Neutron elastic scattering in magnetic media: Refracted-wave scattering approach

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The use of the refracted-wave scattering approach allowed encompassing some aspects of neutron scattering in magnetic media that are not described in the standard approach to scattering. The corresponding mathematical formalism has been introduced. The Born approximation and the Larmor precession approximation have been modified to take into account the refractive properties of magnetic media. [S0163-1829(99)10009-2]

I. INTRODUCTION

The interaction of a neutron with magnetic field is described by the Schroedinger equation, the exact solution of which, quite often, cannot be found. Therefore, such standard approaches as the Born approximation (BA) and the Larmor precession approximation (LA) are used (e.g., Ref. 1). The neutron interaction is treated in the majority of works in the frame of these approaches.

In both approaches the refraction effect of magnetic medium is ignored. Indeed, two velocities should be attributed to a neutron in magnetic field, $\mathbf{v}_+$ and $\mathbf{v}_-$, respectively, for the neutron states with the spins ''up'' $(+)$ and ''down'' $(-)$ the field. This difference in velocities is lost in LA, as it means transition to the coordinate system moving with the neutron on assumption that $\mathbf{v}_+=\mathbf{v}_-$. It is lost also in BA, as the neutron state, which is a superposition of the states with the spin projections $+1/2$ $(\uparrow)$ and $-1/2$ $(\downarrow)$ onto a quantization axis, is described at infinity by an asymptotic based on a linear combination of the wave functions with one and the same wave vector $\mathbf{k}$:

$$\Psi_\uparrow=\exp(i\mathbf{kr})\begin{pmatrix}1\\0\end{pmatrix},\quad \Psi_\downarrow=\exp(i\mathbf{kr})\begin{pmatrix}0\\1\end{pmatrix}.$$

However, as the beam is divergent and the scattering region is finite, another asymptotic behavior at infinity should be generally used (see Sec. III for more details).

In some polarized neutron experiments, such as neutron spin echo spectroscopy,[2] it is essential to follow exactly the details of the neutron interaction with magnetic fields. The same is true for neutron depolarization and SANS techniques when neutron scattering at large-scale inhomogeneities, i.e., with extremely low energy transfers, is essential. In addition, even the minutest details of neutron magnetic interaction may be essential for ultracold neutrons. We also mention the latest observations[3–6] in polarized neutron reflectometry in which subtle neutron optical effects of magnetic fields do play significant roles. Therefore, the detailed analysis of elastic neutron scattering in magnetic media seems to be of practical interest.

The use of the refracted-wave scattering approach (RWSA), outlined earlier,[7] allowed encompassing some aspects of neutron scattering in magnetic media that are not described in the standard approach to scattering (SA). LA and BA have been modified to take into account the refractive properties of magnetic media. It may also be said that the present work develops the qualitative consideration[8] (''minimal theory'') by introduction of a mathematical formalism, which is a modification of SA.

It follows from RWSA that, in contrast to SA where all representations are equivalent, only the representations with the quantization axis Z collinear to the mean field $\langle\mathbf{B}\rangle$ can be used to calculate magnetic scattering cross sections for certain spin states, in full compliance with the intrinsic anisotropy introduced by the mean field. Consequently, the NSF and SF scattering probabilities, strictly speaking, are defined only for $Z\|\langle\mathbf{B}\rangle$.

II. SCHEME OF NEUTRON ELASTIC MAGNETIC SCATTERING

When the refraction properties of a magnetic medium are taken into account, the wave vectors for the neutron states with the spin ''up'' $(+)$ and ''down'' $(-)$ the field turn out to be different in length and, generally, in direction. Therefore, the neutron scattering scheme for the interaction region with a mean magnetic field $\langle\mathbf{B}\rangle\neq0$ (an external field and/or that induced by magnetization of the sample) should include two incident wave vectors, $\mathbf{k}_a^+$ for neutrons with the spin ''up'' the field $\langle\mathbf{B}\rangle$ and $\mathbf{k}_a^-$ for neutrons with the opposite spin (in general, the directions of these vectors may depend on the neutron path and configuration of magnetic fields before the interaction region). They produce two Ewald spheres of radii k_a^+ and k_a^- in the reciprocal space. Transitions between the Ewald spheres are possible owing to spin-flip (SF) elastic scattering with momentum transfers $\mathbf{q}_{+-}$ and $\mathbf{q}_{-+}$, whereas non-spin-flip (NSF) elastic scattering leaves the resultant wave vectors on a given Ewald sphere, the corresponding momentum transfers being $\mathbf{q}_{++}$ and $\mathbf{q}_{--}$. The incident wave vectors in a general scheme of elastic scattering give rise to all possible transitions defined by the two Ewald spheres.

For further analysis, introduce an ''elementary'' elastic scattering scheme with four momentum transfers $\mathbf{q}_{\sigma_a\sigma_b}$ (σ_a and σ_b designate the initial and final neutron spin projections onto the quantization axis) defined for transitions from $\mathbf{k}_a^+$ and $\mathbf{k}_a^-$ to $\mathbf{k}_b^+$ and $\mathbf{k}_b^-$, i.e., to the wave vectors for neutrons scattered, respectively, in the $(+)$ and $(-)$ spin states (Fig. 1). The lengths of the wave vectors in the interaction region

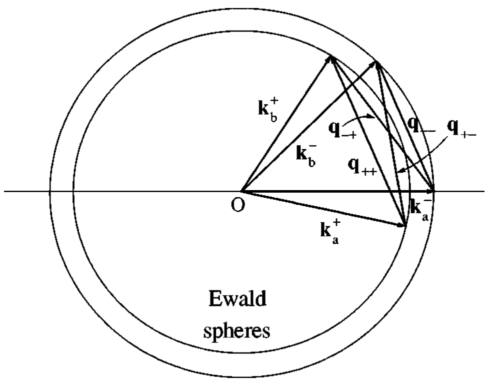

FIG. 1. The ''elementary'' scheme of scattering in a mean magnetic field $\langle \mathbf{B}\rangle \neq 0$. For the sake of simplicity, the wave vectors $\mathbf{k}_a^{\pm}$ and $\mathbf{k}_b^{\pm}$ are assumed to lie in one plane.

are (E is the neutron total energy, $\langle V_n\rangle$ is the mean nuclear potential)

$$k_a^{\pm}=k_b^{\pm}=\sqrt{\frac{2m_n}{\hbar^2}[E-(\langle V_n\rangle \pm |\mu_n\langle \mathbf{B}\rangle|)]}. \tag{1}$$

For a given scattering angle the effect of refraction in the case of the NSF scattering boils down to a proportional change of the length of the wave vectors of the incident and scattered neutrons and that of the momentum transfer. Therefore, the direction of the NSF momentum transfer is not changed and the relative change of its length is

$$(\Delta q/q)_{\mathrm{NSF}}=(\Delta k/k)\cong 5.6\times 10^{-7}(\langle V_n\rangle \pm 60\langle B\rangle)\lambda^2 \tag{2}$$

(here V_n [neV], λ [nm] and B [T]). For the nuclear and magnetic potentials are small ($\sim$100 neV or less), the refraction effects should be taken into account only in the case of UCN in sufficiently strong magnetic fields.

