Domain-wall and domain-structure dynamics in weak ferromagnets

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(Received 14 September 1998)

Domain-wall drift motion in two-sublattice weak ferromagnets of rare-earth type induced by an external oscillating magnetic field is studied. The dependences of the drift velocity on the amplitude, frequency, and polarization of the field are obtained for two possible types of 180-degree domain walls. The possibility of the drift of stripe domain structures is considered. [S0163-1829(99)08509-4]

I. INTRODUCTION

There is a steady interest in investigations of the dynamic properties of domain walls (DW) in magnetically ordered crystals. Principal attention is paid, in both theoretical and experimental studies, to two main types of DW motions: (1) translational DW motion in a constant external magnetic field and (2) vibrational DW motion in an oscillating external field. The theoretical dependences of the DW velocity on the external field, the maximum steady velocities of DW, and the nonlinear regime of wall oscillations have been found for all principal magnet types (ferro-, antiferro-, weak ferro-, and ferrimagnets).

Experiment[1,2] has revealed one more type of DW motion—wall drift, i.e., the onset of a constant DW velocity component, in an oscillating external magnetic field. A similar effect was observed in Refs. 3 and 4 for another topological soliton form, viz., a Bloch line.

Domain-wall drift in a ferromagnet (FM) was predicted from energy considerations in Refs. 5 and 6. A more consistent analysis of DW drift in FM, based on the solution of the equations of motion averaged over the field oscillation period, was carried out in Ref. 7; an analogous method was used in Ref. 8 to analyze the drift of Bloch lines in DW. However, the results of Refs. 7 and 8 are valid only for frequencies ω substantially higher than the ferromagnetic-resonance frequency.

The most adequate approach for this class of problems, based on a specific perturbation theory for solitons, has been proposed in Ref. 9 when analyzing the drift of Bloch lines in the simplest model of one-sublattice FM. A similar approach was used in Refs. 10 and 11 to study DW drift in FM and antiferromagnets under the influence of external magnetic fields of different polarizations. The drift motion of kinks in the framework of the well-known ''φ^4 model'' has been studied in Ref. 12

The present paper is devoted to a study of DW drift in another type of magnet—two-sublattice weak ferromagnets (WFM). Dynamic properties of WFM differ substantially from those in a one-sublattice FM.[13,14] In particular, the velocity limit of steady DW motion, which is determined only by exchange interactions, and the DW mobility in an external magnetic field, greatly exceed the corresponding values in FM. One should therefore expect the DW drift velocity in an oscillating field to be also substantially higher than in an FM.

As an example, we consider a WFM of the type of rare-earth orthoferrites, the DW dynamics of which has been studied in detail both theoretically and experimentally (see, e.g., the review in Ref. 14 and the literature cited there).

II. GENERAL EQUATIONS AND DOMAIN WALLS

We consider a model of a two-sublattice weak ferromagnet whose state is determined by two sublattice-magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$; $M_0=|\mathbf{M}_{1,2}|=\text{const}$. For rare-earth orthoferrites, characterized by a symmetry $2_x^- 2_z^-$ (the Cartesian axes *x, y*, and *z* are oriented along the *a, b*, and *c* axes of the crystal, respectively), an energy of the magnet can be written in the form:

$$W=\int d\mathbf{r}\left\{\frac{\delta}{2}\mathbf{M}^2+\frac{\alpha}{2}(\nabla\mathbf{L})^2+\mathbf{d}[\mathbf{M},\mathbf{L}]+\frac{\beta_1}{2}L_z^2+\frac{\beta_2}{2}L_y^2-\mathbf{MH}\right\}, \tag{1}$$

where $\mathbf{M}=\mathbf{M}_1+\mathbf{M}_2$, $\mathbf{L}=\mathbf{M}_1-\mathbf{M}_2$ are the vectors of weak ferromagnetism and of antiferromagnetism, respectively; $\delta>0$ is the constant of homogeneous exchange between sublattices, α is the inhomogeneous exchange constant, β_1 and β_2 are the anisotropy constants, $\mathbf{H}$ is the external magnetic field, $\mathbf{d}=d\mathbf{e}_y$, $\mathbf{e}_y$ is the unit vector along the *y* axis, *d* is the exchange-relativistic Dzyaloshinskii constant. In energy (1) we omit the small terms associated with nonantisymmetric Dzyaloshinskii interaction, which are of importance only for relaxation processes in WFM,[15] and the anisotropy terms of the fourth order, which should be taken into account only in the vicinity of the spin reorientation region.[16]

The WFM spin dynamics can be described by means of the equations of motion for the sublattice magnetization vectors $\mathbf{M}_{1,2}$ (Landau-Lifshitz equations) or of the respective equations for vectors $\mathbf{M}$ and $\mathbf{L}$. The latter can be considerably simplified using the fact that the homogeneous exchange interaction is much greater than other interactions involved in the energy (1). In the main approximation with respect to the small parameter $\delta^{-1} \ll 1$, the magnetization vector **M** can be expressed via vector **L**,[13]

$$\mathbf{M} = \frac{1}{\delta}\left\{ 2[\mathbf{l}[\mathbf{H}\mathbf{l}]] + \frac{2}{g}[\dot{\mathbf{l}}\mathbf{l}] + M_0[\mathbf{d}\mathbf{l}] \right\} \tag{2}$$

where $\mathbf{l} = \mathbf{L}/|\mathbf{L}|$ is the antiferromagnetism unit vector, $\mathbf{l}^2 = 1$; g is the gyromagnetic ratio, a superior dot marks a derivative with respect to time. As shown in Refs. 13 and 17, it enables one to describe the nonlinear macroscopic dynamics of a two-sublattice WFM on the basis of the closed equation for the vector **l**, which is the variational equation for the Lagrange function $L\{\mathbf{l}\}$,

