Liquid-crystal phases of quantum Hall systems

Eduardo Fradkin

Department of Physics, University of Illinois, Urbana, Illinois 61801-3080

Steven A. Kivelson

Department of Physics, U. C. L. A., Los Angeles, California 90095

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Mean-field calculations for the two-dimensional electron gas in a large magnetic field with a partially filled Landau level with index $N \geq 2$ consistently yield ''stripe-ordered'' charge-density wave ground states, for much the same reason that frustrated phase separation leads to stripe-ordered states in doped Mott insulators. We have studied the effects of quantum and thermal fluctuations about such a state and show that they can lead to a set of electronic liquid crystalline states, particularly a stripe-nematic phase, which is stable at $T>0$. Recent measurements of the longitudinal resistivity of a set of quantum Hall devices have revealed that these systems spontaneously develop, at low temperatures, a very large anisotropy. We interpret these experiments as evidence for a stripe nematic phase, and propose a general phase diagram for this system. [S0163-1829(99)06711-9]

There are many condensed matter systems in which the charge degrees of freedom form regular spatial patterns commonly known as ''stripes.'' These structures are typically the result of the competition between short-range attractive forces, which give rise to a condensed (usually insulating) phase, and the largely unscreened, long-range Coulomb interactions. Specifically, in the condensed phase, and in the absence of long-range repulsive forces between like charges, the charge degrees of freedom have a tendency to form clumps, that is to phase separate. This tendency to phase separation is frustrated by the long-range repulsive Coulomb interactions and the result is the spontaneous organization of the charge degrees of freedom in low-dimensional structures.[1] In a large class of quasi-two-dimensional strongly correlated materials, i.e., doped Mott insulators such as the copper oxides, the nickelates, and the manganates, stripe phases have recently been observed experimentally.[2,3]

However, these structures should not be peculiar to doped Mott insulators, but should also arise in other electronic systems in which the same sort of competition is present. It has been realized for some time that there is a strong tendency for the two-dimensional electron gas (2DEG) in a high-magnetic field to condense into incompressible quantum Hall liquid states with quantized ''filling factor'' ν. Thus, in the presence of long-range Coulomb interactions, at electron densities intermediate between two quantized values it is natural to expect the system to form an inhomogeneous state with a periodic array of stripes of the two incompressible liquids. Indeed, some time ago Kulakov, Fogler, and Shklovskii[4] and Moessner and Chalker[5] showed that a two-dimensional electron gas in a perpendicular magnetic field can have a stripe or charge-density wave (CDW) ground state in which the charge density in a partially-filled high Landau level exhibits periodic oscillations along one spatial direction. According to these calculations, which are based on a Hartree-Fock approach, the electrons in a partially occupied Nth Landau level form stripes in which the Landau level is alternately full or empty. The stripe pattern is a pe-riodic modulation of the local Hall conductance between the quantized values $2Ne^2/h$ and $(2N+1)e^2/h$.

In this paper, we present a theoretical description of the 2DEG in a moderately large magnetic field, and show that this system actually behaves like a set of dynamical conducting stripes. We have recently[6] developed a theory of the phase diagram of fluctuating conducting stripes for doped Mott insulators. In the present context, the ''edge states'' at the interface between two regions of differently quantized Hall conductance play the same role as the conducting stripes. However, there are two important differences between these edge states and the fluctuating stripes we studied in the context of doped Mott insulators: (i) because of the high-magnetic field, the edge states are intrinsically chiral (with alternating chirality, as shown in Fig. 2), and (ii) the high-temperature phase has full $U(1)$ rotational symmetry, as opposed to the discrete rotational (point-group) symmetry found in doped insulators. We will show here that the quantum-mechanical fluctuations about the Hartree-Fock state of Refs. 4 and 5 lead to a variety of new phases. Based on these results, we propose the qualitative $T=0$ phase diagram in the absence of disorder shown in Fig. 1.

This phase diagram includes the following *electronic liquid crystalline* and true crystalline phases:

1. *A quantum smectic*, which has charge-density wave order, which breaks translational and rotational symmetry, but in which the liquid-like (metallic) behavior of the chiral edge states is preserved.

2. *A quantum nematic*, in which quantum fluctuations of the stripe order are sufficiently strong to restore translational symmetry, i.e., to melt the CDW order, but still small enough that local orientational order of the stripes persists, thus breaking rotational symmetry.

3. *A quantum isotropic fluid phase*, in which the 2DEG is invariant under both rotations and translations.

