducting tunnel junctions and later by Schön and one of the present authors[7,8] in a somewhat broader context of a metallic system with dissipation. Although the microscopic Hamiltonian describing electrons in a metal[6–8] is quite different from the one used in the model,[4] the final expressions for the influence functionals for various metallic systems and tunnel junctions obtained in[6–8] turn out to be similar or even completely equivalent to those considered in Refs. 4,5. This equivalence is just an illustration of the property discussed above: the effect of physically different environments is indistinguishable provided they are described by the same influence functional.

In Refs. 6–8 quantum dynamics of a certain collective variable of interest (the phase) was considered. This variable was extracted ''from interaction,'' after that electronic degrees of freedom were integrated out and the effective action for the phase was derived. One can also generalize this procedure and describe quantum dynamics of superconductors considering the phase as a quantum field.[9] In all these cases the collective variable is intimately linked to the electronic bath, quantum dynamics of the former does not exist without the latter at all.

In this paper we will analyze a somewhat different situation. Namely, we will study quantum dynamics of an electron propagating in a disordered metal and interacting with other electrons which play the role of an effective environment. It is well known that quantum interference of electrons scattered on impurities lead to quantum corrections to the classical Drude conductivity.[10,11] These so-called weak-localization corrections have been extensively discussed in the literature (see, e.g., Refs. 12,13 for review). The magnitude of these corrections is known to be determined by the time within which electrons in a metal can be described by a phase coherent wave function. At times exceeding this so-called decoherence time τ_φ quantum coherence is destroyed, quantum interference is not anymore possible, and therefore the classical diffusion picture is restored.

The decoherence time τ_φ or the decoherence length $L_\varphi = \sqrt{D\tau_\varphi}$ in a disordered metal can be determined by various physical processes, such as electron-electron and electron-phonon interactions, electron scattering on magnetic impurities, etc. (see, e.g., Refs. 12,13). It was shown by Altshuler, Aronov, and Khmelnitskii[14] that at not very low temperatures the effect of electron-electron interaction on the decoherence time is equivalent to that of classical Nyquist noise in a disordered conductor. In this case one finds[14–16] $\tau_\varphi \propto T^{2/(d-4)}$, where d is the effective system dimension. This result demonstrates that the decoherence effect of the electron-electron interaction becomes weaker as the temperature is lowered.

Down to which temperature does the above result remain correct? Or, more generally, does L_φ increase with decreasing T at all temperatures thus going to infinity at $T\to 0$? A positive answer on the latter question would mean that at $T=0$ the electron in a disordered metal can be described by the phase-coherent wave function even in the presence of Coulomb interaction with other electrons.

It is sometimes believed that this answer can be obtained without any calculation. One can argue that at $T\to 0$ in equilibrium all quantum states below the Fermi energy are occupied. Thus scattering into any of these states is forbidden due to the Pauli principle and the electron energy cannot change. Since purely elastic processes do not destroy the coherence of the wave function one can conclude that L_φ should diverge at $T\to 0$.

Here the role of the Pauli principle is merely to provide the energy constraint. Hence, the above argument is not specific for fermions and should also apply to a low-energy Bose particle interacting with a collection of harmonic oscillators. If initially all the oscillators were in their *noninteracting* ground states and the particle kinetic energy E was small $E\to 0$ after the scattering process this energy cannot change and remains equal to zero because none of the oscillators can either be excited or give energy to a particle. Therefore, the coherence of the particle wave function is preserved. Similar arguments were used by the authors[17] who argued against an attempt[18] to relate the low-temperature saturation of τ_φ to the effect of zero-point fluctuations of impurities.

The above arguments can be applied to a scattering problem for which the total energy of the system is fixed to be the sum of energies of *noninteracting* particles. In this case—provided the interaction is small—a standard Golden rule perturbation theory is usually sufficient. It is obvious, however, that the above physical situation does not account for the equilibrium properties of an *interacting* system. In the latter case the above arguments are *not correct*. As a simple example one can consider a quantum particle (with mass m and coordinate q) interacting with the Caldeira-Leggett bath of oscillators (with the high-frequency cutoff ω_c). This model can be solved exactly and for the reduced density matrix of the particle q one obtains[19]

$$\rho(q_1,q_2)\propto \exp[-m\langle E\rangle (q_1-q_2)^2], \tag{1}$$

where $\langle E\rangle$ is the expectation value of the particle kinetic energy. At high temperatures one has $\langle E\rangle \approx T/2$ while at lower temperatures this expectation value saturates and is determined by interaction $\langle E\rangle = (\gamma/2\pi)\ln(\omega_c/\gamma) > 0$, where the parameter γ is the effective coupling strength between the particle and the bath. These results do not depend on the initial conditions and can also be obtained, e.g., by an exact diagonalization of the initial Hamiltonian.[20] *Any* perturbation of the density matrix will relax to the same equilibrium form (1) and the information about the initial phase of the particle wave function is lost as a result of interaction even at $T=0$. All these results cannot be derived within the simple Golden-rule-type perturbation theory in the interaction.

At this point we would like to emphasize that in the above example we (on purpose) consider the basis of ‘‘noninteracting’’ eigenstates of the system. It is obvious that the full wave function of the total system with interaction as well as each of the eigenmodes obtained by an exact diagonalization always stay coherent. However, if the behavior of the particle q (and not that of the eigenmodes) is of interest, the reduced density matrix $\rho(q_1,q_2)$ should be studied. The decay of the off-diagonal elements of ρ on the length scale $L_d \sim 1/\sqrt{m\langle E\rangle}$ just implies that the bath in some conventional sense ‘‘measures’’ the particle position.[20] In principle the off-diagonal elements of $\rho(q_1,q_2)$ (and thus the coherence of the particle q) can be suppressed completely (L_d tends to zero if one, e.g., chooses $\omega_c\to\infty$), while the eigenmodes of the total system obviously remain fully coherent. Therefore, the question about the presence or absence of quantum coherence in the interacting many-body system can be discussed only after the physical quantity of interest is defined.

An illustration for this point is provided, e.g., by the well-known problem of single electron (or Cooper pair) tunneling in the presence of an effective environment produced by other electrons.[8,21–25] If one takes into account only the effective charging energy of a tunnel junction and ignores the effect of the electronic environment one arrives at the conclusion that at $T=0$ and small voltages noninteracting (and therefore fully coherent) electrons stay at each side of a tunnel barrier and do not tunnel. Tunneling is blocked due to Coulomb interaction at the junction and no current can flow in the system. It is well known, however, that interaction with other electrons (equivalent to the presence of an effective impedance) lifts the Coulomb blockade of tunneling even at $T=0$ leading to a nonzero current in the system at any nonzero voltage. Tunneling becomes *incoherent* due to interaction with the effective electronic environment. It is also important to emphasize that this effect cannot be described perturbatively in the interaction, the Golden-rule-type expansion in the effective impedance is insufficient and yields the incorrect result in this case. The validity of the above theoretical picture was confirmed in experiments (see, e.g., Refs. 26,27) the results of which can be interpreted as a direct evidence for the low temperature saturation of the expectation value of the electron energy $\langle E\rangle$ and the effective length L_d due to interaction.

The problem to be considered here is similar to the above example in several important aspects. In both cases we are dealing with interacting electrons in a disordered metal and therefore the Pauli principle should be explicitly accounted for. In both cases an electron propagating in a disordered metal interacts with a fluctuating electric field produced by other electrons which play the role of an effective dissipative environment.[28] In both cases the Golden rule perturbation theory in the interaction is not sufficient for calculation of the system conductance and a nonperturbative analysis should be developed. This analysis is presented below.

The paper is organized as follows. In Sec. II we make use of the general formalism of the Green-Keldysh functions[29] and derive a formally exact equation of motion for the electron density matrix in the presence of Coulomb interaction. This equation explicitly accounts for the Pauli principle and allows for a clear understanding of its role in the process of electron-electron interaction in a metal. In Sec. III the effective action (or the influence functional) for the fluctuating scalar potential in a metallic conductor is derived. We also demonstrate that in the equilibrium this influence functional satisfies the fluctuation-dissipation theorem[30] and establish the relation with the real time effective action derived in the Caldeira-Leggett models.[4,5,8,31] In Sec. IV with the aid of these general results we will derive the real time effective action for the electron propagating in a metal and determine the decoherence time τ_φ and the weak localization correction to conductivity at low temperatures. Our formalism naturally includes both electron-electron and electron-phonon interactions and allows us to establish the corresponding contributions to τ_φ from each of these processes. In Sec. V we derive the quasiclassical kinetic equation and demonstrate the relation of our analysis to the standard kinetic approach which allows us to evaluate the inelastic scattering time τ_i. We also derive the quasiclassical Langevin equation which under certain conditions can be used to describe propagation of electrons in a disordered metal. Discussion of the results is given in Sec. VI. We also briefly compare our theoretical predictions with the experimental data.[1] Some details of our calculation are presented in Appendixes.

II. DENSITY MATRIX

We will consider a standard Hamiltonian describing electrons in a disordered metal

$$\boldsymbol{H}_{\rm el}=\boldsymbol{H}_0+\boldsymbol{H}_{\rm int}, \tag{2}$$

where

$$\boldsymbol{H}_0=\int d\boldsymbol{r}\psi^{+}_{\sigma}(\boldsymbol{r})\left[-\frac{\nabla^2}{2m}-\mu+U(\boldsymbol{r})\right]\psi_{\sigma}(\boldsymbol{r}), \tag{3}$$

$$\boldsymbol{H}_{\rm int}=\frac{1}{2}\int d\boldsymbol{r}\int d\boldsymbol{r}'\,\psi^{+}_{\sigma}(\boldsymbol{r})\psi^{+}_{\sigma'}(\boldsymbol{r}')e^2v(\boldsymbol{r}-\boldsymbol{r}')\psi_{\sigma'}(\boldsymbol{r}')\psi_{\sigma}(\boldsymbol{r}). \tag{4}$$

Here μ is the chemical potential, $U(\boldsymbol{r})$ accounts for a random potential due to nonmagnetic impurities, and $v(\boldsymbol{r})=1/|\boldsymbol{r}|$ represents the Coulomb interaction between electrons.

Let us define the generating functional for the electron Green-Keldysh functions in terms of the path integral over the Grassmann fields $\overline{\psi}$ and ψ

$$J[\eta,\eta^*]=\frac{\displaystyle\int\mathcal{D}V\int\mathcal{D}\overline{\psi}\int\mathcal{D}\psi\exp\left(iS_{\rm eff}[\overline{\psi},\psi,V]+i\int_K dt\int d\boldsymbol{r}[\overline{\psi}(t,\boldsymbol{r})\eta(t,\boldsymbol{r})+\psi(t,\boldsymbol{r})\eta^*(t,\boldsymbol{r})]\right)}{\displaystyle\int\mathcal{D}V\int\mathcal{D}\overline{\psi}\int\mathcal{D}\psi\exp(iS_{\rm eff}[\overline{\psi},\psi,V])}, \tag{5}$$

where $S_{\rm eff}$ is the effective action

$$\begin{aligned}S_{\rm eff}[\overline{\psi},\psi,V]=&\int_K dt\left(\int d\boldsymbol{r}[i\overline{\psi}(t,\boldsymbol{r})\partial_t\psi(t,\boldsymbol{r})\right.\\&\left.-e\overline{\psi}(t,\boldsymbol{r})\psi(t,\boldsymbol{r})V(t,\boldsymbol{r})]-\boldsymbol{H}_0[\overline{\psi},\psi]\right)\\&+\frac{1}{2}\int_K dt\int d\boldsymbol{r}\int d\boldsymbol{r}'\,V(t,\boldsymbol{r})\\&\times v^{-1}(\boldsymbol{r}-\boldsymbol{r}')V(t,\boldsymbol{r}'),\end{aligned} \tag{6}$$

where $v^{-1}(\boldsymbol{r}-\boldsymbol{r}')=-\nabla^2/4\pi$. Integration over time t in Eq. (6) goes along the Keldysh contour K which runs in the forward and then in the backward time directions.[29] In Eqs. (5),(6) we performed a standard Hubbard-Stratonovich transformation introducing the path integral over a scalar potential field V in order to decouple the ψ^4 interaction in Eq. (4).

