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Cyclotron-Stark-phonon resonances in semiconductor superlattices under terahertz irradiation

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The influence of strong terahertz irradiation on quantum transport in semiconductor superlattices under parallel magnetic and dc electric fields is studied within a quantum-kinetic approach. The consideration is focused on field strengths at which Wannier-Stark and Landau quantization occur. The current density comprises two different contributions. One is due to scattering-induced delocalization of carriers. The other one is negative, and results mainly from photon absorption processes. Within a microscopic as well as a simple relaxation-time approach, the electric- and magnetic-field dependencies of combined cyclotron-Stark-phonon and Stark-photon resonances are studied. [S0163-1829(99)06811-3]

I. INTRODUCTION

The dynamics of charged particles in semiconductor superlattices (SL's) subject to a time-dependent external electric field has been the subject of intense research. The advent of free-electron lasers that are continuously tunable in the terahertz (THz) range allows a systematic study of effects, which result from alternating fields in the THz domain. A variety of time-dependent phenomena have been predicted to occur, such as Bloch oscillations,[1] self-induced transparency,[2] negative differential conductivity,[3] absolute negative conductance,[4] photon-assisted transport,[5] and other effects. Under the conditions that only the lowest miniband of the SL is occupied and collisions can be neglected, carriers execute Wannier-Stark oscillations under the action of strong dc electric fields. They are localized by moving along closed orbits, and do not take part in the dc transport. The carrier transport is only activated by scattering processes. If an ac field is applied to the SL, a second transport channel opens up due to delocalization of carriers in the irradiation field accompanied by resonant photon absorption. In this paper we will systematically study these two sources of carrier transport under the influence of dc and ac electric fields applied parallel to the SL axis.

Intense dc and ac electric fields lead to interesting features in the current-voltage (I-V) characteristics. Near zero bias a THz field tends to localize the electrons, which oscillate with a finite amplitude, and the current density is suppressed. This is a manifestation of dynamical localization,[6] which is a specific resonance of oscillations with the frequency ω of the THz field and the ac Bloch frequency $\Omega_{\mathrm{ac}}=eE_{\mathrm{ac}}d/\hbar$ (where E_{ac} is the ac electric field strength, and d the SL period). When the dc bias increases until the ac and dc field strengths become comparable, ac and dc Bloch oscillations interfere resonantly, which can lead to a gain of one field. This gain is independent of collisions that govern the dc transport. At $\Omega_{\mathrm{dc}}=k\omega$ (where $\Omega_{\mathrm{dc}}=eE_{\mathrm{dc}}d/\hbar$ is the dc Bloch frequency, and k an integer) the carriers tend to escape to infinity. On the other hand, under this condition tunneling against the field direction by absorbing a photon becomes resonant. Therefore, there is a competition between dynamical delocalization,[2] which leads to an increase of the current and a tendency that the carriers use the energy of the irradiation field in order to tunnel against the dc bias, which may lead to an absolute negative current. Quantum-mechanical transport occurs when the THz frequency is larger than some reciprocal collision time.[7] Otherwise all resonance structures are completely smeared out by collisions, and the current is represented by some time-averaged quantities.

We will restrict our consideration to SL's with sufficiently low carrier densities. Otherwise dynamical instabilities, including transition to chaos, are predicted to occur under the influence of an ac field due to the formation of field domains[8] or the self-consistent interplay between the internal and external fields.[9,10] The related cooperative oscillations lead to a highly multistable behavior of the electronic system.

The main objective of our paper is to study quantum transport in SL's under strong dc and ac electric fields. Quantum effects are strongly enhanced by a magnetic field, which we therefore include in our consideration. When electrons are subject to a magnetic field,[11] applied parallel to the SL axis, the in-plane motion also becomes quantized. The related gradual confinement of the lateral electron motion by the magnetic field allows a simultaneous treatment of photon-assisted tunneling between Wannier-Stark (WS) ladder states in SL's and quantum-box SL's. WS localization and Landau quantization of the in-plane motion lead to a completely discrete energy spectrum. As a consequence, lifetime broadening becomes fundamentally important. Resonances associated with dc and ac Bloch frequencies, and THz and cyclotron frequencies, as well as scattering on polar-optical phonons, give rise to a number of peaks in the field-dependent current density. We want to stress that this field configuration enables experimental and theoretical studies of resonances and nonlinear dynamical phenomena in artificial semiconductor SL's, which have been accessed up to now only in atoms and molecules.

Most previous theoretical work treated the dynamics of charge carriers in SL's within a one-dimensional transport model on the basis of the Boltzmann equation in a constant relaxation-time approximation.[5,2,12–17] This simplified model provides only a crude description of scattering processes, and neglects the quantum nature of the transport as well as the heating of the lateral electron motion described by the three-dimensional electron distribution function. Like the balance-equation approach,[18] this quasiclassical approximation neglects the quantization of eigenstates associated with the high electric field. However, quantum-mechanical effects have been unambiguously identified in SL transport measurements.[19–21] Calculations of the current within the sequential tunneling model[22,23] do not cope with the statistical properties of the SL system under strong electric fields, because they use equilibrium distribution functions. To our knowledge there is no rigorous theoretical study of quantum effects in SL transport that accounts for the influence of a strong THz field. It is the main objective of our paper to fill this gap, and to present a microscopic quantum-kinetic approach that accounts for quantum transport in SL's under THz irradiation. We will focus on the influence of strong THz irradiation on electronic transport in semiconductor SL's in the presence of parallel dc electric and magnetic fields applied perpendicular to the layers in the WS and Landau quantized regimes, where the electric and magnetic fields cannot be considered as small perturbations. The current density is calculated within a microscopic quantum-kinetic approach that takes into account the heating of the lateral electron motion via the nonequilibrium electron distribution function. Intracollisional field effects due to both the dc electric and magnetic fields are included. This makes it possible to treat simultaneously WS localization and Landau quantization of the in-plane motion manifest in a completely discrete energy spectrum. Under this condition the consideration of lifetime broadening effects is essential. The importance of dissipation and line broadening has also been stressed in Ref. 5, where up to four photon resonances have been identified. The discrete energy spectrum leads to cyclotron-Stark-phonon and Stark-photon resonances, which are systematically studied both in their electric and magnetic-field dependences. Particular emphasis is put on the magnetic-field dependence of such interesting features in the I-V characteristics as dynamic localization, resonant delocalization, and the absolute negative current induced by photon-assisted tunneling against the dc electric field.

It has been claimed in the literature[22] that the main experimental findings[7,24,25] can be well described within the so-called standard theory of photon-assisted tunneling,[26,27] which expresses the I-V characteristics under irradiation in terms of the dc current. On the basis of our microscopic approach, we arrive at the conclusion that this approach is not able to cope with quantum transport. We will demonstrate that the carrier transport under THz irradiation is not completely determined by the expression for the dc current. Instead, the time-averaged I-V characteristics are composed of two different contributions, namely, one which is due to scattering of carriers on phonons, and another one which results from phononless delocalization of electrons in the radiation field. This result is obtained from a full microscopic quantum-kinetic approach, and will be compared with conclusions drawn from a simple relaxation-time approximation.

The paper is organized as follows. In Sec. II we shall derive explicit quantum-kinetic equations that describe the high-field transport. The relaxation-time approximation is treated in Sec. III. Numerical results will be presented and discussed in Sec. IV. Finally Sec. V provides a summary of our work.