4. *An insulating stripe-crystal* phase is also possible, when the partially filled Landau level is not half filled (i.e., is not particle-hole symmetric). This phase is characterized by
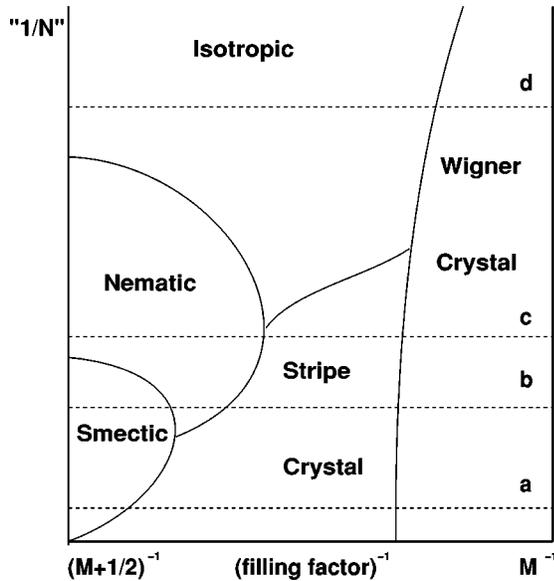
FIG. 1. Qualitative $T=0$ phase diagram for a clean 2DEG. The vertical axis measures the strength of the quantum fluctuations (which is roughly inversely proportional to the Landau level index, N) and the horizontal axis is the inverse filling factor (over a range in which the partial filling of the highest spin-polarized Landau level varies between M and $M+1/2$, where $M=2N$ or $M=2N+1$. Lines a, b, c, and d are three realizations for increasing the strength of the quantum fluctuations. The smectic and nematic are compressible while the isotropic phase may either be compressible or incompressible. The stripe crystal and the Wigner crystal phases are insulating and exhibit plateau behavior with the same quantized Hall conductance, $\sigma_{xy}=e^2 M/h$.
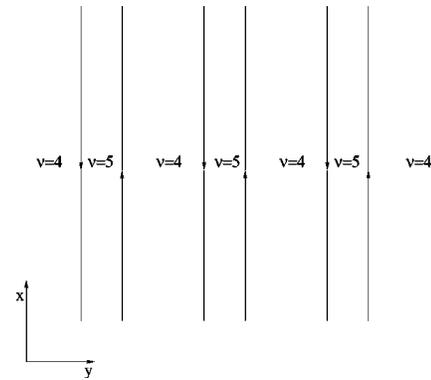

FIG. 2. Schematic view of the smectic phase: In this picture, we have taken the filling factor somewhere between $\nu=9/2$ and $\nu=4$. The system is compressible with the fraction of the sample with filling fraction $\nu=5$ decreasing with increasing magnetic field. As this progresses, nearby pairs of edge states become strongly coupled and the result is a stripe (smectic) phase of nonchiral Luttinger liquids separating regions with $\nu=4$. This phase exists only at $T=0$.

the same CDW order perpendicular to the stripe direction as the smectic, but in addition the density-wave fluctuations along neighboring stripes phase lock to each other, forming a true, insulating, two-dimensional electron crystal.

5. *A Wigner Crystal* phase is also expected, especially at partial filling of the Landau level near 0 or 1. This phase is also insulating, but differs from the insulating stripe crystal in its crystal structure, and its degree of isotropy.

The smectic and the nematic phases both break rotational symmetry; because of the conducting character of the chiral edge states, both liquid crystalline phases possess highly anisotropic conductivity tensors, with principal axes parallel to (''x direction'') and perpendicular to (''y direction'') the preferred stripe orientational direction. Both phases are compressible and have a nonquantized Hall conductance. The stripe and Wigner crystal phases are ''insulating,'' since the crystals are easily pinned by impurities or boundary effects. Of course, what this means is that the full conductivity tensor at low temperature is that of the lower-lying, full Landau levels, so these states are actually quantized Hall states. The isotropic fluid is actually a set of phases, including the fractional quantum Hall and compressible Hall metal phases which are familiar from previous studies.

Because $d=3$ is the lower critical dimension for smectic order,[7] the electron smectic only exists at $T=0$; at finite temperature, it is indistinguishable from the nematic. Thermal fluctuations also eliminate true long-ranged orientational nematic order, but quasi-long-ranged (power-law) order survive up to a finite temperature transition. Wigner crystalline

order is destroyed by thermal fluctuations in the standard (2D melting) fashion, i.e., either there is a single, first-order melting transition, or a sequence of two continuous transitions and an intermediate temperature ''hexatic'' phase, with short-range positional and sixfold orientational quasi-long-range order.[8] The stripe-crystal order is destroyed in a similar fashion, although the intermediate state phase in this case is a ''biatic,'' which from a symmetry point of view is indistinguishable from the finite temperature nematic. (See Fig. 3.)

The paper is organized as follows. In Sec. I, we summarize the main results of the mean-field theory of Refs. 4 and 5. Here we argue that this state should be viewed as an electron smectic. In Sec. II, we discuss in detail the effects of quantum fluctuations and their implications for the $T=0$ phase diagram. We focus on two main effects: fluctuations of the geometry (or shape) of the stripes and intrastripe charge-density fluctuations. We show that intrastripe fluctuations alone always produce an instability of the smectic to a stripe-crystal phase. In contrast, small shape fluctuations tend to stabilize the smectic.[6] Furthermore, we argue that large shape fluctuations lead to a two stage quantum melting of the smectic through an intermediate nematic phase into an isotropic electron fluid. In this section, we also characterize the various phases in the phase diagram. In Sec. III, we discuss the effects of thermal fluctuations and the fate of the $T=0$ phases. In Sec. IV, we discuss on general grounds the expected behavior of the conductivity tensor in the various phases, especially its behavior and anisotropies at low temperatures. Section V contains a brief and highly incomplete discussion of the effects of quenched randomness (disorder) on the principal findings of this paper. Finally, in Sec. VI we discuss the relation between this theoretical picture and the recent experiments of Lilly, Cooper, Eisenstein, Pfeiffer, and West.[9]