The electron Green-Keldysh function $\hat{\boldsymbol{G}}$ can be determined from Eq. (5) by taking the derivatives with respect to the source fields η and η^*:

$$\hat{\boldsymbol{G}}(t,\boldsymbol{r};t',\boldsymbol{r}')=i\frac{\delta}{\delta\eta^*(t,\boldsymbol{r})}\frac{\delta}{\delta\eta(t',\boldsymbol{r}')}J[\eta,\eta^*]\Big|_{\eta=\eta^*=0}. \tag{7}$$

Making use of Eqs. (5),(7) and the definition of the Green-Keldysh function for an electron interacting with the field V

$$\hat{G}_V(t,\boldsymbol{r};t',\boldsymbol{r}')=-i\frac{\displaystyle\int\mathcal{D}\overline{\psi}\int\mathcal{D}\psi\,\psi(t,\boldsymbol{r})\overline{\psi}(t',\boldsymbol{r}')\exp(iS_{\rm eff}[\overline{\psi},\psi,V])}{\displaystyle\int\mathcal{D}\overline{\psi}\int\mathcal{D}\psi\exp(iS_{\rm eff}[\overline{\psi},\psi,V])}, \tag{8}$$

it is easy to prove the identity

$$\hat{\boldsymbol{G}}=\frac{\displaystyle\int \mathcal{D}V_1\mathcal{D}V_2\,\hat{G}_V\,e^{i\boldsymbol{S}[V_1,V_2]}}{\displaystyle\int \mathcal{D}V_1\mathcal{D}V_2\,e^{iS[V_1,V_2]}}, \tag{9}$$

where

$$i\boldsymbol{S}[V_1,V_2]=2\,\mathrm{Tr}\ln\hat{G}_V^{-1}+i\int_0^t dt'\int d\boldsymbol{r}\;T\frac{(\nabla V_1)^2-(\nabla V_2)^2}{8\pi}. \tag{10}$$

The factor 2 in front of the trace comes from the summation over a spin index. In Eqs. (9),(10) we explicitly defined the fields $V_1(t)$ and $V_2(t)$ equal to $V(t)$, respectively, on the forward and backward parts on the Keldysh contour K. Analogously $\hat{\boldsymbol{G}}$ and $\hat{G}_V\equiv\hat{G}[V_1,V_2]$ are the 2×2 matrices in the Keldysh space:

$$\hat{\boldsymbol{G}}=\begin{pmatrix}\boldsymbol{G}_{11} & -\boldsymbol{G}_{12}\\ \boldsymbol{G}_{21} & -\boldsymbol{G}_{22}\end{pmatrix}. \tag{11}$$

The matrix function $\hat{G}_V$ obeys the equation

$$\left(i\frac{\partial}{\partial t_1}-\hat{H}_0(\boldsymbol{r}_1)+e\hat{V}(t_1,\boldsymbol{r}_1)\right)\hat{G}_V=\delta(t_1-t_2)\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2); \tag{12}$$

where

$$\hat{H}_0=H_0\hat{1}=\begin{pmatrix}\dfrac{-\nabla^2}{2m}-\mu+U(\boldsymbol{r}) & 0\\ 0 & \dfrac{-\nabla^2}{2m}-\mu+U(\boldsymbol{r})\end{pmatrix};$$

$$\hat{V}=\begin{pmatrix}V_1(t,\boldsymbol{r}) & 0\\ 0 & V_2(t,\boldsymbol{r})\end{pmatrix}. \tag{13}$$

Note that the function $\hat{G}_V$ is to some extent similar to the Green-Keldysh function of an electron in an external field. However, there exists an important difference: in our case the electron interacts with a fluctuating (quantum) field V. Formally this implies that the fields $V(t,\boldsymbol{r})$ on two parts of the Keldysh contour differ $V_1(t,\boldsymbol{r})\neq V_2(t,\boldsymbol{r})$, while for the external field one always has $V_1(t,\boldsymbol{r})\equiv V_2(t,\boldsymbol{r})$.

The general solution of the Eq. (12) can be expressed in the form

$$\hat{G}_V(t_1,t_2)=-i\hat{U}_V(t_1,t_2)\times[\theta(t_1-t_2)\hat{a}-\theta(t_2-t_1)\hat{b}+\hat{f}_V(t_2)]. \tag{14}$$

Here we defined

$$\hat{a}=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix},\quad \hat{b}=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, \tag{15}$$

and $\hat{U}_V(t_1,t_2)$ is the matrix evolution operator

$$\hat{U}_V(t_1,t_2)=\begin{pmatrix}u_1(t_1,t_2) & 0\\ 0 & u_2(t_1,t_2)\end{pmatrix}, \tag{16}$$

which consists of the scalar evolution operators

$$\begin{aligned}u_{1,2}(t_1,t_2)&=\mathbf{T}\exp\left[-i\int_{t_1}^{t_2}dt'[H_0-eV_{1,2}(t')]\right]\\ &=\int_{\boldsymbol{r}(t_1)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_2)=\boldsymbol{r}_f}\mathcal{D}\boldsymbol{r}(t')\\ &\quad\times\exp\left[i\int_{t_1}^{t_2}dt'\left(\frac{m\dot{\boldsymbol{r}}^2}{2}-U(\boldsymbol{r})+eV_{1,2}(t',\boldsymbol{r})\right)\right],\end{aligned} \tag{17}$$

T being the time ordering operator. In Eq. (14) and below we always imply integration over the internal coordinate variables in the product of operators, whereas integration over time is written explicitly. For the sake of brevity we also do not indicate the coordinate dependence in Eq. (14) and many subsequent expressions. This dependence can be trivially restored if needed.

Note that Eq. (14) is completely equivalent to the standard representation of the Green-Keldysh matrix which elements can be expressed in terms of retarded, advanced and Keldysh-Green functions. The representation (14) defines a general solution of the linear differential Eq. (12): the term $\hat{U}_V(t_1,t_2)\hat{f}_V(t_2)$ with an arbitrary matrix operator $\hat{f}_V(t_2)$ represents a general solution of the homogeneous equation, while the terms with θ functions give a particular solution of the inhomogeneous equation. The operator function $\hat{f}_V(t_2)$ in Eq. (14) is fixed by the Dyson equation

$$\hat{G}_V(t_1,t_2)=\hat{G}_0(t_1,t_2)-\int_0^t dt'\,\hat{G}_0(t_1,t')e\hat{V}(t')\hat{G}_V(t',t_2). \tag{18}$$

The matrix $\hat{G}_0$ is the electron Green-Keldysh function without the field. This function is defined by Eqs. (14)–(17) with $V_{1,2}(t,\boldsymbol{r})\equiv 0$ and $\hat{f}_0(t_2)$ has the form

$$\hat{f}_0(t_2)=\begin{pmatrix}-\rho_0(t_2) & \rho_0(t_2)\\ 1-\rho_0(t_2) & \rho_0(t_2)\end{pmatrix}, \tag{19}$$

where $\rho_0(t)=e^{-iH_0t}\rho(0)e^{iH_0t}$ is the electron density matrix for $V_{1,2}=0$ at a time t.

Equation (18) can be solved perturbatively in $e\hat{V}$. Combining this solution with Eq. (9) one reproduces the standard Keldysh diagrams. This way of treating the problem is quite complicated in general and becomes particularly nontransparent in the interesting limit of low temperatures.

We will proceed differently. It is well known that the 1,2 component of the Green-Keldysh matrix $\hat{\boldsymbol{G}}$ is directly related to the exact electron density matrix

$$\boldsymbol{\rho}(t;\boldsymbol{r},\boldsymbol{r}')=-i\boldsymbol{G}_{12}(t,t;\boldsymbol{r},\boldsymbol{r}'), \tag{20}$$

which contains all necessary information about the system dynamics in the presence of interaction. Analogously one can define the ''density matrix'' $\rho_V(t)\equiv f_{1,2}(t)$ related to the 1,2 component of the matrix $\hat{G}_V$ by the equation equivalent to Eq. (20). Our strategy is as follows. First we will derive the exact equation of motion for the density matrix $\rho_V(t)$ which turns out to have a very simple and transparent form. Already at this stage we will clarify the role of the fluctuating fields $V_{1,2}$ and the Pauli principle in our problem. Then we will evaluate the influence functional $S[V_1,V_2]$ (10) and find the density matrix $\boldsymbol{\rho}$ from the equation

$$\boldsymbol{\rho}(t;\boldsymbol{r},\boldsymbol{r}')=\langle\rho_V(t;\boldsymbol{r},\boldsymbol{r}')\rangle_{V_1,V_2}, \tag{21}$$

where the average over fields V_1 and V_2 is defined in Eq. (9).

The derivation of the equation for the density matrix $\rho_V(t)$ is straightforward. Let us perform the time integration in the last term of Eq. (18). Integrating by parts and making use of Eq. (12) after a simple algebra (see Appendix A) we obtain

$$\hat{G}_0(t_1,t)\hat{G}(t,t_2)-\hat{G}_0(t_1,0)\hat{G}(0,t_2)=0. \tag{22}$$

Substituting the representation (14) into Eq. (22) we arrive at the matrix equation which relates the matrix $\hat{f}_V(t)$, the evolution operator $\hat{U}_V(t)$, and the initial density matrix $\rho_0(0)$ defined for $V_{1,2}=0$. With the aid of this equation one determines the 1,2 component of the matrix $\hat{f}_V(t)$ and thus the density matrix $\rho_V(t)$. The details of this calculation are presented in Appendix A. As a result we find

$$[1-\rho_0(t)(u_2(t,0)u_1(0,t)-1)]\rho_V(t)=\rho_0(t). \tag{23}$$

One can also rewrite this result in the form of the differential equation describing the time evolution of the density matrix:

$$i\frac{\partial\rho_V}{\partial t}=[H_0,\rho_V]-(1-\rho_V)eV_1\rho_V+\rho_V eV_2(1-\rho_V),$$

$$\rho_V(0)=\rho_0(0). \tag{24}$$

Equation (24) is the main result of this section. We would like to emphasize that our derivation was performed *without any approximation*, i.e., the result (24) *is exact*. It contains all information about the system dynamics hidden in the four components of the Green-Keldysh matrix. In the absence of the fluctuating field $V_{1,2}=0$ Eq. (24) reduces to the standard equation for the electron density matrix with the Hamiltonian H_0. In the presence of the field $V_{1,2}$, Eq. (24) exactly accounts for the Pauli principle. This is obvious from our derivation which automatically takes care about the Fermi statistics through the integration over the Grassmann fields ψ and $\bar{\psi}$. This is also quite clear from the form of the last two terms in the right-hand side of Eq. (24). In Sec. V we will demonstrate that within the quantum kinetic analysis these terms are responsible for the standard in- and out-scattering terms in the collision integral.

In order to understand the role of the Pauli principle let us rewrite Eq. (24) in the form

$$i\frac{\partial\rho_V}{\partial t}=[H_0-eV^+,\rho_V]-(1-\rho_V)\frac{eV^-}{2}\rho_V-\rho_V\frac{eV^-}{2}(1-\rho_V), \tag{25}$$

where we defined $V^+=(V_1+V_2)/2$ and $V^-=V_1-V_2$. It is quite obvious from Eq. (25) that the field $V^+(t,\boldsymbol{r})$ plays the same role as an external field. All electrons move collectively in this field. The Pauli principle does not play any role here. Below we will demonstrate that quantum fluctuations of the field V^+ are responsible for the low-temperature saturation of the decoherence time τ_φ and the weak localization correction to conductivity in disordered metals.

The field V^- is, on the contrary, very sensitive to the Pauli principle. It will be shown below that this field is responsible for damping due to radiation of an electron which moves in a metal. The corresponding energy losses can be only due to electron transitions into lower energy states. At $T=0$ in equilibrium all such states are already occupied by other electrons, therefore such processes are forbidden and the electron energy remains unchanged due to the Pauli principle. We will demonstrate, however, that these processes are irrelevant for the decay of the off-diagonal elements of the electron density matrix and, therefore, the Pauli principle can hardly affect quantum decoherence in a disordered metal even at $T=0$.

III. INFLUENCE FUNCTIONAL FOR THE FIELD

Let us now derive the expression for the influence functional (effective action) $S[V_1,V_2]$ for the field V. A formally exact action $\boldsymbol{S}[V_1,V_2]$ obtained by integration over all electron degrees of freedom is given by Eq. (10). Let us expand this expression up to the second order in $V_{1,2}$. The first-order terms of this expansion vanish because the Green function $\hat{G}_0$ corresponds to a zero current and zero charge-density state of the system. In the second order we obtain

$$i\delta S^{(2)}=-\mathrm{Tr}(\hat{G}_0e\hat{V}\hat{G}_0e\hat{V})=-e^2\,\mathrm{tr}\Big[G_{11}V^+G_{11}V^+-G_{12}V^+G_{21}V^+-G_{21}V^+G_{12}V^++G_{22}V^+G_{22}V^+G_{11}V^+G_{11}V^-$$
$$-G_{12}V^+G_{21}V^-+G_{21}V^+G_{12}V^--G_{22}V^+G_{22}V^-\frac{1}{4}G_{11}V^-G_{11}V^-+\frac{1}{4}G_{12}V^-G_{21}V^-+\frac{1}{4}G_{21}V^-G_{12}V^-$$
$$+\frac{1}{4}G_{22}V^-G_{22}V^-\Big]. \tag{26}$$

Here the subscript $_0$ for the Green functions is omitted for the sake of simplicity, all the Green functions here and below in this section are defined for $V_{1,2}=0$. Expression (26) can be simplified with the aid of the identities

$$G_{11}=G_{12}+G^R=G_{21}+G^A, \quad G_{22}=G_{12}-G^A=G_{21}-G^R, \tag{27}$$

which allow us to exclude the functions G_{11} and G_{22} from the action (26). The terms containing V^+V^+ are reduced to

$$\mathrm{tr}\,(G^RV^+G^RV^+ + G^AV^+G^AV^+). \tag{28}$$

Making use of the expressions

$$G^R=-i\theta(t_1-t_2)u_0(t_1,t_2), \quad G^A=i\theta(t_2-t_1)u_0(t_1,t_2), \tag{29}$$

and writing the traces (28) in the time-space representation, we immediately observe that the product $\theta(t_1-t_2)\theta(t_2-t_1)=0$ appears under the integral, and the whole combination (28) vanishes. In other words, the terms V^+V^+ give no contribution to the action (26). The remaining terms in combination with the last two terms in Eq. (10) describing the free field action yield

$$\begin{aligned} iS[V_1,V_2]=&\,i\int_0^t dt'\int d\boldsymbol{r}\,\frac{(\nabla V^+\nabla V^-)}{4\pi} \\ &-\int_0^t dt_1\int_0^t dt_2\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \\ &\times[iV^-(t_1,\boldsymbol{r}_1)\chi(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)V^+(t_2,\boldsymbol{r}_2) \\ &+V^-(t_1,\boldsymbol{r}_1)\,\eta(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)V^-(t_2,\boldsymbol{r}_2)], \end{aligned} \tag{30}$$

where

$$\begin{aligned} \chi(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=&-ie^2\{G^R(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)[G_{21}(t_2,t_1,\boldsymbol{r}_2,\boldsymbol{r}_1) \\ &+G_{12}(t_2,t_1,\boldsymbol{r}_2,\boldsymbol{r}_1)]+[G_{12}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2) \\ &+G_{21}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)]G^A(t_2,t_1,\boldsymbol{r}_2,\boldsymbol{r}_1)\}, \end{aligned} \tag{31}$$

$$\begin{aligned} \eta(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=&\frac{e^2}{2}[G_{12}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)G_{21}(t_2,t_1,\boldsymbol{r}_2,\boldsymbol{r}_1) \\ &+G_{21}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)G_{12}(t_2,t_1,\boldsymbol{r}_2,\boldsymbol{r}_1)]. \end{aligned} \tag{32}$$