$$L = M_0^2 \left\{ \frac{\alpha}{2}\left[\frac{1}{c^2}\dot{\mathbf{l}}^2 - (\nabla \mathbf{l})^2 \right] - \frac{1}{2}(\beta_1 l_z^2 + \tilde{\beta}_2 l_y^2) + \frac{4}{\delta g M_0^2}[\mathbf{H}[\dot{\mathbf{l}}\mathbf{l}]] - \frac{2}{\delta M_0^2}(\mathbf{l}\mathbf{H})^2 + \frac{2d}{\delta M_0}(l_z H_x - l_x H_z) \right\}, \tag{3}$$

where $c = g M_0 (\alpha\delta)^{1/2}/2$ is the characteristic velocity (it coincides with the minimal spin-wave phase velocity), and $\tilde{\beta}_2 = \beta_2 + d^2/\delta$. The dynamic stopping of the DW, due to dissipative processes, will be taken into account by using the dissipative function Q,

$$Q = \frac{\lambda M_0}{2g}\dot{\mathbf{l}}^2, \tag{4}$$

where λ is the relaxation constant.

Since the components of the vector **l** are connected by the relation $\mathbf{l}^2 = 1$, it is convenient to rewrite the Lagrange function (3) in term of two independent angle variables θ and φ which parametrize the unit vector **l**:

$$l_x + i l_z = \sin\theta \exp(i\varphi), \quad l_y = \cos\theta. \tag{5}$$

The equation of motion for the variables θ and φ, allowing for the relaxation terms, takes the form

$$\alpha\left(\Delta\theta - \frac{1}{c^2}\ddot{\theta} \right) + \sin\theta\cos\theta \times \left[\frac{\alpha}{c^2}\dot{\varphi}^2 - \alpha(\nabla\varphi)^2 + \tilde{\beta}_2 - \beta_1 \sin^2\varphi \right] + \frac{2d}{\delta}(h_x \sin\varphi - h_z\cos\varphi)\cos\theta - \frac{4}{\delta}(h_x \sin\theta\cos\varphi + h_y\cos\theta + h_z\sin\theta\sin\varphi)(h_x\cos\theta\cos\varphi - h_y\sin\theta + h_z\cos\theta\sin\varphi) + \frac{4}{\delta g M_0}[\dot{h}_x\sin\varphi - \dot{h}_z\cos\varphi + 2\dot{\varphi}\sin^2\theta(h_x\cos\varphi + h_z\sin\varphi) + h_y\dot{\varphi}\sin 2\theta] = \frac{\lambda}{g M_0}\dot{\theta}, \tag{6}$$

$$\alpha\nabla[\sin^2\theta(\nabla\varphi)] - \frac{\alpha}{c^2}(\dot{\varphi}\sin^2\theta)^{\cdot} - \beta_1\sin^2\theta\sin\varphi\cos\varphi + \frac{2d}{\delta}\sin\theta(h_x\cos\varphi + h_z\sin\varphi) + \frac{4}{\delta g M_0}\left[\frac{\sin 2\theta}{2}(\dot{h}_x\cos\varphi + \dot{h}_z\sin\varphi) - \dot{h}_y\sin^2\theta - 2\dot{\theta}\sin^2\theta(h_x\cos\varphi + h_z\sin\varphi) - h_y\dot{\theta}\sin 2\theta \right] + \frac{4\sin\theta}{\delta}(h_x\sin\theta\cos\varphi + h_y\cos\theta + h_z\sin\theta\sin\varphi)(h_x\sin\varphi - h_z\cos\varphi) = \frac{\lambda}{g M_0}\dot{\varphi}\sin^2\theta, \tag{7}$$

where $\mathbf{h} = \mathbf{H}/M_0$.

If $\beta_1, \tilde{\beta}_2 > 0$, the vector **l** in the absence of an external field is collinear with the x axis (a axis of the crystal) in the homogeneous ground state. It can be easily seen in this case the equations of motion have two particular classes of nontrivial solutions describing two types of 180-degree DW can exist then in the magnet under consideration: the vector **l** rotates in the (XZ) plane in one of them (this type of DW will be referred as DW1) and in the (XY) plane in the other (DW2).

A stability analysis of these two types of DW (Refs. 13 and 16) showed that in the case $\tilde{\beta}_2 > \beta_1$, the stable DW is the one with rotation of **l** in the (XZ) plane (DW1). This DW corresponds to $\theta = \theta_0 = \pi/2$, and the angle variable $\varphi_0 = \varphi_0(y)$ satisfies the equation

$$\alpha\varphi_0'' - \beta_1\sin\varphi_0\cos\varphi_0 = 0 \tag{8}$$

(we shall assume that the magnetization distribution in the DW is nonuniform along the y axis; a prime denotes differentiation with respect to this coordinate). A static 180-degree DW1 in which the functions $\varphi_0(y)$ satisfy the boundary conditions

$$\varphi_0(-\infty) = 0, \quad \varphi_0(+\infty) = \pi, \quad \varphi_0'(\pm\infty) = 0 \tag{9}$$

is described by the relations

$$\varphi_0' = \frac{1}{y_0}\sin\varphi_0 = \frac{1}{y_0}\operatorname{sech}\left(\frac{y}{y_0}\right), \quad \cos\varphi_0 = -\tanh\left(\frac{y}{y_0}\right), \tag{10}$$

where $y_0 = (\alpha/\beta_1)^{1/2}$ is the wall thickness.

