# **Surface-acoustic-wave absorption by quantum-dot arrays**

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We have investigated the absorption of surface acoustic waves  $(SAW's)$  by arrays of quantum dots as a function of electron concentration, temperature, and magnetic field. With no illumination, the measured transmitted SAW amplitude was largely independent of magnetic field, in all sizes of dots, suggesting that sidewall depletion had removed most of the free carriers. After additional carriers were introduced via an infrared light-emitting diode a broad amplitude minimum developed near  $B=0$  in all cases, while the amplitude at high fields  $~(-10 T)$  approaches that of the unilluminated sample. The high-field  $~(\geq 0.1 T)$  behavior is reasonably well described using a Fermi's golden rule model of resonant absorption when the magnetic-field dependence of the signal is proportional to the density of single-electron level crossings. Surprisingly, our measurements of transmitted SAW amplitude versus magnetic field were found to be strongly hysteretic upon reversal of the magnetic-field sweep direction. Experiments in tilted magnetic fields indicated that the *two-dimensional* quantum confinement of the dot is a key parameter and we propose that there are two components to the magneticfield dependence of the SAW amplitude, a genuine field dependence of the attenuation for a dot with fixedelectron concentration, and a change due to a field-induced reduction in the electron concentration. The linear temperature dependence of the SAW attenuation coefficient at low temperature and zero magnetic field, appears to be consistent with recent theoretical predictions by Knäbchen *et al.* [Europhys. Lett. **39**, 419 (1997)] that, in our measurement regime, absorption will be dominated by Debye relaxation. When we analyze our results in this light we find the measured absorption to be three orders of magnitude larger than the theoretical calculation. [S0163-1829(99)07711-5]

### **INTRODUCTION**

The ability to fabricate semiconductor structures in which electrons are fully quantum confined in all three dimensions holds out the promise of dramatic improvements in device performance.<sup>1</sup> The discrete electronic level spectrum of such quantum dots leads, for example, to a strong suppression of carrier scattering as well as very sharp electron-hole recombination lines. With the recent demonstration of selfassembled quantum dots during epitaxial growth, the realization of such devices has come an important step closer, and a detailed understanding of their electronic structure is now a key priority. A number of optical techniques such as photoluminescence<sup>2</sup> and Raman spectroscopy<sup>3</sup> have been used to probe the electronic structure of quantum dots, yet such studies are relatively invasive due to the large photon excitation energies used. Recent transport studies<sup>4</sup> have succeeded in measuring both the phase and amplitude of the transmission coefficient for tunnelling through a quantum dot, yet the need for leads to electronic ''reservoirs'' means that such structures are not truly isolated. In contrast, the use of surface acoustic wave  $(SAW)$  attenuation is an excellent contactless technique to probe quantum nanostructures in piezoelectric materials and is largely noninvasive due to the very small SAW energy quantum ( $\hbar \omega$  ~ 0.3  $\mu$ eV, where  $\omega$  is the SAW frequency). Moreover, the long SAW wavelengths  $(\sim 40 \mu m$  at 70 MHz) rule out possible diffraction effects which could arise in samples composed of large periodic arrays of quantum dots. SAW's have extensively been used to study the magnetoconductivity of two-dimensional electron gases  $(2DEG's)$ ,<sup>5,6</sup> most recently in the context of the fractional quantum Hall effect and composite Fermions.<sup>7,8</sup> We have also previously studied anisotropic SAW attenuation in arrays of quantum wires and highlighted the role played by momentum conservation rules. $9$  Since such rules are completely relaxed in quantum dots and screening of the piezoelectric coupling will be rather weak, one would anticipate quite strong SAW absorption in large array samples. We describe here a range of studies on arrays of etched quantum dots, of various dimensions, as a function of magnetic field, temperature, and carrier concentration. Data interpretation is complicated by the unexpected observation of finite electron recombination effects following the use of illumination to increase the dot electron densities at low temperatures. Nevertheless, we are able to draw some important conclusions about the SAW attenuation process as well as the mechanism of electron recombination.