Expressions (30)–(32) define the influence functional for the field V in terms of the Green-Keldysh functions for the (in general nonequilibrium) electron subsystem. It is easy to check that the above expressions satisfy the requirement of causality: in the $V^-(t_1)V^+(t_2)$ terms the time t_1 is always larger than t_2 due to analytic properties of retarded and advanced Green functions (29). It is also straightforward to demonstrate (see Appendix B) that in thermodynamic equilibrium the kernels $\eta(\omega)$ and $\mathrm{Im}\,\chi(\omega)$ (Fourier transformed with respect to the time difference t_1-t_2) satisfy the equation

$$\eta(\omega,\boldsymbol{r}_1,\boldsymbol{r}_2)=-\frac{1}{2}\coth\left(\frac{\omega}{2T}\right)\mathrm{Im}\,\chi(\omega,\boldsymbol{r}_1,\boldsymbol{r}_2). \tag{33}$$

The latter equation is just the fluctuation-dissipation theorem.[30]

Finally, for a homogeneous system one can also perform the Fourier transformation with respect to $\boldsymbol{r}_1-\boldsymbol{r}_2$ and find

$$\begin{aligned} iS[V_1,V_2]=&\,i\int\frac{d\omega d^3k}{(2\pi)^4}V^-(-\omega,-k)\frac{k^2\epsilon(\omega,k)}{4\pi}V^+(\omega,k) \\ &-\frac{1}{2}\int\frac{d\omega d^3k}{(2\pi)^4}V^-(-\omega,-k)\frac{k^2\,\mathrm{Im}\epsilon(\omega,k)}{4\pi} \\ &\times\coth\left(\frac{\omega}{2T}\right)V^-(\omega,k), \end{aligned} \tag{34}$$

where $\epsilon(\omega,k)$ is the dielectric susceptibility of the system. For a homogeneous electron gas it is given by the standard random-phase approximation (RPA) formula:

$$\epsilon(\omega,k)=1+\frac{e^2}{\pi^2k^2}\int d^3p\frac{n_{p+k}-n_p}{\omega-\xi_{p+k}+\xi_p+i0}. \tag{35}$$

Equation (35) includes only the electron contribution to the susceptibility. In general the effect of ions should also be accounted for. Here we will describe this effect within a very simple approximation which is, however, sufficient for our analysis. Namely, the ion contribution to the susceptibility will be taken in the form $\delta\epsilon_i=-\omega_{pi}^2/\omega^2$, where ω_{pi} is the ion plasma frequency. Then the phonon spectrum is determined by the equation $\epsilon(\omega,k)=0$. In the long-wave limit this approximation works sufficiently well for longitudinal phonons which mainly interact with electrons. The effect of transverse phonons cannot be described within this simple model. But such phonons are weakly coupled to the elecrons anyway, and therefore their effect can be safely ignored. If needed, further generalizations of this simple model can be also incorporated into our analysis.

In the relevant case of a disordered metal a standard calculation of polarization bubbles yields

$$\epsilon(\omega,k)=1+\frac{4\pi\sigma}{-i\omega+Dk^2}-\frac{\omega_{pi}^2}{\omega^2}. \tag{36}$$

Here $\sigma=2e^2N_0D$ is the classical Drude conductivity, N_0 is the metallic density of states, and $D=v_Fl/3$ is the diffusion coefficient.

Expression (36) is valid for wave vectors smaller than the inverse elastic mean free path $k\lesssim 1/l$ and for small frequencies $\omega\lesssim 1/\tau_e$, where $\tau_e=l/v_F$ is the elastic mean free time. Note, that if one neglects the effect of phonons and considers only nearly uniform in space ($k \approx 0$) fluctuations of the field V one immediately observes that Eqs. (34),(36) *exactly* coincide with the real time version of the Caldeira-Leggett action[4,5,7,8] in this limit. For a disordered metal this action was derived by means of the quasiclassical Eilenberger equations in Refs. 7,8. Our analysis (see, also Ref. 9) reproduces these results and generalizes them to the case of nonuniform fluctuations of the field V.

Taking into account only uniform fluctuations of the electric field one can also derive the Caldeira-Leggett action expressed in terms of the *electron coordinate only*. In the quasi-one-dimensional situation one should simply write down the electron action on the Keldysh contour, take into account the potential energy $-Ex$ of the electron with the coordinate $x(t)$ in the fluctuating electric field $E(t)$ and add the action for the field (34),(36) [with the last term in Eq. (36) being dropped]. After identification $E(\omega)=-ikV(\omega,k)\rightarrow -V(\omega)/L$ (L is the sample length) and integration over the fluctuating field V, one arrives at the Caldeira-Leggett action for the electron coordinate $x(t)$. In this case the effective viscosity in the Caldeira-Leggett influence functional is proportional to $1/\sigma$ (in contrast to the effective viscosity for the field V which is proportional to σ). The whole procedure is completely analogous to that discussed in details in Ref. 31 where we considered the real time effective action for a dissipative system characterized by two collective degrees of freedom (the phase and the charge). Integrating over the charge variable one arrives at the Caldeira-Leggett action in the ''phase-only'' representation. The same can be done here if we use a formal analogy of $x(t)$ with the phase and $V/i\omega$ with the charge (as defined in Ref. 31).

For our present purposes it is not sufficient to restrict ourselves to uniform fluctuations of the collective coordinate V of the electron environment. We will see that fluctuations with nonzero k play an important role and should be taken into account in the quantitative analysis. The corresponding effective action will be derived in the next section. However, the main message is clear already from the simple example considered above: in a disordered metal the effect of Coulomb interaction of the electron with other electrons is equivalent to that of an effective *dissipative* environment with the correspondent effective viscosity governed by the Drude conductivity σ.

IV. DECOHERENCE TIME AND CONDUCTANCE

A. Conductance and electron effective action

In order to evaluate the system conductance we will determine the single-particle density matrix $\boldsymbol{\rho}$ in the presence of an external electric potential $V_x(\boldsymbol{r})$ applied to the metal. Generalization of the results obtained in the previous sections to the case $V_x \neq 0$ is straightforward. The density matrix is determined by the equation

$$\boldsymbol{\rho}(t)=\langle \rho_V(t,V_x)\rangle_{V_1,V_2}=\frac{\int \mathcal{D}V_1\mathcal{D}V_2\,\rho_V(t,V_x)e^{i\boldsymbol{S}_{V_x}[V_1,V_2]}}{\int \mathcal{D}V_1\mathcal{D}V_2\,e^{i\boldsymbol{S}_{V_x}[V_1,V_2]}}, \tag{37}$$

where the effective action $\boldsymbol{S}_{V_x}[V_1,V_2]$ reads

$$i\boldsymbol{S}_{V_x}[V_1,V_2]=2\,\mathrm{Tr}\ln \hat{G}^{-1}_{V+V_x}+i\int_0^t dt'\int d\boldsymbol{r}\,\frac{(\nabla V_1)^2-(\nabla V_2)^2}{8\pi}, \tag{38}$$

where the subscript $V+V_x$ indicates the shift of the fields $V_{1,2}\rightarrow V_{1,2}+V_x$. Here we use the formally exact expression for the effective action, the approximation (34) will be introduced after the expansion in V_x will be carried out.

The density matrix $\rho_V(t,V_x)$ obeys Eq. (25) with $V^+\rightarrow V^+ + V_x$. Assuming the field V_x is sufficiently small one can linearize the equation for $\rho_V(t,V_x)=\rho_V(t)+\delta\rho_V(t,V_x)$ and get

$$i\frac{\partial \delta\rho_V}{\partial t}=H_1\delta\rho_V-\delta\rho_V H_2-[eV_x,\rho_V], \tag{39}$$

where

$$H_1=H_0-eV^+-\frac{1}{2}(1-2\rho_V)eV^-,$$

$$H_2=H_0-eV^++\frac{1}{2}eV^-(1-2\rho_V). \tag{40}$$

The formal solution of Eq. (39) can be easily found:

$$\delta\rho_V(t)=i\int_0^t dt'\,U_1(t,t')[eV_x,\rho_V(t')]U_2(t',t), \tag{41}$$

where

$$U_{1,2}(t_1,t_2)=\mathbf{T}\exp\left[-i\int_{t_1}^{t_2}dt'\,H_{1,2}(t')\right]. \tag{42}$$

The operators $H_{1,2}$ Eq. (40) are nonlocal (since they contain the density matrix), therefore the path-integral representation for the evolution operators (42) contains an additional integration over momentum. The operators (40) can be written in the form

$$H_1(\boldsymbol{p},\boldsymbol{r})=\frac{\boldsymbol{p}^2}{2m}+U(\boldsymbol{r})-eV^+(t,\boldsymbol{r})-\frac{1}{2}[1-2n(H_0(\boldsymbol{p},\boldsymbol{r}))]eV^-(t,\boldsymbol{r}),$$

$$H_2(\boldsymbol{p},\boldsymbol{r})=\frac{\boldsymbol{p}^2}{2m}+U(\boldsymbol{r})-eV^+(t,\boldsymbol{r})+\frac{1}{2}eV^-(t,\boldsymbol{r})[1-2n(H_0(\boldsymbol{p},\boldsymbol{r}))], \tag{43}$$

where $n(\xi)=1/[\exp(\xi/T)+1]$ is the Fermi function. In deriving Eq. (43) from Eq. (40) we set ρ_V to be an equilibrium density matrix. We also neglected the effect of Coulomb interaction in the expression for $\rho_V(t)$. This approximation is justified as long as Coulomb interaction is sufficiently weak.

It is consistent with the RPA approximation for the electromagnetic part of the action (34). Note that the same approximation for $\rho_V(t)$ can be used in Eq. (41).

The evolution operators (42) acquire the form

$$U_{1,2}(t_1,t_2;\boldsymbol{r}_f,\boldsymbol{r}_i)$$

$$=\int_{\boldsymbol{r}(t_1)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_2)=\boldsymbol{r}_f}\mathcal{D}\boldsymbol{r}(t')\int\mathcal{D}\boldsymbol{p}(t')$$

$$\times\exp\left[i\int_{t_1}^{t_2}dt'(\boldsymbol{p}\dot{\boldsymbol{r}}-H_{1,2}(\boldsymbol{p},\boldsymbol{r}))\right]. \qquad (44)$$

For the sake of generality we note that in the presence of interaction there exists an additional (linear in the field) correction to the density matrix. In order to see that let us expand the action (38) to the first order in V_x:

$$i\delta\boldsymbol{S}[V_x]=-2e\,\mathrm{tr}((G_{11}-G_{22})V_x). \qquad (45)$$

This correction to the action gives an additional contribution to the density matrix (37). Expressing the functions G_{11} and G_{22} in terms of the density matrix ρ_V and the evolution operators $u_{1,2}$ and combining this correction to the density matrix with one defined in Eq. (41) we find

$$\delta\boldsymbol{\rho}(t)=\langle\delta\rho_V\rangle_{V^+,V^-}+\langle\delta\rho_{\mathrm{int}}\rangle_{V^+,V^-}, \qquad (46)$$

where $\delta\rho_V$ is given by Eq. (41) and $\delta\rho_{\mathrm{int}}$ has the form

$$\delta\rho_{\mathrm{int}}=-2i\rho_V(t)\int_0^t dt'\,\mathrm{tr}(u_1(t,t')$$

$$\times[eV_x(t'),\rho_V(t')]u_2(t',t)). \qquad (47)$$

In the limit of weak interaction between electrons the averaging in Eq. (46) may be performed with the approximate action $S[V_1,V_2]$ Eq. (34).

It is easy to observe that the second term in Eq. (46) is small in the limit of weak interaction and vanishes completely if interaction is neglected. The weak localization correction is described by the first term in Eq. (46) which will be only considered further below.

