Frictional drag between spatially separated two-dimensional electron gases mediated by virtual-phonon exchange

Samvel M. Badalyan
Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany and Department of Radiophysics, Yerevan State University, 375049 Yerevan, Armenia

Ulrich Rössler
Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany
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We have calculated the temperature dependence of the frictional drag between spatially separated quantum wells with parallel two-dimensional electron gases due to interlayer electron-electron interaction mediated by virtual exchange of acoustic phonons due to piezoelectric and deformation potential interaction. It is shown that the frictional drag is dominated by the piezoelectric coupling. According to our calculations the temperature dependence of the drag scattering rate divided by T^2 exhibits a pronounced peak that for the experimental situation and in agreement with the finding of Gramila *et al.*, Phys. Rev. B **47**, 12 957 (1993), is obtained at approximately $T \approx 2.1$ K. We ascribe the appearance of this peak to a change from small to large angle scattering in the virtual phonon exchange. [S0163-1829(98)06247-X]

I. INTRODUCTION

Double layer two-dimensional (2D) carrier systems as realized in semiconductor nanostructures are of interest with respect to fundamental phenomena such as the fractional quantum Hall effect and superconductivity because of the interlayer carrier interaction.[1] Depending on the width of the potential barrier that separates the two layers one has to distinguish the regime of quantum-mechanical coupling via tunneling for small barrier widths from that of interlayer coupling mediated by (direct or effective) electron-electron interaction for sufficiently large barriers. In the former case, the frictional aspects of superconductivity[2,3] and of the fractional quantum Hall effect[4,5] have been studied. In the latter case, when the large barrier allows independent control of electron conduction in the two layers, the frictional drag effect manifests itself when a current driven along one layer (layer 1) induces, via momentum transfer, a drag voltage in the second layer (layer 2) under conditions such that no current flows in this well.[6,7] The drag effect has been studied experimentally for two 2D electron systems (2DES's) and two 2D hole systems (2DHS), but also in situations with layers 1 and 2 being a 2DES or 2DHS, respectively.[8–16] Recently the effect of a magnetic field perpendicular to the layers has been investigated.[17,18] Theoretical work on the frictional drag effect was devoted to elucidating the microscopic mechanism of the momentum transfer.[19–21] The calculated temperature dependence of the frictional drag due to direct Coulomb electron-electron interaction shows a strong dependence on the interlayer separation Λ (Refs. 22 and 23) while in the experiment by Gramila *et al.*,[13] the temperature dependence of the observed frictional drag for separations at least up to $\Lambda \sim 50$ nm exhibits almost no dependence on Λ, and has been interpreted as being due to the exchange of virtual phonons. The frictional drag accomplished by the exchange of virtual phonons has been studied theoretically in Ref. 24. This work, however, because of its conceptual complexity requires involved numerical computations and does not lead to analytical expressions for the dependence of the frictional drag on the system parameters. Therefore, we present here an analytical derivation of the drag scattering rate for the particular case of asymmetric two-layer systems ($n_1 \neq n_2$) coupled by virtual exchange of acoustic phonons including its temperature dependence. We thus model the experimental situation investigated by T. J. Gramila *et al.*[13] and find by using the corresponding system parameters the characteristic T dependence of the drag scattering rate in quantitative agreement with these experimental data.

II. THEORETICAL CONCEPT

Our model as depicted in Fig. 1 consists of two parallel electron sheets separated by a distance Λ with electron densities n_1 and n_2 and the layer extensions d_1 and d_2. According to experimental observations, a current I_1 driven along the layer 1 by applying a voltage V_1 leads by frictional drag to a voltage V_2 in the layer 2. The drag scattering rate in a double-layer system can be defined as[16]

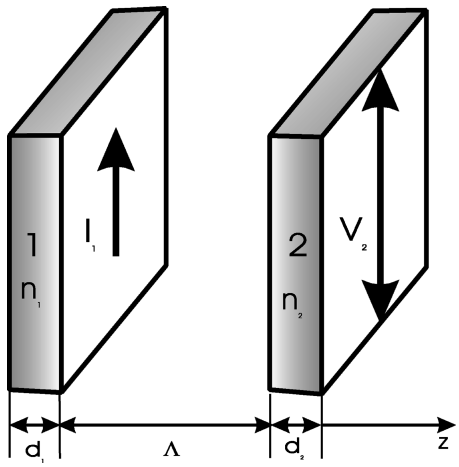

FIG. 1. Schematic view of the double-layer system with the characterizing parameters (described in the text).