I. MEAN-FIELD THEORY AND THE SMECTIC PHASE

We take as our starting point a mean-field state, which consists of alternating stripes with filling fraction $\nu=M$ and

$\nu = M + 1$ as shown in Fig. 2 for the case $M = 4$. Here, we have assumed, for simplicity, that the cyclotron and Zeeman energies are sufficiently large that electrons sequentially fill individual spin-polarized Landau levels as B decreases, and $M = 2N$ or $2N + 1$ depending on whether the partially filled spin-polarized level is spin up or spin down. The striped-CDW state is characterized by an order parameter Δ_{CDW}, which describes a charge-density modulation with wavelength λ. In the Hartree-Fock description of Refs. 4 and 5, the single-particle states in the Nth Landau level near the "crest" of the CDW are filled while those near the "trough" are empty. The wavelength λ of the CDW is of the order of R_c, the cyclotron radius in the Nth Landau level, $\lambda = AR_c = A\ell_0\sqrt{N}$, where $\ell_0 = \sqrt{\hbar c/eB}$ is the magnetic length. Here A is a constant determined by the details of the interaction and A increases smoothly as the interactions become progressively screened. For $N \gg 1$, the Fermi wavelength λ_F is small, $\lambda_F \ll R_c$.

It follows from general hydrodynamic principles that there exist gapless edge states at the boundary between two regions of differently quantized Hall conductance.[10–12] In the case of a boundary between two integer quantized Hall states, these hydrodynamic edge modes can be simply constructed as particle-hole excitations, which propagate with a velocity, $v = cE_{edge}/B$, which is proportional to the strength of the electric field at the edge, E_{edge}. Thus, in the absence of interactions between edges, these excitations form *chiral Fermi liquids*, and intraedge electron-electron interactions only renormalize the velocity v. In an ordered stripe phase, there are two such chiral edge states per unit cell with opposite chirality, as shown in Fig. 2.

However, there is an important distinction between the edge states that occur on the boundaries of quantum Hall devices, which lie along equal potential contours defined by an externally applied gate voltage, and the internal edge states in a stripe phase where the edges are self-consistently generated. In the latter case, in addition to the intraedge excitations described above, there is a second class of low-energy excitations associated with deformations of the effective potential itself, or in other words with the "shape" (and even topology) of the stripe structure. Formally, the Hartree-Fock state can be thought of as a saddle-point solution of an imaginary time-path integral in which an effective potential has been introduced as a Hubbard-Stratonovich field, $\Delta_{CDW}(\vec{r},t)$, which is just the local CDW order parameter. The intraedge excitations occur with fixed $\Delta_{CDW}(\vec{r},t)$, while the shape excitations involve deformations of $\Delta_{CDW}(\vec{r},t)$, itself. A uniform order parameter Δ_{CDW} defines an ordered CDW state with wavelength λ. Because this is a state of spontaneously broken symmetry, the transverse excitations are Goldstone modes, and hence gapless. These are the stripe deformations referred to above. (It is sometimes useful to think of the intraedge excitations as the "quasi-Goldstone modes" associated with an almost broken translational symmetry, i.e., the quasi-long-range order along the stripe direction.[13])

When $\nu = M + 1/2$, the system is particle-hole symmetric, which means that exactly half of the area is occupied by regions of $\nu = M$ and half by regions of $\nu = M + 1$ integer quantum Hall liquid. As the magnetic field is increased so

that ν varies between $M + 1/2 \geqslant \nu \geqslant M$, the ratio of areas of the two locally coexisting quantum Hall liquids varies from 0 to 1. (The range $M + 1 \geqslant \nu \geqslant M + 1/2$ is related to the range $M + 1/2 \geqslant \nu \geqslant M$ by particle-hole symmetry.) If we denote by D_M the width of each strip of $\nu = M$, then the ratio of D_M/D_{M+1} is determined by the filling fraction according to

$$\nu - \left(M + \frac{1}{2}\right) = \frac{D_M - D_{M+1}}{D_M + D_{M+1}}. \tag{1}$$

The length scale $\lambda = D_M + D_{M+1}$ is determined by the competition between the short- and long-range pieces of the Coulomb potential, and so depends on the details of the short-distance screening—in the schematic phase diagram in Fig. 1, the y axis signifies appropriate changes in the short-range piece of the Coulomb interaction, which on a phenomenological level we roughly associate with changes in Landau-level index N.

In the next section, we will consider the effects of interactions between the edge states. As ν is decreased from $M + 1/2$, the edge states (of opposite chiralities) on either side of each $\nu = M + 1$ stripe begin to approach each other; D_{M+1} decreases. Consequently, the interactions between these pairs of edges grow stronger and the interactions among the electrons in different pairs of edges (separated by D_M) grow weaker. For filling fractions $\nu \neq M + \frac{1}{2}$, rather then thinking of individual, chiral edges, we should consider the excitations of an array of nonchiral one-dimensional structures, each constituted from a *pair* of chiral edge states.