#### **THEORY**

A surface acoustic wave propagating in a piezoelectric medium like GaAs carries a longitudinal electric field that

couples into any electronic systems in its path. The piezoelectric interaction between SAW's and two-dimensional electrons is usually described in terms of a classical relaxation model, where the SAW attenuation is a nonmonotonic function of the diagonal component of the conductivity tensor  $(\sigma_{xx})$  and has a maximum near a characteristic conductivity  $\sigma_M$  ( $\approx$  4 × 10<sup>-7</sup>  $\Omega$ <sup>-1</sup>).<sup>6</sup> In the case of small quantum dots  $(d<1 \mu m)$ , however, one is dealing with a set of discrete electronic levels and the problem demands a full quantum-mechanical treatment. Moreover, when the mean energy-level spacing  $(\Delta)$  is very much larger than the SAW energy quantum  $(\hbar \omega)$  this discreteness is of key importance and SAW absorption is critically dependent on electron states lying very close to the Fermi energy. Since the dots in a sample will all be slightly different due to variations in the electrostatic confinement potential and/or impurity distribution a statistical approach should be used to calculate absorption due to the whole array. The statistical properties of energy levels in quantum systems that exhibit chaotic classical dynamics have been extensively studied in recent years and it is now well known that their distribution can be described by random matrix theory<sup>10</sup> (RMT). Their properties can be expressed in terms of a level correlation function  $R(\varepsilon)$  (the probability that two energy states are separated by an energy  $\varepsilon$ ) which depends only on the global symmetries of the system

$$
R(\varepsilon) = \frac{1}{\Delta} \begin{cases} c_{\beta} |\varepsilon/\Delta|^{\beta}, & \text{for } |\varepsilon| \leq \Delta \\ 1 & \text{for } |\varepsilon| \geq \Delta, \end{cases}
$$
 (1)

where  $c_{\beta} \approx 1$  is a constant and  $\beta = 1$  and 2 for the orthogonal  $(B=0)$  and unitary  $(B \neq 0$ , broken time-reversal symmetry) ensembles, respectively. Note that the probability of finding two degenerate levels is zero, which is a statement of the level repulsion characteristic of classically chaotic systems.

Recently, the problem of SAW absorption by an array of small quantum dots has been addressed directly in a paper by Knäbchen *et al.*<sup>11</sup> using the level correlation function as a starting point. These authors treat the limit of small dots (*d*  $\leq$ 1  $\mu$ m) at low temperatures such that the level broadening due to phase breaking  $(\hbar / \tau_{\phi})$  is very much less than the mean-level spacing  $(\Delta)$ , while the SAW energy quantum  $(\hbar \omega)$  is much smaller still. Three main absorption mechanisms are considered.

The simplest of these is resonant absorption when a SAW excites a transition between two closely spaced electronic levels. The attenuation coefficient per unit length  $(\Gamma)$  is therefore proportional both to  $R(\hbar \omega)$  and the screened piezoelectric potential arising from the SAW and is given by

$$
\Gamma_r = A \frac{\omega}{s} \tanh\left(\frac{\hbar \omega}{2kT}\right) \hbar \omega R(\hbar \omega),\tag{2}
$$

where *s* is the sound velocity and  $A=(N/g)(a_B\gamma_q/\hbar s)^2$ . Here, *N* is the areal density of dots, *g* is the dimensionless conductance,  $a_B$  the Bohr radius, and  $\gamma_q$  is the piezoelectric interaction vertex.

The second possible mechanism is the excitation of electrons between the overlapping tails of two broadened levels. In this case, the attenuation depends on the phase-breaking

rate  $[\tau_{\phi}^{-1}(T)]$ , but not the level correlation function (and is therefore independent of applied magnetic field) and is given by

$$
\Gamma_t = A \frac{\omega}{s} \left( \frac{\hbar \omega}{\Delta} \right)^3 \frac{1}{\omega \tau_{\phi}(kT)}.
$$
 (3)