Not only a changed length, but also a changed direction of the momentum transfer corresponds to the SF scattering in magnetic medium for a given scattering angle. It is related to refraction in the mean field (the refractive indices for a neutron before and after spin flipping are different). The corresponding angular deviations δ (Fig. 2) can be found from the formula

$$\sin^2(\alpha\pm\delta)=\sin^2(\alpha)\pm 1.47\times 10^{-4}\langle B\rangle\lambda^2, \tag{3}$$

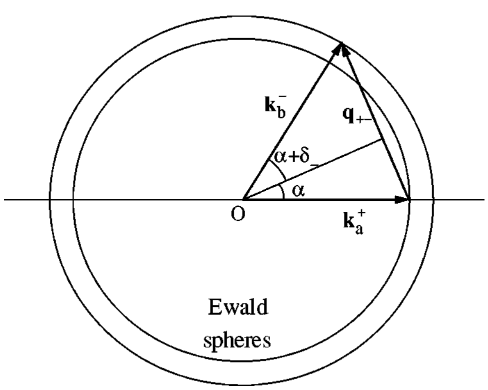

FIG. 2. The angular deviation δ due to refraction under $(+-)$ elastic scattering in a mean magnetic field $\langle \mathbf{B}\rangle \neq 0$.

where α is the angle between $\mathbf{k}_a^+$ ($\mathbf{k}_a^-$) and the plane perpendicular to $\mathbf{q}+-$ $(\mathbf{q}-+)$ (λ [nm] and B [T]). When $\alpha \ll 1$ and $\delta \ll \alpha$, one obtains from Eq. (3) that

$$\delta \cong \frac{1.47\times 10^{-4}\langle B\rangle\lambda^2}{2\alpha}. \tag{4}$$

Usually the difference between k_a^+ and k_a^- is very small even for maximum magnetic fields available, so refraction under the SF scattering may be neglected almost in the whole range of momentum transfers. If $|\mu_n\langle B\rangle| \ll E$, the refraction under magnetic scattering with spin flip is noticeable only at very small scattering angles when the momentum transfer is comparable with

$$q_p=k_b^- - k_b^+ \cong k\frac{|\mu_n\langle B\rangle|}{E}=\frac{\omega_L}{v}=\frac{q_m^2}{k}, \tag{5}$$

where $\omega_L \equiv 2|\mu_n\langle B\rangle|/\hbar$ and v are, respectively, the Larmor precession frequency in the field $\langle \mathbf{B}\rangle$ and the neutron velocity.

A physically unusual situation, when the vector $\mathbf{q}_{+-}$ becomes perpendicular to $\mathbf{k}_a^+$ and the vector $\mathbf{q}_{-+}$ is perpendicular to $\mathbf{k}_b^+$, arises when the momentum transfer is equal to

$$q_m=\sqrt{\frac{2m_n}{\hbar^2}2|\mu_n\langle \mathbf{B}\rangle|}=K\sqrt{\langle B\rangle} \tag{6}$$

(with $K=7.61\times 10^{-2}$, if q_m [nm^{-1}] and B [T], e.g., for $\langle B\rangle=0.001$, 0.01, 0.1, 1, 2 [T], one finds, respectively, $q_m=2.4\times 10^{-3}, 7.6\times 10^{-3}$ 2.4×10^{-2}, 7.6×10^{-2}, 1.08×10^{-1} [nm^{-1}] and the corresponding characteristic sizes $a_m=2\pi(q_m)^{-1}=2600$, 830, 260, 83, 58 [nm]). Then the $(+-)$ scattering originates from the planes parallel to the velocity of the incident neutrons, whereas the velocity of the $(+-)$ scattered neutrons lie within the scattering (perpendicilar to $\mathbf{q}_{-+}$) planes.

It can be easily seen from formulas (1) and (6) that the condition $|\mu_n\langle B\rangle| \ll E=\hbar^2k^2/2m_n$, coincides with $q_m/k_a^{\pm} \cong q_m/k \ll 1$, whereas the characteristic scales under elastic magnetic scattering are related to each other as $q_p/q_m=q_m/k$, so the condition of large neutron energies coincides with the condition $q_p/q_m \ll 1$. The SF scattering within the cone $q<q_m$, i.e., $\alpha<\arccos(k_a^+/k_a^-)$, possesses unusual properties.

(a) It is ''piercing'': the scattered neutrons turn out to be ''behind'' the planes perpendicilar to the momentum transfer $\mathbf{q}_{\mathrm{SF}}$ ($\mathbf{q}_{+-}$ or $\mathbf{q}_{-+}$), but not ''in front'' of them, as is usually the case under elastic scattering. Such scattering is possible owing to the difference in the kinetic energy before and after scattering that is equal to the Zeeman splitting $2|\mu_n\langle B\rangle|$.

(b) On reduction of the length of the momentum transfer $\mathbf{q}_{\mathrm{SF}}$, its direction is changed from perpendicular to parallel to the incident neutron velocity. In scattering according to the conventional scheme (SA), on the contrary, $\mathbf{q}$ in the limit $q\to 0$ becomes perpendicular to the incident neutron velocity.

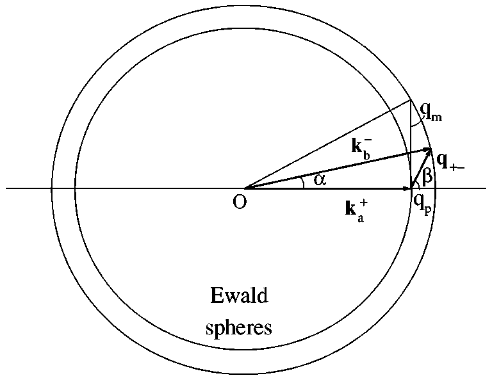

FIG. 3. The scheme of $(+-)$ elastic scattering with a momentum transfer $q<q_m$ in a mean magnetic field $\langle \mathbf{B} \rangle \neq 0$.

(c) Extremely small changes in the scattering angle α may correspond to a significant change in the direction of the momentum transfer (the angle β in Fig. 3). In the approximation $k_a^+ \cong k_a^- \cong k$:

$$\sin\alpha/\sin\beta \cong q_{\mathrm{SF}}/k \quad (q<q_m). \tag{7}$$

Thus, a change in the scattering angle by a fraction of a second may correspond to a change in the direction of the momentum transfer by tens of degrees. Approximating the circumferences (Fig. 3) by parabolas, one finds for a given $\mathbf{q}_{\mathrm{SF}}$ the relation between an arbitrary β and a small α:

$$\tan\beta = \frac{k_a^{\pm}\alpha}{q_p \pm k_a^{\pm}\alpha^2/2} \tag{8}$$

[the upper and lower signs correspond to the $(-+)$ and $(+-)$ scattering modes, respectively], as well as the magnitude of the momentum transfer

$$q_{\mathrm{SF}} = \frac{k_a^{\pm}\alpha}{\sin\beta} = (q_p \pm k_a^{\pm}\alpha^2/2)/\cos\beta. \tag{9}$$

(d) The momentum transfer under SF scattering q_{SF} in the presence of a mean field may not be smaller than q_p. Hence, it is not correct, say, to go to the limit $q_{\mathrm{SF}} \to 0$, if the mean field is not equal to 0.