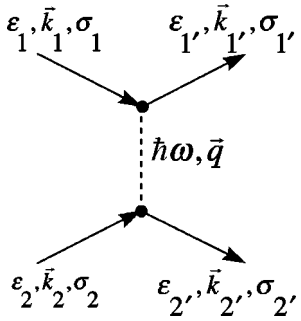

FIG. 2. Diagram of the effective electron-electron interaction between layers 1 and 2 as mediated by exchange of virtual acoustic phonons.

$$\frac{1}{\tau_{Drag}}=\frac{E_2}{\tau_1 E_1} \tag{1}$$

where E_1, E_2 are the electric fields connected with the voltages V_1, V_2, and τ_1 is the transport relaxation time that determines via the mobility $\mu=e\tau_1/m^*$ the current in the layer 1; m^* is the electron effective mass. Following Ref. 23, we obtain by using the linearized Boltzmann equation for the drag scattering rate

$$\frac{1}{\tau^{\Upsilon}_{Drag}}=\frac{\hbar^2}{4mTn_1L^2}\sum_{\sigma_1,\sigma_2,\sigma_{1'}\sigma_{2'}}\sum_{\vec{k}_1,\vec{k}_2,\vec{k}_{1'},\vec{k}_{2'}} \times q_\perp^2 W^{\Upsilon}_{1,2\to 1',2'} f_1 f_2 (1-f_{1'})(1-f_{2'}), \tag{2}$$

where L is the normalization length, $\vec{k}$ the electron in-plane momentum, σ the electron spin, $\vec{q}_\perp=\vec{k}-\vec{k}'$ the transferred momentum, f the Fermi distribution function determined by temperature T, and Fermi energy $\varepsilon_F=\hbar^2k_F^2/2m^*$ with the Fermi wave vector related to the areal electron concentration n, by $k_F^2=2\pi n$. $W^{\Upsilon}_{1,2\to 1',2'}$ is the transition probability of two electrons from the states $|1\rangle=|\sigma_1,\vec{k}_1\rangle$ and $|2\rangle=|\sigma_2,\vec{k}_2\rangle$ with energies ε_1 and ε_2, respectively, into the states $|1'\rangle=|\sigma_{1'},\vec{k}_{1'}\rangle$ and $|2'\rangle=|\sigma_{2'},\vec{k}_{2'}\rangle$ with energies $\varepsilon_{1'}$ and $\varepsilon_{2'}$. The index Υ refers to the type of effective electron-electron interaction that causes this transition.

In this work we consider effective electron-electron interaction mediated by the exchange of virtual acoustic phonons related to the piezoelectric ($\Upsilon=$PA) and deformation ($\Upsilon=$DA) potential coupling. Using Fermi's golden rule, $W^{\Upsilon}_{1,2\to 1',2'}$ can be written in the form

$$W^{\Upsilon}_{1,2\to 1',2'}=\frac{2\pi}{\hbar}|T^{\Upsilon}_{1,2\to 1',2'}|^2\delta(\varepsilon_1+\varepsilon_2-\varepsilon_{1'}-\varepsilon_{2'}), \tag{3}$$

where $T^{\Upsilon}_{1,2\to 1',2'}$ is the transition matrix element. Phonon mediated electron-electron interaction appears in second-order perturbation theory with respect to the electron-phonon coupling. The transition matrix element of this process is visualized by the diagram in Fig. 2. For intrasubband scattering the vertex part of this diagram corresponds to

$$\Gamma^{\Upsilon}(\vec{q})=\sqrt{B^{\Upsilon}(q)}\,\delta_{\vec{k}_1,\vec{k}_{1'}+\vec{q}_\perp}\int dz\rho(z)\exp(iq_z z) \tag{4}$$

with the subband electron density $\rho(z)$ and $\vec{q}=(\vec{q}_\perp,q_z)$. $B^{\Upsilon}(q)$ depends only on the absolute value of $\vec{q}$ (Ref. 25)

$$B^{PA}(q)=B_0^{PA}q^{-1},\quad B_0^{PA}=\frac{\hbar(e\beta)^2}{2\varrho s}, \tag{5}$$