According to Eq. (2), the magnetization vector **M** in a static DW1 also rotates in the (XZ) plane, $|\mathbf{M}|$ being constant:

$$\mathbf{M} = \frac{d M_0}{\delta}(\mathbf{e}_x\sin\varphi_0 - \mathbf{e}_z\cos\varphi_0).$$

In the case $\beta_1 > \tilde{\beta}_2$, the stable DW is the one with rotation of **l** in the (XY) plane (DW2). In this DW2 $\varphi = \varphi_0 = 0$, and

$$\theta_0' = \frac{1}{\tilde{y}_0}\cos\theta_0 = \frac{1}{\tilde{y}_0}\operatorname{sech}\left(\frac{y}{\tilde{y}_0}\right), \quad \sin\theta_0 = -\tanh\left(\frac{y}{\tilde{y}_0}\right), \tag{11}$$

where $\tilde{y}_0 = (\alpha/\tilde{\beta}_2)^{1/2}$ is the thickness of DW2. In opposite to DW1, the magnetization vector **M** in a static DW2 does not rotate but changes its value:

$$\mathbf{M}=-\frac{dM_0}{\delta}\mathbf{e}_z \sin\theta_0$$

III. INDUCED MOTION OF DOMAIN WALLS: LINEAR APPROXIMATION

Let us consider now the solutions of the equations of motion in an external magnetic field. For definiteness, we shall consider first a forced motion of a DW1. The behavior of DW2 can be studied similarly, and the results for DW2 will be discussed in Sec. V.

Under the influence of a constant external field of definite orientation (in our case along the z axis), a DW moves with a fixed velocity determined by the balance of the magnetic pressure acting on the DW and the dynamic stopping force.[13] In an oscillating field, the wall oscillates at the field frequency[18] and we shall show below that its center drifts with a certain velocity. In addition, the presence of the field distorts the shape of the DW.

Assuming the external field amplitude to be small enough, we determine the drift velocity of the DW and the distortion of its form, following,[9,10] by one of the perturbation-theory versions for solitons. To this end we introduce a collective variable $Y(t)$, which has the meaning of the coordinate of the DW center at the instant t, and seek a solution of Eqs. (6) and (7) in the form

$$\theta=\frac{\pi}{2}+\vartheta(\xi,t), \quad \varphi=\varphi_0(\xi)+\psi(\xi,t), \tag{12}$$

where $\xi=y-Y(t)$. The function $\varphi_0(\xi)$ describes the motion of an undistorted DW1 [the structure of $\varphi_0(\xi)$ is the same as that of $\varphi_0(y)$ in the static solution (10)]. The wall drift velocity is defined as the instantaneous DW velocity $V(t)=\dot{Y}(t)$ averaged over the oscillation period, $V_{\rm dr}=\overline{V(t)}$ (the bar denotes averaging over the external-field oscillation period).

We represent the functions $\vartheta(\xi,t)$ and $\psi(\xi,t)$, describing the distortion of the DW shape, as well as the wall velocity $V(t)$, by series in powers of the field amplitude, recognizing that we are interested only in stimulated DW motion:

$$\vartheta(\xi,t)=\vartheta_1(\xi,t)+\vartheta_2(\xi,t)+\ \dots,$$

$$\psi(\xi,t)=\psi_1(\xi,t)+\psi_2(\xi,t)+\ \dots,$$

$$V=V_1+V_2+\ \dots, \tag{13}$$

where the subscripts $n=1,2,\dots$ denote the order of smallness of the quantity to the field amplitude $\psi_n,\vartheta_n,V_n\sim h^n$. We substitute the expansions (13) in Eqs. (6)–(7) and separate terms of different orders of smallness. Obviously, in the zeroth approximation we obtain Eq. (8), which describes a DW1 at rest.

The first-order perturbation-theory equation can be written in the form

$$(\hat{L}+\hat{T})\psi_1=\frac{2d}{\beta_1\delta}[h_x\cos\varphi_0(\xi)+h_z\sin\varphi_0(\xi)] -\frac{4\dot{h}_y}{gM_0\beta_1\delta}+\frac{\alpha}{y_0\beta_1 c^2}(\dot{V}_1+\omega_r V_1)\sin\varphi_0(\xi) \tag{14}$$

$$(\hat{L}'+\hat{T}+\sigma)\vartheta_1=\frac{4}{\beta_1\delta g M_0}[\dot{h}_x\sin\varphi_0(\xi)-\dot{h}_z\cos\varphi_0(\xi)], \tag{15}$$

where $\omega_0=c/y_0=gM_0(\beta\delta)^{1/2}/2$ is the activation frequency of the lower spin-wave mode, $\omega_r=\lambda\delta gM_0/4$ is the characteristic relaxation frequency, $\sigma=(\tilde{\beta}_2-\beta_1)/\beta_1>0$,

$$\hat{T}=\frac{1}{\omega_0^2}\frac{d^2}{dt^2}+\frac{\omega_r}{\omega_0^2}\frac{d}{dt}.$$

The operator $\hat{L}$ takes the form of a Schrodinger operator with a nonreflecting potential:

$$\hat{L}=-y_0^2\frac{d^2}{d\xi^2}+1-\frac{2}{\cosh^2(\xi/y_0)}. \tag{16}$$

The spectrum and the wave functions of the operator $\hat{L}$ (16) are well known. It has one discrete level with eigenvalue $\lambda_0=0$ corresponding to a localized wave function

$$f_0(\xi)=\frac{1}{(2y_0)^{1/2}\cosh(\xi/y_0)}, \tag{17}$$

and also to a continuous spectrum $\lambda_k=1+k^2y_0^2$ corresponding to the eigenfunctions

$$f_k(\xi)=\frac{1}{b_k L^{1/2}}[\tanh(\xi/y_0)-iky_0]e^{ik\xi}, \tag{18}$$

where $b_k=(1+k^2y_0^2)^{1/2}$ and L is the crystal length.