The fact that $\mathbf{k}_b^+$ and $\mathbf{k}_b^-$ are generally different not only in magnitude but also in direction affects the behavior of polarization of the scattered neutron beam. Assume that the incident neutrons are polarized either up or down the mean field $\langle \mathbf{B} \rangle$. The ‘‘elementary’’ scattering scheme of Fig. 1 yields then two wave vectors (for NSF and SF scattering). Consequently, precession of $\mathbf{P}_b$, the polarization vector of neutrons scattered in this scheme, is due to the difference $\mathbf{k}_b^+ - \mathbf{k}_b^-$. The front of such a precession (i.e., the beam cross section in which the polarization vector is constant in direction) is perpendicular to $\mathbf{k}_b^+ - \mathbf{k}_b^-$, rather than to $\mathbf{k}_b^{\pm}$. Therefore, nonfrontal precession[9] is intrinsic to magnetic scattering in which NSF and SF processes combine. Generally, the neutron spin is inclined to $\langle \mathbf{B} \rangle$. The incident neutron spin state is then a superposition of the spin components ‘‘up’’ and ‘‘down’’ $\langle \mathbf{B} \rangle$, both yielding a nonfrontal precession. The behavior of the polarization can be described then as a superposition of two nonfrontal precessions about $\langle \mathbf{B} \rangle$. Moreover, there will be a superposition of many ‘‘elementary’’ scattering schemes leading to a rather intricate behavior of the polarization. Consequently, if the incident beam is perfectly collimated and polarized, then $P_b = 1$ at each point, but averaging in a cross section of the scattered beam will lead to a decrease in (or elimination of) the perpendicular-to-$\langle \mathbf{B} \rangle$ component of $\mathbf{P}_b$ (reversible depolarization). Only point-by-point polarization analysis could restore the scattered beam polarization pattern. In a real experiment the incident beam is divergent and $\mathbf{P}_b$ results from averaging over numerous independent scattering events. As a rule, the perpendicular-to-$\langle \mathbf{B} \rangle$ component of $\mathbf{P}_b$ is then averaged to 0 and the SF scattering leads just to (irreversible) depolarization.

Such effects as nonfrontal neutron spin precession and angular splitting are usually hidden and lost in the bulk of numerous scattering events. Nevertheless, they may be important for understanding details of magnetic scattering and depolarization mechanism and essential in analysis of experimental data for very low momentum transfers in strong magnetic fields. Besides, the use of a mirror makes their observations possible in specular or even in diffuse scattering. Numerous events of scattering (reflection) are then distributed on the surface of the mirror, i.e., spatially correlated in a definite way, the result for the specular reflection being that the NSF and SF scattering vectors are parallel to the surface normal. As a consequence, if the incident neutrons are polarized either up or down the guide field, the precession fronts inside a layered structure of the mirror and in the specularly reflected and transmitted beams are parallel to the sample surface, i.e., almost parallel to neutron trajectories in the corresponding beams. Angular splitting under specular reflection[10,11] in a strong magnetic field has been recently demonstrated.[3]

Observations of the angular splitting and the nonfrontal precession related to that splitting could not be done simultaneously. Indeed, if the scattered beam is perfectly collimated, then $\mathbf{k}_b^+$ and $\mathbf{k}_b^-$ are strictly parallel and the precession front is perpendicular to the neutron velocities. It also implies that the precession front of the transmitted beam, into which only neutrons transmitted without scattering or scattered with extremely small momentum transfers contribute, remains to be perpendicular to neutron trajectories.

III. NEUTRON ELASTIC MAGNETIC SCATTERING: REFRACTED WAVE SCATTERING APPROACH

In order to encompass the new features in magnetic scattering, the refracted wave scattering approach (RWSA) outlined earlier[7] can be used. The starting point of this approach is the following asymptotic behavior of the wave function operator at $r \to \infty$:

$$\hat{\Psi}(\mathbf{r}) = \exp(i\hat{\mathbf{k}}_a\mathbf{r}) + \frac{\exp(i\hat{k}_b r)}{r}\hat{F}(\Omega_b), \tag{10}$$

where the phase operators are used instead of scalar phases and the quantities defining the amplitude operator $\hat{F}$ are assumed to be known from experiment or theory. Thus, the main distinction of the given approach from the conventional one (e.g., Refs. 12 and 13) is the use of, instead of the wave vectors, their operator equivalents, $\hat{\mathbf{k}}_a$ and $\hat{\mathbf{k}}_b$, for the incident and scattered waves, respectively. The latter implies that the potential is an operator, the eigenvalues of which at infinity are generally nonzero constants. This means in the case of interaction of a neutron with magnetic field that the target (scatterers) is in a space filled with a homogeneous field. The physical meaning of such a suggestion is discussed later.

The use of a standard scheme,[12,13] dealing with scalar wave vectors, for the solution of type (10) yields the following scattering operator:

$$\hat{S}=1+2i\hat{k}\hat{F}. \tag{11}$$

Note that in the conventional S operator (see, e.g., Refs. 12,13) a scalar wave number k is used instead of the operator $\hat{k}$. It follows from the flux conservation that $\hat{S}\hat{S}^{\dagger}=1$ and, consequently ($\hat{k}$ and $\hat{F}$ commute),

$$\hat{F}-\hat{F}^{\dagger}=2i\hat{k}\hat{F}\hat{F}^{\dagger}. \tag{12}$$

This relation leads to the optic theorem for elastic scattering of spin particles in media. The use of the reversibility of the solutions of the Schrödinger equation with respect to time yields

$$\hat{K}^{-1}\tilde{\hat{S}}\hat{K}=\hat{S} \tag{13}$$

where $\hat{K}$ is the time-reversal operator. Substituting Eq. (11) into Eq. (13), one obtains the equation

$$\hat{K}^{-1}\widetilde{\hat{k}\hat{F}}\hat{K}=\hat{k}\hat{F} \tag{14}$$

that can be used to find the relations between the scattering amplitudes of the direct and reversed scattering schemes in RWSA. Note that the scalar wave number is simply cancelled out in SA. Just to illustrate the difference between two approaches, mention without proof that in the case of a layered structure, in which all magnetic induction vectors and the external field (the quantization axis is assumed to be along this field) lie in one plane, RWSA yields the relation $k_{\perp}^{-}r_{+-}=k_{\perp}^{+}r_{-+}$ between nondiagonal elements of the specular reflection matrix ($k_{\perp}^{\pm}$ are the eigenvalues of the normal-to-the-surface component of the incident neutron wave vector operator), whereas SA predicts $r_{+-}=r_{-+}$. More detailed analysis of formulas (11)–(14) will be given elsewhere.