II. KINETIC EQUATION

A simultaneous description of WS localization and Landau quantization requires an exact treatment of the external electric and magnetic fields. It is expedient to include these fields into the unperturbed part H_0 of the total Hamiltonian. Using the so-called WS representation, which diagonalizes H_0, one can derive a kinetic equation for the density matrix by treating scattering as a small perturbation.[28,29] We start from this formulation, and switch in the course of calculation in an exact manner to the Wigner representation by considering the symmetry properties of correlation functions in the presence of electric and magnetic fields. The derivation of a quantum-kinetic equation for the electron distribution function of a spatially homogeneous system within this framework is well documented in the literature,[30–32] and its generalization to time-dependent phenomena is straightforward.[33] Restricting consideration to electrons in the lowest miniband, which interact with phonons, and assuming that the homogeneous electric field is aligned along the z axis perpendicular to the layers of the SL, in the limit of low electron densities we obtain the following kinetic equation for the time-dependent distribution function $f(\boldsymbol{k}|t)$:

$$\frac{\partial f(\boldsymbol{k}|t)}{\partial t}+\frac{eE_z(t)}{\hbar}\frac{\partial f(\boldsymbol{k}|t)}{\partial k_z}-\frac{i}{\hbar}[\varepsilon(\boldsymbol{k}+\boldsymbol{A}(i\nabla_k))-\varepsilon(\boldsymbol{k}-\boldsymbol{A}(i\nabla_k))]f(\boldsymbol{k}|t)=4\sum_{\boldsymbol{q},\lambda} M_{\boldsymbol{q}\lambda}g(q_z)[F_{\boldsymbol{q}}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}|t)-F_{\boldsymbol{q}}(\boldsymbol{k},\boldsymbol{k}-\boldsymbol{q}|t)], \tag{1}$$

in which $E_z(t)=E_{\mathrm{dc}}+E_{\mathrm{ac}}\cos\omega t$ is the time-dependent electric field and $\boldsymbol{A}(\boldsymbol{k})$ the vector potential of the magnetic field in the symmetric gauge. $\varepsilon(\boldsymbol{k})=\hbar^2\boldsymbol{k}_\perp^2/2m^*+\Delta(1-\cos k_z d)/2$ describes the tight-binding band structure of the lowest miniband with the width Δ, and $M_{\boldsymbol{q}\lambda}$ is the electron-phonon coupling matrix element for phonons of wave vector $\boldsymbol{q}$ in branch λ. $g(q_z)$ is the form factor of the SL miniband calculated in the extreme tight-binding limit.[34] The electron-phonon interaction is considered as Lei, Horing, and Cui[34,35] treated it in their studies of the SL miniband transport. We use a simple bulk-phonon model that disregards effects of confined and interface phonons. This approach neglects details of the electron-phonon interaction in a SL, which are not of great relevance for the high-field transport studies and in the context of our present interest. The correlation function $F_{\boldsymbol{q}}$, which enters Eq. (1), is expressed by the expectation value

$$F_{\boldsymbol{q}}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}|t)=\frac{i}{\hbar}\sum_{\varkappa}\langle b_{\boldsymbol{q}\lambda}a^{+}_{\boldsymbol{k}+\boldsymbol{q}-\varkappa/2}a_{\boldsymbol{k}+\varkappa/2}\rangle_t \tag{2}$$

of electron ($a_{\boldsymbol{k}}$) and phonon ($b_{\boldsymbol{q}\lambda}$) field operators. This correlation function satisfies itself a kinetic equation of the form

$$\frac{\partial F_{\boldsymbol{q}}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}|t)}{\partial t}+\frac{eE_z(t)}{\hbar}\frac{\partial F_{\boldsymbol{q}}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}|t)}{\partial k_z}-\frac{i}{\hbar}[\varepsilon(\boldsymbol{k}+\boldsymbol{A}(i\nabla_k))-\varepsilon(\boldsymbol{k}-\boldsymbol{A}(i\nabla_k))-\hbar\omega_{\boldsymbol{q}\lambda}]F_{\boldsymbol{q}}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}|t)$$

$$=\frac{1}{\hbar^2}M_{\boldsymbol{q}\lambda}g(q_z)[(N_{\boldsymbol{q}\lambda}+1)f(\boldsymbol{k}+\boldsymbol{q}|t)-N_{\boldsymbol{q}\lambda}f(\boldsymbol{k}|t)], \tag{3}$$

with $N_{\boldsymbol{q}\lambda}$ being the equilibrium phonon distribution function and $\omega_{\boldsymbol{q}\lambda}$ the phonon frequency. The expression on the right-hand side of Eq. (3) has been obtained by a decoupling of the electronic four-point function. Applying the method of characteristics,[36] Eq. (3) can be solved exactly with the result

$$\frac{\partial f(\boldsymbol{k}|t)}{\partial t}+\frac{eE_z(t)}{\hbar}\frac{\partial f(\boldsymbol{k}|t)}{\partial k_z}-\frac{i}{\hbar}[\varepsilon(\boldsymbol{k}+\boldsymbol{A}(i\nabla_k))-\varepsilon(\boldsymbol{k}-\boldsymbol{A}(i\nabla_k))]f(\boldsymbol{k}|t)=\sum_{\boldsymbol{k}'}\int_{t_0}^{t}dt'\,f(\boldsymbol{k}'|t')W(\boldsymbol{k}',\boldsymbol{k}|t',t), \tag{4}$$

where at time t_0 the external fields are switched on. The scattering probability W is given by[37]

$$W(\boldsymbol{k}',\boldsymbol{k}|t',t)=\frac{4}{\hbar^2}\sum_{\boldsymbol{q},\lambda}|M_{\boldsymbol{q}\lambda}|^2|g(q_z)|^2\ \mathrm{Re}[(N_{\boldsymbol{q}\lambda}+1)e^{-i\omega_{\boldsymbol{q}\lambda}(t-t')}+N_{\boldsymbol{q}\lambda}e^{i\omega_{\boldsymbol{q}\lambda}(t-t')}]$$

$$\times\left\{P\left(\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\boldsymbol{q}\middle|t',t\right)-P\left(\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\boldsymbol{q}\middle|t',t\right)\right\}, \tag{5}$$

where the field-dependent Green's function

$$P(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{q}|t',t)=\exp\left\{-\frac{i}{\hbar}\int_0^{t-t'}dt''\left[\varepsilon\left(\boldsymbol{k}'+\frac{\boldsymbol{q}}{2}+\frac{e}{\hbar}\int_{t'}^{t'+t''}d\tau\boldsymbol{E}(\tau)+\boldsymbol{A}(i\nabla_{k'})\right)\right.\right.$$

$$\left.\left.-\varepsilon\left(\boldsymbol{k}'-\frac{\boldsymbol{q}}{2}+\frac{e}{\hbar}\int_{t'}^{t'+t''}d\tau\boldsymbol{E}(\tau)-\boldsymbol{A}(i\nabla_{k'})\right)\right]\right\}\delta_{\boldsymbol{k}',\boldsymbol{k}+(e/\hbar)\int_t^{t'}d\tau\boldsymbol{E}(\tau)} \tag{6}$$

has been introduced. From Eqs. (5) and (6) it is seen that the scattering probability W depends on the magnetic and electric fields. These intracollisional field effects have been treated in an exact manner. As a consequence, during the scattering process neither the energy nor the quasimomentum are conserved in the considered $\boldsymbol{k}$ representation.

