COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in **Physical Review B.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Crossover exponents in percolating superconductor–nonlinear-conductor mixtures"

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(Received 14 November 1997)

The purpose of this comment is to point out that results appearing in a paper by Zhang [Phys. Rev. B **53**, 20 (1996)] on the crossover exponents in superconductor–nonlinear-normal-conductor composites are incorrect and to clarify the relation between the correlation length ξ and the size L of the system. [S0163-1829(99)10001-8]

In a recent paper,[1] Zhang studied the crossover exponents on superconductor–nonlinear-conductor composites (S/N) in the case of arbitrary nonlinearity. This is an important problem because previous studies are only restricted to cubic nonlinearity.[2–4] In Ref. 1, a d-dimensional hypercubic lattice with the fraction p of superconductors and the fraction $(1-p)$ of normal conductors is considered. The conductors are assumed to have a current density **J** and electric field **E** response of the form $\mathbf{J}=\sigma_1\mathbf{E}+\chi_1|\mathbf{E}|^{\beta}\mathbf{E}$, where σ_1 and χ_1 are the linear conductivity and nonlinear susceptibility, respectively, β is the nonlinear exponent, and $\beta>0$. (Here, β is equivalent to $\beta-1$ in Ref. 1.) The nonlinear term is assumed to be weak, i.e., $\chi_1|\mathbf{E}|^{\beta}/\sigma_1\ll 1$. The crossover electric field E_c is defined as the electric field at which the linear response and the nonlinear response become comparable and has

$$E_c=\left(\frac{\sigma_e}{\chi_e}\right)^{1/\beta}, \tag{1}$$

the corresponding crossover current density

$$J_c=2\sigma_e E_c. \tag{2}$$

Below the percolation threshold of the superconductors, E_c and J_c are found to behave as

$$E_c\sim(p_c-p)^{M(\beta)},\quad J_c\sim(p_c-p)^{W(\beta)}, \tag{3}$$

where crossover exponents $M(\beta)$ and $W(\beta)$ are β dependent, and since $\sigma_e\sim(p_c-p)^{-s}$, we have $W(\beta)=M(\beta)-s$. A crude estimate can be obtained by using an effective medium approximation, $M(\beta)=1/2$, $W(\beta)=-1/2$ for all spatial dimensions d and arbitrary $\beta(>0)$.[5] By using the connection of this nonlinear response to the conductance fluctuation of the corresponding linear composite, we can obtain the expressions of $M(\beta)$ and $W(\beta)$.

It is found that the effective third-order nonlinear susceptibility is in proportion to the mean-square fluctuation of the effective linear conductivity σ_e in an effective linear composite[2]

$$\chi_e(\beta=2)\sim L^d\delta\sigma_e^2, \tag{4}$$

where L is the size of the system. This relation can be generalized to the effective nonlinear susceptibility $\chi_e(\beta)$ in the case of arbitrary nonlinear exponent β (Refs. 1 and 6),

$$\chi_e(\beta)\sim L^d\langle\delta\sigma_e^{(\beta+2)/2}\rangle_c, \tag{5}$$

where $\langle\delta\sigma_e^{(\beta+2)/2}\rangle_c$ indicates the high-order $[(\beta+2)/2]$th cumulant. The high-order relative fluctuation, which is the ratio of high-order $[(\beta+2)/2]$th cumulant to the $[(\beta+2)/2]$th order effective linear conductivity, can be expressed as[1,6]

$$\frac{\langle\delta\sigma_e^{(\beta+2)/2}\rangle_c}{\sigma_e^{(\beta+2)/2}}\sim L^{d[1-(\beta+2)/2]}(p_c-p)^{-\kappa'[(\beta+2)/2]}, \tag{6}$$

where $\kappa'[(\beta+2)/2]$ denotes the divergence of the high-order relative fluctuation and can be reduced to the noise exponent $(\beta=2)$. Combining Eq. (5) with Eq. (6), we can easily get

$$\chi_e(\beta)\sim L^{(2-\beta)d/2}(p_c-p)^{-(\beta+2)s/2-\kappa'[(\beta+2)/2]}. \tag{7}$$