At zero temperature, mean-field stripe-ordered states have a stripe spacing that varies continuously with ν, so they are clearly compressible. They spontaneously break the $U(1)$ rotational invariance of the 2DEG as well as translation invariance along one direction. Thus, these states have both orientational and translational long-range order (in one direction); this is an *electron smectic*. As a consequence, we expect $\sigma_{xy} \sim nec/B$ to be unquantized, and $\sigma_{xx} > \sigma_{yy}$; indeed, we show below that, in the absence of disorder, $\sigma_{xy} = \sigma_{yx} = e^2\nu/h$, σ_{xx} diverges, and σ_{yy} vanishes as $T \to 0$. In addition, at precisely $\nu = M + \frac{1}{2}$, where the system has an exact particle-hole symmetry (in the half-filled spin-polarized Landau level), this discrete $\mathbf{Z}_2$ symmetry is also spontaneously broken.

II. EFFECTS OF QUANTUM FLUCTUATIONS

So far we have ignored the effects of quantum and thermal fluctuations around the mean-field state. Two distinct sorts of quantum fluctuation effects can fundamentally change the character of the ground state: (1) fluctuations of the interacting one-dimensional metallic intraedge degrees of freedom (induced by electron-electron interactions), and (2) shape fluctuations in the positions, and ultimately even the connectivity, of the edges themselves.[6]

The fluctuations of the metallic edge degrees of freedom can be described most simply using standard bosonization methods. At $\nu = M + \frac{1}{2}$, the low-energy-charged degrees of freedom that are active in the smectic phase are the fluctuations of the "edge states," which are described by an array (with alternating chiralities) of Fermi liquids, i.e., chiral bosons with unit compactification radius (or Luttinger pa-

rameter). Well away from $\nu = M + \frac{1}{2}$, the charged degrees of freedom of each *pair* of close-by edges form a *nonchiral Luttinger liquid* with a Luttinger parameter $K < 1$, which is a smooth function of the strength of the intrapair Coulomb repulsion, which is in turn a function of the mean separation between the edges in a given pair, i.e., of D_M for $\nu < M + 1/2$. Note[6] that for $K < 1$, the zero-temperature density fluctuations associated with an isolated pair of edges exhibit quasi-long-range order,

$$\langle \mathcal{O}_j^\dagger(x)\mathcal{O}_j(0)\rangle \sim \cos(2k_F^{eff}x + \theta_0)/|x|^{2K}, \qquad (2)$$

where $\mathcal{O}_j$ is the $2k_F^{eff}$ piece of the charge-density[14] operator on the jth stripe (i.e., the jth *pair* of edge states). Consequently the intrapair CDW susceptibility diverges as the temperature $T \to 0$ as

$$\chi_{CDW} \sim T^{-2(1-K)}, \qquad (3)$$

where the CDW has a period determined by an effective value of

$$2k_F^{eff} = D_M / \ell_0^2. \qquad (4)$$

Direct electron tunneling, and even pair tunneling between pairs of ideal straight edges are forbidden by momentum conservation. However, for ν not too close to $M + 1/2$, the Coulomb interactions between neighboring pairs of edges couples the intrapair CDW fluctuations; schematically, this makes a contribution to the Hamiltonian density

$$\mathcal{H}_{Coul} = V\{\mathcal{O}_j^\dagger(x)\mathcal{O}_{j+1}(x) + \text{H.c.}\}. \qquad (5)$$

The scaling dimension of the operator $\mathcal{H}_{Coul}$, which represents the coupling of the $2k_F^{eff}$ CDW order parameters on neighboring stripes is $d_{CDW} = 2K$. For repulsive Coulomb interactions $K < 1$, χ_{CDW} diverges and $d_{CDW} < 2$. In the renormalization group sense, this coupling is relevant. It produces an instability of the smectic phase, analogous to one that is commonly encountered in *quasi*-one-dimensional materials, toward the formation of an insulating, stripe crystal phase with long-range CDW order both along and transverse to the stripe direction.

This is not the whole story, since, as discussed above, the stripes are spontaneously generated, so their *shapes* are also dynamically fluctuating low-energy degrees of freedom.[6] Moreover, the couplings between these geometric degrees of freedom and the (bosonized) charge fluctuations are nonlinear and involve many derivatives.[15] It is somewhat technical but nevertheless possible to show[16] that backscattering processes assisted by weak-shape fluctuations of the stripes leads only to further renormalizations of the Luttinger parameter. Where even weak-shape fluctuations can be qualitatively important is through their effect on the CDW fluctuations on neighboring pairs of edges. Since the fluctuating CDW order oscillates with an effective Fermi wavelength $\lambda_F = 2\pi/2k_F^{eff}$ along the *locally defined* stripe direction, the slightly different geometries defined by neighboring stripes means that the CDW fluctuations on those stripes are geometrically dephased when the arc lengths differ by an amount of order λ_F. Formally, these fluctuations induce an additional phase factor,

$$\mathcal{O}_j^\dagger(x)\mathcal{O}_{j+1}(x) \to \mathcal{O}_j^\dagger(x)\mathcal{O}_{j+1}(x)\exp\{i2k_F^{eff}\Delta_j L(x)\} \quad (6)$$

in the expression for $\mathcal{H}_{Coul}$, where $L_j(x)$ is the arc length to position x measured along the jth stripe. Here, we have defined $\Delta_j L(x) = L_j(x) - L_{j+1}(x)$. In Ref. 6, we showed that this sort of fluctuation renders the coupling between CDW fluctuations on neighboring stripes irrelevant, and this produces a first-order transition as a function of the magnitude of the shape fluctuations from an insulating stripe-crystal phase for small fluctuations to a conducting smectic phase for larger ones. Naturally, the smaller λ, or equivalently the closer ν is to $M + 1/2$, the more sensitive the CDW ordering is to small amplitude shape fluctuations. As a consequence, we generally expect the smectic phase to be more stable for ν near $M + 1/2$ and the stripe-crystal phase to be more stable away from this value.