Making use of a standard definition of the current density $\boldsymbol{j}$:

$$\boldsymbol{j}(t,\boldsymbol{r})=\frac{ie}{m}[\nabla_{r_1}\delta\boldsymbol{\rho}(t,\boldsymbol{r}_1,\boldsymbol{r}_2)-\nabla_{r_2}\delta\boldsymbol{\rho}(t,\boldsymbol{r}_1,\boldsymbol{r}_2)]|_{\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}}, \qquad (48)$$

combining it with the above equations and assuming the external electric field to be constant in space and time, $V_x=-\boldsymbol{E}\boldsymbol{r}$, we arrive at the expression for the system conductance

$$\sigma=\frac{e^2}{3m}\int_{-\infty}^{t}dt'\int d\boldsymbol{r}_{i1}d\boldsymbol{r}_{i2}(\nabla_{r_{1f}}-\nabla r_{2f})|_{\boldsymbol{r}_{1f}=\boldsymbol{r}_{2f}}$$

$$\times J(t,t';\boldsymbol{r}_{1f},\boldsymbol{r}_{2f};\boldsymbol{r}_{1i},\boldsymbol{r}_{2i})(\boldsymbol{r}_{1i}-\boldsymbol{r}_{2i})\rho_0(\boldsymbol{r}_{1i},\boldsymbol{r}_{2i}). \qquad (49)$$

Here we have shifted the initial time to $-\infty$. The function J is the kernel of the operator

$$\boldsymbol{J}=\sum_V U|V\rangle\langle V|U^+,$$

where the sum runs over all possible states of the electromagnetic environment. This function can be expressed in terms of the path integral

$$J(t,t';\boldsymbol{r}_{1f},\boldsymbol{r}_{2f};\boldsymbol{r}_{1i},\boldsymbol{r}_{2i})$$

$$=\int_{\boldsymbol{r}_1(t')=\boldsymbol{r}_{1i}}^{\boldsymbol{r}_1(t)=\boldsymbol{r}_{1f}}\mathcal{D}\boldsymbol{r}_1\int_{\boldsymbol{r}_2(t')=\boldsymbol{r}_{2i}}^{\boldsymbol{r}_2(t)=\boldsymbol{r}_{2f}}\mathcal{D}\boldsymbol{r}_2\int\mathcal{D}\boldsymbol{p}_1\mathcal{D}\boldsymbol{p}_2$$

$$\times\langle e^{iS_0[\boldsymbol{r}_1,\boldsymbol{p}_1]-iS_0[\boldsymbol{r}_2,\boldsymbol{p}_2]+i\int_{t'}^{t}dt''\int d\boldsymbol{r}(f^-V^++f^+V^-)}\rangle_{V^+,V^-}. \qquad (50)$$

Here the action $S_0[x,p]$ has the form

$$S_0[\boldsymbol{r},\boldsymbol{p}]=\int_{t'}^{t}dt''\left(\boldsymbol{p}\dot{\boldsymbol{r}}-\frac{\boldsymbol{p}^2}{2m}-U(\boldsymbol{r})\right), \qquad (51)$$

and the ''charge densities'' f^-,f^+ are defined by the equations:

$$f^-(t,\boldsymbol{r})=e\delta(\boldsymbol{r}-\boldsymbol{r}_1(t))-e\delta(\boldsymbol{r}-\boldsymbol{r}_2(t)),$$

$$f^+(t,\boldsymbol{r})=\frac{1}{2}\{e[1-2n(\boldsymbol{p}_1(t),\boldsymbol{r}_1(t))]\delta(\boldsymbol{r}-\boldsymbol{r}_1(t))$$

$$+e[1-2n(\boldsymbol{p}_2(t),\boldsymbol{r}_2(t))]\delta(\boldsymbol{r}-\boldsymbol{r}_2(t))\}. \qquad (52)$$

Averaging over V^+,V^- in Eq. (50) amounts to calculating Gaussian path integrals with the action (34) and can be easily performed. We obtain

$$J(t,t';\boldsymbol{r}_{1f},\boldsymbol{r}_{2f};\boldsymbol{r}_{1i},\boldsymbol{r}_{2i})$$

$$=\int_{\boldsymbol{r}_1(t')=\boldsymbol{r}_{1i}}^{\boldsymbol{r}_1(t)=\boldsymbol{r}_{1f}}\mathcal{D}\boldsymbol{r}_1\int_{\boldsymbol{r}_2(t')=\boldsymbol{r}_{2i}}^{\boldsymbol{r}_2(t)=\boldsymbol{r}_{2f}}\mathcal{D}\boldsymbol{r}_2\int\mathcal{D}\boldsymbol{p}_1\mathcal{D}\boldsymbol{p}_2$$

$$\times\exp\{iS_0[\boldsymbol{r}_1,\boldsymbol{p}_1]-iS_0[\boldsymbol{r}_2,\boldsymbol{p}_2]-iS_R[\boldsymbol{r}_1,\boldsymbol{p}_1,\boldsymbol{r}_2,\boldsymbol{p}_2]$$

$$-S_I[\boldsymbol{r}_1,\boldsymbol{r}_2]\}, \qquad (53)$$

where

$$S_R[\boldsymbol{r}_1,\boldsymbol{p}_1,\boldsymbol{r}_2,\boldsymbol{p}_2]=\frac{e^2}{2}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2\{R[t_1-t_2,\boldsymbol{r}_1(t_1)-\boldsymbol{r}_1(t_2)][1-2n(\boldsymbol{p}_1(t_2),\boldsymbol{r}_1(t_2))]-R[t_1-t_2,\boldsymbol{r}_2(t_1)-\boldsymbol{r}_2(t_2)]$$

$$\times[1-2n(\boldsymbol{p}_2(t_2),\boldsymbol{r}_2(t_2))]+R[t_1-t_2,\boldsymbol{r}_1(t_1)-\boldsymbol{r}_2(t_2)][1-2n(\boldsymbol{p}_2(t_2),\boldsymbol{r}_2(t_2))]$$

$$-R[t_1-t_2,\boldsymbol{r}_2(t_1)-\boldsymbol{r}_1(t_2)][1-2n(\boldsymbol{p}_1(t_2),\boldsymbol{r}_1(t_2))]\}, \qquad (54)$$

and

$$S_I[\boldsymbol{r}_1,\boldsymbol{r}_2]=\frac{e^2}{2}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2\{I[t_1-t_2,\boldsymbol{r}_1(t_1)-\boldsymbol{r}_1(t_2)]$$

$$+I[t_1-t_2,\boldsymbol{r}_2(t_1)-\boldsymbol{r}_2(t_2)]$$

$$-I[t_1-t_2,\boldsymbol{r}_1(t_1)-\boldsymbol{r}_2(t_2)]$$

$$-I[t_1-t_2,\boldsymbol{r}_2(t_1)-\boldsymbol{r}_1(t_2)]\}. \tag{55}$$

At the scales $|\boldsymbol{r}|\gtrsim l$ the functions R and I are defined by the equations

$$R(t,\boldsymbol{r})=\int\frac{d\omega d^3k}{(2\pi)^4}\frac{4\pi}{k^2\epsilon(\omega,k)}e^{-i\omega t+i\boldsymbol{k}\boldsymbol{r}}, \tag{56}$$

$$I(t,\boldsymbol{r})=\int\frac{d\omega d^3k}{(2\pi)^4}\mathrm{Im}\left(\frac{-4\pi}{k^2\epsilon(\omega,k)}\right)\coth\left(\frac{\omega}{2T}\right)e^{-i\omega t+i\boldsymbol{k}\boldsymbol{r}}. \tag{57}$$

If necessary, more general expressions for R and I for the whole range $|\boldsymbol{r}|\gtrsim 1/p_F$ can be derived. We will avoid doing this here because the approximation (57) is already sufficient for our present purposes.

Note that the expression in the exponent of Eq. (53) defines the real time effective action of the electron propagating in a disordered metal and interacting with other electrons. The first two terms represent the electron action S_0 Eq. (51) on two branches of the Keldysh contour while the last two terms S_R and S_I determine the influence functional which comes from the effective electron (and/or phonon) environment. As can be seen from Eqs. (54)–(57) this influence functional is not identical to one derived in the Caldeira-Leggett model. However, on a qualitative level the similarity is obvious: in both models the influence functionals describe the effect of a certain effective *dissipative* environment.

Let us now analyze expression (49) and establish connection with other conductivity calculations. It is convenient to introduce the Wigner function $n(\boldsymbol{p},\boldsymbol{r})=\int d\boldsymbol{r}^- e^{-i\boldsymbol{p}\boldsymbol{r}^-}\rho(\boldsymbol{r}+\boldsymbol{r}^-/2,\boldsymbol{r}-\boldsymbol{r}^-/2)$ instead of the density matrix. For homogeneous systems it does not depend on $\boldsymbol{r}$ at scales exceeding the mean free path. With this in mind we obtain

$$\sigma=-\frac{2e^2}{3m}\int_{-\infty}^{t}dt'\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mathbf{p}\hat{W}(t,t')\frac{\partial n(\mathbf{p})}{\partial\mathbf{p}}, \tag{58}$$

where $\hat{W}(t,t')$ is the evolution operator for the Wigner function:

$$n(t,x,\mathbf{p})=\hat{W}(t,t')n(t',x,\mathbf{p}). \tag{59}$$

The kernel of this operator and that of J Eq. (53) are related by means of the Fourier transformation with respect to $\boldsymbol{r}_{1i,1f}-\boldsymbol{r}_{2i,2f}$. In equilibrium one has $n(\mathbf{p})=1/[\exp(\xi/T)+1]$, and therefore, at small T one finds

$$\frac{\partial n(\mathbf{p})}{\partial\mathbf{p}}\simeq-\mathbf{v}_F\delta(\xi),$$

and arrives at the standard result (cf., e.g., Ref. 13)

$$\sigma=\frac{2e^2N_0}{3}\int_{-\infty}^{t}dt'\langle\mathbf{v}(t)\mathbf{v}(t')\rangle. \tag{60}$$

B. Weak localization correction and decoherence time

Let us analyze the structure of the function J Eq. (53) in the same spirit as it has been done in Ref. 13. In the zero-order approximation one can neglect the terms S_R and S_I describing the effect of Coulomb interaction. Then in the quasiclassical limit $p_Fl\gg 1$ the path integral Eq. (53) is dominated by the saddle-point trajectories for the action S_0 which are just classical paths determined by the Hamilton equations

$$\dot{\boldsymbol{p}}=-\frac{\partial H_0(\boldsymbol{p},\boldsymbol{r})}{\partial\boldsymbol{r}},\quad \dot{\boldsymbol{r}}=\frac{\partial H_0(\boldsymbol{p},\boldsymbol{r})}{\partial\boldsymbol{p}} \tag{61}$$

with obvious boundary conditions $\boldsymbol{r}_1(t')=\boldsymbol{r}_{1i}$, $\boldsymbol{r}(t)=\boldsymbol{r}_{1f}$ for the action $S_0[\boldsymbol{r}_1,\boldsymbol{p}_1]$ and $\boldsymbol{r}_2(t')=\boldsymbol{r}_{2i}$, $\boldsymbol{r}_2(t)=\boldsymbol{r}_{2f}$ for the action $S_0[\boldsymbol{r}_2,\boldsymbol{p}_2]$. Substituting these saddle-point trajectories into Eq. (53) and integrating out small fluctuations around them one finds

$$J(t,t';\boldsymbol{r}_{1f},\boldsymbol{r}_{2f};\boldsymbol{r}_{1i},\boldsymbol{r}_{2i})=\sum_{\boldsymbol{r}_1}A_{\boldsymbol{r}_1}\sum_{\boldsymbol{r}_2}A^*_{\boldsymbol{r}_2}\exp[iS_0(t,t';\boldsymbol{r}_{1f},\boldsymbol{r}_{1i})$$

$$-iS_0(t,t';\boldsymbol{r}_{2f},\boldsymbol{r}_{2i})], \tag{62}$$

where the actions $S_0(t,t';\boldsymbol{r}_{1,2f},\boldsymbol{r}_{1,2i})$ are taken on the classical paths $\boldsymbol{r}_{1,2}(t)$ and

$$A_{\boldsymbol{r}_1}=\sqrt{\frac{i^3}{8\pi^3}\left|\det\left(\frac{\partial^2S_0(\boldsymbol{r}_{1f},\boldsymbol{r}_{1i})}{\partial\boldsymbol{r}_{1f}\partial\boldsymbol{r}_{1i}}\right)\right|}. \tag{63}$$

The value $A^*_{\boldsymbol{r}_2}$ is defined analogously.

Since in a random potential $U(\boldsymbol{r})$ there is in general no correlation between different classical paths $\boldsymbol{r}_1(t)$ and $\boldsymbol{r}_2(t)$ these paths give no contribution to the double sum (62): the difference of two actions in the exponent of Eq. (62) may have an arbitrary value and the result averages out after summation. Thus only the paths for which $S_0[\boldsymbol{r}_1,\boldsymbol{p}_1]\simeq S_0[\boldsymbol{r}_2,\boldsymbol{p}_2]$ provide a nonvanishing contribution to Eq. (62). Two different classes of such paths can be distinguished (see, e.g., Ref. 13):

(i) The two classical paths are almost the same: $\boldsymbol{r}_1(t'')\simeq\boldsymbol{r}_2(t'')$, $\boldsymbol{p}_1(t'')\simeq\boldsymbol{p}_2(t'')$ [see Fig. 1(a)]. For such pairs we obviously have $\boldsymbol{r}_{1i}\simeq\boldsymbol{r}_{2i}$ and $\boldsymbol{r}_{1f}\simeq\boldsymbol{r}_{2f}$. In other words, in the path integral (53) one integrates only over trajectories with $|\boldsymbol{r}_1(t'')-\boldsymbol{r}_2(t'')|\lesssim 1/p_F$. Physically this corresponds to the picture of electrons propagating as nearly classical particles

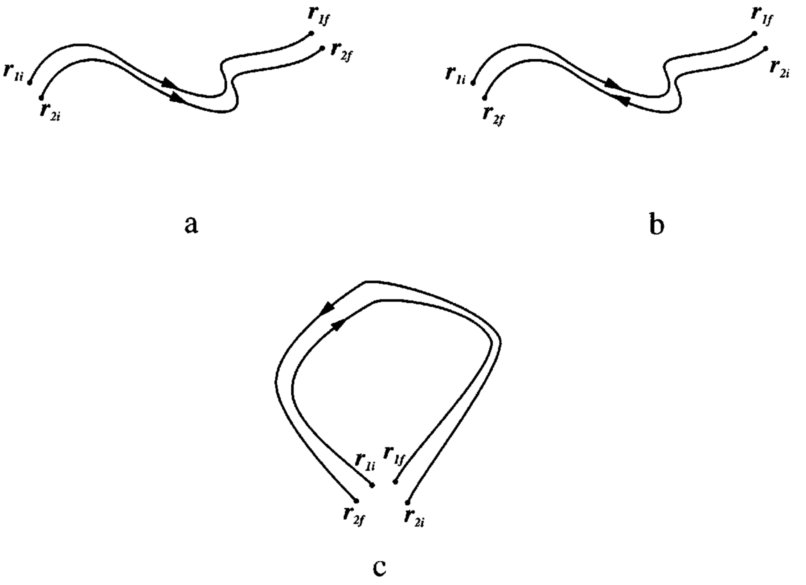

FIG. 1. Different quasiclassical trajectories: (a) $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are close to each other; (b) time-reversed paths; (c) time-reversed paths returning to the vicinity of the initial point, these paths determine the weak localization correction to the conductivity.

which can be described by the diagonal elements of the density matrix only. In the diffusive limit these paths give rise to diffusons (see, e.g., Ref. 12) and yield the standard Drude conductance.