Substituting Eq. (7) into the definition of crossover electric field and current density, we obtain

$$E_c\sim L^{(d/2)(1-2/\beta)}(p_c-p)^{\{s/2+\kappa'[(\beta+2)/2]/\beta\}} \tag{8}$$

and

$$J_c \sim L^{(d/2)(1-2/\beta)}(p_c-p)^{\{-s/2+\{\kappa'[(\beta+2)/2]/\beta\}}. \quad (9)$$

The above relations are correct for $L>\xi$, i.e., in the Euclidean regime; thus Eqs. (6)–(9) are not only valid for finite system, but also correct in the thermodynamic limit and below the percolation threshold p_c (which includes $p \to p_c^-$). In thermodynamic limits case, those physical parameters, which are similar to a system conductance such as $G\sim(p_c-p)^{-s}L^{d-2}$, depend on both p_c-p and L.[7] Therefore, the crossover exponents $M(\beta)$ and $W(\beta)$, which describe the dependence of the crossover electric field and current density on (p_c-p), are given as follows:

$$M(\beta)=\frac{s}{2}+\frac{\kappa'[(\beta+2)/2]}{\beta} \quad (10)$$

and

$$W(\beta)=-\frac{s}{2}+\frac{\kappa'[(\beta+2)/2]}{\beta}; \quad (11)$$

when $\beta=2$, the above results will be reduced to the well-known results $M(2)=[\kappa'(2)+s]/2$ and $W(2)=[\kappa'(2)-s]/2$, respectively.

Here we must emphasize that the above results differ from Eqs. (9) and (10) in Ref. 1. If we replace L by the correlation length ξ in Eqs. (8) and (9), which diverges as $\xi\sim(p_c-p)^{-\nu}$ near the percolation threshold, we can easily get Eqs. (9) and (10) in Ref. 1. Unfortunately, such substitution is unreasonable. It is known that L and ξ are two different physics parameters. In the vicinity of p_c, ξ is the function of p_c-p, while L is a fixed quantity for a given system (in the thermodynamic limit L is taken as ∞) and is independent of p_c-p. While at the percolation threshold, however, the percolation correlation length ξ diverges and the above relations are not approached; the whole system is always in the fractal or self-similar regime. In this case, these physical parameters will only depend on L and can be obtained by making $(p_c-p)\sim\xi^{-1/\nu}=L^{-1/\nu}$, i.e., replacing ξ with L (not L with ξ). In order to demonstrate that L cannot be replaced by ξ, we also give some concrete examples. (i) Hui remarked that "$[\delta\sigma_e^2/\sigma_e^2\sim L^{-d}(p_c-p)^{-\kappa'(2)}]$, the relative fluctuation, which is the ratio of the mean-square fluctuation to the square of the effective conductivity, behaves as $(p_c-p)^{-\kappa'(2)}$ in a S/N mixture for p approaching to p_c from below,"[3] not $\delta\sigma_e^2/\sigma_e^2\sim L^{-d}(p_c-p)^{-\kappa'(2)}\sim\xi^{-d}(p_c-p)^{-\kappa'(2)}=(p_c-p)^{\nu d-\kappa'(2)}$ according to Zhang's wrong substitution. (ii) Equation (7) shows that the critical exponent, which describes the divergence of $\chi_e(\beta)$ on p_c-p, should be $\kappa'[(\beta+2)/2]+[(\beta+2)/2]s$,[8] not $k'[(\beta+2)/2]+(\beta+2/2)+[(2-\beta)/2]d\nu$, which is the wrong result of replacing L with ξ. (iii) Kolek *et al.* mentioned that "The critical behavior of $p>p_c$ given by the law $G\propto(p-p_c)^t$ $[G\sim(p-p_c)^t L^{d-2}]$, where G is the network conductance, t is the critical exponent, and p_c is the percolation threshold ...,"[7] and so on. In a word, L cannot be replaced by ξ in any case.

In Zhang's work, the relation between ξ and L is confused, and L is replaced by ξ in those equations, such as Eqs. (4) and (5) in Ref. 1, incorrectly. Their so-called crossover exponents characterize by no means the physical parameters dependence on (p_c-p).