So far, we have only considered the case in which the shape fluctuations are sufficiently small that they do not damage the basic stripe order of the mean-field ground state. When the shape fluctuations grow in magnitude to be comparable to the spacing between edges, backscattering interactions assisted by nonlinear retarded shape fluctuations induce operators that break up the stripes. In other words, these operators generate *dislocations* in the smectic stripe order. These operators are irrelevant at weak coupling. The strength of this coupling is a measure of the effects of quantum fluctuations on the stripe structure and it decreases with increasing stripe rigidity.[6] When these operators become relevant, the system undergoes a *quantum phase transition* from the *smectic* state to the *nematic* state in which dislocations proliferate. In this state rotational long-range order is still present but translation invariance is restored. Since this phase is far from the mean-field state from which we started, our knowledge of its properties is less certain. However, because in two spatial dimensions, the smectic to nematic phase transition is expected, from Landau theory, to be continuous, we can imagine that substantial local stripe order persists well into the nematic phase. As a consequence we expect this phase to be compressible, to posses a nonquantized Hall conductance, and an anisotropic longitudinal conductivity with $\sigma_{xx} > \sigma_{yy}$.

A schematic $T = 0$ phase diagram that summarizes the above considerations is shown in Fig. 1. Here, the x axis is the partial filling of the highest occupied spin-polarized Landau level and the y axis is a microscopic quantum parameter, related to the strength of the short-range piece of the Coulomb interaction, which determines the typical magnitude of shape fluctuations in units of the stripe width. (Roughly, since fluctuations about the mean-field state are thought to become less severe with increasing Landau index N we have identified this coordinate with $1/N$, with shape fluctuations of the stripes being increasingly important the larger $1/N$.)

The general structure of the phase diagram along its edges is completely determined by general principles and the above considerations. In the vicinity of the $\nu = M$ axis, the system can be thought of as consisting of dilute quasiparticles, which thus necessarily form a triangular lattice quasiparticle Wigner crystal.[17] This is separated by a line of first-order transitions from the various phases discussed in this paper. The instability of the smectic phase to stripe-crystal order in the absence of shape fluctuations means that along the x axis,

the system is always crystalline; a stripe crystal for ν near $M+1/2$ and a Wigner crystal for smaller ν. For $\nu=M+1/2$, the smectic phase is marginally stable, due to particle-hole symmetry, or equivalently, due to the fact that the edge states here are chiral Fermi liquids. However, as the quantum fluctuations become more severe, the smectic phase melts, first to form a stripe nematic and then an isotropic liquid phase. Presumably, for $\nu=M+1/2$, this isotropic state is the famous Hall metal. Finally, for large quantum parameter and variable ν, the system is dominated by the familiar liquid states, including various Hall metal and fractional quantum Hall liquid states; we group all these states into one region of the phase diagram labeled ''isotropic.'' Within these constraints, we have used artistic license to complete the schematic phase diagram in a consistent manner. The resulting phase diagram bears strong similarities with the phase diagram we constructed previously for doped antiferromagnets.[6]

Associated with the various broken continuous symmetries of the $T=0$ phases are a set of Goldstone modes whose character can be deduced from general principles. Detecting these modes may, ultimately, be the most direct way of unambiguously identifying the various broken symmetry phases experimentally.

The existence of Goldstone modes follows directly from the generalized elastic theory of the electron liquid-crystal phases. However, since the magnetic-field affects the dynamics of charge motion, the modes studied here have quite different character than those of the corresponding phases in zero magnetic field. Moreover, because of the high density of low-energy charge excitations associated with the edge states, dissipation may play a significant role in the dynamics of the Goldstone modes.

We expect the Goldstone modes of the crystalline phases to be fairly standard. In particular, the Wigner crystal phase has been extensively studied in the literature.[18] The stripe-crystal phase should be analogous except for differences due to the different point group symmetries of the two crystalline structures.

The liquid-crystal phases have not been studied so far. The transverse fluctuations of the CDW structure are the Goldstone modes. These modes per se do not couple directly to charge fluctuations. However, the locus of the nodes of the CDW modulate the local charge density profile and through it they determine the local structure of the edge states. These two sets of low-energy modes, local fluctuations of the stripes and charge fluctuations along each stripe, govern the low-energy physics of the smectic phase. In a separate publication we will present a theory of these modes.

III. FINITE TEMPERATURE EFFECTS

At finite temperature, continuous symmetries cannot be broken in two dimensions, so both translation symmetry (which is spontaneously broken at $T=0$ in the smectic and the two crystalline phases) and rotational symmetry (in all liquid crystalline and true crystalline phases) are clearly restored for $T>0$. However, quasi-long-range order is possible, even at finite temperature, so phases can still be distinguished according to what power-law order they possess.