(ii) The pairs of time reversed paths. In this case $\boldsymbol{r}_{1i} \simeq \boldsymbol{r}_{2f}$, $\boldsymbol{r}_{1f} \simeq \boldsymbol{r}_{2i}$ [Fig. 1(b)]. In the path integral (53) the trajectories $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are related as $\boldsymbol{r}_2(t'') \simeq \boldsymbol{r}_1(t+t'-t'')$ and $\boldsymbol{p}_2(t'') \simeq -\boldsymbol{p}_1(t+t'-t'')$. In other words, in Eq. (53) one integrates over paths with $|\boldsymbol{r}_2(t'')-\boldsymbol{r}_1(t+t'-t'')| \lesssim 1/p_F$, however, the difference $|\boldsymbol{r}_1(t'')-\boldsymbol{r}_2(t'')|$ may be arbitrarily large in this case. In a disordered metal these paths essentially determine the dynamics of off-diagonal elements of the electron density matrix. They correspond to Cooperons and give rise to the weak localization correction to the conductivity. This correction is defined by the following equation:[11–13]

$$\delta\sigma = -\frac{2e^2D}{\pi}\int_{\tau_e}^{\infty} dt\, W(t) = -\frac{2e^2D}{\pi}\int_{-\infty}^{t-\tau_e} dt'\, W(t-t'). \tag{64}$$

Here we changed the parameter of integration in order to make the relation with the Eq. (49) more transparent. The quantity $W(t)$ represents the effective probability for the diffusive path to return to the same point after the time t. Note that $W(t)$ contains the contribution from time-reversed paths and therefore differs from a classical probability. However, in the absence of any kind of interaction which breaks the time-reversal symmetry this value coincides with the classical return probability and is given by the formula $W_0(t) = (4\pi Dt)^{-d/2}a^{-(3-d)}$, where d is the system dimension and a is the transverse sample size (the film thickness for $d=2$ and the square root of the wire cross section for $d=1$).

The weak localization correction (64) diverges for $d \leqslant 2$. This divergence can be cured by introducing the upper limit cutoff for the integral (64) at a certain time τ_φ. This time is usually referred to as decoherence time. As we have already discussed, in a disordered metal the time τ_φ is determined by electron-electron, electron-phonon, and other types of interaction which may destroy quantum coherence. From Eq. (64) one finds[12,13]

$$\delta\sigma_d = \begin{cases} -\dfrac{\sqrt{3}e^2}{2\pi^{3/2}l}, & d=3, \\ -\dfrac{e^2}{2\pi^2}\ln\left(\dfrac{\tau_\varphi}{\tau_e}\right), & d=2 \\ -\dfrac{e^2}{\pi}\sqrt{D\tau_\varphi}, & d=1. \end{cases} \tag{65}$$

Here and below $\sigma_d = \sigma a^{3-d}$ is the Drude conductance of a d-dimensional sample.

To evaluate τ_φ we first note that the functions R and I Eq. (57) change slowly at distances of the order of the Fermi wavelength $1/p_F$. Therefore, we may set $\boldsymbol{r}_1(t'') = \boldsymbol{r}(t'')$, $\boldsymbol{r}_2(t'') = \boldsymbol{r}_t(t'') \equiv \boldsymbol{r}(t+t'-t'')$. Here $\boldsymbol{r}(t'')$ is a classical trajectory with the initial point $\boldsymbol{r}(t')=0$ (its position can be chosen arbitrarily) and the final point $|\boldsymbol{r}(t)| \lesssim l$ [Fig. 1(c)]. In other words, we consider trajectories which return to the vicinity of the initial point. Then the return probability $W(t)$ can be written in the form

$$W(t-t') \simeq W_0(t-t')\left\langle e^{-iS_R[t,t';\boldsymbol{r},\boldsymbol{p};\boldsymbol{r}_t,\boldsymbol{p}_t]-S_I[t,t';\boldsymbol{r},\boldsymbol{r}_t]}\right\rangle_r, \tag{66}$$

where the average is taken over all diffusive paths returning to the initial point. The average in Eq. (66) decays exponentially in time, therefore, we may define τ_φ as follows:

$$e^{-(t-t')/\tau_\varphi} = \left\langle e^{-iS_R[t,t';\boldsymbol{r},\boldsymbol{p};\boldsymbol{r}_t,\boldsymbol{p}_t]-S_I[t,t';\boldsymbol{r},\boldsymbol{r}_t]}\right\rangle_r \simeq e^{\langle -iS_R[t,t';\boldsymbol{r},\boldsymbol{p};\boldsymbol{r}_t,\boldsymbol{p}_t]-S_I[t,t';\boldsymbol{r},\boldsymbol{r}_t]\rangle_r}, \quad (t-t') \to +\infty; \tag{67}$$

where the average is again taken over all classical paths returning to the same point at the time t.

Let us consider the term S_R Eq. (54). Making use of the obvious relations $R(t,-\boldsymbol{r}) = R(t,\boldsymbol{r})$, $n(-\boldsymbol{p},\boldsymbol{r}) = n(\boldsymbol{p},\boldsymbol{r})$ after a trivial algebra we get

$$S_R[t,t';\boldsymbol{r},\boldsymbol{p};\boldsymbol{r}_t,\boldsymbol{p}_t]=\frac{e^2}{2}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2\{2R[t_1-t_2,\boldsymbol{r}(t_1)-\boldsymbol{r}(t_2)][n(\boldsymbol{p}(t_1),\boldsymbol{r}(t_1))-n(\boldsymbol{p}(t_2),\boldsymbol{r}(t_2))]$$
$$+R[t_1+t_2-t-t',\boldsymbol{r}(t_1)-\boldsymbol{r}(t_2)][1-2n(\boldsymbol{p}(t_2),\boldsymbol{r}(t_2))]-R[t_1+t_2-t-t',\boldsymbol{r}(t+t'-t_1)-\boldsymbol{r}(t+t'-t_2)]$$
$$\times[1-2n(\boldsymbol{p}(t+t'-t_2),\boldsymbol{r}(t+t'-t_2))]\}. \tag{68}$$

It is clear that the difference of the occupation numbers n in the first term of Eq. (68) vanish: n depends only on the energy and not on time because the energy is conserved along the classical path (61). Thus the first term in the integral (68) is identically zero already before averaging over diffusive paths. The difference of two other terms vanishes after averaging over such paths. It can be easily seen if we again bear in mind that the occupation numbers do not depend on time and make use of the fact that the average $\langle R[t_3,\boldsymbol{r}(t_1)-\boldsymbol{r}(t_2)]\rangle_r$ is a function of the time t_3 and the absolute value of the time difference $|t_1-t_2|$. This implies that after averaging the two last terms in Eq. (68) are equal and cancel each other exactly. As a result the whole functional S_R does not give any contribution to $1/\tau_\varphi$.

The same analysis can be applied to S_I. We find

$$S_I[t,t';\boldsymbol{r};\boldsymbol{r}_t]=\frac{e^2}{2}\int_{t'}^{t}dt_1\int_{t'}^{t}dt_2\{2I[t_1-t_2,\boldsymbol{r}(t_1)-\boldsymbol{r}(t_2)]$$
$$-I[t_1+t_2-t-t',\boldsymbol{r}(t_1)-\boldsymbol{r}(t_2)]$$
$$-I[t_1+t_2-t-t',\boldsymbol{r}(t+t'-t_1)$$
$$-\boldsymbol{r}(t+t'-t_2)]\}. \tag{69}$$

Averaging over diffusive paths $\boldsymbol{r}(t)$ and taking the limit $t-t'\to+\infty$, we observe that τ_φ is determined by the first term under the integral, the other terms grow slower in time and therefore are irrelevant. Thus we get

$$\frac{1}{\tau_\varphi}=e^2\int_{-\infty}^{+\infty}dt\langle I[t,\boldsymbol{r}(t)-\boldsymbol{r}(0)]\rangle_r . \tag{70}$$

To find the average over the diffusive paths, we introduce the Fourier transform of the function $I(t,\boldsymbol{r})$ and replace $\langle e^{-ik(\boldsymbol{r}(t)-\boldsymbol{r}(t'))}\rangle_r$ by $e^{-Dk^2|t-t'|}$. Then we obtain

$$\frac{1}{\tau_\varphi}=\frac{e^2}{a^{3-d}}\int_{-\infty}^{+\infty}dt\int\frac{d\omega d^dk}{(2\pi)^{d+1}}$$
$$\times\mathrm{Im}\left(\frac{-4\pi}{k^2\epsilon(\omega,k)}\right)\coth\left(\frac{\omega}{2T}\right)e^{-i\omega t-Dk^2|t|}. \tag{71}$$

As it was already discussed, Eq. (71) includes the effect of both electron-electron and electron-phonon interactions.

To evaluate τ_φ we use expression (36) for the dielectric susceptibility. For typical metallic systems one can usually neglect the first term in the expression for $\epsilon(\omega,k)$ Eq. (36). Then we find

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)=\frac{\omega}{4\pi\sigma}\left[1+\frac{c^4k^4-c^4\omega^2/D^2}{(\omega^2-c^2k^2)^2+c^4\omega^2/D^2}\right]. \tag{72}$$

Here $c=\omega_{pi}(D/4\pi\sigma)^{1/2}$ is the speed of sound in our system. Possible corrections to Eq. (72) may become important only in special cases of one-dimensional (1D) and (2D) semiconductor systems, where the capacitance may become important. We will come back to this point later.

Expression (72) can be significantly simplified for $c/D\lesssim k\lesssim 1/l$. In this limit we obtain

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)\simeq\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)_{\mathrm{ee}}+\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)_{\mathrm{eph}}, \tag{73}$$

where

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)_{\mathrm{ee}}=\frac{\omega}{4\pi\sigma}, \tag{74}$$

and

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)_{\mathrm{eph}}=\frac{Dck^3}{8\sigma}[\delta(\omega-ck)-\delta(\omega+ck)]. \tag{75}$$

Phonons with small wave vectors $k\lesssim c/D$ are strongly damped. For such k we may set $\mathrm{Im}(-1/\epsilon(\omega,k))_{\mathrm{eph}}\simeq 0$.

C. Results

With the aid of the above results we can now calculate the decoherence time τ_φ. Let us first take into account only electron-electron contribution to ϵ and obtain the result for a quasi-one-dimensional system with $a\lesssim l$. Substituting Eq. (74) into Eq. (71) and integrating over time and the wave vector we arrive at the integral over ω which diverges at both low and high frequencies. The low-frequency divergence is cured in a standard manner[32] by neglecting the effect of environmental fluctuations with frequencies below $1/\tau_\varphi$. At high frequencies the integral should be cut at the scale of the order of the inverse transport time because at higher ω the approximation of electron diffusion becomes incorrect. Then we obtain

$$\frac{1}{\tau_\varphi(T)}=\frac{e^2\sqrt{2D}}{\sigma_1}\int_{1/\tau_\varphi}^{1/\tau_e}\frac{d\omega}{2\pi}\frac{\coth(\omega/2T)}{\sqrt{\omega}}. \tag{76}$$

Equation (76) yields

$$\frac{1}{\tau_\varphi}=\frac{e^2}{\pi\sigma_1}\sqrt{\frac{2D}{\tau_e}}[2T\sqrt{\tau_e\tau_\varphi}+1]. \tag{77}$$

At sufficiently high temperature the first term dominates and the standard result[14] $\tau_\varphi\sim(\sigma_1/e^2D^{1/2}T)^{2/3}$ is recovered. This is a classical contribution to τ_φ. As T is lowered the number of classical (low frequency) modes decreases and eventually vanishes in the limit $T\to 0$. At $T\lesssim T_q^{(1)}\sim 1/\sqrt{\tau_\varphi\tau_e}$ expression (77) is dominated by the second term and τ_φ saturates at the value

$$\tau_\varphi\approx\pi\sigma_1/e^2 v_F. \tag{78}$$

The estimate for the crossover temperature $T_q^{(1)}$ is obvious from Eqs. (77),(78):

$$T_q^{(1)}\approx e v_F/2\sqrt{\sigma_1 l}. \tag{79}$$

Making use of Eqs. (65),(78) it is also easy to find the weak localization correction $\delta\sigma_1$ to the Drude conductance in the limit $T=0$. For $T\lesssim T_q^{(1)}$ we obtain

$$\frac{\delta\sigma_1}{\sigma_1}=-\frac{e^2}{\pi\sigma_1}\sqrt{D\tau_\varphi}\approx-\frac{1}{p_F s^{1/2}}, \tag{80}$$

i.e., $\delta\sigma_1\approx-\sigma_1/\sqrt{N_{\rm ch}}$, where $N_{\rm ch}\sim p_F^2 s$ is the effective number of conducting channels in a 1D mesoscopic system.