Following the classical schemes[12,13] further, one finds for the solution of type (10) the following relation between the differential cross section of scattering in the direction Ω_b and the scattering amplitudes (the quantities defining $\hat{F}$):

$$\frac{d\sigma}{d\Omega_b}(\Omega_b)=\frac{\langle s_a|\hat{F}^{\dagger}(\Omega_b)\hat{k}_b\hat{F}(\Omega_b)|s_a\rangle_{\sigma}}{\langle s_a|\hat{k}_a|s_a\rangle_{\sigma}}, \tag{15}$$

where $|s_a\rangle$ is the spinor specifying the spin state of the incident neutron at a point far from the target, the subindex σ indicates that the scalar product is defined in the spin subspace, and the scattering amplitude operator for a spin particle of mass m is

$$\hat{F}(\hat{\mathbf{k}}_a\rightarrow\hat{\mathbf{k}}_b)\equiv-\frac{m}{2\pi\hbar^2}\int e^{-i\hat{\mathbf{k}}_b\mathbf{r}}\hat{V}(\mathbf{r})\hat{\Psi}^{(+)}(\hat{\mathbf{k}}_a,\mathbf{r})d\mathbf{r}, \tag{16}$$

where $\hat{\Psi}^{(+)}(\hat{\mathbf{k}}_a,\mathbf{r})$ is the exact solution of the Schrödinger equation with a potential $\hat{V}(\mathbf{r})$ for the incident neutron the state of which is described by $\hat{\mathbf{k}}_a$, ($\hat{\mathbf{k}}_a\rightarrow\hat{\mathbf{k}}_b$) designates transitions between the states described by the eigenvalues of the corresponding wave vector operators.

It is worth noting that formula (15) implies that the ''elementary'' elastic scattering scheme includes four (two NSF and two SF scattering) transitions from arbitrarily directed wave vectors $\mathbf{k}_a^{+}$ and $\mathbf{k}_a^{-}$ to parallel wave vectors $\mathbf{k}_b^{+}$ and $\mathbf{k}_b^{-}$, pointing in the direction Ω_b. As the spatial dependence of the phase difference between the upper and lower spin components of the incident neutron spinor is taken into account in the mathematical formalism introduced (k_a^{+} and k_a^{-} are different), the angular deviations owing to refraction are automatically described by Eqs. (15) and (16) (see the notion of the pseudograting in Ref. 11). In addition, mention that two terms will arise in the numerator that are proportional to k_b^{+} and k_b^{-}. They are results of interference (cross interference[11]) between the waves scattered without SF and with SF into one spin state, respectively, $(+)$ and $(-)$. Thus, RWSA takes precession of the neutron spin in the mean field during the scattering (in the scattering region) into account. The result of the cross interference depends on neutron spin precession in the scatering region. The interference terms usually vanish for divergent beams, because of averaging over different scattering events. However, the cross interference is observable with specular reflection from magnetically noncollinear layers (see the precession term in the reflectivity[11]).

More generally, in terms of the density operator $\hat{\rho}_a$ for the incident neutron beam,

$$\frac{d\sigma}{d\Omega_b}(\Omega_b)=\frac{\mathrm{Tr}\{\hat{\rho}_a\hat{F}^{\dagger}(\Omega_b)\hat{k}_b\hat{F}(\Omega_b)\}}{\mathrm{Tr}\{\hat{\rho}_a\hat{k}_a\}}. \tag{17}$$

This expression for the differential cross section differs from the conventional one[14] by introduction of $\hat{k}_a$ and $\hat{k}_b$, in accordance with solution (10). In the conventional approach these quantities are scalar, and one obtains the well-known factor k_b/k_a which is equal to 1 for the elastic scattering.

It has been concluded[9] that $\mathbf{P}_n$, the polarization of neutrons in a beam (related to the probability densities), and $\mathbf{P}_b$, the neutron beam polarization (related to the current densities), are generally not equivalent. Another conclusion is that the intensities of neutrons in states with the spin inclined to the magnetic field (i.e., measured simultaneously with the probability of the spin projection onto a quantization axis inclined to the magnetic field) are not defined unambiguously. This means that, strictly speaking, the absolute quantities in magnetic field are only the total differential cross section defined in Eq. (17) and the differential cross sections for the scattered neutron states with the spin up $(+)$ and down $(-)$ the mean field direction

$$\left(\frac{d\sigma}{d\Omega_b}(\Omega_b)\right)_{\pm}=\frac{\mathrm{Tr}\{\hat{\rho}_a\hat{F}^{\dagger}\hat{k}_b(\hat{\sigma}\langle\pm\mathbf{B}\rangle/\langle B\rangle)\hat{F}\}}{\mathrm{Tr}\{\hat{\rho}_a\hat{k}_a\}}, \quad (18)$$

($\hat{\sigma}\langle\pm\mathbf{B}\rangle/\langle B\rangle$, the operator of the spin projection onto $\langle\pm\mathbf{B}\rangle$, commutes with both $\hat{k}_b$ and $\hat{F}$). Consequently, one may rigorously define SF and NSF scattering differential cross sections only when SF and NSF processes are related to the quantization axis collinear to $\langle\mathbf{B}\rangle$ (unlike in BA where all representations are equivalent). The differential cross sections $(d\sigma/d\Omega_b)_{++}$, $(d\sigma/d\Omega_b)_{+-}$, $(d\sigma/d\Omega_b)_{-+}$, $(d\sigma/d\Omega_b)_{--}$ can be obtained from Eq. (18) for these four processes.

As the operator of the current density and the operator of the spin projection onto a direction inclined to the magnetic field do not commute, no general expression for the neutron beam polarization exists (more details in Ref. 9) and one may speak only about its projection onto $\langle\mathbf{B}\rangle$:

$$\mathbf{P}_b\langle\mathbf{B}\rangle/\langle B\rangle=\frac{(d\sigma/d\Omega_b)_+-(d\sigma/d\Omega_b)_-}{(d\sigma/d\Omega_b)_++(d\sigma/d\Omega_b)_-}\ . \quad (19)$$

On the other hand, the polarization of neutrons scattered in the direction Ω_b is the well-defined quantity

$$\mathbf{P}_n(\Omega_b)=\frac{\mathrm{Tr}\{\hat{\rho}_a\hat{F}^{\dagger}(\Omega_b)\hat{\sigma}\hat{F}(\Omega_b)\}}{\mathrm{Tr}\{\hat{\rho}_a\hat{F}^{\dagger}(\Omega_b)\hat{F}(\Omega_b)\}}, \quad (20)$$

The suggestion introduced above about the whole space filled with a homogeneous field is a mathematical abstraction. It implies that in each scattering event it is important only what happens in the scattering region. Indeed, for example, it has been experimentally demonstrated:[15] if the coherent illumination region is much smaller than the domains in a demagnetized thin-film mirror, each of the large domains reflects as a single mirror with its own neutron optical potential; on the other hand, if the domains are small, the effective potential is obtained by averaging over numerous domains in the coherent illumination region. The angular splitting observed under specular reflection[3] can also be explained only with a similar suggestion. Therefore, if the mean field in the interaction region is not zero, the scattering should be described in the refracted wave scattering approach. For a complete solution of the real physical problem, it is necessary, as usually (e.g., Ref. 16), to take into account the change of polarization of the incident and scattered beams in transmission, respectively, from the polarizer to the scattering region and from the scattering region to the analyzer. Inside the sample, to speak nothing of the regions outside the sample, the magnetic fields may differ from $\langle\mathbf{B}\rangle$. The problem is solved for each specific configuration of magnetic fields by taking the method of measurement of the projections of $\mathbf{P}_b$ into account.