$$B^{DA}(q)=B_0^{PA}q,\quad B_0^{DA}=\frac{\hbar\Xi^2}{2\varrho s}. \tag{6}$$

Here we describe electron-phonon interaction in the framework of the isotropic model with s being the longitudinal sound velocity, ϱ the crystal mass density, $e\beta$ and Ξ the piezoelectric and deformation potential constants, respectively, averaged over the directions of the vector $\vec{q}$ and the phonon polarizations.[26,27] We assume also that all elastic parameters of the sample are the same so that phonons are not reflected from the interfaces separating different materials (for instance between GaAs and $Al_xGa_{1-x}As$). The phonon propagator is

$$D(q)=\frac{2\hbar^{-1}\omega_q}{(\omega^2+i/2\tau_q)^2-\omega_q^2} \tag{7}$$

with $\hbar\omega=\varepsilon_1-\varepsilon_{1'}$, $\omega_q=sq$ and a summation over phonon momenta $\vec{q}$ corresponds to the internal phonon line (the dashed line in the diagram). In the above expression of the phonon propagator, τ_q is the lifetime of the intermediate phonon states with respect to all processes that are destroying these intermediate states. For virtual intermediate states there is no energy conservation at the vertices of the diagram, i.e., $\varepsilon_1+\varepsilon_2=\varepsilon_{1'}+\varepsilon_{2'}$ can be different from $\hbar\omega$ and we may assume infinite lifetime τ_q for the intermediate phonons when calculating the drag rate. Then all integrals with respect to the energy parameters in Eq. (2) are to be understood in the sense of the principal value. Taking the summation over intermediate momenta $\vec{q}$, we obtain for the squared modulus of the transition matrix elements for the two considered mechanisms

$$|T^{PA}_{1,2\to 1',2'}|^2=\frac{1}{\tau_D^{PA}}\frac{\hbar^3}{2m}\frac{(2\pi\Lambda)^2}{L^4}|I^{PA}(\alpha,\Lambda)|^2, \tag{8}$$

$$|T^{DA}_{1,2\to 1',2'}|^2=\frac{1}{\tau_D^{DA}}\frac{\hbar^3}{2m}\frac{(2\pi\Lambda)^2}{L^4}\frac{1}{(2k_F\Lambda)^4}|I^{DA}(\alpha,\Lambda)|^2, \tag{9}$$

where

$$|I^{PA}(\alpha,\Lambda)|^2=\frac{e^{-2\alpha\Lambda}}{(\alpha\Lambda)^2}\frac{\sinh^2(\alpha d/2)}{(\alpha d/2)^2[1+(\alpha d/2\pi)^2]^2}, \tag{10}$$

$$|I^{DA}(\alpha,\Lambda)|^2=\frac{e^{-2\alpha\Lambda}}{(\alpha\Lambda)^2}\frac{(\omega\Lambda/s)^4\sinh^2(\alpha d/2)}{(\alpha d/2)^2[1+(\alpha d/2\pi)^2]^2} \tag{11}$$

and α is defined generally as $\alpha(q_\perp,\omega+i\xi)=[q_\perp^2-(\omega+i\xi)^2/s^2]^{1/2}$ where ξ is the positive infinitesimal, and the branch cut for the square root is assumed to lie along the negative real axis. It is easy to check that with such a definition of α, the expressions of the form factors (10) and (11) are valid both for $q_\perp^2-\omega^2/s^2>0$ and $q_\perp^2-\omega^2/s^2<0$. In cal-

culating the form factors I^{PA} and I^{DA} we have assumed that electrons are localized in symmetric infinitely high quantum wells with width d, thus having $\rho_1(z)=(2/d)\sin(\pi z/d)^2$ and $\rho_2(z)=\rho_1(z+\Lambda)$ as explicit forms for the electron density functions in layers 1 and 2, respectively. In Eqs. (8) and (9) we introduce also nominal scattering times

$$\frac{1}{\tau_D^{PA}}=\frac{2m(e\beta)^4}{\pi^2\hbar^3\varrho^2 s^4}\frac{ms^2}{\varepsilon_F}\approx\frac{1}{0.5\mu\mathrm{s}}, \tag{12}$$

$$\frac{1}{\tau_D^{DA}}=\frac{m\Xi^4(2k_F)^4}{2\pi^2\hbar^3\varrho^2 s^4}\frac{ms^2}{\varepsilon_F}\approx\frac{1}{1.4\mu\mathrm{s}} \tag{13}$$

with the numerical values obtained for a GaAs quantum well with the electron concentration $n_1=n_2=n\approx 1.5\times 10^{11}$ cm^{-2} as in Ref. 13. Substituting Eqs. (3), (8), and (9) into Eq. (2) and taking integrals over $\vec{k}_{2'}$ and over directions of the vector $\vec{k}_2$ by exploiting δ functions, we can represent the drag scattering rate in the form