In order to analyze the properties of crossover exponents, we must look for the expression of $\kappa'[(\beta+2)/2]$.

When $L<\xi$, i.e., in the fractal regime, Eq. (6) can be written as

$$\frac{\langle\delta\sigma_e^{(\beta+2)/2}\rangle_c}{\sigma_e^{(\beta+2)/2}}\sim L^{\{d[1-(\beta+2)/2]+k'[(\beta+2)/2]/\nu\}}. \quad (12)$$

On the other hand, the above relation can also be expressed as[6]

$$\frac{\langle\delta\sigma_e^{(\beta+2)/2}\rangle_c}{\sigma_e^{(\beta+2)/2}}\sim\frac{\langle\delta G^{(\beta+2)/2}\rangle_c}{G^{(\beta+2)/2}}\sim L^{\{\psi_G[(\beta+2)/2]-[(2+\beta)/2]\zeta_G\}/\nu}, \quad (13)$$

where $\psi_G[(\beta+2)/2]$ characterizes the scaling of the $[(\beta+2)/2]$th cumulant of the global conductance distribution that comes from the local conductance fluctuation.[9,10] The macroscopic conductance G behaves as $G\sim\sigma_e L^{d-2}\sim(p_c-p)^{-s}L^{d-2}\sim L^{s/\nu+d-2}\sim L^{\zeta_G/\nu}$ for $L<\xi$, where $\zeta_G=s+(d-2)\nu$.

Comparing Eq. (12) with Eq. (13), we can obtain the relation between $\kappa'[(\beta+2)/2]$ and $\psi[(\beta+2)/2]$, i.e.,

$$\kappa'\left(\frac{\beta+2}{2}\right)+d\nu=\psi_G\left(\frac{\beta+2}{2}\right)+\frac{\beta+2}{2}(d\nu-\zeta_G). \quad (14)$$

The above equation is the same as Eq. (7) in Ref. 1, but it is deduced wrongly, so that the results of Eqs. (9) and (10) in Ref. 1 are incorrect. Incidentally, according to Zhang's idea, if L could be replaced by ξ, Eqs. (3)–(5) and Eq. (11) in Ref. 1 would be enough to obtain $M(\beta)$ and $W(\beta)$ and it seems unnecessary to spare much effort in introducing $\kappa'[(\beta+2)/2]$ and obtaining the relation between $\kappa'[(\beta+2)/2]$ and $\psi_G[(\beta+2)/2]$ [see Eqs. (6) and (7) in Ref. 1]. But, the introduction of $k'[(\beta+2)/2]$ is a must.[6]

Substituting Eq. (14) into Eqs. (10) and (11), we have

$$M(\beta)=\frac{1}{\beta}\left[\psi_G\left(\frac{\beta+2}{2}\right)-\zeta_G\right]+\nu \quad (15)$$

and

$$W(\beta)=\frac{1}{\beta}\left[\psi_G\left(\frac{\beta+2}{2}\right)-\zeta_G\right]+\nu-s. \quad (16)$$

Analytic and numerical results of the $\psi_G[(\beta+2)/2]$ have been obtained on a two-dimensional random resistor network in the vicinity of the percolation threshold p_c.[8,9] Making use of the same procedure of calculation and parameters for $d=2$ as in Ref. 1, and noting $\psi_G(1)=\zeta_G$, we have following results: (i) $\lim M(\beta\to0^+)\cong1.131>0$ and $\lim W(\beta\to0^+)\cong-0.166<0$. (ii) $\lim M(\beta\to+\infty)\cong1.333>0$ and $\lim W(\beta\to+\infty)\cong0.036>0$.

Based on so-called "single disconnected bonds picture," Hui gives the upper bound for the crossover exponent $W(\beta)\cong\frac{1}{3}$ for arbitrary β (Ref. 11) in the two-dimensional case; we can easily get the upper bound for $M(\beta)\cong\frac{4}{3}$ independent of β, while Zhang gives $M(\beta\rightarrow 0^{+})=+\infty$ and $W(\beta\rightarrow 0^{+})=+\infty$, which largely exceed such bounds.