IV. REFRACTED WAVE BORN APPROXIMATION (RWBA)

The scattering of neutrons in a magnetic sample is described in terms of the spin-dependent scattering amplitude operator of type (e.g., Ref. 14)

$$\hat{F}=F_1+\hat{\sigma}\mathbf{F}_2. \quad (21)$$

In BA (the first-order Born approximation) $F_1=F_n$ and $\mathbf{F}_2=\mathbf{F}_m$, where F_n and $\hat{\sigma}\mathbf{F}_m$, are the Fourier transforms of $V_n(\mathbf{r})$ and $-\mu_n\hat{\sigma}\mathbf{B}(\mathbf{r})$, the nuclear and magnetic potentials, correspondingly [$\mathbf{B}(\mathbf{r})$ is the magnetic induction at a position $\mathbf{r}$]. Particularly, for a given momentum transfer $\mathbf{q}$ (W is the region of interaction in a scattering event):

$$F_n(\mathbf{q})=-\frac{m_n}{2\pi\hbar^2}\int_W V_n(\mathbf{r})\exp(-i\mathbf{qr})d\mathbf{r}=-\frac{m_n}{2\pi\hbar^2}\tilde{V}_n(\mathbf{q}),$$

$$\mathbf{F}_m(\mathbf{q})=\frac{m_n\mu_n}{2\pi\hbar^2}\int_W \mathbf{B}(\mathbf{r})\exp(-i\mathbf{qr})d\mathbf{r}=\frac{m_n\mu_n}{2\pi\hbar^2}\tilde{\mathbf{B}}(\mathbf{q}), \quad (22)$$

where m_n and μ_n are, respectively, neutron mass and neutron magnetic moment.

Using expression (16), introduce the refracted wave Born approximation (RWBA) as a generalization of the Born approximation (BA) by the assumption that

$$\hat{F}(\hat{\mathbf{k}}_a\to\hat{\mathbf{k}}_b)\cong-\frac{m}{2\pi\hbar^2}\int_W e^{-i\hat{\mathbf{k}}_b\mathbf{r}}\hat{V}(\mathbf{r})e^{i\hat{\mathbf{k}}_a\mathbf{r}}d\mathbf{r}, \quad (23)$$

(the first-order approximation is meant in this paper, though higher orders can be also introduced in the RWSA). The basic distinction between the RWBA and BA is that the operator exponents (exponential operators) $\hat{\Phi}_{a,b}=\exp(i\hat{\mathbf{k}}_{a,b}\mathbf{r})$ are used, which generally do not commute with the operator potential $\hat{V}(\mathbf{r})$ to directly yield a momentum transfer $\mathbf{q}$. Thus, one obtains for a neutron of energy E that

$$\hat{F}(\hat{\mathbf{k}}_a\to\hat{\mathbf{k}}_b)=-\frac{m_n}{2\pi\hbar^2}\int_W e^{-i\hat{\mathbf{k}}_b\mathbf{r}}[V_n(\mathbf{r})-\mu_n\hat{\sigma}\mathbf{B}(\mathbf{r})]e^{i\hat{\mathbf{k}}_a\mathbf{r}}d\mathbf{r}, \quad (24)$$

where the initial (a) and resultant (b) states are described by the wave vector operators $\hat{\mathbf{k}}_a$ and $\hat{\mathbf{k}}_b$. Their eigenvalues, $\mathbf{k}_a^{\pm}$ and $\mathbf{k}_b^{\pm}$, are the wave vectors for the spin components up (+) and down (−) the mean magnetic induction $\langle\mathbf{B}\rangle$ (the mean nuclear potential $\langle V_n\rangle$ and $\langle\mathbf{B}\rangle$ are defined for the interaction region W).

If the quantization axis Z is parallel to $\langle\mathbf{B}\rangle$, the representative matrices of the exponential operators are diagonal:

$$\Phi_{a,b}=\begin{pmatrix}\exp(i\mathbf{k}_{a,b}^{+}\mathbf{r}) & 0\\ 0 & \exp(i\mathbf{k}_{a,b}^{-}\mathbf{r})\end{pmatrix}. \quad (25)$$

One obtains from Eqs. (24) and Eq. (25) in this representation that

$$\hat{F}(\hat{\mathbf{k}}_a \to \hat{\mathbf{k}}_b) = -\frac{m_n}{2\pi\hbar^2}\begin{pmatrix} \tilde{V}_n(\mathbf{q}_{++}) + |\mu_n|\tilde{B}_{\parallel\langle\mathbf{B}\rangle}(\mathbf{q}_{++}) & |\mu_n|\tilde{B}_{\perp\langle\mathbf{B}\rangle}(\mathbf{q}_{-+})\exp[-i\varphi(\mathbf{q}_{-+})] \\ |\mu_n|\tilde{B}_{\perp\langle\mathbf{B}\rangle}(\mathbf{q}_{+-})\exp[i\varphi(\mathbf{q}_{+-})] & \tilde{V}_n(\mathbf{q}_{--}) - |\mu_n|\tilde{B}_{\parallel\langle\mathbf{B}\rangle}(\mathbf{q}_{--}) \end{pmatrix}, \tag{26}$$

where $\mathbf{q}_{\sigma_a\sigma_b} = \mathbf{k}_b^{\sigma_b} - \mathbf{k}_a^{\sigma_a}$ are the momentum transfer vectors corresponding to the four scattering modes (σ_a and σ_b designate the initial and final neutron spin projections onto the quantization axis), $\tilde{V}_n$, $\tilde{B}_{\parallel\langle\mathbf{B}\rangle}$, and $\tilde{B}_{\perp\langle\mathbf{B}\rangle}$ are the Fourier transforms of, respectively, the nuclear potential, the magnetic induction components parallel and perpendicular to $\langle\mathbf{B}\rangle$, $\varphi = \varphi(\mathbf{q})$ is the axial angle of $\tilde{\mathbf{B}}(\mathbf{q}) = \tilde{\mathbf{B}}_{\perp\langle\mathbf{B}\rangle}(\mathbf{q}) + \tilde{\mathbf{B}}_{\parallel\langle\mathbf{B}\rangle}(\mathbf{q})$ in a reference frame $(X,Y,Z\parallel\langle\mathbf{B}\rangle)$. This implies that the scattering amplitudes in RWBA are calculated as in the standard Born approximation (BA), but for the momentum transfers corrected for refraction.