For the considered alignment of the electric and magnetic fields parallel to the SL axis the distribution function $f(\boldsymbol{k}|t)$ depends only on the norm of the perpendicular momentum $k_\perp=|\boldsymbol{k}_\perp|$, so that the third term on the left-hand side of Eq. (4) vanishes. For a sinusoidal ac electric field with a frequency ω, the distribution function is periodic in time $[f(\boldsymbol{k}|t+T)=f(\boldsymbol{k}|t)$, with $T=2\pi/\omega]$. By making use of the Fourier transformation

$$f(\boldsymbol{k}|t)=\sum_{m=-\infty}^{\infty}e^{im\omega t}f_m(\boldsymbol{k}), \tag{7}$$

the kinetic equation (4) is rewritten in the form

$$im\omega f_m(\boldsymbol{k})+\frac{eE_{\mathrm{dc}}}{\hbar}\frac{\partial f_m(\boldsymbol{k})}{\partial k_z}+\frac{eE_{\mathrm{ac}}}{2\hbar}\left(\frac{\partial f_{m+1}(\boldsymbol{k})}{\partial k_z}+\frac{\partial f_{m-1}(\boldsymbol{k})}{\partial k_z}\right)=\sum_{\boldsymbol{k}'}\sum_{m'}f_{m'}(\boldsymbol{k}')W_{m'm}(\boldsymbol{k}',\boldsymbol{k}), \tag{8}$$

where in the limit $t_0\to-\infty$ the scattering probability is calculated by Eq. (5), and

$$W_{m'm}(\boldsymbol{k}',\boldsymbol{k})=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}dt\ e^{i(m'-m)\omega t}\int_0^{\infty}dt'\,e^{-im'\omega t'}W(\boldsymbol{k}',\boldsymbol{k},t-t',t). \tag{9}$$

In the derivation of a kinetic equation we will neglect the influence of the ac electric field on the scattering. This is expected to be a reasonable approximation when the Bloch frequency $\Omega_{\rm ac}$ of the ac electric field is much smaller than $\Omega_{\rm dc}$. In this case we have

$$P(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{q}|t',t)=P(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{q}|t'-t), \tag{10}$$

from which we obtain for the kinetic equation (8)

$$im\omega f_m(\boldsymbol{k})+\frac{eE_{\rm dc}}{\hbar}\frac{\partial f_m(\boldsymbol{k})}{\partial k_z}+\frac{eE_{\rm ac}}{2\hbar}\left(\frac{\partial f_{m+1}(\boldsymbol{k})}{\partial k_z}+\frac{\partial f_{m-1}(\boldsymbol{k})}{\partial k_z}\right)=\sum_{\boldsymbol{k}'} f_m(\boldsymbol{k}')W_m(\boldsymbol{k}',\boldsymbol{k}), \tag{11}$$

with

$$W_m(\boldsymbol{k}',\boldsymbol{k})=\int_0^\infty dt\; e^{-st-im\omega t}\sum_{q,\lambda}\;\mathrm{Re}\,\Phi_{\boldsymbol{q}\lambda}(t)\left[P\left(\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\boldsymbol{q}\middle|t\right)-P\left(\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\boldsymbol{q}\middle|t\right)\right] \tag{12}$$

and

$$\Phi_{\boldsymbol{q}\lambda}(t)=\frac{4}{\hbar^2}|M_{\boldsymbol{q}\lambda}|^2|g(q_z)|^2[(N_{\boldsymbol{q}\lambda}+1)e^{-i\omega_{\boldsymbol{q}\lambda}t}+N_{\boldsymbol{q}\lambda}e^{i\omega_{\boldsymbol{q}\lambda}t}]. \tag{13}$$

In Eq. (12), s is a phenomenological parameter that accounts for a finite level width.

For parallel electric and magnetic fields aligned perpendicular to the SL layers there is no direct coupling between the electric and magnetic fields, so that the Green's function P factorizes exactly:

$$P(\boldsymbol{k}',\boldsymbol{k},\boldsymbol{q}|t)=P_\perp(\boldsymbol{k}'_\perp,\boldsymbol{k}_\perp,\boldsymbol{q}_\perp|t)P_z(k'_z,k_z,q_z|t). \tag{14}$$

The lateral electron motion is spherically symmetric. This suggests to use the following ansatz for the distribution function

$$f_m(\boldsymbol{k})=2\pi l_B^2 n_s e^{-k_\perp^2 l_B^2}\sum_{n=0}^{\infty}(-1)^n L_n(2k_\perp^2 l_B^2)f_{mn}(k_z), \tag{15}$$

in which l_B is the magnetic length, n_s the electron sheet density, and L_n are Laguerre polynomials. Equation (15) has already been exploited in our previous studies.[38,37] Inserting the ansatz (15) into Eq. (11) the following set of integro-differential equations is derived for the unknown functions $f_{mn}(k_z)$

$$\begin{aligned}&im\omega f_{mn}(k_z)+\frac{eE_{\rm dc}}{\hbar}\frac{\partial f_{mn}(k_z)}{\partial k_z}+\frac{eE_{\rm ac}}{2\hbar}\left(\frac{\partial f_{m+1n}(k_z)}{\partial k_z}+\frac{\partial f_{m-1n}(k_z)}{\partial k_z}\right)\\&=\sum_{k'_z}\sum_{n'=0}^{\infty}f_{mn'}(k'_z)\int_0^\infty dt\; e^{-st-im\omega t}\sum_{q,\lambda}\;\mathrm{Re}\,\Phi_{\boldsymbol{q}\lambda}(t)\left\{P_z\left(k'_z+\frac{q_z}{2},k_z-\frac{q_z}{2},q_z\middle|t\right)P_-(n',n,\boldsymbol{q}_\perp|t)\right.\\&\left.-P_z\left(k'_z+\frac{q_z}{2},k_z+\frac{q_z}{2},q_z\middle|t\right)P_+(n',n,\boldsymbol{q}_\perp|t)\right\},\end{aligned} \tag{16}$$

with the abbreviations

$$P_\pm(n',n,\boldsymbol{q}_\perp|t)=8\pi l_B^2\frac{(-1)^{n+n'}}{(2\pi)^4}\int d^2\boldsymbol{k}_\perp d^2\boldsymbol{k}'_\perp e^{-(k_\perp^2+k_\perp'^2)l_B^2}L_{n'}(2k_\perp'^2 l_B^2)L_n(2k_\perp^2 l_B^2)P_\perp\left(\boldsymbol{k}'_\perp+\frac{\boldsymbol{q}_\perp}{2},\boldsymbol{k}_\perp\pm\frac{\boldsymbol{q}_\perp}{2},\boldsymbol{q}_\perp\middle|t\right). \tag{17}$$