The functions $\{f_0,f_k\}$ form a complete orthonormalized set, and it is natural to seek the first-approximation solutions of Eqs. (14) and (15) in the form of an expansion in this set. For a monochromatic external field of frequency ω we put

$$\vartheta_1(\xi,t)=\mathrm{Re}\left\{\left[\sum_k c_k f_k(\xi)+c_0 f_0(\xi)\right]e^{i\omega t}\right\}, \tag{19}$$

$$\psi_1(\xi,t)=\mathrm{Re}\left\{\left[\sum_k d_k f_k(\xi)+d_0 f_0(\xi)\right]e^{i\omega t}\right\}. \tag{20}$$

One important remark is in order here. The first-approximation Eqs. (14)–(15) describes excitation of linear spin waves against a DW1 background. The last term in the expansion of the function $\psi_1(\xi,t)$ corresponds to the Goldstone mode, i.e., to DW motion as a whole. The corresponding degree of freedom of the system, however, has already been taken into account by introducing the collective coordinate $Y(t)$ into the definition of the variable ξ. The Goldstone mode should therefore be left out of the expansion (20), i.e., one must put $d_0=0$ (for a detailed discussion of this question see Rajaraman's book[19]). This condition leads to the requirement that the right-hand side of Eq. (14) be orthogonal to the function $f_0(\xi)$, which determines in turn the equation for the DW velocity $V_1(t)$ in the approximation linear in the external field:

$$\dot{V}_1+\omega_r V_1=-\frac{2dc^2y_0}{\alpha\delta}h_z+\frac{2\pi c^2 y_0}{\alpha\delta g M_0}\dot{h}_y. \tag{21}$$

The solution of Eq. (21) describes the DW oscillations in an oscillating external field and, as can easily seen, does not lead to a DW drift, i.e., $\overline{V_1(t)}=0$.

If $h_y=0$, Eq. (21) agrees, apart from the notation, (in the limit of low velocities $V\ll c$) with the equation obtained for the DW velocity in Ref. 18 by a somewhat different method in the framework of the adiabatic approximation. The presence in the right-hand side of Eq. (18) of a second term not connected with the Dzyaloshinskii interaction attests to the possibility of exciting stimulated DW oscillations in a ''pure'' antiferromagnet in which $d=0$. This effect was first noted in Ref. 20.

The coefficients c_k, c_0, and d_k in the expansions (19) and (20) can be found in standard fashion multiplying the right-hand sides of Eqs. (14) and (15) by $f_k^*(\xi)$ and $f_0^*(\xi)$ and integrating with respect to the variable ξ. For a monochromatic external field of frequency ω, with all three components different from zero and with arbitrary phase shifts,

$$H_x=H_{0x}\cos\omega t,\quad H_y=H_{0y}\cos(\omega t+\chi_1),$$
$$H_z=H_{0z}\cos(\omega t+\chi) \tag{22}$$

we obtain from Eqs. (14)–(15)

$$\vartheta_1(\xi,t)=2\,\mathrm{Re}\{a_1(t)\sin\varphi_0(\xi)+a_2(t)\cos\varphi_0(\xi)\},$$
$$\psi_1(\xi,t)=2\,\mathrm{Re}\{a_3(t)\cos\varphi_0(\xi)+a_4(t)G(\xi)\}. \tag{23}$$

We have introduced here the notation

$$a_1(t)=\frac{2i\omega}{\beta_1\delta g M_0}\frac{h_{0x}e^{i\omega t}}{(\sigma-q_1+iq_2)},$$

$$a_2(t)=-\frac{2i\omega}{\beta_1\delta g M_0}\frac{h_{0z}e^{i(\omega t+\chi)}}{(1+\sigma-q_1+iq_2)},$$

$$a_3(t)=\frac{d}{\beta_1\delta}\frac{h_{0x}e^{i\omega t}}{(1-q_1+iq_2)},$$

$$a_4(t)=-\frac{2i\omega}{\beta_1\delta g M_0}h_{0y}e^{i(\omega t+\chi_1)},$$

$$G(\xi)=\frac{y_0}{2}\int_{-\infty}^{+\infty}dk\frac{[\tanh(\xi/y_0)\sin k\xi-ky_0\cos k\xi]}{b_k^2(\lambda_k-q_1+iq_2)\sinh(\pi k y_0/2)}, \tag{24}$$

where $q_1=(\omega/\omega_0)^2$, $q_2=(\omega\omega_r/\omega_0^2)$.

It follows from Eqs. (23) and (24) that the external-field components H_x and H_z excite bulk oscillations only with $k=0$, whereas the presence of the field component H_y makes possible excitation of bulk spin waves with $k\neq 0$.

IV. SECOND APPROXIMATION: DW DRIFT

Let us go to an analysis of the second approximation in the external magnetic field amplitude. We shall not write down the pertinent equations in general form, since they are extremely cumbersome, but only an equation, averaged over the period of the oscillations, which follows from Eq. (7):

$$\hat{L}\Phi_2(\xi)=\frac{\lambda}{gM_0}\varphi_0'(\xi)\overline{V}_2+\overline{N(\xi,t)}, \tag{25}$$

where $\Phi_2(\xi)=\overline{\psi_2(\xi,t)}$, and the function $N(\xi,t)$ is defined as

$$N(\xi,t)=\frac{\alpha}{c^2}(\dot{V}_1+\omega_r V_1)\psi_1'-\frac{\alpha}{c^2}V_1^2\varphi_0''-2\alpha\varphi_0'\vartheta_1\vartheta_1'$$
$$+\beta_1\psi_1^2\sin 2\varphi_0-\frac{2d}{\delta}(h_x\sin\varphi_0-h_z\cos\varphi_0)$$
$$-\frac{4}{\delta}[(h_y^2-h_x^2)\sin\varphi_0\cos\varphi_0+h_xh_z\cos 2\varphi_0]$$
$$-\frac{4}{\delta g M_0}[(\dot{h}_x\cos\varphi_0+\dot{h}_z\sin\varphi_0)\vartheta_1$$
$$+2(h_x\cos\varphi_0+h_z\sin\varphi_0)\dot{\vartheta}_1. \tag{26}$$

The second equation of the system, which follows from Eq. (6) and defines the function $\vartheta_2(\xi,t)$, has a similar structure, but contains no second-order term in the expansion of the DW velocity (V_2) and will therefore be of no interest.