For 2D and 3D systems the same analysis yields

$$\frac{1}{\tau_\varphi}=\frac{e^2}{4\pi\sigma_2\tau_e}[1+2T\tau_e\ln(T\tau_\varphi)],\quad 2\text{D},$$

$$\frac{1}{\tau_\varphi}=\frac{e^2}{3\pi^2\sigma\sqrt{2D}\,\tau_e^{3/2}}[1+6(T\tau_e)^{3/2}],\quad 3\text{D}. \tag{81}$$

We would like to emphasize that the validity of the high-frequency cutoff procedure $\omega_c\sim 1/\tau_e$ in Eq. (76) and the results (77)–(81) derived on its basis is essentially linked to the assumption that a simple approximation (74) holds up to frequencies of order $1/\tau_e$. This assumption may not be justified for relatively strongly disordered metals with small τ_e, in which case the effective high-frequency cutoff ω_c in Eq. (76) and the corresponding results should be modified accordingly.

Note that in the 2D case we again assumed $a\lesssim l$. Provided this condition is satisfied, the above results for 1D and 2D systems are valid for the whole temperature range. At sufficiently high temperatures this condition can be softened because in this case τ_φ is determined by the low-frequency fluctuations of the environment. Then the system can be considered as a quasi-1D (quasi-2D) one if its transversal dimension is smaller that the corresponding phase breaking length $a\ll L_\varphi\sim\sqrt{D\tau_\varphi}$. However, at low temperatures high-frequency modes become important and the situation changes. E.g., in the limit $a\gg l$ the diffusion process has two (and sometimes even three) stages: at short times (i.e., at frequencies higher than D/a^2) diffusion is obviously 3D, whereas for longer times it can be 1D or 2D.

For $a\gg L_\varphi$ the system is obviously 3D at all T. In the intermediate case $l\ll a\ll L_\varphi$, one should use the corresponding 1D or 2D formulas for τ_φ at high temperatures $T\gg T_{qa}^{(d)}$ and the 3D result (81) in the low-temperature limit $T\ll T_{qa}^{(d)}$. The crossover temperature $T_{qa}^{(d)}$ can be determined either directly from the integral (71) or just by comparison of the corresponding expressions for τ_φ. We get $T_{qa}^{(d)}\approx T_q^{(d)}(a/\pi l)^{3-d}$.

Now let us analyze the effect of the electron-phonon interaction on the decoherence time τ_φ. Substituting Eq. (75) into Eq. (71) and applying the same cutoff procedure after a simple integration one obtains

$$\frac{1}{\tau_\varphi^{\rm eph}}=\frac{e^2c}{4\pi^2\sigma_1}\ln\left(\frac{v_F}{3c}\right)+\frac{e^2D}{4\pi\sigma_1 c}T,\quad 1\text{D},$$

$$\frac{1}{\tau_\varphi^{\rm eph}}=\frac{e^2c}{8\pi^2\sigma_2 v_F\tau_e}+\frac{e^2}{4\pi^2\sigma_2}T\ln\left(\min\left\{\frac{v_F}{3c},\frac{DT}{c^2}\right\}\right),\quad 2\text{D}, \tag{82}$$

$$\frac{1}{\tau_\varphi^{\rm eph}}=\frac{e^2c}{16\pi^3\sigma v_F^2\tau_e^2}+\frac{e^2}{4\pi^3\sigma v_F\tau_e}T\min\left\{1,\frac{Tl}{c}\right\},\quad 3\text{D}.$$

Comparing these expressions with the above results for τ_φ we find $\tau_\varphi^{\rm ee}/\tau_\varphi^{\rm eph}\sim c/v_F\ll 1$. Obviously this estimate will not hold for systems where the effective high-frequency cutoff ω_c in Eq. (76) should be chosen smaller than $1/\tau_e$. The temperature-dependent term in the expression for the inverse dephasing time due to phonons obtained within our simple model is of the same order as the corresponding electro-electron term in 1D and 2D, and can be bigger in 3D at not very low temperatures.

The above results in 1D and 2D are valid if the number of conducting channels $N_{\rm ch}$ in the system is sufficiently big. In typical metallic systems this condition is usually well satisfied. However, in semiconductors one can, in principle, achieve the situation with $N_{\rm ch}\lesssim 10$. In this case the first term in the expression for ϵ Eq. (36) cannot be neglected in general. Moreover for 1D and 2D samples with small $N_{\rm ch}$ the energy of the electromagnetic field outside the sample may also give a substantial contribution. In order to account for this effect we introduce the effective capacitance of the system C. Then the influence functional for the field V has the form (see, also, Ref. 9)

$$iS_C=i\int\frac{d\omega d^dk}{(2\pi)^{d+1}}V^-(-\omega,-k)\left[C(\omega,k)+\frac{k^2[\epsilon(\omega,k)-1]}{4\pi}\right]\times V^+(\omega,k)-\frac{1}{2}\int\frac{d\omega d^3k}{(2\pi)^4}V^-(-\omega,-k)\times\frac{k^2\,{\rm Im}\,\epsilon(\omega,k)}{4\pi}\coth\left(\frac{\omega}{2T}\right)V^-(\omega,k), \tag{83}$$

where $C(\omega,k)\simeq[1+\epsilon_s(\omega)]/4\ln(1/ka)$ for a 1D wire and $C(\omega,k)=[1+\epsilon_s(\omega)]k/8\pi$ for a $2D$ film. Here $\epsilon_s(\omega)$ is the dielectric susceptibility of the substrate. The Fourier transform of the function $I(t,\boldsymbol{r})$ takes the form

$$I_{\omega,k}=\langle|V_{k,\omega}^+|^2\rangle=\frac{\omega\coth(\omega/2T)}{[\omega C(\omega,k)]^2/\sigma_d k^2+\sigma_d q^2[1+C(\omega,k)D/\sigma_d]^2}. \tag{84}$$

Substituting this expression into Eq. (70) we get for 1D wire

$$\frac{1}{\tau_\varphi(T)}=\frac{e^2\sqrt{2D}}{\sigma_1}\int_{1/\tau_\varphi}^{1/\tau_e}\frac{d\omega}{2\pi}\,\frac{\coth(\omega/2T)}{\sqrt{\omega}[1+2f(\omega)]}\times\left(1-\sqrt{\frac{f(\omega)}{1+f(\omega)}}\right),\tag{85}$$

where $f(\omega)=[1+\epsilon_s(\omega)]D/4\sigma_1\ln(1/k_0a)$ and the value k_0a is roughly of order one. We estimate

$$f(\omega)\sim\frac{1+\epsilon_s(\omega)}{e^2N_0s}\sim\frac{[1+\epsilon_s(\omega)]p_Fr_B}{N_{\mathrm{ch}}},\tag{86}$$

where $r_B=1/me^2\simeq0.5$ Å is the Bohr radius. For metallic wires $p_Fr_B\sim1$, $N_{\mathrm{ch}}\gg1$ and the function $f(\omega)$ is usually small unless $\epsilon_s(\omega)\gg1$ at frequencies of the order of $1/\tau_e$. However, for semiconductors $f(\omega)$ may be large and τ_φ may become significantly longer than one could expect from Eq. (77).

The same analysis can be carried out for 2D films. In this case the effect of capacitance is described by the function

$$f_2(\omega)=\frac{[1+\epsilon_s(\omega)]\sqrt{D\omega}}{8\pi\sigma_2}\sim\frac{[1+\epsilon_s(\omega)]\sqrt{\omega\tau_e}r_B}{p_Fla}.$$

Again one can conclude that this effect is typically negligible for metallic films. For semiconductors with small N_{ch} the above effect might cause an increase of τ_φ.

V. QUANTUM KINETIC APPROACH AND LANGEVIN EQUATION

Let us now demonstrate how the usual quantum kinetic description can be derived from our analysis. We start from the equation for the density matrix ρ_V Eq. (25). Rewriting this equation in the "interaction representation," i.e., substituting $\rho_V\to e^{-iH_0t}\rho_Ve^{iH_0t}$ we find

$$i\frac{\partial\rho_V}{\partial t}=-e\hat{V}^+(t)\rho_V+\rho_Ve\hat{V}^+(t)-\frac{e}{2}[(1-\rho_V)\hat{V}^-(t)\rho_V+\rho_V\hat{V}^-(t)(1-\rho_V)],\tag{87}$$

where $\hat{V}^\pm(t)=e^{iH_0t}V^\pm(t)e^{-iH_0t}$. Let us integrate this equation over time, then substitute the resulting expression for ρ_V into the right-hand side of Eq. (87) and average over $V^\pm$. If the Coulomb interaction is sufficiently weak one can proceed perturbatively in V and neglect the dependence of the density matrix ρ_V on this field in the right-hand side of the resulting equation. Then the result of averaging can be expressed in terms of the correlation functions $\langle VV\rangle$. More precisely, two such functions turn out to be important:

$$\langle V^+(t_1,\boldsymbol{r}_1)V^+(t_2,\boldsymbol{r}_2)\rangle=I(t_1-t_2,\boldsymbol{r}_1-\boldsymbol{r}_2),$$

$$\langle V^+(t_1,\boldsymbol{r}_1)V^-(t_2,\boldsymbol{r}_2)\rangle=iR(t_1-t_2,\boldsymbol{r}_1-\boldsymbol{r}_2).\tag{88}$$

The function $\langle V^+(t_1,\boldsymbol{r}_1)V^-(t_2,\boldsymbol{r}_2)\rangle$ differs from zero only for $t_1>t_2$. The correlation function $\langle V^-V^-\rangle$ is zero for all times. Taking this into account we obtain

$$\begin{aligned}\frac{\partial\boldsymbol{\rho}}{\partial t}=e^2\int_{-\infty}^{t}dt'\Big\langle&-\hat{V}^+(t)\hat{V}^+(t')\rho(t')+\hat{V}^+(t)\rho(t')\hat{V}^+(t')+\hat{V}^+(t')\rho(t')\hat{V}^+(t)-\rho(t')\hat{V}^+(t')\hat{V}^+(t)\\&-\frac{1}{2}\hat{V}^+(t)[1-\rho(t')]\hat{V}^-(t')\rho(t')-\frac{1}{2}\hat{V}^+(t)\rho(t')\hat{V}^-(t')[1-\rho(t')]+\frac{1}{2}[1-\rho(t')]\hat{V}^-(t')\rho(t')V^+(t)\\&+\frac{1}{2}\rho(t')\hat{V}^-(t')[1-\rho(t')]V^+(t)\Big\rangle_{V^+,V^-},\end{aligned}\tag{89}$$

where $\rho(t)=\rho_{V=0}(t)$. Let us replace $\rho(t')$ by $\rho(t)$ under the integral. This is the standard way of derivation of a kinetic equation, which is often referred to as the first Bloch approximation. It is fully equivalent to the approach based on the Fermi golden rule. We would like to emphasize that this method is not applicable at very low temperatures, in which case one cannot neglect the dependence of $\rho(t)$ on V^+,V^- in Eq. (89).

For simplicity let us consider a clean metal. Making use of the momentum conservation one can significantly simplify Eq. (89). In this case the density matrix is given by $\rho(\boldsymbol{r}_1-\boldsymbol{r}_2)=\int[d^3p/(2\pi)^3]n_pe^{i\boldsymbol{p}(\boldsymbol{r}_1-\boldsymbol{r}_2)}$. The operator e^{-iH_0t} reduces to $e^{-i\xi_pt}$. Performing the averaging with the aid of Eqs. (88),(56),(57) we find

$$\frac{dn_p}{dt}=\frac{e^2}{\pi^2}\int d\omega d^3k\,\mathrm{Im}\left(\frac{-1}{k^2\epsilon(\omega,k)}\right)\delta(\omega+\xi_{p-k}-\xi_p)\left[\coth\left(\frac{\omega}{2T}\right)(n_{p-k}-n_p)-n_p(1-n_{p-k})-n_{p-k}(1-n_p)\right].\tag{90}$$

The right-hand side of this equation represents the standard collision integral which vanishes in equilibrium, i.e., for $n_p=1/[\exp(\xi_p/T)+1]$.

Equation (90) can be also rewritten in the following form:

$$\begin{aligned}\frac{dn_p}{dt}=\frac{2e^2}{\pi^2}\int_0^{+\infty}d\omega\int d^3k\,\mathrm{Im}\left(\frac{-1}{k^2\epsilon(\omega,k)}\right)\{&\delta(\omega+\xi_{p-k}-\xi_p)N_\omega n_{p-k}(1-n_p)-\delta(\omega+\xi_{p-k}-\xi_p)(1+N_\omega)n_p(1-n_{p-k})\\&+\delta(\omega+\xi_p-\xi_{p-k})(1+N_\omega)n_{p-k}(1-n_p)-\delta(\omega+\xi_p-\xi_{p-k})N_\omega n_p(1-n_{p-k})\},\end{aligned}\tag{91}$$

where $N_\omega=1/[\exp(\omega/T)-1]$ is the Bose function. This equation describes the standard photon absorption and emission processes and thus establishes a transparent relation between our approach and one describing the kinetics of an electron interacting with the quantized electromagnetic field. In our case the field V is due to fluctuations of conducting electrons (or lattice ions—see below). It is quite clear, however, that the physical nature of this field is not important for the electron dynamics, at least as long as this Bose field remains in equilibrium.