In a rigorous treatment the matrix element of the electron-phonon coupling is dynamically screened, which smears out the steep $\boldsymbol{q}$ dependence at $q\to 0$. We will not go into the details of a many-body approach that accounts for screening effects. This complication will be circumvented by using a simple model that neglects the $\boldsymbol{q}$ dependence of the electron-phonon matrix element and the dispersion of optical phonons:

$$|M_{\boldsymbol{q}\lambda}|^2|g(q_z)|^2\to\omega_0^2\Gamma. \tag{18}$$

ω_0 is the dispersionless frequency of polar-optical phonons. Although this approximation is probably not very realistic for polar-optical phonons in SL's, it is sufficient to demonstrate all basic physical features that we want to display. Relying on this approximation, all momentum integrals in Eq. (16) can be calculated analytically. Describing the lateral electron motion by a parabolic dispersion relation the $\boldsymbol{q}_\perp$ integral over $P_\pm(n',n,\boldsymbol{q}_\perp|t)$ can be calculated with the result[37]

$$\sum_{\boldsymbol{q}_\perp} P_-(n',n,\boldsymbol{q}_\perp|t)=\frac{1}{2\pi l_B^2}e^{i\omega_c t(n'-n)},\quad \sum_{\boldsymbol{q}_\perp} P_+(n',n,\boldsymbol{q}_\perp|t)=\frac{1}{2\pi l_B^2}\frac{\delta_{n,n'}}{2i\sin(\omega_c t/2)}e^{i\omega_c t(n+1/2)}, \tag{19}$$

where ω_c is the cyclotron frequency. These equations are used to derive the following simplified version of the kinetic Eq. (16):

$$\begin{aligned} &im\omega f_{mn}(k_z)+\frac{eE_{\rm dc}}{\hbar}\frac{\partial f_{mn}(k_z)}{\partial k_z}+\frac{eE_{\rm ac}}{2\hbar}\left(\frac{\partial f_{m+1n}(k_z)}{\partial k_z}+\frac{\partial f_{m-1n}(k_z)}{\partial k_z}\right)\\ &\quad=\frac{1}{2\pi l_B^2}\sum_{k_z'}\sum_{n'=0}^{\infty}\int_0^\infty dt\, e^{-st-im\omega t}\text{Re}\,\Phi(t)[f_{mn'}(k_z')e^{i\omega_c t(n'-n)}P_{z-}(k_z',k_z|t)-f_{mn}(k_z')e^{i\omega_c t(n-n')}P_{z+}(k_z',k_z|t)], \end{aligned} \tag{20}$$

in which we used the abbreviations

$$P_{z\pm}(k_z',k_z|t)=\sum_{q_z} P_z\left(k_z'+\frac{q_z}{2},k_z\pm\frac{q_z}{2},q_z\middle|t\right) \tag{21}$$

and

$$\Phi(t)=\frac{4\omega_0^2\Gamma}{\hbar^2}[(N_0+1)e^{-i\omega_0 t}+N_0e^{i\omega_0 t}]. \tag{22}$$

In the final step of our derivation of a kinetic equation the periodicity of the distribution function[39] along the field direction is exploited $[f_{mn}(k_z+2\pi/d)=f_{mn}(k_z)]$. Making use of the Fourier transformation

$$f_{mn}(k_z)=\sum_{l=-\infty}^{\infty} f_{mn}^{\,l}e^{ilk_zd}. \tag{23}$$

Equation (20) can be cast into a set of linear equations for the unknown numbers $f_{mn}^{\,l}$

$$\begin{aligned} (im\omega+il\Omega_{\rm dc})f_{mn}^{\,l}+\frac{i}{2}l\Omega_{\rm ac}(f_{m-1n}^{\,l}+f_{m+1n}^{\,l})&=\sum_{n'=0}^{\infty}\sum_{l'=-\infty}^{\infty}A_{m,nn'}^{ll'}f_{mn'}^{\,l'}\\ &=\frac{1}{2\pi l_B^2 d}\sum_{n'=0}^{\infty}\sum_{l'}\left(\frac{d}{2\pi}\right)^2\int_0^{2\pi/d}dk_zdk_z'e^{il'k_z'd-ilk_zd}\int_0^\infty dt\, e^{-st-im\omega t}\\ &\quad\times[f_{mn'}^{\,l'}\text{Re}\,\Phi(t)e^{i\omega_c t(n'-n)}P_{z-}(k_z',k_z|t)\\ &\quad-f_{mn}^{\,l'}\text{Re}\,\Phi(t)e^{i\omega_c t(n-n')}P_{z+}(k_z',k_z|t)]. \end{aligned} \tag{24}$$

In this equation $A_{m,nn'}^{ll'}$ are the matrix elements of the scattering-in and -out contributions. The set of homogeneous linear equations (24) has to be solved while taking into account the normalization condition

$$\sum_{n=0}^{\infty} f_{0n}^{\,0}=1. \tag{25}$$

Throughout this paper we refer to large dc electric-field strengths, where WS localization occurs: $\Omega_{\rm dc}\tau\gg 1$. In this case the perturbational treatment of scattering, which induces the current, has become a unique method of determining the high-field transport.[39] Under the condition $\Omega_{\rm dc}\tau\gg 1$ and high THz irradiation, it is sufficient to retain only the main Fourier components of the distribution function on the right-hand side of Eq. (24), which implies replacing the numbers $f_{mn'}^{\,l'}$ by $\delta_{m,0}\delta_{l',0}f_{0n'}^{\,0}$ in this term. In addition, it is necessary to account for damping of resonances that stem from the THz field. To that end we collect scattering-out contributions with $l=l'$. which dominate in the limit $\Delta/\hbar\Omega_{\rm dc}\ll 1$. Putting all this together, from Eq. (24) we obtain the following quantum-kinetic equation valid for high electric fields

$$\begin{aligned} &(im\omega+il\Omega_{\rm dc})f_{mn}^{\,l}+B_n^lf_{mn}^{\,l}\\ &\quad+\frac{i}{2}l\Omega_{\rm ac}(f_{m-1n}^{\,l}+f_{m+1n}^{\,l})=\delta_{m,0}D_n^l, \end{aligned} \tag{26}$$

where $B_n^{-l}=(B_n^l)^*$ and $D_n^{-l}=(D_n^l)^*$. The matrix elements B_n^l that microscopically account for the damping of current resonances induced by the irradiation field are given by

$$B_n^l = \frac{1}{2\pi l_B^2 d} \sum_{n'=0}^{\infty} \sum_{l'=-\infty}^{\infty} F_{l'} \int_0^{\infty} dt\; e^{-st-i(l+l')\Omega_{\mathrm{dc}}t} \times \mathrm{Re}\;\Phi(t) e^{i\omega_c(n-n')t}. \tag{27}$$

As we will show below, the current density is proportional to the matrix elements $D_n^{l=-1}$, which are expressed by

$$D_n^{l=-1} = -\frac{1}{2\pi l_B^2 d} \frac{\hbar\Omega_{\mathrm{dc}}}{\Delta} \sum_{n'=0}^{\infty} \sum_{l'} l' F_{l'} \int_0^{\infty} dt e^{-st} \times [f_{0n'}^0 \Phi(t) e^{i\omega_c(n'-n)t - il'\Omega_{\mathrm{dc}}t} - f_{0n}^0 \Phi(t) \times e^{-i\omega_c(n'-n)t + il'\Omega_{\mathrm{dc}}t}]. \tag{28}$$

The miniband width Δ enters the kinetic equation (26) only via the function F_l:

$$F_l = \frac{1}{\pi} \int_0^{\pi} dz\; J_l^2\left(\frac{\Delta}{\hbar\Omega_{\mathrm{dc}}} \sin z\right). \tag{29}$$

The t integrals in Eqs. (27) and (28) are elementary, and lead to poles in the complex frequency plane.