Just as in the first-approximation equation (14), we must stipulate that the expansion of the function $\Phi_2(\xi)$ in terms of the eigenfunction of the operator $\hat{L}$, contains no shear mode, i.e., it is necessary that the right-hand side of Eq. (25) be orthogonal to $f_0(\xi)$ (17). This yields an expression for the DW drift velocity $V_{dr}=\overline{V}_2$:

$$V_{dr}=-\frac{gM_0y_0}{2\lambda}\int_{-\infty}^{+\infty}d\xi\overline{N(\xi,t)}\varphi_0'(\xi). \tag{27}$$

Substituting the functions $\psi_1(\xi,t)$ and $\vartheta_1(\xi,t)$ (23)–(24) calculated in the preceding section in Eq. (26), and integrating over the oscillation period and integrating in Eq. (27), we obtain for the drift velocity V_{dr}:

$$V_{dr}=\nu_{xz}(\omega,\chi)H_{0x}H_{0z}+\nu_{xy}(\omega,\chi_1)H_{0x}H_{0y}, \tag{28}$$

where $\nu_{xz}(\omega;\chi)$ and $\nu_{xy}(\omega;\chi_1)$ are the certain functions of the frequency and of the phase shifts, which we shall call nonlinear mobilities of the domain wall (their structure will be given below).

It follows from Eq. (28) that the DW1 drift occurs only if at least two components of the magnetic field differ from zero—either $H_x\neq 0$ and $H_z\neq 0$ or $H_x\neq 0$ and $H_y\neq 0$. This fact can be interpreted in the following manner: the z or y component of the field, as follows from Eq. (21), cause DW1 oscillations, while the x component ensures different values of the wall's linear mobility as it moves in the positive and negative y direction. If, however, the field is oriented in the (YZ) plane, there is no DW1 drift.

We consider next DW drift in a field polarized separately in the (*XZ*) or (*XY*) plane.

A. Field in (*XZ*) plane

The nonlinear mobility ν_{xz} which determines the drift velocity in an oscillating field polarized (in general, elliptically) in the (*XZ*) plane [see Eq. (22) for $H_y=0$] is of the form

$$\nu_{xz}(\omega,\chi)=\nu_0[D(\omega,\chi)+A(\omega,\chi)], \tag{29}$$

where

$$\nu_0=\frac{\pi g^2 y_0}{4\omega_r},$$

$$D(\omega,\chi)=\frac{d^2}{\beta_1\delta}\frac{[(q_1-1)\cos\chi+q_2\sin\chi]}{(q_1-1)^2+q_2^2},$$

$$A(\omega,\chi)=q_1q_2\frac{q_2\cos\chi+[(q_1-\sigma)(q_1-\sigma-1)+q_2^2]\sin\chi}{[(q_1-\sigma)^2+q_2^2][(q_1-\sigma-1)^2+q_2^2]}. \tag{30}$$

We see from Eqs. (29) and (30) that the nonlinear mobility ν_{xz} is determined by two terms of different type. The first one, $D(\omega,\chi)$, is connected with the presence Dzyaloshinskii interaction in the WFM, while the second term, $A(\omega,\chi)$, differs from zero even in a ''pure'' antiferromagnet.

To compare the contributions of both terms at different values of the frequency and phase shift χ and to obtain a numerical estimate of the drift velocity we use the values of the parameters indicative of the typical and well-investigated WFM—yttrium orthoferrite $YFeO_3$ (see, e.g., Ref. 14):

$$\sigma\approx 2;\quad y_0\approx 10^{-6}\ \text{cm};\quad g\approx 1.76\times 10^7\ \text{s}^{-1}\,\text{Oe}^{-1};$$

$$\omega_0=c/y_0\approx 2\times 10^{12}\ \text{s};\quad d^2/\beta_1\delta\sim 1.$$

The relaxation frequency ω_r can be calculated from the experimentally known linear mobility μ of a DW in a static field:[13]

$$\omega_r=\frac{g^2 y_0 H_d}{\mu}$$

where H_d is the Dzyaloshinskii field. For $YFeO_3$ we have:[14] $H_d=1.4\times 10^5$ Oe, $\mu\approx 6.2\times 10^3$ cm/s Oe, whence $\omega_r=0.7\times 10^{10}\,\text{s}^{-1}$ (this value of relaxation frequency corresponds to a dimensionless relaxation constant $\lambda\sim 10^{-3}$). This yields an estimate of the characteristic nonlinear mobility ν_0:

$$\nu_0=\frac{\pi\mu}{4H_d}\approx 3.5\times 10^{-2}\ \text{cm/s Oe}^2. \tag{31}$$

It follows from Eq. (31) that the characteristic DW drift velocity $V_0\sim\nu_0 H_0^2$ is much lower than the DW stationary velocity in a static field of the same strength (by an approximate factor H_0/H_d). For an oscillating field of amplitude $H_0\sim 10$ Oe we obtain $V_0\sim 3.5$ cm/s. The DW drift velocity increases substantially at resonance frequencies. Consider, for example, the drift in a linear polarized field ($\chi=0$). From Eq. (30) we have for $\chi=0$:

$$D(\omega,0)=\frac{d^2}{\beta_1\delta}\frac{(\omega^2/\omega_0^2-1)}{[(\omega^2/\omega_0^2-1)^2+(\omega\omega_r/\omega_0^2)^2]}, \tag{32}$$

$$A(\omega,0)=\frac{\omega^2}{\omega_0^2}\frac{(\omega\omega_r/\omega_0^2)^2}{[(\omega^2/\omega_0^2-\sigma)^2+(\omega\omega_r/\omega_0^2)^2][(\omega^2/\omega_0^2-\sigma-1)^2+(\omega\omega_r/\omega_0^2)^2]}. \tag{33}$$

Numerical estimates of the frequencies ω_0 and ω_r yield for all frequencies of the external field, up to optical, a value $q_2=(\omega\omega_r/\omega_0^2)\sim 10^{-15}\omega\ll 1$, so that the contribution of the term $A(\omega,0)\sim q_2^2$ to Eq. (29) is small at practically all frequencies, and the principle role is assured by the term $D(\omega;0)$ connected with the Dzyaloshinskii interaction.