The monotonicity is the important property of $M(\beta)$ and $W(\beta)$; we have

$$\frac{d[M(\beta)]}{d\beta}=\frac{d[W(\beta)]}{d\beta}=\frac{1}{\beta^{2}}\left[\frac{\beta}{2}\frac{d\{\psi_{G}[(\beta+2)/2]\}}{d[(\beta+2)/2]}-\psi_{G}\left(\frac{\beta+2}{2}\right)+\zeta_{G}\right]. \quad (17)$$

Note that $\psi_{G}[(\beta+2)/2]>0$, $d\psi_{G}[(\beta+2)2]d[(\beta+2)/2]<0$, and $\zeta_{G}=\psi_{G}(1)$ takes the maximum value of $\psi[(\beta+2)/2]$; we have $d[M(\beta)]/d\beta=d[W(\beta)]/d\beta>0$ for any $\beta>0$. This conclusion is also in agreement with the numerical results. Both $M(\beta)$ and $W(\beta)$ are monotonically increasing functions with the increase of β; $M(\beta)>0$ for any $\beta(>0)$, while $W(\beta)$ may take positive, zero, and negative values.

Since $M(\beta)>0$ and $dM(\beta)/d\beta>0$, it shows that the crossover field E_c vanishes faster for larger β; the larger the nonlinearity β, the smaller the electric field that is needed to stimulate a remarkable nonlinear response. This can be understood, for the nonlinear term $\chi_e|E_c|^{\beta}$, which can be compared with the linear term σ_e; with the increase of β, E_c will be needed to reduce and the nonlinear region will increase accordingly.

On the other hand, in a smaller β such as $\beta\rightarrow 0^{+}$, then $E_c\sim(p_c-p)^{1.130}$ takes the maximum for finite p_c-p. In fact, as $\beta\rightarrow 0^{+}$, the nonlinear component has become a linear component; thus the system possesses the largest linear region.

As to the crossover current density J_c, we have the following results:

$$\begin{aligned} &J_c\rightarrow+\infty, \quad W(\beta)<0,\ \beta<\beta_c,\\ &J_c\rightarrow\text{const}, \quad W(\beta)=0,\ \beta=\beta_c, \quad (18)\\ &J_c\rightarrow 0, \quad W(\beta)>0,\ \beta>\beta_c, \end{aligned}$$

where $\beta_c\approx 8.15$ is a critical value at which $W(\beta)=0$.

Our conclusions are perfectly opposite to that in Ref. 1. We believe that our results are more reliable from the viewpoint of the physics meaning.

Finally, other mistakes in Ref. 1 also exist. Equation (1) is a wrong form and Eq. (2) should be J_c not $I_c\,(=L^{d-1}J_c)$. Both Eq. (4) and Eq. (5) are wrong, and they are contrary to each other because $\sigma_e\sim(p_c-p)^{-s}$.

Summarily, our conclusions for a two-dimensional S/N composite are (1) $M(\beta)>0$ for arbitrary $\beta>0$ (this is same as that in Ref. 1; it implies that the nonlinear response of the S/N composite becomes remarkable in the vicinity of percolation threshold, and a small electric field can lead to an enhancement of nonlinear response. $M(\beta)$ increases monotonically with the increase of β (this result is opposite to that in Ref. 1); for a large β, we may predict that a somewhat smaller electric field is enough to stimulate a remarkable nonlinear response. (2) $W(\beta)$ may take positive, zero, and negative values (this conclusion is same as that in Ref. 1), and monotonic increase with the increase of β (this conclusion is opposite to that in Ref. 1), so that J_c shows a complex behavior such as diverging or keeping invariance or vanishing with the increase of β as $p\rightarrow p_c^{-}$. We can conclude that for $\beta<\beta_c(=8.15)$, when $p\rightarrow p_c^{-}$, J_c will diverge, and only E_c can be used to describe the crossover effect. While for $\beta>\beta_c$, both E_c and J_c will vanish as $p\rightarrow p_c^{-}$ and can be used to describe the crossover effect.

This project was supported by the National Natural Science Foundation of China under Grant No. 19974042.

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