Our final set of linear equations (26) for the components f_{mn}^l of the electron distribution function can be solved exactly. This is most conveniently done by transforming back to the time representation. This results in a first-order differential equation

$$\frac{df_n^l(t)}{dt} + i\frac{dA_l(t)}{dt} f_n^l(t) = D_n^l, \tag{30}$$

with

$$\frac{dA_l(t)}{dt} = l(\Omega_{\mathrm{dc}} + \Omega_{\mathrm{ac}} \cos \omega t) + B_n^l / i, \tag{31}$$

which is easily solved. The time-averaged solution is

$$f_{0n}^l = D_n^l \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt\; e^{-iA_l(t)} \left[C + \int_0^t d\tau\; e^{iA_l(\tau)} \right], \tag{32}$$

where the integration constant C is determined from the periodic boundary condition $f_n^l(t+2\pi/\omega) = f_n^l(t)$. In order to calculate the remaining t integrals in Eq. (32), the following expansion in a series of Bessel functions is used:

$$\exp\left(il \frac{\Omega_{\mathrm{ac}}}{\omega} \sin \omega t \right) = \sum_{k=-\infty}^{\infty} J_k\left(l \frac{\Omega_{\mathrm{ac}}}{\omega} \right) e^{ik\omega t}. \tag{33}$$

A solution of the kinetic equation (26) refers to the case of high dc electric-field strengths ($\Omega_{\mathrm{dc}}\tau \gg 1$). It should fulfill the essential requirement that the nonequilibrium distribution function of the dc case is reestablished in the limit $\Omega_{\mathrm{ac}} \to 0$. This is achieved by making use of the replacement $D_n^l \to D_n^l(1 - iB_n^l / l\Omega_{\mathrm{dc}})$ in Eq. (32), which is in accordance with our assumption $\Omega_{\mathrm{dc}}\tau \gg 1$. Finally, we obtain the following solution of Eq. (26):

$$f_{0n}^l = \frac{D_n^l}{il\Omega_{\mathrm{dc}}} \sum_{k=-\infty}^{\infty} J_k^2\left(l \frac{\Omega_{\mathrm{ac}}}{\omega} \right) \frac{il\Omega_{\mathrm{dc}} + B_n^l}{i(l\Omega_{\mathrm{dc}} - k\omega) + B_n^l}, \tag{34}$$

which indeed reduces to the nonequilibrium distribution function of the dc case $[(f_n^l)_{\mathrm{dc}} = D_n^l / il\Omega_{\mathrm{dc}}]$, when Ω_{ac} vanishes. Solution (34) has been obtained under the condition that the influence of an ac field on scattering can be neglected. Which effects an ac field exerts during scattering events is an open question which deserves further investigations.

Once the distribution function has been calculated the current density is readily obtained from the equation

$$j_z(t) = \frac{e}{\hbar V} \sum_{\boldsymbol{k}} \frac{\partial \varepsilon(\boldsymbol{k})}{\partial k_z} f(\boldsymbol{k}|t). \tag{35}$$

Inserting the considered cosine tight-binding dispersion relation into Eq. (35), from Eqs. (7), (12), and (23) we obtain the following explicit expression for the time-averaged current density:

$$j_z = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt\; j_z(t) = \frac{en_s\Delta}{4\hbar} \frac{1}{2i} \sum_{n=0}^{\infty} (f_{0n}^{l=-1} - f_{0n}^{l=1}) = \frac{en_s\Delta}{4\hbar} \mathrm{Im} \sum_{n=0}^{\infty} f_{0n}^{l=-1}. \tag{36}$$

From Eqs. (34) and (36) the current density is expressed by a sum of two quite different contributions, which correspond to the above mentioned mechanisms of delocalization:

$$j_z = j_z^{(\mathrm{phonon})} + j_z^{(\mathrm{photon})} = \frac{en_s\Delta}{4\hbar\Omega_{\mathrm{dc}}} \sum_{n=0}^{\infty} \sum_{k=-\infty}^{\infty} \frac{J_k^2(\Omega_{\mathrm{ac}}/\omega)}{(\Omega_{\mathrm{dc}} - k\omega)^2 - 2\Omega_{\mathrm{dc}}\; \mathrm{Im}\; B_n^{l=-1} + |B_n^{l=-1}|^2} \times \{ \mathrm{Re}(D_n^{l=-1}) [\Omega_{\mathrm{dc}}(\Omega_{\mathrm{dc}} - k\omega) - 2\Omega_{\mathrm{dc}}\; \mathrm{Im}\; B_n^{l=-1} + |B_n^{l=-1}|^2] + \mathrm{Im}(D_n^{l=-1}) k\omega\; \mathrm{Re}\; B_n^{l=-1} \}. \tag{37}$$

If the ac electric field vanishes ($\Omega_{\rm ac}=0$) only the $k=0$ term survives in the k sum, and it remains, as it should be, the scattering induced current density $j_z^{\rm dc}=en_s\Delta\Sigma_n\ {\rm Re}\ D_n^{l=-1}/(4\hbar\Omega_{\rm dc})$ at the presence of a strong dc electric field. This current contribution is calculated from the antisymmetric part of the nonequilibrium distribution function $(f_n^l)_{\rm dc}$. If an additional ac field is applied to the SL this part, denoted by $j_z^{(\rm phonon)}$, is modified as indicated. There is a second contribution to the current density, $j_z^{(\rm photon)}$, which results from photon-mediated delocalization of carriers. This radiation-induced current contribution is strongly manifested at resonances stemming from the interplay between Bloch oscillations and the THz field. It is proportional to the symmetric part of the dc distribution function, which is connected with the mean energy of a simple cosine band $[\tilde{\varepsilon}(k_z)=(\Delta/2)\cos k_z d]$ via the equation

$$\langle\tilde{\varepsilon}\rangle=\frac{1}{V}\sum_{\boldsymbol{k}}\tilde{\varepsilon}(k_z)f_{\rm dc}(\boldsymbol{k})=-\frac{n_s\Delta}{4\Omega_{\rm dc}d}\sum_{n=0}^{\infty}{\rm Im}\ D_n^{l=-1}. \tag{38}$$

This current contribution describes the effect that carriers absorb the energy transferred into the SL by the radiation field and tunnel against the field direction. $j_z^{(\rm photon)}$ is negative, and goes to zero at high dc electric fields or vanishing ac frequency. On the basis of the relaxation-time approximation, we obtain essentially the same physical picture in Sec. III.

III. RELAXATION-TIME APPROXIMATION

The main physical results obtained in Sec. II on the basis of a microscopic quantum-transport theory are now reproduced and discussed within a simple relaxation-time model. In addition, we attempt to clarify a number of issues which have previously been treated in a rather nonrigorous fashion on the basis of the Boltzmann equation in the relaxation-time approximation. Quite similar to former theoretical studies,[5,2,12–17] in this section we will introduce a constant relaxation time, thereby making it possible to solve the kinetic equation analytically. This simplified approach is intended to provide a better physical understanding of how the nonlinear high-field transport is affected by the THz irradiation. Within the constant relaxation-time approximation, Eq. (26) has the form

$$(im\omega+il\Omega_{\rm dc})f_{mn}^l+\frac{i}{2}l\Omega_{\rm ac}(f_{m-1n}^l+f_{m+1n}^l)=-\frac{1}{\tau_{\rm ac}}(f_{mn}^l-\delta_{m,0}\tilde{f}_n^l), \tag{39}$$

where the parameter $\tau_{\rm ac}$ describes the relaxation of the ac signal within the SL. Equation (39) has been solved in the literature[5,2,12–17] by identifying the unknown function $\tilde{f}_n^l$ with the equilibrium distribution function. The validity of this replacement, i.e., the assumption that the scattering-out term is associated with an equilibrium distribution function even at high dc electric fields, has to be strongly questioned. Here we propose a completely different and more natural approach by requiring that the normalized function $\tilde{f}_n^l$ is chosen so as to ensure that at vanishing ac field intensity ($\Omega_{\rm ac}\to 0$) a distribution function is recovered that governs the dc transport. This is achieved by relating $\tilde{f}_n^l$ to the nonequilibrium distribution function $(f_n^l)_{\rm dc}$, which determines the carrier transport under constant bias:

$$\tilde{f}_n^l=(i\tau_{\rm ac}l\Omega_{\rm dc}+1)(f_n^l)_{\rm dc}. \tag{40}$$

$\tilde{f}_n^l$ is the formal solution of Eq. (39) for $\Omega_{\rm ac}=$ and $m=0$ and fulfills, as $(f_n^l)_{\rm dc}$ does, the sum rule (25). Similar to the calculation in Sec. II the Boltzmann equation (39) can be solved exactly by transforming it back to the time representation. The time-averaged solution has the form

$$f_{0n}^l=(f_n^l)_{\rm dc}\sum_{k=-\infty}^{\infty}J_k^2\left(l\frac{\Omega_{\rm ac}}{\omega}\right)\times\frac{l\Omega_{\rm dc}-i/\tau_{\rm ac}}{l\Omega_{\rm dc}+k\omega-i/\tau_{\rm ac}}, \tag{41}$$

which indeed restores $(f_n^l)_{\rm dc}$ when $\Omega_{\rm ac}\to 0$, because in this case only the $k=0$ term survives. Our solution (41) constitutes a novelty, to our knowledge, as previous approaches did not recognize the requirement that at vanishing ac field the nonequilibrium distribution function of the remaining dc field should be reproduced and not the equilibrium function, which cannot adequately describe the nonlinear transport under constant bias.

From Eqs. (41) and (36), the current density can be calculated. Again it is composed of two parts:

$$\begin{aligned}j_z&=j_z^{(\rm phonon)}+j_z^{(\rm photon)}\\&={\rm Im}\ \mathcal{J}_{\rm dc}\sum_{k=-\infty}^{\infty}J_k^2\left(\frac{\Omega_{\rm ac}}{\omega}\right)\frac{\Omega_{\rm dc}(\Omega_{\rm dc}-k\omega)+1/\tau_{\rm ac}^2}{(\Omega_{\rm dc}-k\omega)^2+1/\tau_{\rm ac}^2}\\&\quad-{\rm Re}\ \mathcal{J}_{\rm dc}\frac{1}{\tau_{\rm ac}}\sum_{k=-\infty}^{\infty}J_k^2\left(\frac{\Omega_{\rm ac}}{\omega}\right)\frac{k\omega}{(\Omega_{\rm dc}-k\omega)^2+1/\tau_{\rm ac}^2},\end{aligned} \tag{42}$$

where $\mathcal{J}_{\rm dc}=en_s\Delta\Sigma_n(f_n^{-1})_{\rm dc}/4\hbar$ is a complex function, the imaginary part of which is nothing but the dc current density. As in Sec. II and quite similar to the ac hopping conductivity in disordered systems,[40] the current originates both from phonon- and photon-induced transitions between Stark ladder states. If there is no THz field ($\Omega_{\rm ac}=0$), Eq. (42) reduces to $j_z={\rm Im}\ J_{\rm dc}$, that describes the nonlinear I-V characteristics under constant bias. The second current contribution in Eq. (42) is photon induced, and vanishes when $\Omega_{\rm ac}$ or ω goes to zero. It gives rise to δ-like peaks in the I-V characteristics whenever the Bloch frequency $\Omega_{\rm dc}$ is an integer multiple of the ac frequency ω under the condition that the scattering time goes to infinity ($\tau_{\rm ac}\to\infty$). This contribution is always negative since photon absorption dominates over photon emission. Whereas photon absorption becomes resonant, emission does not. $j_z^{(\rm photon)}$ describes the effect that Bloch oscillating carriers mainly absorb the radiation energy to move the Stark ladder upwards, which results in a negative current contribution. To our knowledge this is a completely new explanation for the fact that the SL current can become negative under THz irradiation. This effect has a mechanical analogy. Consider a vibrating system with an eigenfrequency Ω_{dc} under the influence of a driving oscillating force with a frequency ω. At resonance the system acquires energy from the external time-dependent force, which leads to a resonance enhancement of the vibrational amplitude. Resonant depression does not occur.

The contributions $j_z^{(\mathrm{phonon})}$ and $j_z^{(\mathrm{photon})}$ depend both on the THz field and the electron-phonon scattering. The first term in Eq. (42) leads to sharp peaks in the current density at cyclotron-Stark-phonon resonance positions, whereas the second term, which is proportional to Re J_{dc}, depends on the principal values of the denominators and is, therefore, associated with virtual transitions between Landau- and Stark-ladder states. With respect to the photon resonances the situation is just reversed. Here $j_z^{(\mathrm{photon})}$ becomes resonant when $\Omega_{dc}=k\omega$, whereas $j_z^{(\mathrm{phonon})}$ is related to the principal values of these resonances.

Now we will demonstrate how the so called standard theory of photon-assisted tunneling is obtained within our approach. To this end we calculate the quantity $(f^{\,l}_{\,n})_{dc}$ by disregarding the ac field $(\omega=\Omega_{ac}=0,\tau_{ac}\rightarrow\tau)$ and replacing $\tilde{f}_n^l$ by the equilibrium distribution function in Eq. (39). In this approximation the current is independent of the magnetic field. The resulting Boltzmann equation was solved many years ago[41] to determine the temperature dependence of the current within the Esaki-Tsu model.[42] We obtain

$$\sum_{n=0}^{\infty}(f^{\,l}_{\,n})_{dc}=\frac{I_l\left(\dfrac{\Delta}{2k_BT}\right)}{I_0\left(\dfrac{\Delta}{2k_BT}\right)}\,\frac{1}{il\tau\Omega_{dc}+1},\tag{43}$$

where I_l are modified Bessel functions. Inserting this solution into Eq. (42), the contributions proportional to Re $\mathcal{J}_{dc}$ and Im $\mathcal{J}_{dc}$ combine, and result in a simple equation

$$j_z(\Omega_{dc},\Omega_{ac})=\sum_{k=-\infty}^{\infty}J_k^2\left(\frac{\Omega_{ac}}{\omega}\right)j_z^{(\mathrm{ET})}(\Omega_{dc}+k\omega),\tag{44}$$

with $j_z^{(\mathrm{ET})}$ being the dc current density of the Esaki-Tsu model.[41] Equation (44), known as the Tien-Gordon[26] or Tucker[27] theory, has been widely used to study photon-assisted tunneling in SL's under dc bias.[5,2,12–17] However, its obvious generalization, replacing $j_z^{(\mathrm{ET})}$ by any other expression for the dc current to account for quantum effects, is not correct. If a more detailed analysis of the tunneling current is desired, it is not sufficient to know only the expression for the dc current density (Im $\mathcal{J}_{dc}$) but also the related real part (Re $\mathcal{J}_{dc}$), which is given by the symmetric part of the nonequilibrium distribution function. In this more general case one has to go back to Eq. (42) or if the damping of the THz signal is described within a microscopic approach to a formula like Eq. (37). Such a generalization is necessary, e.g., if one wants to study the behavior of cyclotron-Stark-phonon resonances under THz irradiation, since WS localization is not included within the quasi-classical Esaki-Tsu model. Moreover, the importance of a microscopic description of the dc current for the study of photon-assisted tunneling in SL's was recently addressed by Wacker *et al.*[22]