The smectic phase is destroyed at finite temperature due to a proliferation of dislocations; three is the lower critical dimension for smectic order.[7] This is even true in the presence of long-range Coulomb interactions.[19]

A nematic phase, with power-law orientational order, survives to finite temperature. Indeed, we expect that everywhere in the $T=0$ phase diagram where smectic or nematic order exists, quasi-long-ranged orientational order will survive for small $T>0$. In addition, by analogy with the theory of hexatic phases in two dimensions,[8] there should be a finite temperature phase transition to a fully isotropic 2DEG mediated by unbinding of disclinations. Above this temperature, all tensor quantities, such as the conductivity, should be isotropic, while below it, anisotropies will develop which will grow with decreasing temperature.

Both crystalline phases will melt at finite T by one of the more or less standard routes for two-dimensional melting, that is either via a first order transition, or by a sequence of two transitions. In the latter case, there will be a low-temperature solid phase with power-law positional and orientational order, and a nonvanishing shear modulus. This solid phase melts via a dislocation unbinding transition to an intermediate (liquid crystalline) state with power-law orientational and short-range positional order. In the case of the melting of the stripe crystal, this ''biatic'' phase is not fundamentally distinct from the finite temperature nematic phase discussed above, although in practice, the melted crystal may still be fairly insulating, whereas the nematic is moderately conducting. In the case of the Wigner crystal, the intermediate phase is a ''hexatic,'' in which the power-law orientational order has a sixfold rotational symmetry, rather than the twofold symmetry of the nematic. As indicated above, these intermediate phases give way to a fully isotropic high-temperature phase via a continuous disclination unbinding transition.

In all critical phases, which is to say all the finite temperature phases described above, the effects of symmetry breaking fields are particularly dramatic. At low temperatures, and in the absence of such symmetry breaking fields, there is no true broken symmetry and no order parameter. In these phases the system is actually in a critical region terminating at a Kosterlitz-Thouless phase transition at a critical temperature T_c. Naturally, this phase transition is rounded by a symmetry-breaking field. However, below T_c, even a small symmetry-breaking field h produces a large response. This intuition is made precise in the sense that the exponent $\delta > 1$ where

$$m \sim |h|^{1/\delta}, \tag{7}$$

where m is the value of the order parameter. For instance, for the nematic phase, we can define $m = \sigma_{xx} - \sigma_{yy}/\sigma_{xx} + \sigma_{yy}$, and h then is a dimensionless measure of the underlying anisotropy of the substrate. Near the critical temperature, δ approaches the universal value 15, and δ diverges as $T \to 0$. As a function of temperature, this exponent can be computed exactly in terms of the anomalous dimension η of the XY model (or, with the same result, from a self-consistent phonon approximation)

$$\delta = \frac{4}{\eta} - 1 = \frac{2\pi\kappa(T)}{T} - 1, \tag{8}$$

where η approaches 1/4 as $T \to T_c$. Here, $\kappa(T)$ is the long-wavelength helicity modulus, which approaches a constant as $T \to 0$, and the universal value $\kappa(T_c) = 4T_c/\pi$ as $T \to T_c$. (There is a factor of $4 = 2^2$ difference here than in the usual XY model since the vortices in a nematic have half the usual topological charge.) As $T \to T_c$, δ reaches the universal value $\delta = 15$. The large values of δ imply that very small microscopic anisotropies have an enormous orienting effect.

IV. THE CONDUCTIVITY TENSOR

In this section, we discuss the expected behavior of the conductivity tensor in the various parts of the phase diagram in Fig. 1. It follows directly from the Kubo formula, so long as there are no singularities (such as are found in a superconductor) associated with the $\vec{k}$ and $\omega \to 0$ limit, that $\sigma_{xy} = \sigma_{yx}$ (and, consequently, that $\rho_{xy} = \rho_{yx}$). This statement is true independent of whether or not the system is rotationally symmetric. In general, in a state with a fourfold rotational symmetry (which, of course, includes all isotropic liquid states) $\sigma_{xx} = \sigma_{yy}$. Conversely, in any state which is not fourfold rotationally symmetric and has a finite Hall conductance, there is no *a priori* reason to expect this equality to hold, and therefore there is every reason to expect it not to hold. Thus, in all the crystalline and liquid crystalline states discussed here, we expect $\sigma_{xx} \neq \sigma_{yy}$, although for the two crystalline states, since they are insulating, we expect that both diagonal components of the conductivity tensor will be small at low temperatures.

A. The smectic phase

At $T = 0$ and in the absence of impurities, the smectic phase is boost invariant in the stripe direction. As a consequence, under conditions in which the electric field is perpendicular to the stripe direction, it is possible to go to a co-moving frame in which the electric field vanishes. It therefore follows that the Hall conductance tracks the change in the filling fraction, $\sigma_{xy} = \nu e^2/h$, and that $\sigma_{yy} = 0$. On the other hand, because there are a finite density of conducting channels, and because of the irrelevance of all back-scattering interactions in the smectic phase, the longitudinal conductivity in the stripe direction, σ_{xx}, diverges in the limit $T \to 0$. Impurities will, of course, alter these conclusions, but for weak disorder and low but nonzero temperature, one would still expect $\sigma_{xy} \sim e^2 \nu/h$ and $\sigma_{xx} \gg e^2/h \gg \sigma_{yy}$.