The following expressions are obtained in RWBA from Eqs. (15) and (26) for the differential cross sections of the four scattering modes $(Z\parallel\langle\mathbf{B}\rangle)$:

$$\frac{d\sigma_{++}}{d\Omega_b}(\mathbf{q}_{++}) = \frac{d\sigma_{++}^{BA}}{d\Omega_b}(\mathbf{q}_{++}),$$

$$\frac{d\sigma_{-+}}{d\Omega_b}(\mathbf{q}_{-+}) = \frac{k_b^+}{k_a^-}\frac{d\sigma_{-+}^{BA}}{d\Omega_b}(\mathbf{q}_{-+}),$$

$$\frac{d\sigma_{+-}}{d\Omega_b}(\mathbf{q}_{+-}) = \frac{k_b^-}{k_a^+}\frac{d\sigma_{+-}^{BA}}{d\Omega_b}(\mathbf{q}_{+-}),$$

$$\frac{d\sigma_{--}}{d\Omega_b}(\mathbf{q}_{--}) = \frac{d\sigma_{--}^{BA}}{d\Omega_b}(\mathbf{q}_{--}), \tag{27}$$

where BA designates the standard Born cross sections. The SF cross sections in Eq. (27) and those obtained from BA differ by refraction factors related to the flux conservation law. It is worth noting that $(d\sigma_{-+}^{\mathrm{BA}}/d\Omega_b)(\mathbf{q}) = (d\sigma_{+-}^{\mathrm{BA}}/d\Omega_b(-\mathbf{q})$, hence the reciprocity theorem is not violated both in BA and RWBA.

Unlike in BA where all representations are equivalent, only the representations with the quantization axis Z collinear to the mean field $\langle\mathbf{B}\rangle$ can be used in RWBA to calculate magnetic scattering cross sections for certain spin states: the refraction corrections are defined only in such a representation. On the other hand, any representation can be used to calculate the full differential cross section as defined in Eq. (15) or Eq. (17).

V. NEUTRON TRANSMISSION IN MAGNETIC MEDIA

In accordance with the scheme of Fig. 1, when $\langle\mathbf{B}\rangle \neq 0$, the SF scattering "straight forward" (into zero angle) with momentum transfers $q_{+-} = q_{-+} = q_p$ becomes possible. The characteristic size of the structural features along the neutron trajectory is equal to the period of precession in the field $\langle\mathbf{B}\rangle$:

$$D = 2\pi q_p^{-1}. \tag{28}$$

For the spin flipping $(Z\parallel\langle\mathbf{B}\rangle)$ the field variations $\mathbf{B}_1$ $(\mathbf{B} \equiv \langle\mathbf{B}\rangle + \mathbf{B}_1)$ should have a component perpendicular to $\langle\mathbf{B}\rangle$. Assuming that on the length $L = DN$ (N is the number of periods) the neutron velocity, $\langle\mathbf{B}\rangle$ and $\mathbf{B}_1$ are mutually perpendicular, $\mathbf{B}_1$ being constant in magnitude but directed oppositely in two semiperiods, one obtains the magnetic field configuration used in the Drabkin wiggler[17] (DW). The performance of DW is described in the frame of the theory of the r.f. flipper, with transition from spatial to time dependence of the magnetic fields. Though such an approach turned out to be justified, it is not formally correct, because it suggests inelastic interaction under static spin flipping.

The scattering scheme considered in the present paper gives a purely static explanation of the features of the spatial spin resonance. Substituting q_p from Eq. (5) into Eq. (28), one obtains the known condition on the magnitude of the constant field for the spatial spin resonance. The appearance of the higher orders in DW at larger wavelengths, rather than at smaller wavelengths as the case is in the ordinary Bragg scattering at periodic structures (crystals, multilayers, etc.), conforms with formula (5): the momentum transfer q_p is inversely proportional to k, whereas it is proportional to k under ordinary elastic scattering. When DW is rotated by several degrees, the momentum transfer related to a given structural feature, rotates by the same angle, but its length remains almost unchanged [see Fig. 2 and formula (9]. The latter explains the stability of the work of DW with respect to its rotation by several degrees or the corresponding increase in the beam divergence. The magnitude of B_1 for which the Zeeman splitting is equal to the energy uncertainty ΔE related to the uncertainty in momentum, $\Delta k = 2\pi/L$ in a region of length L, yields the second condition for the spatial spin resonance. Indeed, for the given Δk, one obtains $\Delta E/E = 2\lambda/L$ and, substituting $\Delta E = 2\mu_n B_1$, $\lambda = h/(m_n v)$ and $E = m_n v^2/2$, one finds the condition for B_1,

$$\frac{2\mu_n B_1}{h} = \frac{v}{L}, \tag{29}$$

obtained from purely static considerations. The relation between B_1 and ΔE is not too evident, yet it is not likely to be fortuitous.

If the inhomogeneities significantly exceed the neutron spatial coherency region and scattering at large angles can be neglected, one may pass from a three-dimensional (3D) description of the neutron interaction (scattering of the waves) to a one-dimensional description in which the interaction potential is a function of one coordinate (along the neutron trajectory). The solution of the one-dimensional problem may be found by one of the approaches known in specular polarized neutron reflectometry. If the energy of neutrons much exceeds their potential energy, they pass the interaction region without being reflected and one obtains from the generalized Airy's fromalism[18] the value of the wave function (spinor) at an arbitrary point z along the neutron trajectory from a known value of the spinor at a point z_0:

$$|s(z)\rangle = e^{i\hat{k}(z)[z-z_{N-1}]}\cdots e^{i\hat{k}(z_2)[z_2-z_1]}e^{i\hat{k}(z_1)[z_1-z_0]}|s(z_0)\rangle, \tag{30}$$

where the magnetic medium between the points z_0 and z is represented by a sequence of N homogeneous layers, the condition of scattering ''straight forward'' implying that the flat boundaries between these layers are infinite and perpendicular to the incident neutron velocity (put in other words, the neutron traverses the magnetic medium without being deflected and experiences the field along its trajectory point-by-point, i.e., only the field value at one point of the trajectory is effective at any instant). It is to be emphasized that the solution obtained is the coherent sum of the ''transmitted'' (NSF) and the ''scattered straight forward'' (SF) waves. The coincidence of the directions of these waves lead to their interference and requires that they should be considered as a single beam.

The succession of the exponentials in formula (30) is important, because they generally do not commute. It is easy to show that each component describes a neutron spin precession about the field in the corresponding layer. Therefore, the change of the spinor along the neutron trajectory is described by the equation $|s(z+dz)\rangle = e^{i\hat{k}(z)dz}|s(z)\rangle$, or

$$\frac{d}{dz}|s(z)\rangle = i\hat{k}(z)|s(z)\rangle, \tag{31}$$

which is a stationary analog of the classical Bloch equation

$$\frac{d\mathbf{P}_n}{dt} = \gamma[\mathbf{P}_n \times \mathbf{B}], \tag{32}$$

describing the motion of the neutron polarization vector in a time-dependent field. Equation (31) cannot be obtained from Eq. (32).

We remind the reader that the Bloch equation (32) is obtained by transition to the neutron rest frame. However, such a transition is always approximate and, moreover, contradictory. In the first place, if the neutron velocity is vanishing, its wavelength grows infinite. Consequently, the neutron is everywhere and the neutron trajectory makes no sense. In addition, even the minutest interaction would drastically change the neutron wave function everywhere. In the second place, the neutron velocity changes along the trajectory according to the change in the potential. Therefore, different rest frames should correspond not only to different points along the neutron trajectory, but also to different spin components in a spin-dependent magnetic potential. Nevertheless, one reference frame is used to describe the neutron behavior and derive the Bloch equation. In such a reference frame a change in the potential is transformed into the corresponding change in the neutron total energy, rather than to its kinetic energy (as it should be). This energy transfer contradicts to the static nature of interaction and is not observed experimentally. Particularly, to explain the spatial spin resonance on the analogy with the ''rotating field approximation'' in the theory of the r.f. flipper, one is inevitably led to introduction of two worlds turning oppositely around the device (DW). Only the use of a purely static consideration, as given in the present paper, provides a more reasonable explanation.