Here we will give an example of such a study, and consider cyclotron-Stark resonances due to scattering on ionized impurities.[43] We want to stress that it is not sufficient to restrict the consideration on elastic scattering, since a finite dc current is only obtained if dissipation via inelastic scattering is taken into account. Here we circumvent this problem in a rather nonrigorous way by introducing a phenomenological broadening parameter s. The expression, we will use to characterize the dc transport, was derived in Ref. 37:

$$\begin{Bmatrix}\mathrm{Re}\\ \mathrm{Im}\end{Bmatrix}\mathcal{J}_{dc}=\frac{2en_sn_i}{\hbar^2}\left(\frac{4\pi e^2}{\varepsilon}\right)^2\int_0^{\infty}\frac{dq_\perp}{(2\pi)^2}q_\perp\int_0^{2\pi/d}dq_z\frac{|g(q_z)|^2}{(q_\perp^2+q_z^2+q_{\mathrm{TF}}^2)^2}\sum_{l=1}^{\infty}l\begin{Bmatrix}\sinh\\ \cosh\end{Bmatrix}\left(\frac{\hbar\Omega_{dc}}{2k_BT}l\right)J_l^2\left(\frac{\Delta}{\hbar\Omega_{dc}}\sin\frac{q_zd}{2}\right)\int_0^{\infty}dt$$
$$\times\exp(-st)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(l\Omega_{dc}t)\exp\left\{-\frac{q_\perp^2l_B^2}{2\sinh(\hbar\omega_c/2k_BT)}[\cosh(\hbar\omega_c/2k_BT)-\cos\omega_ct]\right\}.\tag{45}$$

Here n_i is the impurity density, ε the dielectric constant, and $1/q_{\mathrm{TF}}$ the Thomas-Fermi screening length. Numerical results derived from Eqs. (42) and (45) will be presented and discussed in Sec. IV.

IV. NUMERICAL RESULTS AND DISCUSSION

At first we will present and discuss numerical results obtained by the phenomenological treatment of the THz field presented in Sec. III. The current density is calculated from Eq. (42) together with the quantity $\mathcal{J}_{dc}$ in Eq. (45) that characterizes the transport properties of an electron in a SL under strong magnetic and dc electric fields. We considered elastic scattering on ionized impurities where dissipation was phenomenologically introduced by a dc broadening parameter $\tau_{dc}=1/s$. Equations (42) and (45) allow us to study the influence of strong THz irradiation on cyclotron-Stark resonances in the SL transport. Numerical results for the dc electric field dependence of the current density are shown in Fig. 1 for $\Omega_{ac}=0$ (thick dashed line) compared with the case when a strong THz field is applied $(\Omega_{ac}/\omega=2)$. In addition, phonon- and photon-induced current contributions that are given by the first and second terms of the right-hand side of Eq. (42) are displayed by thin solid lines (a) and (b), respectively. The positions of cyclotron-Stark resonances at $l\Omega_{dc}=m\omega_c$ are marked by vertical dashed lines, while one-, two-, and three-photon transitions at $\Omega_{dc}=k\omega$ are indicated by vertical solid lines in Fig. 1. Below 20 kV/cm the *I-V* characteristics changes profoundly due to the presence of the ac electric field. As already discussed in Secs. II and III the photon-induced current contribution (curve *b*), which is proportional to the symmetric part of the nonequilibrium distribution function $(f_n^l)_{dc}$, is always negative, and describes carrier transport against the field direction via photon absorption. This contribution exhibits sharp photon resonances at $\Omega_{dc}=k\omega$, whereas its magnitude vanishes strongly in the limits $\tau_{ac}\rightarrow\infty$, $\Omega_{ac}\rightarrow 0$, or $\Omega_{dc}\rightarrow\infty$. Contrary, the first contribution on the right-hand side of Eq. (42) (curve *a*) depends on the principal value, and leads to a slight shift of the photon resonances. This contribution may exhibit sharp peaks at cyclotron-Stark resonance positions (indicated by vertical dashed lines).

The magnetic-field dependence of the current density is shown in Fig. 2, where the vertical dashed lines mark again the positions of cyclotron-Stark resonances. Such resonances have been identified in recent experiments.[19,20] The thick dashed line exhibits the current density for $\Omega_{ac}=0$, whereas the thick solid line has been calculated with $\Omega_{ac}/\omega=2$. Again the thin solid lines *a* and *b* are the results obtained from the two current contributions in Eq. (42). The scattering induced current contribution (curve *a*) follows mainly the magnetic-field dependence of the dc current (dashed line), whereas the second one, which is proportional to the principal value of cyclotron-Stark resonances, leads to a shift of resonance peaks and drives the current appreciably towards negative values.

Qualitatively the same results are obtained within the microscopic quantum approach derived in Sec. II, where we treated the effect of the THz irradiation on cyclotron-Stark-phonon resonances, which are due to carrier scattering on polar optical phonons. Figure 3 shows the current density in units of $j_{z0}=en_s m^*\omega_0^2\Gamma/2\pi\hbar^3 d$ (m^* is the effective mass) as a function of the dc electric field for $\Omega_{ac}=0$ (dashed line) and $\Omega_{ac}/\omega=4$ (thick solid line) as calculated from Eq. (37) together with Eqs. (27)–(29). Again the two current contributions of Eq. (37) are denoted by *a* and *b*. The frequency positions of cyclotron-Stark-phonon resonances, marked by vertical lines in Fig. 3, do not change significantly with varying Ω_{ac}. This is a consequence of the fact that the dc and ac contributions in Eq. (37) essentially factorize. Above 30 kV/cm dynamical delocalization leads to an enhancement of the current density under the influence of THz radiation. The interplay between dynamical localization and delocalization is also seen in the magnetic-field dependence of the current density displayed in Fig. 4 for $\Omega_{ac}=0$ (dashed line), $\Omega_{ac}/\omega=2$ (upper solid line), and $\Omega_{ac}/\omega=4$ (lower solid line). With an increasing strength of the ac field the current first increases slightly for almost all magnetic-field values, until a further increase of Ω_{ac} leads to a drastic reduction of the current density. The details of this current change depend sensitively on the dc electric-field strength.

Figure 5 shows the temperature dependence of the current for $\Omega_{ac}=0$ (dashed line), $\Omega_{ac}/\omega=2$ (upper solid line), and $\Omega_{ac}/\omega=4$ (lower solid line). The fact that the current in-

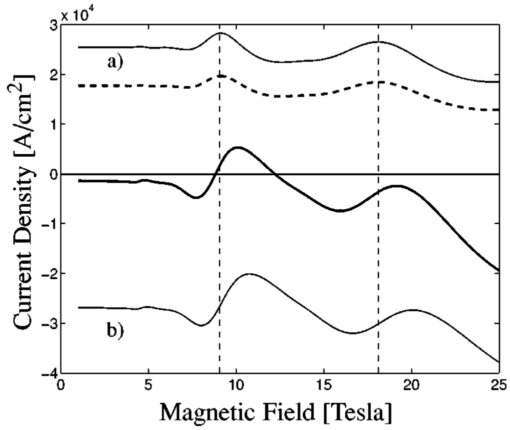

FIG. 2. Magnetic-field dependence of the current density for $\Omega_{ac}=0$ (thick dashed line) and $\Omega_{ac}/\omega=2$ (thick solid line) and $E=15$ kV/cm. The same parameters as in Fig. 1 have been used.