Finally, absorption can occur via Debye relaxation processes when the periodic motion of the energy levels in response to the SAW electrostatic potential leads to a nonequilibrium occupation. In this case, energy dissipation is due to inelastic relaxation mechanisms, which try to restore instantaneous equilibrium occupancy of the levels. This attenuation is a function of both the energy relaxation rate  $\left[ \tau_{\varepsilon}^{-1}(T) \right]$ and the level correlation function and is given by

$$
\Gamma_D = A \frac{\omega}{s} \frac{\omega \tau_\varepsilon(kT)}{1 + \omega^2 \tau_\varepsilon^2(kT)} \hbar \omega R(kT). \tag{4}
$$

In our experiments the following inequality is expected to hold  $\Delta \sim kT \gg \hbar \tau_{\phi}^{-1} \geq \hbar \tau_{\epsilon}^{-1} \geq \hbar \omega$ , and we calculate that attenuation due to Debye relaxation is many orders of magnitude larger than the other two mechanisms. As  $\Gamma_D$  is a function of the level correlation function it should be strongly sensitive to the global symmetries of the system. At very low temperatures, therefore, the application of a small magnetic field sufficient to break time-reversal symmetry  $(\sim 0.1 \text{ T})$ should lead to a dramatic reduction in attenuation, and hence, an increase in the measured SAW intensity  $[I(B)]$  $=I_0e^{-\Gamma(B)L}$ , where *L* is the length of the interaction region]. Since  $\Delta \sim kT$  in our experiments, however, the most interesting limiting case of Eq.  $(1)$  does not apply and we would expect this effect to be almost entirely suppressed. Assuming, therefore, that  $R(kT)$  is approximately constant the temperature dependence of  $\Gamma_D$  arises principally from that of  $\tau_{\epsilon}(T)$  and provides a unique opportunity to investigate this quantity.

The results of Knäbchen *et al.* assume that the distribution of energy levels is not perturbed by an applied magnetic field and are only valid at very low fields  $(\leq 0.1$  T). As a first estimate of the attenuation at higher fields we have calculated the resonant absorption rate for a system of noninteracting electrons with parabolic electrostatic confinement for which the following analytic expression for the level spectrum exists $^{12}$ 

$$
E^{l,m}(B) = (2m + |l| + 1)\hbar \sqrt{\omega_0^2 + (\omega_c/2)^2} + l\hbar \omega_c/2, \quad (5)
$$

where *l, m* are azimuthal and radial quantum numbers that index the levels,  $\omega_c$  is the cyclotron frequency, and  $\hbar \omega_0$  is the characteristic confinement energy. Using Fermi's golden rule it is straight forward to write down the scattering rate for resonant SAW mediated transitions. Moreover, since the SAW energy quantum is very much smaller than all other energy scales in the problem it is sufficient to only consider quasielastic processes at the Fermi energy. In this case, we find the scattering rate to be given by



FIG. 1. The density of single-electron level crossings (negated) per unit magnetic-field interval for a cylindrical dot with parabolic electrostatic confinement ( $\hbar \omega_0 = 1$  meV) containing 210 electrons.

$$
\tau_r^{-1} \propto \left[ \frac{q^2}{q^2 + q_0^2(B)} \right]^2 \sum_{\substack{m,m',l,l' \\ m \neq m',l \neq l'}} |\rangle \psi_{m'l'}| e^{iqx} |\psi_{ml} \langle |^2
$$
  
 
$$
\times \delta(E_F - E_{ml}, E_F - E_{m'l'}), \qquad (6)
$$

where *q* is the SAW wave vector, and  $q_0(B)$  the magneticfield-dependent inverse Thomas-Fermi screening length. The energy delta function in Eq.  $(6)$  restricts absorption to magnetic fields where two or more energy levels are degenerate. Hence, neglecting the magnetic-field dependence of screening, a measure of the absorption is given by the density of single-electron level crossings per unit magnetic-field interval. We have calculated this quantity as a function of magnetic field for a cylindrical dot with parabolic electrostatic confinement ( $\hbar \omega_0 = 1$  meV) containing 210 electrons, and this is plotted in Fig. 1. The calculation reveals a strongly peaked (and highly structured) density of level crossings at about 1 T, which falls off rapidly to zero with increasing field. There is also a narrow peak at zero field reflecting the multiple degeneracies of our highly symmetric model dot. Surprisingly, the Landau level-like ''fan'' structure seen in two-dimensional systems is still clearly reflected in this plot for  $B > 1$  T. Finally, we note that at fields where the magnetic length is much smaller than the dot diameter  $(\geq 0.5 \text{ T})$ one would expect Eq.  $(5)$  to be invalid due to the formation of compressible and incompressible edge channels at the perimeter of the dot. $^{13}$  Understanding this regime will require both a reliable self-consistent model of the electronic structure of the dot as well as insight into how SAW's interact with edge stripes, neither of which currently exist.