In the limiting case of low frequencies ($\omega\ll\omega_0$) the drift velocity is equal to

$$V_{dr}\approx -V_0=-\nu_0 H_{0x}H_{0z} \tag{34}$$

(a negative V_{dr} means that the DW1 moves in the negative direction of the y axis). At high frequencies ($\omega\gg\omega_0$) the drift velocity is positive and decreases as ω^{-2}:

$$V_{dr}(\omega)\approx V_0\left(\frac{d^2}{\beta_1\delta}\right)\left(\frac{\omega_0}{\omega}\right)^2\sim\omega^{-2}. \tag{35}$$

In the frequency region near ω_0 (activation frequency of the lower mode of the bulk spin waves)-a behavior of the function $D(\omega,0)$ is of the ''resonance-antiresonance'' type. The maximum (in absolute value) drift velocity is realized at $\omega=\omega_0\pm\omega_r/2$ and reaches a value of the order of $(\omega_0/\omega_r)V_0\sim 3\times 10^2\,V_0$. Therefore even in relatively weak (~10 Oe) fields the drift velocity at resonance is of the order of 10 m/s.

The second term $A(\omega,0)$ has the two usual resonances at the frequencies $\omega_1=\omega_0\sigma^{1/2}$ and $\omega_2=(1+\sigma)^{1/2}\omega_0$, which coincide respectively with the activation frequencies of the localized (on the DW) and upper bulk spin-wave modes. At the resonances we have $A(\omega_{1,2},0)\sim 1$, which is comparable with $D(\omega_{1,2},0)$. The dependence of the drift velocity on the frequency at $\chi=0$ (normalized to the characteristic value of V_0) is shown schematically in Fig. 1(a).

At phase shifts χ that differ from zero ($0<\chi<\pi/2$) but are not too close to $\pi/2$ the frequency dependence of V remains approximately the same as for $\chi=0$. The function

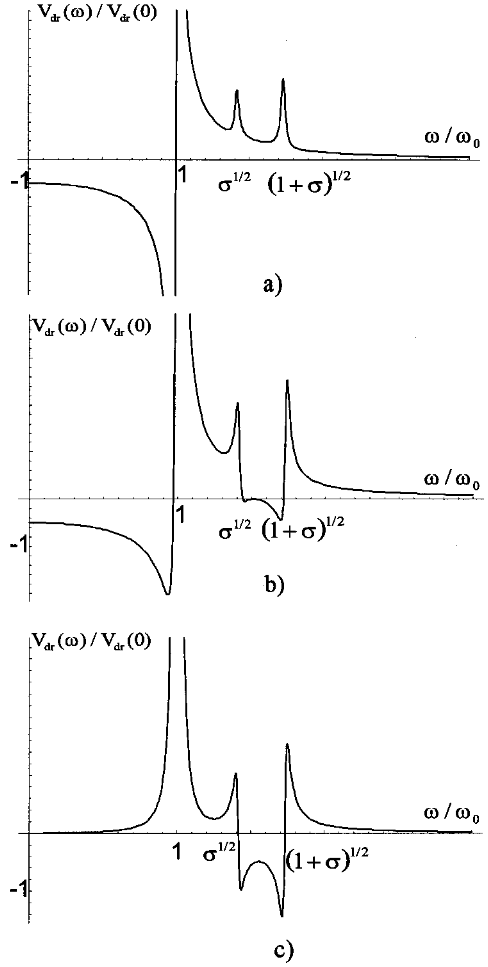

FIG. 1. Frequency dependence of the drift velocity of DW1 at different values of the phase shift χ (schematically): $\chi=0$ (a), $\chi=\pi/3$ (b), $\chi=\pi/2$ (c).

$D(\omega,\chi)$ has as before a resonance-antiresonance behavior near $\omega=\omega_0$, but it becomes asymmetric: the negative resonance amplitude $-(\omega_0/2\omega_r)(1-\sin\chi)$ decreases with increases of χ, whereas the positive resonance amplitude $(\omega_0/2\omega_r)(1+\sin\chi)$ increases.

If $\chi\neq 0$ the function $A(\omega,\chi)$ at $\omega=\omega_{1,2}$ changes its behavior from resonant to resonant-antiresonant one. The amplitude values of the function $A(\omega,\chi)$ at frequencies close to $\omega_1=\omega_0\sigma^{1/2}$ are equal to $\sigma(\cos\chi\pm 1)/2\sim 1$, which is much less than the amplitude of the function $D(\omega_1,\chi)$ at $\omega\approx\omega_0$. These values, however are comparable with and may even exceed the value $D(\omega_1,\chi)$, which decreases with increase of χ: $D(\omega_1,\chi)=D(\omega_1,0)\cos\chi$. At sufficiently large χ the contribution of the antiferromagnetic term $A(\omega,\chi)$ can therefore make the DW drift velocity negative in a narrow frequency interval, of order ω near $\omega=\omega_1$. The similar picture takes place also at frequencies close to $\omega_2=\omega_0(1+\sigma)^{1/2}$. A typical dependence of the drift velocity on the frequency for elliptic polarization of the magnetic field in the (XZ) plane at $\chi=\pi/3$ is shown in Fig. 1(b).