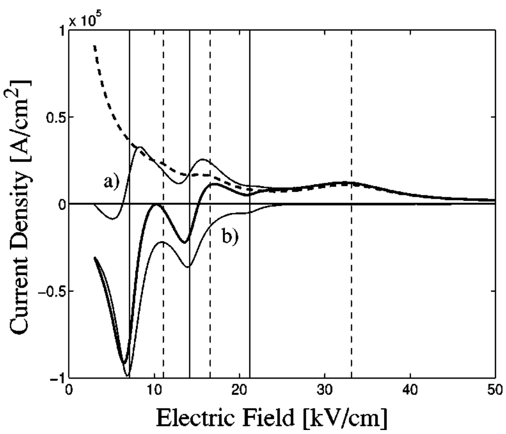

FIG. 1. Electric-field dependence of the current density for $\Omega_{ac}=0$ (thick dashed line) and $\Omega_{ac}/\omega=2$ (thick solid line) and $\Delta/\hbar\omega_0=1$, $\hbar\omega_0/k_BT=1$, $\omega/\omega_0=0.2$, and $H=20$ T (the SL period *d* is 10 nm). The thin solid lines (a) and (b) denote the first and second contributions in Eq. (42). The following parameters $1/\omega_0\tau_{dc}=0.2$ and $1/\omega_0\tau_{ac}=0.05$ have been used, where τ_{dc} is the relaxation time of the dc transport. The impurity concentration is $n_i=10^{18}/\mathrm{cm}^3$.

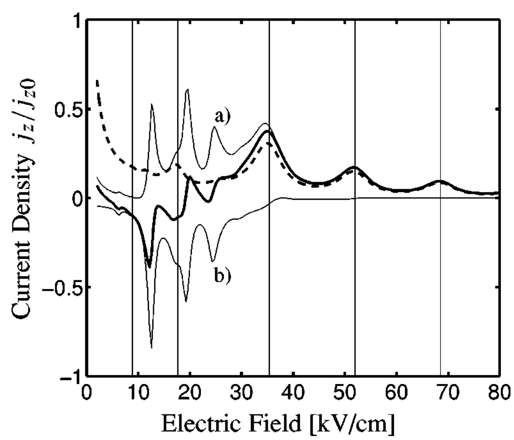

FIG. 3. Electric-field dependence of the current density for $\Omega_{ac}=0$ (dashed line) and $\Omega_{ac}/\omega=4$ (solid line). The following parameters have been used: $\Delta/\hbar\omega_0=0.5$, $\hbar\omega_0/k_BT=5$, $\omega/\omega_0=0.2$, $\delta=0.1$, and $H=10$ T. The thin solid lines (a) and (b) denote the first and second contributions in Eq. (37). Positions of cyclotron-Stark-phonon resonances are marked by thin vertical lines.

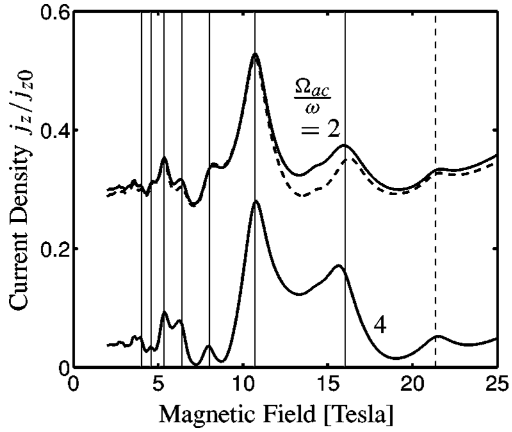

FIG. 4. Magnetic-field dependence of the current density for $\Omega_{ac}=0$ (dashed line), $\Omega_{ac}/\omega=2$ (upper solid line), and $\Omega_{ac}/\omega=4$ (lower solid line). The dc electric-field strength is $E_{dc}=17.7$ kV/cm ($eE_{dc}d=\hbar\omega_0/2$) and $\hbar\omega_0/k_BT=2$. All other parameters are the same as in Fig. 3. Positions of cyclotron-Stark-phonon resonances for $l=1(l=2)$ are marked by thin solid (dashed) vertical lines.

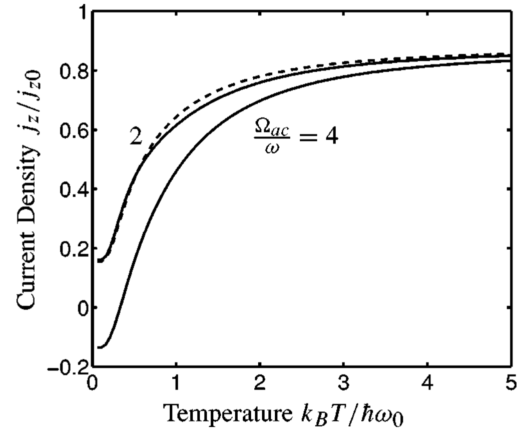

FIG. 5. Temperature dependence of the current density for $\Omega_{ac}=0$ (dashed line), $\Omega_{ac}/\omega=2$ (upper solid line), and $\Omega_{ac}/\omega=4$ (lower solid line). The dc electric-field strength is $E_{dc}=17.7$ kV/cm, and $H=10$ T. All other parameters are the same as in Fig. 3.

creases with increasing temperature clearly underlines the hopping character of the SL transport. The ac field-induced shift of the current density towards negative values is most pronounced at low temperatures.

V. SUMMARY

Studies of nonlinear SL transport under THz irradiation have recently occupied a great deal of attention both theoretically[5,2,12–17] and experimentally.[7,24,25] Most previous theoretical studies and the analysis of experiments relied on an analytical solution of the Boltzmann equation in the relaxation-time approximation. This approach suffers from two serious deficiencies: (i) it cannot account for quantum effects due to WS localization; and (ii) it introduces an equilibrium distribution function, which has to be strongly questioned when a strong dc electric field is applied. It was the main objective of our paper to overcome these serious disadvantages of former theoretical approaches. Within a microscopic quantum-mechanical picture we studied the influence of strong THz radiation on cyclotron-Stark and cyclotron-Stark-phonon resonances, which appear in the nonlinear SL transport when high magnetic and dc electric fields are applied parallel to the SL axis. This approach has been compared with a phenomenological treatment of ac field effects on the SL transport, which is intended to provide a better understanding of the main physical aspects. Our theory is constructed in such a way as to ensure the reappearance of the nonequilibrium dc electron distribution function when the ac field vanishes. An equilibrium distribution function, which has no physical meaning if a strong dc electric field is applied to the system, is completely absent in our approach. The calculated current density is composed of two contributions, one of which is proportional to the dc current density and describes scattering-induced carrier transport, whereas the other one is due to delocalization in the THz field. The latter current contribution is related to the symmetric part of the nonequilibrium dc distribution function, and disappears when the ac field vanishes. It becomes prominent when the dc and ac electric-field strengths become comparable in magnitude. This term, which is always negative, describes carrier transport against the field direction due to photon absorption. It exhibits sharp minima at photon-Stark resonance positions $\Omega_{dc}=k\omega$, and is proportional to the principal value of cyclotron-Stark-phonon resonances. The second transport channel, opened by carrier collisions, is mainly determined by the dc current density modified by a factor that depends on the THz field and the principal value of photon-Stark resonances.

In conclusion we focused on the influence of intense THz fields on the nonlinear SL transport, thereby making it possible to study combined cyclotron-Stark-phonon and Stark-photon resonances. Further progress is expected from experimental studies of the nonlinear transport in SL's under strong THz irradiation, where quantum-mechanical effects are strongly manifest.

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