It is important to emphasize that the effect of electron-phonon interaction is also taken into account in Eq. (91). The phonon spectrum is determined by the equation $\epsilon(\omega,k)=0$, i.e., the function $-1/\epsilon(\omega,k)$ has a pole at $\omega=\omega_{\mathrm{ph}}(k)-i0$. Therefore, one can write

$$\frac{-1}{\epsilon(\omega,k)}=-\frac{A(k)}{\pi[\omega-\omega_{\mathrm{ph}}(k)+i0]}+\cdots,\qquad \mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)=A(k)\,\delta[\omega-\omega_{\mathrm{ph}}(k)]+\cdots,\tag{92}$$

where other contributions to ϵ^{-1} are denoted by dots. The value $A(k)$ determines the strength of electron-phonon interaction. Within the simple model (75) one has $A(k)=Dck^3/8\sigma=ck^3/16e^2N_0$.

Substituting expression (92) into Eq. (91) and integrating over ω we reproduce the standard electron-phonon collision integral:

$$\begin{aligned}I_{\mathrm{eph}}=\frac{2e^2}{\pi^2}\int d^3k\frac{A(k)}{k^2}\{&\delta(\omega_{\mathrm{ph}}(k)+\xi_{p-k}-\xi_p)N_{\omega_{\mathrm{ph}}(k)}n_{p-k}(1-n_p)-\delta[\omega_{\mathrm{ph}}(k)+\xi_{p-k}-\xi_p](1+N_{\omega_{\mathrm{ph}}(k)})n_p(1-n_{p-k})\\&+\delta[\omega_{\mathrm{ph}}(k)+\xi_p-\xi_{p-k}](1+N_{\omega_{\mathrm{ph}}(k)})n_{p-k}(1-n_p)-\delta[\omega_{\mathrm{ph}}(k)+\xi_p-\xi_{p-k}]N_{\omega_{\mathrm{ph}}(k)}n_p(1-n_{p-k})\}.\end{aligned}\tag{93}$$

This result demostrates that the function $\epsilon(\omega,k)$ correctly describes both electron-phonon and electron-electron interactions. It is not surprising, because this function just accounts for the collective effect of the environment. Electrons propagating in a metal ''feel'' only the fluctuating electric field produced by the environment, both by electrons and lattice ions. Therefore it is quite natural that both contributions can be successfully treated within the same approach.

Equations (90)–(93) are applicable if the distribution functions n_p and N_ω are close to the equilibrium Fermi and Bose functions. It is not difficult to generalize this approach for stronger deviations from equilibrium. Actually the electron-phonon collision integral (93) remains the same in this case, only the distribution function N_k can deviate far from the Bose function. In order to generalize the electron-electron collision integral we make use of the following nonequilibrium formulas:

$$\begin{aligned}\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,k)}\right)&=-\frac{e^2}{\pi k^2|\epsilon(\omega,k)|^2}\int d^3p\,\delta(\omega-\xi_{p+k}+\xi_p)\\&\quad\times(n_{p+k}-n_p),\\ \langle V^+V^+\rangle_{\omega,k}&=\frac{4e^2}{k^4|\epsilon(\omega,k)|^2}\int d^3p[n_{p+k}(1-n_p)\\&\quad+n_p(1-n_{p+k})]\delta(\omega-\xi_{p+k}+\xi_p),\end{aligned}\tag{94}$$

which can be easily derived from Eqs. (35) and (B6), respectively. Substituting these expressions into Eq. (90) we arrive at the electron-electron collision integral for the degenerate plasma:

$$\begin{aligned}I_{ee}=\int&\frac{d^3k}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\left(\frac{4\pi e^2}{k^2}\right)^2\frac{8\pi}{|\epsilon(\xi_{p+k}-\xi_p,k)|^2}\\&\times\delta(\xi_{p'+k}+\xi_{p-k}-\xi_{p'}-\xi_p)[n_{p'+k}n_{p-k}(1-n_{p'})\\&\times(1-n_p)-n_{p'}n_p(1-n_{p'+k})(1-n_{p-k})].\end{aligned}\tag{95}$$

Thus, the kinetic equation can be written in a standard form:

$$\frac{dn_p}{dt}=I_{\mathrm{eph}}+I_{\mathrm{ee}},\tag{96}$$

where the collision integrals I_{eph} and I_{ee} are defined, respectively, by Eqs. (93) and (95).

In order to estimate the characteristic electron scattering time we have to substitute the function $n_p+\delta n_p$ instead of n_p in the collision integral (90). The inverse inelastic scattering time $1/\tau_i$ is then defined as a coefficient in front of the term δn_p describing deviations from equilibrium. Making use of an obvious identity $1-2n_{p-k}=\tanh[(\xi_p-\omega)/2T]$ we get

$$\begin{aligned}\frac{1}{\tau_i(p)}=\frac{e^2}{\pi^2}\int d\omega d^3k\,\mathrm{Im}\left(\frac{-1}{k^2\epsilon(\omega,k)}\right)\delta(\omega+\xi_{p-k}-\xi_p)\\ \times\left(\coth\frac{\omega}{2T}+\tanh\frac{\xi_p-\omega}{2T}\right).\end{aligned}\tag{97}$$

It is clear from this equation that the time τ_i becomes infinite at zero temperature and at the Fermi energy due to the Pauli principle. The same is true for the inelastic scattering time due to electron-phonon interaction.

The above kinetic equations were derived for the simple case of a clean system and do not account for the effect of elastic scattering. In the case of a disordered metal the electron momentum is not conserved and the whole derivation becomes much more complicated. One can demonstrate (see, e.g., Ref. 12) that in the diffusive limit the result is roughly equivalent to a substitution

$$\delta(\omega+\xi_{p-k}-\xi_p)\rightarrow \mathrm{Re}\left[\frac{1}{i\omega+Dk^2}\right]$$

in expression (97). An extended analysis of the inelastic scattering time in various limits is given in Ref. 33.

As it was already discussed, the analysis presented in this section is essentially equivalent to the Golden rule perturbation theory in the interaction. At low temperatures this perturbation theory becomes insufficient. One can also formulate an alternative approach and derive the quasiclassical Langevin equations describing electron dynamics in a weakly disordered metal at all temperatures. In doing so, we follow the same procedure as the one described in Refs. 5,6,31.

Consider only close electron paths for which the values $\boldsymbol{r}^-=\boldsymbol{r}_1-\boldsymbol{r}_2$ and $\boldsymbol{p}^-=\boldsymbol{p}_1-\boldsymbol{p}_2$ are small. Then we can expand the effective action in the exponent of Eq. (53) in powers of $\boldsymbol{r}^-$ and $\boldsymbol{p}^-$ keeping only the quadratic terms. The action becomes Gaussian in terms of these variables and the integral (53) is dominated by the saddle-point trajectories: $\delta S/\delta \boldsymbol{p}^-=0$ and $\delta S/\delta \boldsymbol{r}^-=0$. The first equation coincides with one without dissipation: $\dot{\boldsymbol{r}}=\boldsymbol{p}/m$. With the aid of this equation the momentum can be easily excluded and we get

$$m\ddot{\boldsymbol{r}}+\nabla U(\boldsymbol{r})+e^2[1-2n(\boldsymbol{r},m\dot{\boldsymbol{r}})]\times\int_{-\infty}^{t}dt'\nabla_r R[t-t',\boldsymbol{r}(t)-\boldsymbol{r}(t')]=-e\boldsymbol{E}(t,\boldsymbol{r}). \tag{98}$$

Here $\boldsymbol{E}(t,\boldsymbol{r})$ is a fluctuating electric field. Equilibrium fluctuations of this field are described by the correlator

$$\langle E_i(t_1,\boldsymbol{r}_1)E_j(t_2,\boldsymbol{r}_2)\rangle=4\pi\delta_{ij}\int\frac{d\omega d^3k}{(2\pi)^4}\mathrm{Im}\left(\frac{-\coth(\omega/2T)}{\epsilon(\omega,k)}\right)\times e^{-i\omega(t_1-t_2)+ik(\boldsymbol{r}_1-\boldsymbol{r}_2)}. \tag{99}$$

If needed, the generalization of Eqs. (98), (99) to a strongly nonequilibrium situation can be also provided. Also more general expressions for the kernel $R(t,\boldsymbol{r}_1,\boldsymbol{r}_2)$ and for the correlation function (99) for the case $|\boldsymbol{r}_1-\boldsymbol{r}_2|<l$ can be derived. Combining these expressions with Eq. (98) one can obtain a quasiclassical description of electron dynamics also at scales $\lesssim l$. However, such details are not important for us here, Eq. (98) is presented merely to illustrate important physical effects.

Equation (98) obviously satisfies the requirement of causality and captures all the essential features of electron dynamics in a metal. E.g., it demonstrates that electrons in a metal cannot infinitely decrease their energy: effective damping due to the presence of the environment [described by the last term in the left-hand side of Eq. (98)] is zero at the Fermi energy ($n=1/2$) and becomes negative below this energy. Thus electrons with the initial energy above μ will lose it before they reach the Fermi level. On the contrary, holes with the initial energy below μ will be pushed up to the Fermi surface. This simple example demonstrates again that our analysis accounts explicitly for the Pauli principle. The corresponding information is contained in the influence functional which depends on the occupation numbers.

The damping term in Eq. (98) depends on the function $R(t,\boldsymbol{r})$ which is determined by the correlation function $\langle V^+V^-\rangle$ [see Eq. (88)]. The physical origin of this damping term is quite transparent: the electron (or the hole) propagating in a metal produces the screened electric potential due to the presence of other electrons and ''feels'' this potential itself. In this sense Eq. (98) is similar to the equation of motion of a high-energy particle (e.g., muon) in a metal. The important difference between these two cases, however, lies in the factor $1-2n$ which is present in our case due to the Pauli principle. Formally this factor enters due to fluctuations of the field V^- which is ''sensitive'' to the Pauli principle. The fluctuating electric field E in the right-hand side of Eq. (98) is, on the contrary, not affected by the Pauli principle because its correlation function depends only on the field V^+.

With the aid of Eq. (98) it is also easy to understand why the real part of the influence functional S_R Eq. (54) does not contribute to the decoherence time. According to Eq. (98) the phase difference acquired by the electron propagating along some classical path can be split into two parts: the regular contribution due to damping (S_R) which depends only on the electron trajectory, and irregular part due to noise (S_I). Considering now the contribution from a pair of time-reversed paths, we observe that the regular contributions are the same and cancel each other because they enter with a different sign. Only irregular contributions due to noise survive and determine τ_φ. For each path the regular contribution may have a different value depending on the path and energy and even vanish (for energies at the Fermi level). However, by no means does this affect the noise terms and thus τ_φ, which always remains finite.

VI. DISCUSSION

With the aid of the Keldysh technique we developed a path-integral formalism which allows us to study quantum dynamics of electrons in a disordered metal in the presence of interactions. Our formalism allows us to proceed beyond the perturbation theory in the interaction and obtain nonperturbative results for the weak localization correction to the conductivity of a disordered metal at low temperatures. Our treatment was carried out with no more assumptions than the usual ones in the weak localization theory: the elastic mean free path was considered large as compared to the Fermi wavelength $p_F l \gg 1$ and interactions were assumed to be sufficiently weak.

Our analysis consists of the two main steps.

(1) The first step is to reformulate the initial many-body problem with the interaction in terms of a single quantum particle interacting with an effective quantum environment. We derived a formally exact equation of motion for the single electron density matrix ρ in the presence of interaction [Eqs. (24-25) for ρ_V]. The matrix ρ is obtained after averaging of ρ_V over the fluctuating fields $V_\pm$ [Eq. (21)] carried out with the (again formally exact) effective action $S[V_+,V_-]$ Eq. (10) derived by integrating out electronic degrees of freedom. No approximations have been made at this stage.

Although the exact expression for $S[V_+,V_-]$ is too complicated to deal with, some important observations can be made already before making approximations. Namely, the fluctuating field V_+ enters the equations just like an external field whereas the field V_- enters in a qualitatively different manner. Fluctuations of the field V_+ are essentially responsible for dephasing.

In order to proceed further we make the first approximation: we evaluate the effective action $S[V_+,V_-]$ within the RPA. As a result the action S becomes quadratic in $V_\pm$ and contains the dielectric susceptibility $\epsilon(\omega,k)$ of the effective environment. After that we easily integrate out the fields $V_\pm$ and arrive at the influence functional F for interacting electrons in a disordered metal. This completes the first part of our analysis. We can add that this approach can be also used in physical situations in which approximations other than RPA are more appropriate. In such cases the action $S[V_+,V_-]$ and the influence functional should be modified accordingly.

(2) As a result of our derivation we arrived at the problem of a quantum particle in a random potential in the presence of the effective environment described by the influence functional F. The Fermi statistics and the Pauli principle are explicitly accounted for in the expression for F. The kinetic energy of a particle E is counted from the Fermi energy μ and the states with $E<0$ are forbidden. The second step of our analysis is to investigate the quantum dynamics of such a particle and to calculate the conductivity of the system σ. The latter quantity is defined by Eq. (49) where the kernel J is expressed in terms of the path integral (53) which includes the influence functional F. This integral is evaluated within the saddle-point approximation which is applicable in the quasiclassical limit $p_F l \gg 1$. This procedure yields the weak localization correction to the conductivity which saturates in the low-temperature limit due to the effect of interactions. This implies a low-temperature saturation of the parameters τ_φ and L_φ extracted from the measurements of the system magnetoconductance.