From a microscopic viewpoint, if an external electric field *perpendicular* to the stripes is applied, every stripe with filling fraction ν_M (ν_{M+1}) has an induced Hall current *parallel* to the direction of the stripe and the Hall conductance of that stripe is $\sigma_{xy}(M)$ [$\sigma_{xy}(M+1)$]. Notice that in this configuration the current of the edge states separating each pair of nearby stripes is part of the Hall current. The Hall current changes continuously and no longitudinal current is induced in this configuration. However, if the external electric field is applied *parallel* to the stripes, the induced Hall current in each stripe is now *perpendicular* to the stripes. (Recall that nearest neighboring stripes have different Hall conductance.) As the boundary between two stripes is approached the current switches from being perpendicular to the stripe to being parallel to the stripe and it is carried by the corresponding edge state. Thus, current is conserved but, in addition to the "bulk" Hall current (which is the same as in the other configuration) there is now a current *parallel* to the external electric field and it is carried entirely by the edge states. At least at the mean-field level, $\sigma_{xx} \sim Ce^2/h$ where C is the number of edges that make it across the system (and so diverges with the size of the system) while $\sigma_{yy} = 0$. This result is correct for a clean system and it is robust against quantum fluctuations provided that they do not destabilize the smectic phase.

B. The nematic phase

Because the nematic phase is featureless, and so boost invariant, it follows that at $T = 0$, $\sigma_{xx} = \sigma_{yy} = 0$ and $\sigma_{xy} = e^2 \nu/h$. For $T > 0$, since the nematic is a critical phase, the zero-temperature result is likely to be strongly modified. On general dimensional grounds, it is reasonable to expect $\sigma_{xx} \sim e^2/h \gg \sigma_{yy}$, and that σ_{xx} is greatest at $\nu = M + 1/2$, where the Luttinger exponent K associated with the edge states is largest, and drops symmetrically (due to particle-hole symmetry) as ν is varied from this value. There is no reason to expect σ_{xy} to be strongly temperature dependent.

V. EFFECTS OF QUENCHED DISORDER

Disorder likely eliminates most of the sharp distinctions between phases, and hence, turns most of the phase transitions discussed above into crossovers. However, if the disorder is sufficiently weak, then the crossovers can be sharply defined, and important local distinctions between the various "phases" should be experimentally detectable. Certainly, neither broken translational nor rotational symmetry survive disorder.

The effects of disorder on the conductivity tensor in the $T \to 0$ limit are likely to be severe and nonperturbative; even weak disorder can cause localization. However, at nonvanishing temperatures, we can expect that in the low-disorder limit, the conductivity tensor will resemble that of the ideal system. Interesting nonlinear effects, involving pinning of the various forms of CDW order, can be expected in the presence of disorder. In the stripe-crystal and Wigner crystal cases, these effects are well studied previously, but for the smectic they may have some novel features. Because the Luttinger exponent $K < 1$, disorder is a relevant perturbation to the one-dimensional edge-state problem in the absence of stripe shape fluctuations. Thus, disorder is likely to produce dramatic decreases in the diagonal matrix elements of the conductivity tensor at sufficiently low temperatures in both electronic liquid crystalline phases. In general, the effects of weak disorder on electronic liquid crystals is an area for future study.

VI. RELATION TO EXPERIMENTS

The study undertaken in the present paper was originally motivated by some very recent and remarkable experiments done by Lilly, Cooper, Eisenstein, Pfeiffer, and West[9] in which large, temperature-dependent anisotropies were discovered in the 2DEG under conditions in which two (or more) Landau levels are full. The experiments were done in ultrahigh mobility GaAs/Al$_x$Ga$_{1-x}$As heterojunctions. That

the samples have very weak disorder is indicated, for instance, by the observation of the quite fragile quantum Hall plateau at $\nu = \frac{5}{2}$ and by the large number of fractional quantum Hall states seen in the lowest Landau level. The salient features of the experiments of Ref. 9 are as follows: For a partially filled third Landau level with ν in the neighborhood of $\nu = \frac{9}{2}$, there is a characteristic temperature $T_0 \approx 150$ mK above which the resistivity tensor is nearly isotropic, and below which there is a rapid crossover to a highly anisotropic compressible state. The resistivity tensor in this state exhibits a nonquantized Hall resistance and an anisotropic longitudinal response, such that ρ_{xx} grows very rapidly with decreasing temperature until it reaches a value of order 1000Ω at the lowest temperatures, $T \approx 25$ mK, while ρ_{yy}, measured by rotating the current by $90°$, becomes very small. Since in the absence of disorder, on theoretical grounds $\rho_{xy} = \rho_{yx} = (h/e^2)\nu^{-1}$, we expect that this relation should hold approximately in this system. Inverting this tensor we find, for the conductivity tensor at low temperatures, $\sigma_{yy} \sim e^2/h$, $\sigma_{xy} = \sigma_{yx} \sim \nu e^2/h$, and σ_{xx} small. The low temperature value of ρ_{xx} as a function of ν exhibits a broad peak centered at $\nu = 9/2$, with a width in ν, which is substantial and approximately temperature independent. Structure is also seen in the "wings" of the Landau level even at 150 mK. Specifically, two very well-defined pairs of quantum Hall plateaus are seen, one pair with $\sigma_{xy} = 4e^2/h$ for ν near 4, and another one with $\sigma_{xy} = 5e^2/h$ for ν near 5. However, the resistivity peak between the two plateaus (with the same quantized Hall conductance) becomes smaller with decreasing temperature, in contrast with the usual critical peaks seen in transitions between plateaus.