If the phase shift is equal to $\pi/2$ the behavior of the function $D(\omega,\pi/2)$ becomes purely resonant (the negative peak vanishes), the amplitude at the maximum being double the corresponding value for $\chi=0$. The function $A(\omega,\pi/2)$ exhibits a symmetric resonance-antiresonance behavior with amplitudes of order 1. Outside the resonance regions, the two terms in Eq. (29) are of the same order and are small: $D(\omega,\pi/2)\sim A(\omega,\pi/2)\sim q_2\ll 1$. The function $V_{dr}(\omega,\pi/2)$ is shown in Fig. 1(c).

B. Field in (*XY*)-plane

The nonlinear mobility $\nu_{xy}(\omega,\chi)$ in the case of an oscillating field (22) (at $H_z=0$) is given by the expression

$$\nu_{xy}(\omega,\chi_1)=-\nu_0\frac{\pi}{4}\left(\frac{d^2}{\beta_1\delta}\right)^{1/2}\frac{\omega}{\omega_0}\times\mathrm{Im}\left\{e^{i\chi_1}\left[\frac{1+P_2}{1-q_1+iq_2}+P_1\right]\right\}, \tag{36}$$

where the function $P_n=P_n(\omega)$, $n=1,2$, are defined by the integrals

$$P_n=\frac{2}{\pi}\int_0^{+\infty}dx\,\frac{x^{2n}}{(1+x^2)(1+x^2-q_1+iq_2)\sinh^2(\pi x/2)} \tag{37}$$

Using the approximation

$$P_n(\omega)\approx\frac{\eta_n}{1-q_1+iq_2},$$

where η_n are certain constants of the order of unity, we obtain from Eq. (36):

$$\nu_{xy}(\omega,\chi_1)=-\nu_0\frac{\pi}{4}\left(\frac{d^2}{\beta_1\delta}\right)^{1/2}\frac{\omega}{\omega_0}\left\{\frac{(1-\eta_1)q_2\cos\chi_1+[(1+\eta_1)(1-q_1)+\eta_2]\sin\chi_1}{(q_1-1)^2+q_2^2}\right\}. \tag{38}$$

It should be noted that the drift of DW1 in the field polarized in the (XY) plane is completely due to the presence of the Dzyaloshinskii interaction and is absent in ''pure'' antiferromagnets. It also follows from Eq. (38) that the frequency dependence of the nonlinear mobility $\nu_{xy}(\omega)$ differs somewhat from $\nu_{xz}(\omega)$ [Eqs. (29) and (30)]. First, $\nu_{xy}\sim\omega$ and vanishes as $\omega\rightarrow 0$; second, at high frequencies we have $\nu_{xy}\sim\omega^{-1}$ in place of $\nu_{xz}\sim\omega^{-2}$. The resonant properties of $\nu_{xy}(\omega,\chi_1)$ are similar to the corresponding properties of the function $A(\omega,\chi)$ (30). In the case $\chi_1=0$, we have at the frequency $\omega=\omega_0$ the usual resonance with amplitude of the order of $\nu_0(\omega_0/\omega)$ at the maximum. In the case $\chi\neq 0$ the function $\nu_{xy}(\omega,\chi_1)$ has an asymmetric resonance-antiresonance behavior, which becomes symmetric at $\chi=\pi/2$. The maximum amplitude of the drift velocity in fields of the order of 10 Oe is $\sim$10 m/s, just as for the field in the (*XZ*) plane.

V. DYNAMICS OF DW2

Let us consider dynamic properties of DW of the second type (DW2), in which the vector **l** rotates in the (*XY*) plane. This type of DW is stable at $\tilde{\beta}_2<\beta_1$; the static magnetization distribution in DW2 is described by Eq. (11).

The analysis of the dynamic behavior of DW2 is similar to that of DW1, therefore below we write down only the main results. In the linear approximation with respect to the external magnetic field, DW2 oscillates with the field frequency, and its velocity $V_1(t)$ is described by the equation

$$\dot{V}_1+\omega_r V_1=-\frac{\tilde{y}_0(gM_0)^2}{2}\left(dh_z+\frac{\pi}{gM_0}\dot{h}_z\right). \tag{39}$$

It should be noted that, in opposite to DW1, these oscillations are excited only by Z component of the external magnetic field.

The DW2 drift occurs only if $H_x\neq 0$ and $H_y\neq 0$,

$$V_{dr}=\tilde{\nu}_{xy}(\omega,\chi_1)H_{0x}H_{0y}, \tag{40}$$

where the nonlinear mobility $\tilde{\nu}_{xy}(\omega,\chi_1)$, similarly to the nonlinear mobility ν_{xz} in Eq. (29), can be presented as a sum of weak ferromagnet term, which is connected to the Dzyaloshinskii interaction, and ‘‘pure antiferromagnet’’ terms:

$$\tilde{\nu}_{xy}(\omega,\chi_1)=\nu_0[\tilde{D}(\omega,\chi_1)+\tilde{A}(\omega,\chi_1)],$$

$$\tilde{D}(\omega,\chi_1)=-\frac{\pi}{2}\left(\frac{d^2}{\tilde{\beta}_2\delta}\right)^{1/2}\tilde{q}_1^{1/2}\left\{\frac{\{[(\tilde{q}_1-\tilde{\sigma})^2+\tilde{q}_2^2](\tilde{\eta}_1-1)+2(\tilde{q}_1-\tilde{\sigma})\}}{[(\tilde{q}_1-\tilde{\sigma})^2+\tilde{q}_2^2][(\tilde{q}_1-\tilde{\sigma}-1)^2+\tilde{q}_2^2]}\tilde{q}_2\cos\chi_1\right.$$
$$\left.+\frac{\{[(\tilde{q}_1-\tilde{\sigma})^2+\tilde{q}_2^2][-(\tilde{q}_1-\tilde{\sigma}-1)\tilde{\eta}_1+\tilde{\eta}_2]+[(\tilde{q}_1-\tilde{\sigma})^2-\tilde{q}_2^2](\tilde{q}_1-\tilde{\sigma}-1)\}}{[(\tilde{q}_1-\tilde{\sigma})^2+\tilde{q}_2^2][(\tilde{q}_1-\tilde{\sigma}-1)^2+\tilde{q}_2^2]}\sin\chi_1\right\},$$