We would like to emphasize that saturation of the parameter L_φ at $T\to 0$ at a finite value determined by the interaction obviously *should not* be interpreted as the absence of coherent eigenmodes in the interacting system. However, the presence of such modes does not yet imply that the physical measurables should necessarily demonstrate the coherent behavior. An example is provided by a quantum particle interacting with the Caldeira-Leggett bath of oscillators. The off-diagonal elements of the density matrix of such a particle always decay on a finite length $L_d \sim 1\sqrt{m\langle E\rangle}$ which is set by interaction. Hence, even at $T=0$ the interference between different Feynman paths is suppressed at a typical scale $\sim L_d$ (see, e.g., Ref. 34). Because of the energy exchange with the bath of oscillators the particle kinetic energy is distributed as $w(E)\propto \exp(-E/2\langle E\rangle)$ with a nonzero expectation value $\langle E\rangle$ which does not depend on temperature at sufficiently low T. The value $\langle E\rangle$ depends on the interaction strength and the bath high-frequency cutoff ω_c. The latter dependence has nothing to do with the excitation of the bath oscillators, it persists even if $\langle E\rangle$ is evaluated in the *true ground state* of the interacting system. A more detailed discussion of the relation of our results to ones obtained for the Caldeira-Leggett-type of models is presented in Ref. 35.

One can also provide an example of a physical system where the interaction induced low-temperature saturation of the parameters $\langle E\rangle$ and L_d as well as the dependence of these parameters on the high-frequency cutoff ω_c of the effective bath can be (and have been) directly measured in experiments. This is the well-known problem of single electron and Cooper pair tunneling in mesoscopic tunnel junctions in the presence of interaction with other electrons. The effect of this interaction is equivalent to that of a dissipative environment and was intensively studied in the literature.[8,21–25] Interaction with the electronic environment destroys the quantum coherence, lifts the Coulomb blockade, and is responsible for *incoherent* electron tunneling across the junction down to $T=0$.

Despite clear differences this problem is similar in many respects (both physically and formally) to the one studied here. In both cases the conductance of a disordered metallic system in the presence of the electron-electron interaction is studied and the Pauli principle should be accounted for. In both cases one can map the problem to that of one electron interacting with a fluctuating quantum field $V(\omega,k)$ which dynamics is determined by fluctuations of other electrons. In both cases the physical quantity of interest is expressed in terms of the correlation function for this field and cannot be correctly described by means of the Golden rule perturbation theory in the interaction. Although the expressions for the correlator $\langle VV\rangle$ are somewhat different in the two problems, at $T\to 0$ they both saturate to a finite value determined by the interaction and the high-frequency cutoff ω_c of the effective electronic environment. In the case of single charge tunneling the low-temperature saturation of the parameter L_d was directly measured in various experiments, see, e.g., Refs. 26,27. We believe the low-temperature saturation of the decoherence length L_φ detected in Ref. 1 and other experiments has exactly the same physical nature: it is caused by the electron-electron interaction.

Finally, we briefly discuss the agreement between our results and the experimental data[1] (see, also, Ref. 28). The comparison between theoretical and experimental values for the decoherence length $L_\varphi=\sqrt{D\tau_\varphi}$ at zero temperature is given in Table I. To calculate L_φ we first estimate the decoherence time with the aid of Eq. (77). At $T=0$ the time τ_φ can be conveniently expressed in terms of measurable quantities:

TABLE I. Theoretical values of the coherence length $L_\varphi=\sqrt{D\tau_\varphi}$ in comparison with the experimental results (Ref. 1).

Sample	w (nm)	t (nm)	L (μm)	R/L (Ω/μm)	D (10^{-3} m^2/s)	$L_\varphi^{\rm exp}$ (μm)	$L_\varphi^{\rm theor}$ (μm)
Au-1	60	25	57.9	29.14	7.8	5.54	1.8
Au-2	110	60	207	1.46	35.5	16	16.5
Au-3	100	35	155	9.31	10.5	5.2	3.6
Au-4	60	25	57.9	31.29	7.3	3.6	1.6
Au-5	190	40	18.9	191.7	0.24	0.35	0.12
Au-6	180	40	155	2.91	16.3	8	8.1

$$\tau_\varphi=\sqrt{6}\,\frac{R_qL}{Rv_F}, \tag{100}$$

where $R_q=\pi/2e^2=6453\Omega$ is a quantum resistance, L is the wire length and R is the total resistance. The Fermi velocity for gold wires was taken to be[36] 1.39×10^6 m/s. The diffusion coefficient D was estimated with the aid of the Drude formula $\sigma=2e^2N_0D$. The density of states for gold is chosen to be[36] $N_0=6\times10^{12}$ s/m^3. Note that the numerical values for D are not identical to those given in Ref. 1.

The width, the thickness, and the length of the wire are denoted, respectively, by w, t, and L; R/L is the resistance per unit length and $L_\varphi^{\rm exp}$, $L_\varphi^{\rm theor}$ are the experimental and theoretical values of the decoherence length. The agreement between both looks reasonable for all samples, especially if one takes into account an uncertainty in a numerical prefactor in our formulas due to the cutoff procedure and possible effects of the sample geometry. It is also important to emphasize that our comparison involves *no* fitting parameters.

Further experiments are desirable for better understanding of the electron-electron interaction effects in disordered low dimensional systems.

Recently, we became aware of a paper by Aleiner, Altshuler, and Gershenzon.[37] In it they developed a perturbative calculation of the weak localization correction to the conductivity and arrived at the result for the decoherence rate $1/\tau_\varphi$ which differs from our result at low T. This difference is not surprising because, as it was discussed above, the Golden rule perturbation theory[37] is not sufficient and a nonperturbative analysis is needed to properly account for the effect of interactions. This analysis is developed in our paper. On a perturbative level all the diagrams[37] are fully reproduced from our path integral formalism. For a more detailed discussion we refer the reader to Ref. 38.

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APPENDIX A: DENSITY MATRIX

With the aid of Eq. (9) the time integral in the last term of the Dyson equation (18) can be transformed as follows:

$$\begin{aligned}
&\int_0^t dt'\,\hat G_0(t_1,t')e\hat V(t')\hat G(t',t_2)\\
&\quad=\hat G_0(t_1,t_2)-\int_0^t dt'\,\hat G_0(t_1,t')\left(i\frac{\partial}{\partial t'}-\hat H_0(t')\right)\\
&\qquad\times\hat G(t',t_2)\\
&\quad=\hat G_0(t_1,t_2)-i\hat G_0(t_1,t)\hat G(t,t_2)+i\hat G_0(t_1,0)\hat G(0,t_2)\\
&\qquad+\int_0^t dt'\left(i\frac{\partial}{\partial t'}+\hat H_0(t')\right)\hat G_0(t_1,t')\hat G(t',t_2)\\
&\quad=\hat G_0(t_1,t_2)-\hat G(t_1,t_2)-i\hat G_0(t_1,t)\hat G(t,t_2)\\
&\qquad+i\hat G_0(t_1,0)\hat G(0,t_2)\,.
\end{aligned}\tag{A1}$$

Here we performed the integration by parts over the time t' and made use of the equation

$$\left(i\frac{\partial}{\partial t'}+\hat H_0(x')\right)\hat G_0(t_1,t')=-\delta(t_1-t')\delta(x_1-x'),$$

which defines the field-free Green-Keldysh function $\hat G_0$. Substituting the result (A1) into Eq. (18) we arrive at Eq. (22).

Let us substitute the representation (14) into Eq. (22). Then we find

$$\begin{aligned}
&\hat U_0(t_1,t)[-\hat b+\hat f_0(t)]\hat U_V(t,t_2)[\hat a+\hat f_V(t_2)]\\
&\quad-\hat U_0(t_1,0)[\hat a+\hat f_0(0)]\hat U_V(0,t_2)[-\hat b+\hat f_V(t_2)]=0.
\end{aligned}\tag{A2}$$

It is now convenient to represent the operator $\hat f_V$ in the form

$$\hat f_V(t_2)=\hat U_V(t_2,0)\hat g(t_2)\hat U_V(0,t_2). \tag{A3}$$

Then for $\hat g$ we get

$$\begin{aligned}
&[1+(\hat b-\hat f_0(0))(\hat U_0(0,t)\hat U_V(t,0)-1)]\hat g(t_2)\\
&\quad=\hat f_0(0)-[\hat b-\hat f_0(0)][\hat U_0(0,t)\hat U_V(t,0)-1]\hat a.
\end{aligned}\tag{A4}$$

Let us set $t_2=t$ and introduce the scattering matrix $S=U_0(0,t)U_V(t,0)=s_1\hat a+s_2\hat b$. This matrix is diagonal because both U_0 and U_V are the diagonal matrices. The matrix elements s_1 and s_2 are defined as $s_{1,2}=u_0(0,t)u_{1,2}(t,0)$. Making use of the above notations and rewriting Eq. (A4) in components we find

$$\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}\rho_0&-\rho_0\\-1+\rho_0&1-\rho_0\end{pmatrix}\begin{pmatrix}s_1-1&0\\0&s_2-1\end{pmatrix}\right]\times\begin{pmatrix}g_{11}&g_{12}\\g_{21}&g_{22}\end{pmatrix}=\begin{pmatrix}-\rho_0&\rho_0\\1-\rho_0&\rho_0\end{pmatrix}-\begin{pmatrix}\rho_0&-\rho_0\\-1+\rho_0&1-\rho_0\end{pmatrix}\times\begin{pmatrix}s_1-1&0\\0&s_2-1\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix}. \tag{A5}$$

Multiplying matrices and keeping only the part of the resulting matrix equation which depends on g_{12} and g_{22} we obtain

$$\begin{pmatrix}1+\rho_0(s_1-1)&-\rho_0(s_2-1)\\-s_1+1+\rho_0(s_1-1)&s_2-\rho_0(s_2-1)\end{pmatrix}\begin{pmatrix}g_{12}\\g_{22}\end{pmatrix}=\begin{pmatrix}\rho_0\\\rho_0\end{pmatrix}. \tag{A6}$$

Substracting the second equation from the first one we get $s_1g_{12}-s_2g_{22}=0$, or $g_{22}=s_2^{-1}s_1g_{12}$. Subsituting this result into the first equation we find

$$[1+\rho_0(s_2^{-1}s_1-1)]g_{12}=\rho_0. \tag{A7}$$

Note that the S matrices enter the above equation only in the combination $s_2^{-1}s_1$ which does not contain the evolution operator u_0, i.e., we have $s_2^{-1}s_1=u_2(0,t)u_1(t,0)$ and

$$\{1-\rho_0(0)[1-u_2(0,t)u_1(t,0)]\}g_{12}(t)=\rho_0(0). \tag{A8}$$

Rewriting Eq. (A3) for the 1,2 component of the matrix $\hat{f}_V(t)$

$$f_{12}(t)=u_1(t,0)g_{12}(t)u_2(0,t),$$

and making use of the identity $\rho_V(t)=f_{12}(t)$ we arrive at the result (37).

APPENDIX B: EFFECTIVE ACTION AND FDT

For $V_{1,2}\equiv 0$ the electron Green functions G_{12} and G_{21} can be expressed in the form

$$G_{12}=iu_0(t_1,0)\rho_0u_0(0,t_2),$$

$$G_{21}=-iu_0(t_1,0)(1-\rho_0)u_0(0,t_2). \tag{B1}$$

In thermodynamic equilibrium we have $u_0(t,0)\rho_0u_0(0,t)=\rho_0$ for any time t.

Let us introduce the basis of the eigenfunctions for the single electron Hamiltonian, $H_0\psi_k=\xi_k\psi_k$. Without loss of generality we can choose these eigenfunctions to be real. The initial density matrix ρ_0 is assumed to be diagonal in the basis ψ_k, namely $\rho_0=\Sigma_k n_k|\psi_k\rangle\langle\psi_k|$. This assumption is justified only for weakly interacting particles. Then the functions G_{12}, G_{21} Eq. (B1), G^R and G^A Eq. (29) can be written in the form

$$G_{12}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=i\sum_k e^{-i\xi_k(t_1-t_2)}n_k\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2),$$

$$G_{21}(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=-i\sum_k e^{-i\xi_k(t_1-t_2)}(1-n_k)\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2),$$

$$G^R(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=-i\theta(t_1-t_2)\sum_k e^{-i\xi_k(t_1-t_2)}\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2),$$

$$G^A(t_1,t_2,\boldsymbol{r}_1,\boldsymbol{r}_2)=i\theta(t_2-t_1)\sum_k e^{-i\xi_k(t_1-t_2)}\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2). \tag{B2}$$

With the aid of these expressions the kernels (31),(32) can be represented as follows:

$$\chi(t,\boldsymbol{r}_1,\boldsymbol{r}_2)=2ie^2\theta(t)\sum_{k,q} e^{-i(\xi_k-\xi_q)t}(n_k-n_q)\times\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_1), \tag{B3}$$

$$\eta(t,\boldsymbol{r}_1,\boldsymbol{r}_2)=\frac{1}{2}e^2\sum_{k,q} e^{-i(\xi_k-\xi_q)t}[n_k(1-n_q)+n_q(1-n_k)]\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_1). \tag{B4}$$

Performing the Fourier transformation in time:

$$\chi(\omega,\boldsymbol{r}_1,\boldsymbol{r}_2)=-2e^2\sum_{k,q}\frac{n_k-n_q}{\omega-\xi_k+\xi_q+i0}\times\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_1), \tag{B5}$$

$$\eta(\omega,\boldsymbol{r}_1,\boldsymbol{r}_2)=\pi e^2\sum_{k,q}\delta(\omega-\xi_k+\xi_q)[n_k(1-n_q)+n_q(1-n_k)]\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_2)\psi_q(\boldsymbol{r}_1), \tag{B6}$$

and substituting the equilibrium distribution function $n_k=1/(e^{\xi_k/T}+1)$ one immediately arrives at Eq. (33).

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