$$\frac{1}{\tau_{Drag}^{\Upsilon}}=\frac{1}{\tau_D^{\Upsilon}}\frac{T^2}{(2\hbar s k_F)^2}\mathcal{F}^{\Upsilon}(T), \tag{14}$$

where

$$\mathcal{F}^{\Upsilon}(T)=\frac{1}{\pi}\frac{(2k_F\Lambda)^2}{T^3}\,\mathrm{p.v.}\int_0^\infty d\varepsilon_1\int_0^\infty d\varepsilon_{1'}\int_{\varepsilon_0(q_\perp)}^\infty d\varepsilon_2 \times\int_0^{2k_F}\frac{dq_\perp}{\sqrt{4k_F^2-q_\perp^2}}\sqrt{\frac{\varepsilon_{q_\perp}}{\varepsilon_2-\varepsilon_0(q_\perp)}}\,|I^{\Upsilon}(\alpha,\Lambda)|^2 \times f(\varepsilon_1)f(\varepsilon_2)[1-f(\varepsilon_{1'})][1-f(\varepsilon_1-\varepsilon_{1'}+\varepsilon_2)]. \tag{15}$$

In this equation $\varepsilon_{q_\perp}=\hbar^2q_\perp^2/(2m^*)$ and the limiting energy $\varepsilon_0(q_\perp)=(\hbar\omega-\varepsilon_{q_\perp})^2/(4\varepsilon_{q_\perp})$ is obtained from the energy and momentum conservation laws. Now making use of the identity

$$f(x)(1-f(x+y))=\frac{f(x)-f(x+y)}{1-\exp(-y)} \tag{16}$$

and taking integrals in Eq. (15) with respect to the energy parameters, we reduce the expression $\mathcal{F}^{\Upsilon}(T)$ for a one-dimensional integral of the form

$$\mathcal{F}^{\Upsilon}(T)=\frac{\sqrt{\pi x_F}}{2}\gamma^2\int_0^1\frac{dx}{\sqrt{1-x}}\frac{\beta^{\Upsilon}(x)}{\sinh^2(\gamma\sqrt{x}/2)} \times(Li_{1/2}(-e^{-x_+})-Li_{1/2}(-e^{-x_-})) \tag{17}$$

where $Li_{1/2}[Li_\nu,z]$ gives the polylogarithm function of the order ν[28]

$$Li_\nu(z)=\frac{z}{\Gamma(\nu)}\int_0^\infty dt\,\frac{t^{\nu-1}}{e^t-z} \tag{18}$$

and

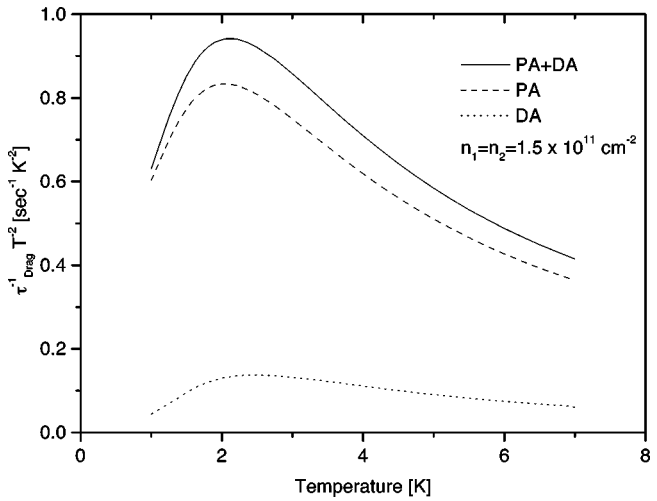

FIG. 3. Drag rate divided by the squared absolute temperature vs temperature calculated for the symmetric GaAs/Al_xGa_{1-x}As double layer system of Ref. 13. The contributions of deformation potential (DA, dotted line) and piezoelectric coupling (PA, dashed line) are shown together with the sum (solid line).

$$x_\pm=x_F\left(\left(\frac{\gamma}{4x_F}\pm\sqrt{x}\right)^2-1\right),\quad \gamma=\frac{2\hbar s k_F}{T}, \tag{19}$$

$$x_F=\frac{\varepsilon_F}{T},\quad \beta^{\Upsilon}(x)=\begin{cases}1, & \text{for PA}\\ x^2, & \text{for DA.}\end{cases} \tag{20}$$

In a recent work by Bonsager *et al.*,[29] these authors showed that even the principal value (pv) of the integral in Eq. (15) of this section has weak divergence. They overcame this difficulty and obtained a finite drag rate in the case of an infinite phonon mean free path systematically including dynamical screening due to electron-electron interaction. We expect that even without screening a finite drag rate could be obtained by using exact phonon Green function in evaluating the diagram of Fig. 2. Instead we used the δ-function regularization near the resonance when going from Eqs. (15) to (17), which can be obtained from the theory of Bonsager *et al.* by switching off screening.