## **EXPERIMENTAL METHOD**

Measurements were performed on arrays of quantum dots fabricated by holographic lithography and reactive ion etching in two different  $AI_xGa_{1-x}As/GaAs$  molecular-beam epitaxial heterostructures. The two wafers were nominally identical and had unstructured mobilities of  $\sim$  550 000  $\text{cm}^2\text{V}^{-1}\text{ s}^{-1}$  and carrier concentrations of  $\sim$  2  $\times$  10<sup>11</sup> cm<sup>-2</sup> at 4.2 K in the dark. By varying the interference angle the lithographic process can be used to produce rather homogeneous arrays of almost square nanostructures of varying sizes. Dots were patterned with geometric widths of approximately 250, 375, and 500 nm (separated by approximately equally sized trenches) and an etch depth less than the distance of the 2DEG below the surface  $({\sim}80 \text{ nm}$  to minimize edge depletion). A final mesa etch left a  $3 \text{ mm} \times 3 \text{ mm}$  square of nanostructures containing typically 30–150 million dots in the center of each sample. Four interdigital transducers were realized at each edge of the chip by thermal evaporation of Cr/Au or Al. The center-to-center spacing of the transducer fingers was 20  $\mu$ m yielding a broad fundamental SAW resonance of about  $70\pm4$  MHz.

Measurements were achieved by exciting 0.4  $\mu$ s long SAW pulses with a repetition frequency of 250 kHz at the transmitter and receiving the time-delayed signal at the opposing transducer. The SAWs propagate either in the  $[110]$ or [10] directions, which are known to have the strongest piezoelectric coupling coefficient and carry a longitudinal electric field. The SAW power at the sample  $({\sim}0.1 \mu W)$  was reduced to well below the level where there was any evidence of heating of the electrons. Our detection system is a phase sensitive detector based on an eight-tap digital delay line and can measure changes in both the amplitude and phase (velocity shift) to better than one part in  $10^4$  and to one part in  $10<sup>5</sup>$ , respectively. Measurements were performed at 1.5 K either in liquid He or under a low pressure of He exchange gas (no qualitative differences were observed) in a 10-Tesla helium cryostat. Unless otherwise stated, the magnetic field was applied perpendicular to the plane of the samples.

### **RESULTS**

Figure 2 presents typical 70 MHz SAW transmission measurements on arrays of 500, 375, and 250 nm dots as the applied magnetic field is swept from 0 to 10 T at  $\sim 0.15$ T/min. Persistent photoconductivity at our 1.5 K measurement temperature was used to increase the electron concentration in the dots  $[100\text{-ms long } 50 \text{ mA pulses from an in-}$ frared light-emitting diode (LED), corresponding to  $\sim 0.5$  $mW/cm<sup>2</sup>$  incident at the sample surface] yielding the families of traces shown in each panel. The transmitted SAW signal was monitored for long periods of time (some tens of minutes) after both the initial cool down and illumination events to establish that any time transients had died away (see inset of Fig. 4 at  $B=0$ ). In all three cases, a large, fairly flat, transmitted amplitude was recorded without illumination suggesting that sidewall depletion has removed most of the free carriers from our dots. After additional carriers are introduced using the LED a broad amplitude minimum develops near  $B=0$  in all three sizes of sample, while the amplitude at high fields approaches that of the unilluminated trace. In all samples the amplitude was observed to be a nonmonotonic function of illumination, with the maximum observed attenuation of the unilluminated amplitude, calculated at *B*  $=0$ , corresponding to attenuation coefficients of 70, 28, and 40  $\mathrm{m}^{-1}$  for the 500, 375, and 250 nm dots, respectively. Surprisingly, for our experimental parameters the maximum theoretical attenuation is predicted to be three orders of magnitude smaller than this, and additionally, our observed attenuation is of the same order as, or larger than, the maximum predicted attenuation  $(30 \text{ m}^{-1})$  for an unstructured



FIG. 2. Measured transmitted SAW amplitude as a function of magnetic field at 1.5 K for arrays of  $(a)$  500 nm quantum dots,  $(b)$  $375$  nm quantum dots and (c)  $250$  nm dots. Each plot is labeled with the illumination time in seconds.