The quantum-mechanical description of behavior of a neutron spin along a neutron trajectory in a static magnetic field is often believed to be exactly equivalent with the description of behavior of a classical magnetic dipole subjected to a magnetic induction $\mathbf{B}(t)$, in the sense that the time dependence of the polarization vector of a neutron beam is described by the Bloch equation. Usually the following proof of this statement is adduced.[19] The interaction between a neutron and a magnetic induction $\mathbf{B}$ is described by the Hamiltonian $\hat{H} = -\mu_n\hat{\sigma}\mathbf{B}$. By definition, $\mathbf{P}_n = \langle\hat{\sigma}\rangle$ and substituting $\hat{H}$ into $d\mathbf{P}_n/dt = \langle\partial\hat{\sigma}/\partial t\rangle + (i/\hbar)\langle[\hat{H},\sigma]\rangle = (i/\hbar)\langle[\hat{H},\sigma]\rangle$ one obtains Eq. (32). However, this derivation of the Bloch equation is not invulnerable. Indeed, the Hamiltonian for a neutron of energy E in static fields is $\hat{H} = \hat{\mathbf{p}}^2/2m_n - \mu_n\hat{\sigma}\mathbf{B} = E$. Since the total neutron energy E is a constant of motion, one obtains $\langle[\hat{H},\sigma]\rangle = 0$ and $d\mathbf{P}_n/dt = 0$. It corresponds to the experimental fact that in static fields the polarization vector $\mathbf{P}_n$ inclined to $\mathbf{B}$ precesses along a neutron trajectory about the field direction with a Larmor frequency, but remains fixed at any point of the trajectory (static precession).

The Bloch equation can be derived from Eq. (31) by approximating the operator

$$\hat{k}(z) = \sqrt{\frac{2m_n}{\hbar^2}[E - V_n(z) \mp \hat{\mu}_n\mathbf{B}(z)]}$$
$$\cong k\{1 - [V_n(z) \pm \hat{\mu}_n\mathbf{B}(z)]/E\}, \tag{33}$$

where $k = (2m_nE/\hbar^2)^{1/2}$, and by the substitution $dz = (\hbar k/m_n)dt = v\,dt$. This derivation of the Bloch equation on the basis of the exact solution for magnetic layered structures[18] is rid of the abovementioned contradictions. Besides, it reveals more details about the approximations used.

Equation (31) describes the behavior of the neutron spin in magnetic media (static magnetic fields) more exactly than Eq. (32). According to the exact QM approach, the origin of the spin precessions is a change in the phase difference between the states with opposite spins (in the relative phase). This phase difference is related to the difference in the velocities of neutrons in the states with the spin ''up'' and ''down'' the field. However, when the Bloch equation is used, the difference in velocities is transformed into the difference in total neutron energies $\delta E_\pm = \pm|\mu_n B|$ of the respective spin states. As the kinetic energy in the reference frame moving with the neutron is equal to 0, the use of the Bloch equation suggests that the total energies $\delta E_\pm$ change as a function of the field magnitude (inelastic interaction). A difference in the total energies of the neutron in the states with the spin ''up'' (+) and ''down'' (−) the field is known to produce not statical, but dynamic precession with a Larmor frequency $|\delta E_+ - \delta E_-|/\hbar = 2|\mu_n B|/\hbar$. In contrast to the Bloch equation, Eq. (31) takes into account such a subtle phenomenon as deviation of the precession frequency from the Larmor frequency due to nonlinearity of the dependence of the neutron wave phase on the magnitude of the potential (see the paper on ''optical precession''[20]).

The effect of a r.f. flipper may be efficiently described by introduction of the time-dependent phases $E_{\pm}t/\hbar$, different for the states with the opposite spins, into the respective eigenvalues of the phase operator ($E_{\pm}=E\pm\delta E$ are the total energies for the corresponding spin states of the neuron, δE is the Zeeman splitting in the static field of the r.f. flipper), as well as into the respective eigenvalues of the operator $\hat{k}$ (corresponding to $E_{+}\neq E_{-}$). Then Eq. (31) may be used to describe not only precession, but also such phenomena (described in Ref. 9) as the spin nutation related to the superposition of the static and dynamic precessions and the spin multiprecession (superposition of numerous spin precessions about several mutually noncollinear axes). The eigenvalues of the wavevector operators may contain the imaginary part, so another advantage of Eq. (31) is that it enables taking into account the beam attenuation (including the spin-dependent attenuation related, say, to the accompanying magnetic scattering at large angles) by means of the optical theorem, i.e., in a most natural way.

As it follows from the consideration given above, the use of either Eq. (31) or Eq. (32) implies that the structure is represented by a sequence of homogeneous layers with flat boundaries perpendicular to a given neutron trajectory. It coincides with the assumptions that neutron trajectories in the real structure are straight and only the field value at one point of the trajectory is effective at any instant. Neither the Bloch equation nor Eq. (31) takes refaction during passage of numerous boundaries along neutron trajectories into account (one-dimensional consideration is no longer valid, if the angles between the neutron trajectories and the gradients of the nuclear and magnetic potentials in the sample are taken into account).

Neither the Bloch equation nor Eq. (31) leads directly to depolarization. The events for different neutron trajectories are considered to be incohererent, and depolarization results from averaging over different neutron paths inside the sample. As it follows from the consideration given above, the classical Larmor approach (LA) to the depolarization theory is equivalent to the refracted-wave scattering approach (RWSA) in the limit when neutron interaction with the structure may be described by the model that the neutrons experience the field point-by-point along each trajectory and traverse the magnetic medium without being deflected. RWSA and the conventional scattering approach[21] (SA) are identical for zero mean fields. On the other hand, the conclusion[22] about the identity of LA and SA for zero mean field (provided that all neutrons enter the ''analyzer +detector''), strictly speaking, can be taken only with reservation. Indeed, calculation of depolarization in LA is reduced to the integration carried out on the (x,y) plane in the reciprocal space, even though in the real space the scattering is assumed to be straightforward (the model of a sequence of homogeneous layers perpendicular to neutron trajectories). On the other hand, in SA the integration is carried out on the Ewald sphere. Of course, if the main contribution into depolarization is from scattering at small angles, the regions of integration in the reciprocal space practically coincide, and LA is equivalent to SA. When the mean field is nonzero, the relation between LA and SA is more complicated. In a certain sense the two approaches are complimentary.[16,23] Yet, whenever the result of integration on the (x,y) plane in the reciprocal space may be expected to coincide with that for integration on the Ewald sphere, one may use LA which is usually more simple and sraightforward for calculations. As usually, the use of LA implies that all neutrons enter the ''analyzer+detector,'' and the model of the structure is supposed to be adequate. Of particular interest is the conclusion that both SA and LA stem from one approach, RWSA. A more detailed depolarization theory may be built in the frame of RWSA; however, it is beyond the scope of the present work.