Our δ-function regularization means that phonons, with a normal momentum in the z direction approximately equal to zero, realize drag between spatially separated 2DEG's. There are two relevant length parameters in the z direction which determine the normal phonon momentum: interlayer separation and phonon mean free path. The normal phonon momentum is subjected to the restriction, it should be much smaller than the momentum scale determined by the inverse interlayer separation. But the only parameter which is smaller than this momentum scale is the momentum scale determined by the inverse phonon mean free path. So that the normal phonon momentum should be of the order of the inverse phonon mean free path which for infinite phonon lifetime limit gives the normal phonon momentum equal to zero. This means if virtual phonons have infinite lifetime, even infinitesimally small normal phonon momentum is enough to realize the in-plane momentum transfer between the spatially separated 2DEG's.

For the special GaAs system with $n\approx 1.5\times 10^{11}$ cm^{-2} we have $\gamma\approx(7.6\ \mathrm{K})/T$ and $x_F\approx(62.4\ \mathrm{K})/T$. One can see from

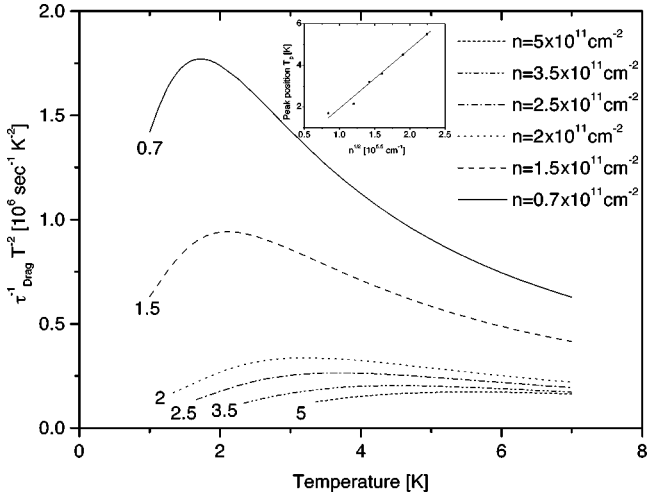

FIG. 4. Drag rate (sum of DA and PA) divided by the squared temperature as a function of temperature plotted for the symmetric $GaAs/Al_xGa_{1-x}As$ double-layer system as in Fig. 3 but with different electron concentrations. Inset shows the peak positions (squares) corresponding to drag rate data from this figure as a function of square root of the matched electron densities. Solid line is a guide to the eye.

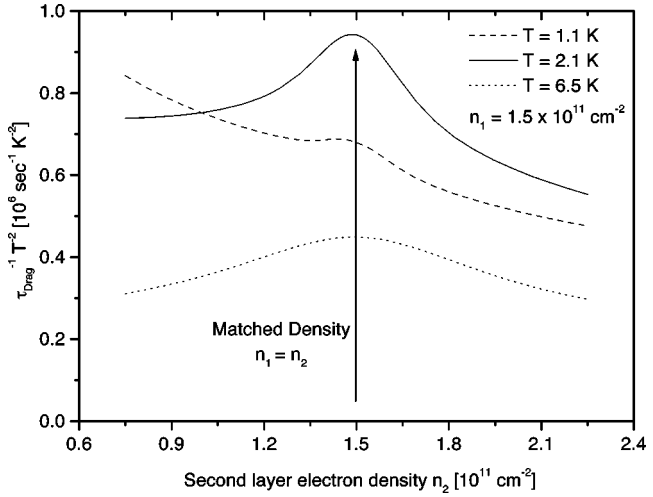

FIG. 5. Drag rate (sum of DA and PA) divided by the squared temperature for fixed first-layer electron density n_1 as a function of the second-layer electron density n_2 at different temperatures.