2DEG.<sup>6</sup> In all three cases the maximum observed change in amplitude as a function of applied field represents 3–4 % of the unilluminated amplitude. The behavior at very low and reversed fields was examined carefully to see whether indications of the abrupt change in attenuation predicted by Eq.  $(4)$  could be observed. In all cases the amplitude was rather flat through  $B=0$  as one should probably expect at our relatively high measurement temperature as discussed earlier. At saturation illumination levels (2 min continuous operation of the LED at 50 mA) the traces for the 500 and 375 nm dots lose the low-field minimum and are both very similar in amplitude and field dependence to the unilluminated ones. In contrast the smallest 250 nm dots reach a maximum attenuation state at saturation with a strong magnetic-field dependence. We tentatively attribute these differences to the formation of a highly disordered parallel electronic system in the *n*-Al<sub>x</sub>Ga<sub>1-x</sub>As layer in the larger dots at high electron concentrations, and this point will be discussed in more detail later.

Surprisingly, the measurements of Fig. 2 are strongly hysteretic upon reversal of the magnetic-field sweep direction. A typical set of data is shown in Fig. 3, for the same 250 nm dots, where the magnetic field has been swept up to 10 T and then immediately back down to the origin at  $\sim 0.3$  T/min. Clearly, although the up and down traces are similar, the final amplitude is considerably higher than the initial one, an effect that cannot be traced to the stability of the electronics or hysteresis in the superconducting solenoid. It appears, moreover, that this hysteresis is related to time transients in



FIG. 3. Measured transmitted SAW amplitude for an array of 250 nm dots after saturation levels of illumination during a complete magnetic field cycle at 1.5 K. The ''down'' sweep almost immediately followed the ''up'' sweep.

the signal that are initiated by high applied magnetic fields. Figure 4 summarizes SAW amplitude data as a function of time for a typical sample  $(500 \text{ nm}$  dots after 8.5 s illumination) undergoing a complex field excursion. Note that a different 500-nm dot sample, fabricated from a different wafer and having Cr/Au rather than Al SAW transducers, was used in this experiment. However, the qualitative behavior of this sample and of the one used to obtain the traces in Fig.  $2(a)$ was identical. Along the sections of the graph indicated by arrows the field has been swept up at  $\sim 0.15$  T/min between the two indicated values, while it has been held fixed on segments in between to reveal any time transients. Even though the signal is extremely stable at  $B=0$ , the application of a magnetic field in excess of  $\sim$ 1.5 T leads to the onset of pronounced time transients in the signal. The amplitude always drifts upwards with time and the original  $B=0$  SAW amplitude can be restored by further illumination. Consequently, we speculate that the transients to higher amplitudes



FIG. 4. Measured transmitted SAW amplitude as a function of time for an array of 500 nm dots at 1.5 K after a total of 8.5 s illumination. The applied magnetic field was initially swept from zero to 2 T, where it was held constant for ten min. This procedure was repeated until the maximum applied field was reached. The inset shows a plot of SAW amplitude as a function of time for the same sample during illumination  $(30 \times 100 \text{ ms} \text{ pulses})$  immediately before the measurement began.



FIG. 5. Measured transmitted SAW amplitude at 1.5 K as a function of magnetic field for an array of 250 nm dots after saturation levels of illumination. Each trace corresponds to a different tilt angle between the applied magnetic field and the normal to the sample surface as shown.

reflect gradual reductions in the electron concentration of the dot.