VI. CONCLUSIONS

The use of the refracted-wave scattering approach (RWSA) allowed us to encompass some aspects of neutron scattering in magnetic media that are not described in the standard approaches. The presence of a nonzero mean field plays a fundamental role under neutron scattering. Even the magnetic dipole scatters neutrons differently in the presence of an external field. The corresponding SF scattering cross sections should be corrected for refraction. The NSF and SF scattering amplitudes are easily obtained in the refracted-wave Born approximation (RWBA), because they are calculated as in the standard Born approximation (BA) (see Ref. 14), but for the corrected-for-refraction momentum transfers.

The latter implies that the selection rule (proved in BA), according to which only the magnetic induction variations perpendicular to the scattering vector are efficient for magnetic scattering, is valid also in RWBA. However, owing to refraction corrections (for momentum transfers and SF scattering cross sections), the magnetic scattering in RWBA is sensitive to the component of the mean field $\langle \mathbf{B}\rangle$ parallel to the scattering vector. We mention also that the angular (owing to refraction) deviations under SF scattering depend on $\langle \mathbf{B}\rangle$, i.e., on all components of $\langle \mathbf{B}\rangle$. Moreover, the selection rule for magnetic scattering is not valid in that classical sense that the cross section for the elastic scattering of monochromatic neutrons into a given direction depends only on a magnetization component lying in one plane (perpendicular to $\mathbf{q}$). Such a plane is absent in the case of RWBA, since from the four scattering vectors (Fig. 1) related to the elastic scattering into a given direction only $\mathbf{q}_{++}$ and $\mathbf{q}_{--}$ are parallel (for the sake of simplicity, the wave vectors $\mathbf{k}_a^{+}$ and $\mathbf{k}_a^{-}$ are assumed to be parallel). Consequently, the full cross section for the elastic scattering into a given direction is sensitive to all components of $\mathbf{B}$. Of course, it plays an essential role only at (usually very small) scattering angles for which the SF momentum transfers are comparable with q_p. In this case the very origin of the NSF and SF scattering into a given direction is different: not only the directions but also the magnitudes of the respective NSF and SF momentum transfers essentially differ. This is due to the change of the kinetic energy under SF scattering. The kinetic energy is changed by a quantity equal to the Zeeman splitting. Such a quantity is transferred to or from the kinetic energy related to the momentum component parallel to the respective SF momentum transfer. It is worth emphasizing that the exchange between the potential and kinetic energies of the neutron in the field $\langle \mathbf{B}\rangle$ does not change its total energy, and the corresponding SF scattering is elastic.

Angular beam splitting and nonfrontal spin precession are shown to be inherent to combined NSF and SF neutron scattering in magnetic media. The angular beam splitting in a uniform mean field (it can be an external field) considered in the present work is not a Stern-Gerlach splitting observed in magnetic field gradients, when no spin flipping is required. Nor the magnetic field variations solely yield the splitting (the same field variations with $\langle \mathbf{B} \rangle = 0$ produce no beam splitting). The effect is a manifestation of the refraction law related to the difference in the refractive indices before and after neutron spin flipping. This is a generalization of the effect of the angular splitting of a specularly reflected beam[10,11] that was observed in a recent publication.[3]

The evolution of the initial plane wave states is envisaged in the qualitative ''minimal theory''[8] as splitting-up into a multitude of waves due to refraction at a sequence of boundaries between homogeneous regions. The author remains ''in the realm of the Stern-Gerlach effect.''[8] The refraction here is due to neutron transmission into an optically different region (see also Ref. 24). The same is true for numerous experiments on polarized neutron reflection from layered magnetic structures. The combined effect of small angle (''straight forward'') spin-flip scattering within a demagnetized layer and refraction at its boundaries is another spectacular observation.[25] Therefore, generally, $\langle \mathbf{B} \rangle$ is not constant in the scattering region. This problem may be considered within RWSA, too. However, it is out of the scope of the present paper. We mention only that, when the gradient of the mean potential is constant in direction, the corresponding theory will resemble that of specular and diffuse scattering at layered structures.

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[1] O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941); I. I. Gurevich and L. V. Tarasov, *Physics of Low Energy Neutrons* (Nauka, Moscow, 1965); F. Mezei, Z. Phys. **255**, 146 (1972); M. Th. Rekveldt, *ibid.* **259**, 391 (1973); S. V. Maleyev, J. Phys. C **43**, 723 (1982).

[2] *Neutron Spin Echo*, edited by F. Mezei (Springer, Heidelberg, 1980).

[3] G. P. Felcher, S. Adenwalla, V. O. de Haan, and A. A. Van Well, Nature (London) **377**, 409 (1995).

[4] C. Fermon, Physica B **213-214**, 910 (1995).

[5] G. P. Felcher, S. Adenwalla, and R. J. Goyette, J. Appl. Crystallogr. (to be published).

[6] F. Ott, C. Fermon, Physica B **234-236**, 522 (1997).

[7] N. K. Pleshanov, Physica B **234-236**, 516 (1997).

[8] F. Mezei, Physica B **151**, 74 (1988).

[9] N. K. Pleshanov, Phys. Lett. A (to be published).

[10] V. K. Ignatovich, *Physics of Ultracold Neutrons* (Nauka, Moscow, 1988).

[11] N. K. Pleshanov, Z. Phys. B **94**, 233 (1994).

[12] L. D. Landau and E. M. Lifshits, *Quantum Mechanics* (Nauka, Moscow, 1963).

[13] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1970), Vol. 2.

[14] S. W. Lovesey, *Theory of Neutron Scattering from Condensed Matter* (Clarendon, Oxford, 1984), Vols. 1 and 2.

[15] V. M. Pusenkov, N. K. Pleshanov, V. G. Syromyatnikov, V. A. Ul'yanov, and A. F. Schebetov, J. Magn. Magn. Mater. **175**, 237 (1997).

[16] B. P. Toperverg and J. Weniger, Z. Phys. B **47**, 105 (1989).

[17] G. M. Drabkin, Sov. Phys. JETP **16**, 781 (1963).

[18] N. K. Pleshanov, Z. Phys. B **100**, 423 (1996).

[19] R. Rosman, Ph.D. thesis, Delft University, 1991.

[20] A. I. Frank, Nucl. Instrum. Methods Phys. Res. A **284**, 161 (1989).

[21] S. V. Maleyev and V. A. Ruban, Sov. Phys. JETP **35**, 222 (1972).

[22] R. Rosman, B. P. Toperverg, and M. Th. Rekveldt, Z. Phys. B **79**, 61 (1990).

[23] M. de Yong, Ph.D. thesis, Delft University, 1996.

[24] S. Sh. Shil'shtein, N. O. Elyutin, V. A. Somenkov, Fiz. Tverd. Tela (Leningrad) **16**, 2008 (1974); O. Schaerpf and H. Strothmann, Phys. Scr. **24**, 58 (1988).

[25] Th. Krist, D. J. Mueller, and F. Mezei, Physica B (to be published).