Eq. (17) that in the framework of the adopted approximation when the lifetime of the intermediate phonon states is infinite, i.e., $\tau_q^{-1}=0$, the drag scattering rate, Eq. (14), is independent of the interlayer spacing Λ. This means that our theory refers to the experimental situation when Λ is smaller than the phonon mean free path, $\Lambda_0=s\tau_q$, associated with the finite phonon lifetime. For $\Lambda\gtrsim\Lambda_0$, we have to consider the finite τ_q in the phonon propagator, which would lead to an exponential dependence of the scattering probability of $W\propto\exp(-\Lambda/\Lambda_0)$.

III. RESULTS AND DISCUSSION

Using Eqs. (14) and (17) we plot the temperature dependence of the drag scattering rate divided by T^2 for identical layers in a symmetric $GaAs/Al_xGa_{1-x}As$ double quantum well structure as in Ref. 13 coupled both by PA- and DA-phonon mediated electron-electron interaction (Fig. 3). It is seen from this figure that the contribution of the DA interaction is smaller (approximately by one order of magnitude) than that of the PA interaction. This is because the scattering probability due to long-range piezoelectric interaction includes an additional factor q^{-4} with respect to the short-range deformation potential interaction. This difference is strongly pronounced at low temperatures[30] where the Pauli exclusion principle restricts electron-phonon scattering processes in a Fermi gas to those with small angles.

Our calculations correspond to the experimental situation of Ref. 13 where the drag rate was measured for samples with the interlayer spacing $\Lambda=17.5$, 22.5, and 50 nm with the electron concentration $n=1.5\times10^{11}$ cm^{-2} in a temperature range from 1 to 7 K. Starting from the lower edge of this interval, where the drag rate is small because it is dominated by small angle scattering connected with a small momentum transfer, the drag rate increases with increasing temperature. Towards the upper edge of the interval, large angle scattering becomes dominant, which reduces the increase of the drag scattering rate in temperature. This gives rise to a peak of the function $1/(\tau_{Drag}^{\Upsilon}T^2)$ for drag caused by the exchange of both DA and PA phonons at approximately the same temperatures as shown in Fig. 3. The maximum drag rate of about 1 s^{-1} K^{-2} due to combined effect of DA and PA phonons is obtained at a temperature near $T_p\approx2.1$ K in good agreement with the experimental results for the samples with the interlayer spacing $\Lambda=17.5$, 22.5, and 50 nm.[13] Our theory is not applicable for much larger interlayer spacing (as, e.g., for the sample with $\Lambda=500$ nm in Ref. 13). We believe that in this sample due to other scattering mechanisms, the intermediate virtual phonon states that effectively realize electron-electron interaction, are destroyed and this leads to the exponential suppression of the drag scattering rate observed in the experiment.

In Fig. 4 we present the temperature dependence of the drag scattering rate for a symmetric double quantum well system as in Fig. 3 but for different values of the matched electron density n. [The second line from the top ($n=1.5\times10^{11}$ cm^{-2}) is identical to the solid line in Fig. 3.] With increasing n the peak structure becomes broader while the maximum value decreases and the position T_p shifts to higher temperature. This shift follows approximately the relation $T_p\propto k_F\propto\sqrt{n}$ (see the inset of Fig. 4) which supports our findings that the peak position is related to the energy $\hbar sk_F$ that separates the regions of small ($\hbar\omega\ll\hbar sk_F$) and large ($\hbar\omega\sim\hbar sk_F$) angle scattering. We have also studied the drag scattering rate as in Fig. 3 with fixed density $n_1=1.5\times10^{11}$ cm^{-2} but changing density n_2 at different temperatures. A peak occurs always at matched densities for which the momentum transfer is most efficient. It is seen for $T=6.5$ K and most clearly pronounced at $T=2.1$ K while for $T=1.1$ K it is only a small feature on an otherwise monotonous function of n_2 (see Fig. 5). This strikingly different behavior for $T=1.1$ K as compared to $T=2.1$ and 6.5 K which is similar to the experimental finding in Refs. 13 and 14, can be attributed to the changing dominance of small and large angle scattering with temperature and n_2.

In conclusion, we have calculated the drag scattering rate

between spatially separated electron layers due to the virtual phonon exchange via DA and PA mechanisms We have shown that the frictional drag due to PA coupling is dominating. Assuming infinite lifetime of the intermediate virtual phonon states, the frictional drag does not depend on the interlayer separation. According to our calculations the temperature dependence of the drag scattering rate divided by T^2 exhibits a pronounced peak both at matched electron density of two layers and at temperatures where a transition from small to large angle scattering occurs. These results are in agreement with the finding of Gramila *et al.*[13]

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