In order to establish the extent to which the SAW signal depends on the orientation of the magnetic field with respect to the plane of the dots a series of experiments in tilted fields was performed. Difficulties associated with magnetic hysteresis were overcome by using the 250 nm dots, after saturation illumination, since this always allowed us to return to *exactly* the same sample state after each measurement. This would have not been possible with the larger dots since they show very little field dependence at saturation illumination. Figure 5 shows measurements of the SAW amplitude as a function of the normal magnetic-field component  $(B_{\perp})$  at three different tilt angles where successive traces have been offset by  $\sim$  -0.2 units on the *y* axis for clarity (in all cases the applied magnetic field was swept upwards). The similarity of the sections of the different traces indicates that the *two-dimensional* quantum confinement of the dot is a key parameter in the problem. Small differences are to be expected due to the different in-plane field components and the slightly different effective sweep rate  $(dB_{\perp}/dt)$  for each measurement. We propose, therefore, that there are two components to the field dependence of the SAW amplitude; a genuine change in the attenuation for a dot with fixed electron concentration and a change due to a field-induced reduction in the electron concentration. This will be discussed in more detail later.

Finally in Fig. 6 we present a measurement of the zero field-temperature dependence of the SAW amplitude for 500 nm dots in both the unilluminated state and after a total of 5.5 and 14.5 s illumination. Before illumination, when we expect the dots to be largely depleted of electrons, we see only very weak temperature dependence of the signal indicating that all nonelectronic sources of attenuation  $(e.g., elas-)$ tic scattering off etched steps) remain rather constant. However, in the illuminated sample we expect the temperature dependence of the SAW attenuation coefficient  $\{\Gamma(T) \propto$  $-\ln[Amplitude(T)]$  to arise from coupling to the electronic system. The approximately linear behavior below 2.5 K appears to be consistent with absorption via Debye relaxation in the regime  $kT \sim \Delta$  and  $\omega \tau$ <sup>2</sup>/<sub>2</sub> and will be discussed in more detail in the next section.



FIG. 6. Temperature dependence of the attenuation coefficient  $\{\Gamma(T) \propto -\ln[A(T)]\}$  of a SAW incident on an array of 500 nm dots after the indicated illumination times.

### **DISCUSSION**

Magnetic-field hysteresis and field-induced time transients make the detailed analysis of these data extremely difficult. However, the various complementary measurements allow us to make some clear statements about SAW attenuation by quantum dot arrays, as well as speculations about the origin of the observed irreversibilities.

Given that the field-induced transients are always to *higher* amplitudes, the fact that the basic shapes of the up and down sweeps of Fig. 3 are similar is very significant. It appears, therefore, that a broad zero-field amplitude minimum that rises smoothly towards the unilluminated level at high field is rather characteristic of SAW attenuation by electrons in arrays of quantum dots of the typical size studied here. Assuming that the preillumination level represents the unattenuated wave (we assume that dots are initially largely depleted of electrons) we conclude that the SAW only interacts weakly with the electron system at high fields. We note that such a behavior is quite consistent with the resonant absorption picture for our model cylindrical dot (Fig. 1) if one assumes that the  $B=0$  degeneracies are broken (our dots are closer to squares than circles) and the Landau-fan-like features are smeared out due to inhomogeneities. The amplitude minima, however, are consistently confined to lower fields than the model prediction and for intermediate illumination levels there is still significant attenuation at high fields suggesting that the agreement is at best qualitative rather than quantitative. This is not surprising as our analytic model is not anticipated to be valid for fields where the magnetic length is much smaller than the dot diameter  $(\geq 0.5 \text{ T})$ , when screening cannot be ignored, and the formation of compressible and incompressible edge channels at the perimeter of the dot becomes important. The low attenuation at the highest applied fields  $(\sim 10 \text{ T})$  is probably expected since the magnetic confinement will now dominate the electrostatics of the nanostructures, and the dot approximates to a very small quasi-two-dimensional system with effective filling factor less than 1. At such a low filling factor there will be no tendency for compressible and incompressible stripes to form and we expect instead to find classical edge states (skipping orbits) at the perimeter of the dots. Provided the interaction between the SAW and these skipping orbits is weak then we expect the overall interaction with the electron system to be weak as it is in a two-dimensional electron gas for noninteger filling factors.