$$\tilde{A}(\omega,\chi_1)=\tilde{q}_1\tilde{q}_2\frac{\tilde{q}_2\cos\chi_1+[(\tilde{q}_1-\tilde{\sigma})(\tilde{q}_1-\tilde{\sigma}-1)+\tilde{q}_2^2]\sin\chi_1}{[(\tilde{q}_1-\tilde{\sigma})^2+\tilde{q}_2^2][(\tilde{q}_1-\tilde{\sigma}-1)^2+\tilde{q}_2^2]}, \tag{41}$$

where $\tilde{q}_1=(\omega/\tilde{\omega}_0)^2$, $\tilde{q}_2=\omega\omega_r/\tilde{\omega}_0^2$, $\tilde{\sigma}=(\beta_1-\tilde{\beta}_2)/\tilde{\beta}_2>0$, $\tilde{\omega}_0=c/\tilde{y}_0=\frac{1}{2}gM_0(\tilde{\beta}_2\delta)^{1/2}$; $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are the constants of the order 1. The structure of the functions $\tilde{D}(\omega,\chi_1)$ and $\tilde{A}(\omega,\chi_1)$ are similar to that of $D(\omega,\chi)$ and $A(\omega,\chi)$ in Eq. (29). It should be only noted that the term $\tilde{D}(\omega,\chi_1)$, in contrast to $D(\omega,\chi)$, is proportional to the field frequency and tends to 0 at $\omega\to 0$.

VI. DRIFT OF A STRIPE DOMAIN STRUCTURE

We consider now the possibility of a drift in an external alternating magnetic field with a stripe domain structure (SDS) consisting of domains with $l_z=1$ and $l_z=-1$ separated by 180-degree DW's. Let us first discuss the case $\tilde{\beta}_2>\beta_1$, in which DW of the first type (DW1) is stable.

It must be borne in mind here that neighboring DW in the SDS have opposite topological charges determined by the boundary conditions (9) of Eq. (8). In addition, the rotation of the vector **l** in various DW can be about either a positive or a negative direction of the Z axis. These two factors determine the DW drift direction in a field of fixed frequency ω and a phase shift χ (or χ_1). An SDS drift is possible, naturally, only when neighboring DW move in one and the same direction.

We define the topological charge $R=\pm 1$ of the DW and the parameter $\rho=\pm 1$ that describes the rotation of the vector **l** in a DW as follows:

$$l_x(\pm\infty)=\mp R,\quad l_z(y=0)=\rho. \tag{42}$$

The domain walls of the first type (DW1) considered above, satisfying the boundary conditions (8) and having a magnetization distribution (10), correspond to $R=\rho=+1$. In the general case we have in lieu of Eq. (10)

$$\varphi_0'=\frac{1}{y_0}R\sin\varphi_0=\frac{1}{y_0}R\rho\,\mathrm{sech}\left(\frac{y}{y_0}\right),\quad \cos\varphi_0=-R\tanh\left(\frac{y}{y_0}\right). \tag{43}$$

Analysis shows that in the general case the drift velocity of DW1 with the given values of the parameters R and ρ is determined by an equation similar to Eq. (28):

$$V_{dr}=R\rho\nu_{xz}(\omega,\chi)H_{0x}H_{0z}+R\nu_{xy}(\omega,\chi_1)H_{0x}H_{0y}, \tag{44}$$

where the nonlinear mobilities ν_{xz} and ν_{xy} are described as before by Eqs. (29) and (36).

We see thus in Eq. (44) that SDS drift in a field polarized in the (*XY*) plane is altogether impossible, since the topological charges R of neighboring DW are different. In a field polarized in the (*XZ*) plane, SDS drift is possible, but provided that neighboring DW have different values of the parameter ρ and of the topological charge R, i.e., the rotation of the vector **l** in neighboring DW must be in the same direction (e.g., clockwise) and orientations of **l** in the centers of the neighboring DW's are opposite to each other.

The drift motion of SDS in the case $\tilde{\beta}_2 < \beta_1$, in which DW2 are stable, can be considered similarly. In this case DW can be charactered by the topological charge $R \pm 1$ and the parameter $\tilde{\rho} = \pm 1$,

$$l_x(\pm\infty) = \mp R, \quad l_y(y=0) = \tilde{\rho}. \tag{45}$$

An analysis shows the drift velocity of DW2 with different values of the parameters R and $\tilde{\rho}$ can be described by Eq. (40) in which with the nonlinear mobility $\tilde{\nu}_{xy}(\omega,\chi_1)$ has the form

$$\tilde{\nu}_{xy}(\omega,\chi_1) = \nu_0[R\tilde{D}(\omega,\chi_1) + R\tilde{\rho}\tilde{A}(\omega,\chi_1)], \tag{46}$$

where the functions $\tilde{D}(\omega,\chi_1)$ and $\tilde{A}(\omega,\chi_1)$ are defined in Eq. (41). We see that two terms in Eq. (46) differently depend on the topological charges R and $\tilde{\rho}$. As the first one (weak-ferromagnet term) is dominant and proportional only to R, SDS with DW2 cannot drift as a whole. Only in the case of a ''pure'' antiferromagnet, in which $\tilde{D} = 0$, a drift of SDS with DW2 is possible under the influence of the external magnetic field polarized in the (XY) plane.

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