The same structure is repeated for ν in the vicinity of 11/2, 13/2, 15/2, and beyond, although it apparently becomes more difficult to resolve beyond $\nu = 15/2$. In particular, the peak value of ρ_{xx} decreases with increasing ν, roughly in such a way that σ_{yy} at $\nu = (2M+1)/2$ remains in the vicinity of e^2/h. It is important to stress that, even at low temperatures, no substantial anisotropy is apparent at the lowest temperatures at smaller values of ν, in particular near $\nu = 7/2$ and 5/2, nor at any magnetic field at temperatures in excess of 100 mK.

The existence of this anisotropy observed in highly pure samples clearly indicates that this effect is driven by electron-electron interactions and that disorder plays a secondary role. This is doubly remarkable since the natural expectation was that precisely in the middle of the plateau there should be a phase transition from a $\nu = 4$ to a $\nu = 5$ quantum Hall liquid. However at the transition between plateaus, which is a quantum phase transition driven by disorder, although one expects a peak in ρ_{xx}, the peak should narrow as $T \to 0$ following a universal scaling law. The results of these experiments suggest that, although the system is indeed compressible, there is no narrowing of the peak as $T \to 0$. Thus, the system is critical for a range of filling factors. Furthermore the transition between plateaus should be essentially isotropic.

The picture presented in this paper gives a natural interpretation[20] of these effects. It is natural to identify the experimentally observed anisotropic state with the finite temperature nematic state discussed above, which is a critical state, and the crossover observed at $T \approx 100$ mK with the
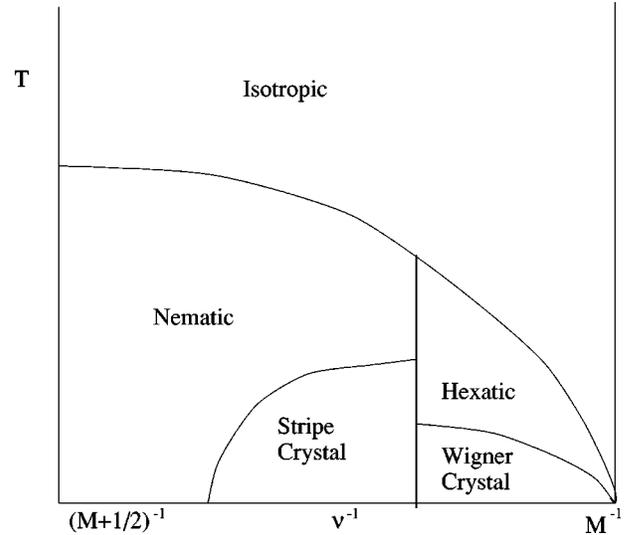

FIG. 3. Schematic finite temperature phase diagram as a function of inverse filling factor along line c of Fig. 1.

disclination unbinding transition, perhaps somewhat rounded due to quenched disorder. A small anisotropy in the heterojunction device is the symmetry-breaking field that picks a preferred orientation for the nematic, and insures that there is actual long-ranged orientational order, as described in Eq. (7). The two quantized Hall states observed in the wings are naturally identified with the stripe-crystal and quasiparticle Wigner crystal phases[21] that appear as $\nu \to M$ in the phase diagrams in Figs. 1 and 3. It would be interesting to test this hypothesis by looking for evidence of a finite temperature melting transition, or characteristic nonlinear I-V's in these ranges of B.

This still does not address the question of whether the ground-state phase near $\nu = 9/2$ is a smectic or nematic. However, since the longitudinal resistivity $\rho_{xx} \sim 1000\Omega$ corresponds to a longitudinal conductivity $\sigma_{yy} \sim e^2/h$, it seems likely that the ground state is either a quantum nematic phase or that the stripe order of the underlying smectic is strongly disrupted due to pinning by impurities. If disorder does not play a dominant role, the characteristic temperature dependence of the nematic order implied by Eq. (8) will govern the resistivity ratio, ρ_{xx}/ρ_{yy}.

It is also worth noting that in the experiments of Willett et al.[22] in which an external modulation was imposed on a 2DEG in the compressible state at $\nu = \frac{1}{2}$, similar phenomena were observed as in the experiments of Lilly et al. On the one hand, this gives us greater confidence in concluding that the observed anisotropies are a consequence of stripe formation. On the other hand, it supports the intuitive notion[23] that the isotropic compressible state at $\nu = 1/2$ still has substantial local stripe correlations, which are simply disordered by quantum fluctuations at long distances—this would rationalize the large susceptibility of this state to the formation of stripes. Similarly, it may be that anomalously broad regions of compressible smectic or nematic phases may be stabilized by an externally potential at the "edge" of a quantum Hall device, producing a form of macroscopic edge reconstruction.[24]

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[24] We thank B. I. Spivak for this observation.