If we accept that we can cautiously assign some significance to the maximum amplitude changes for the three dot sizes in Fig. 2, the fact that they are *all* about 3–4 % of the unilluminated amplitude tells us something about how the absorption per dot must scale. The total number of dots in the arrays varies as  $\sim 1/d^2$  (where *d* is the dot diameter) implying that the attenuation per dot must have approximately the inverse dependence. For a dot with parabolic electrostatic confinement the  $B=0$  spacing of degenerate levels varies approximately as  $\Delta \sim 1/d$  for a fixed Fermi energy. If we assume that the high-field attenuation is close to zero then the maximum amplitude change is characterized by the  $B=0$  value, which must vary roughly as the mean energy level spacing squared ( $\sim 1/\Delta^2$ ). These arguments are not rigorous, however, and it is not surprising that this dependence does not agree with any of the predictions of the RMT model [Eqs.  $(2)$ – $(4)$ ]. In practice the degeneracy of the levels of a cylindrical dot increases rapidly with energy and must be included more carefully. Since less levels are occupied in smaller dots and the Fermi energy will become smaller due to sidewall depletion effects, the mean level spacing will grow faster than 1/*d* as the dot size shrinks. Consequently, a  $1/\Delta$  dependence of the SAW attenuation by a single dot is quite plausible in agreement with the expected result for Debye relaxation  $[Eq. (4)]$ . Assuming that in our measurements  $kT \sim \Delta$ , so that *R*(*kT*) is approximately constant, the temperature dependence of the Debye absorption arises from the temperature dependence of  $\tau_{\varepsilon}$  [we assume  $\tau_{\varepsilon}(T) \propto 1/T$ ]. There are then two limiting cases. When  $\omega \tau_{\varepsilon} \geq 1$ ,  $\Gamma_D$  $\propto 1/\tau_{\epsilon}(T) \propto T$ , and a plot of  $\Gamma$  versus temperature is linear with a positive slope. However, when  $\omega \tau_{\epsilon} \leq 1$ ,  $\Gamma_D \propto \tau_{\epsilon}(T)$  $\propto$  1/*T*, and the same plot decreases with increasing temperature. The linear behavior of Fig. 6 for  $T \leq 2.5$  K is therefore more consistent with Debye absorption in the regime  $\omega \tau_{\epsilon}$  $\geq 1$ , while the tendency to role off at higher temperatures indicates we are moving into a crossover regime where  $\omega \tau$  – 1. The increase of slope with electron concentration in the linear regime also seems to confirm that the attenuation has an electronic origin.

The observation that the low-magnetic field SAW amplitude in the 500 and 375 nm dots almost recovers to the preillumination level upon saturation illumination is certainly surprising. The fact that this does not occur in the very narrowest 250 nm dots, which will suffer more severely from sidewall depletion, suggests that there is a minimum electron concentration for this to occur and it is likely that screening effects are involved. We tentatively attribute this effect to the formation of a rather disordered parallel electronic system in the *n*-Al<sub>x</sub>Ga<sub>1-x</sub>As layer (the equivalent of parallel conduction in a 2DEG) which shields the original quantum dot from the incident SAW. Since this will require substantial screening of the sidewall depletion layer it is more likely to be observed in large dots than smaller ones. We note here that we also observe the complete suppression of the characteristic structures near integer filling factors in unstructured  $2DEG$ 's after very large amounts of illumination, $14$  consistent with this explanation.

The tilted field data shown in Fig. 5 indicate that the

component of magnetic field normal to the dots and the way it modifies the two-dimensional confinement potential is of paramount importance in our system. This will act in two distinct ways. First, it modifies the electronic structure and hence the way in which the SAW interacts with the nanostructures. Second, it increases the Fermi energy of the dots and modifies the dynamics of any electron generation/ recombination processes. The latter point is particularly relevant in view of the apparent magnetic field-induced depopulation of our dots during measurements. Since we use the excitation (by illumination) of electrons trapped at *DX* centers<sup>15</sup> in the *n*-Al<sub>x</sub>Ga<sub>1-x</sub>As layer of the heterostructure to increase the electron concentration it is natural to suppose that the reverse process is responsible for our time transients. In the case of the two-dimensional electron gas it can be demonstrated that the kinetics of this process are unmeasurably slow at low temperatures due to the very large activation energies involved. It is surprising, therefore, that we appear to observe relatively rapid recombination rates in our dots. One concludes that the large amount of etched surface surrounding a dot must be playing a key role, perhaps by providing intermediate electronic states, which allow the carriers to overcome the kinetic barrier. It is also puzzling that the amplitude transients turn on at high fields  $(>1.5 T)$  and it appears that the relative positions of the Fermi energy in the dots and the *DX* centers in  $AI_xGa_{1-x}As$  must be important. Since the  $DX^-$  represents an atomiclike defect it will not be strongly affected by magnetic field while the Fermi energy in the dots will increase by a few millivolts from 0 to 10 T. This would tend to tip the balance of the kinetic equations in favor of recombination, but does not explain why we see *no* transients below 1.5 T. We note that although the majority of the samples we measure show hysteresis, it is not invariably the case. A few rare samples have displayed completely reproducible and reversible behavior with no time transients whatsoever at temperatures in the range 1.5–10 K over a three day measurement period and these data will be reported elsewhere.<sup>16</sup> It appears that either the precise details of the reactive ion etching used to define the dots and/or the sample cool-down cycle play a critical role in establishing the recombination process, but we have so far failed to identify any systematic trends.

Finally, future work might lead to more detailed information regarding the energy relaxation time  $[\tau_{\epsilon}(T)]$  and the phase-breaking time  $[\tau_{\phi}(T)]$  in quantum dots. An absolute value for  $\tau_s(T)$  could be obtained by measuring the SAW attenuation as a function of frequency (perhaps using harmonics of the transducers), at a fixed temperature and in zero magnetic field, as the Debye absorption is predicted to have a maximum when  $\omega \tau_{\epsilon} \sim 1$  [Eq. (4)]. At very high frequencies,  $\sim$  2.5 GHz for our experimental parameters, "tail" rather than Debye absorption is predicted to dominate. In this regime (which we would need new transducers to access) the temperature dependence of  $\tau_{\phi}(T)$  could be measured from the temperature dependence of the SAW attenuation  $[Eq.$ (3)], and the absolute value of  $\tau_{\phi}(T)$  could be calculated if the mean energy level spacing  $(\Delta)$  was known. To fully understand the magnetic field dependence of the SAW absorption the *B* hysteresis would need to be eliminated, perhaps by using surface passivation or dots buried by regrowth.

# **CONCLUSIONS**

In conclusion, we have studied absorption of SAW's by arrays of quantum dots, of various sizes, as a function of magnetic field, temperature, and electron concentration. After carriers were introduced into the dots, via an infared LED, a broad amplitude minimum in the transmitted SAW amplitude develops near  $B=0$  in all cases, while the amplitude at high fields  $(10 T)$  approaches that of the unilluminated sample. The maximum observed attenuation as a function of illumination, at  $B=0$ , was three orders of magnitude larger than the maximum theoretical attenuation predicted for our experimental parameters, and is of the same order as the maximum attenuation predicted for an unstructured 2DEG. This anomalous SAW attenuation has not been previously reported. The high-field behavior qualitatively agrees with theoretical predictions for resonant absorption based on the density of single-electron level crossings, whilst at low fields we find no evidence for indications of the abrupt change in attenuation predicted by the calculations of Knab-

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chen *et al.* At saturation illumination levels in the largest dots, there is no magnetic-field dependence of the amplitude, which we attribute to the formation of a parallel conducting system in the  $n^+$ -Al<sub>x</sub>Ga<sub>1-x</sub>As layer. The surprising hysteresis of the measurements upon reversal of the magnetic-field sweep direction is thought to be due to field-induced depopulation of *DX* centers, which has not been reported previously. However, measurements in tilted magnetic fields demonstrate that there is also a genuine field dependence of the attenuation for a dot with fixed electron concentration. Finally, the linear temperature dependence of the SAW attenuation coefficient below 2.5 K is consistent with recent theoretical predictions of attenuation by Debye relaxation.<sup>11</sup>

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