Phase transitions of electron-hole and unbalanced electron systems in coupled quantum wells in high magnetic fields

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The superfluidity of spatially separated electrons and holes and of an unbalanced two-layer electron system in a high magnetic field is considered. The temperature T_c of the Kosterlitz-Thouless transition to a superfluid state is obtained as a function of magnetic field H and interlayer separation D. The equation of state for the magnetoexciton system in the quasiclassical regime is analyzed. The transition from excitonic phase to electron-hole phase is considered. Possible experimental manifestations of the predicted effects are briefly discussed. [S0163-1829(99)05204-2]

I. INTRODUCTION

Systems of excitons with spatially separated electrons (e) and holes (h) (indirect excitons) in coupled quantum wells (CQW's) in magnetic fields (H) are now the subject of intensive experimental investigations.[1–3] They are of interest, in particular, in connection with the possibility of superfluidity of indirect excitons or e-h pairs, which would manifest itself in the CQW's as *persistent* electrical currents in each well[4] (see also recent articles[5–7]) and also in connection with the quasi-Josephson phenomena in the system (see Ref. 8 and references therein). In high magnetic fields, two-dimensional (2D) excitons survive in a substantially wider temperature region, as the exciton binding energies increase with magnetic field.[9–11] In addition, the 2D e-h system in high fields H is interesting due to the existence, under some conditions, supersymmetry in the system (for the single quantum well) leading to unique exact solutions of the many-body problem (the last corresponding to ideal Bose condensation of magnetoexciton at *any* density).[12]

The superfluid state appears in the system under consideration below the temperature of Kosterlitz-Thouless transition.[13] The latter was studied recently for systems with spatially separated electrons (e) and holes (h) in the absence of magnetic field.[5]

Attempts of experimental investigation of magnetoexciton superfluidity in coupled quantum wells[1] make it essential to study the magnetic-field dependence of the temperature of phase transition to the superfluid state in systems of indirect magnetoexcitons and to analyze the density of the superfluid component. This is the subject of this paper. It will be shown below that increasing of magnetic field at a fixed magnetoexciton density leads to a lowering of the Kosterlitz-Thouless transition temperature T_c on account of the increase of the exciton magnetic mass as a function of H. But it turns out that the highest possible Kosterlitz-Thouless transition temperature increases with H (at small D) due to the rise in the maximum density of magnetoexcitons vs H. Quantum phase transition of a magnetoexciton system to incompressible liquid states is briefly discussed in connection with the problem of maximum density of stable magnetoexciton system.

We show also that contributions to the thermodynamic potential and the state equation connected with interexciton interactions for rare magnetoexciton system at fixed density and temperature vanishes with H increasing.

In a magnetoexciton system in coupled quantum wells at essentially higher temperatures than the transition to the superfluid state the exciton thermal ionization takes place. The dependence of maximal $T_i(H,n)$ is defined by the magnetoexciton binding energy $E_0(H,D)$ and rises with magnetic field and decreases with interlayer separation D (n is the surface density of magnetoexcitons).

The paper is organized in the following way. In Sec. II (which has auxiliary character) we discuss the relation between electric current and magnetic momentum of noninteracting isolated magnetoexcitons, which we shall use in calculation of the density of normal component. In Sec. III, we consider the spectrum of collective excitations for the system of rare indirect excitons in a high magnetic field in the ladder approximation. In Sec. IV, we analyze the dependence of the density of the superfluid component on magnetic field and interlayer distance. In Sec. V, we calculate the dependence of the temperature of the Kosterlitz-Thouless transition to the superfluid phase on magnetic field and interlayer distance. In Sec. VI, we consider thermodynamics and equation of the state of rare magnetoexciton system at high temperatures and discuss the resemblance of the system to the ideal gas in the limiting case of high magnetic field. In Sec. VII, we estimate the magnetoexcitons existence line $T_i(H,n)$ connected with ionization transition (more strictly, this is crossover region). We use the ionization equilibrium condition analogous to Saha relation in the quasiclassical region and also discuss briefly the quantum region of magnetoexciton system stability in relation with the quantum transition of superfluid magnetoexciton system to two-layer Laughlin liquids of electrons and holes. In Sec. VIII, we discuss the phase transitions in the ''dense'' system. In Sec. IX we consider properties of indirect magnetoexcitons in unbalanced two-layer electron system. In Sec. X, we discuss possible experimental manifestations of superfluidity of magnetoexcitons in CQW. In Sec. XI we present our conclusions.

II. ISOLATED MAGNETOEXCITON IN THE SYSTEM OF SPATIALLY SEPARATED ELECTRONS AND HOLES

The total Hamiltonian $\hat{H}$ of an isolated pair of spatially separated e and h in the magnetic field is

$$\hat{H} = \frac{1}{2m_e}(-i\nabla_e + e\mathbf{A}_e)^2 + \frac{1}{2m_h}(-i\nabla_h - e\mathbf{A}_h)^2$$
$$- \frac{e^2}{\sqrt{(\mathbf{r}_e - \mathbf{r}_h)^2 + D^2}}, \tag{1}$$

where m_e, m_h are the effective electron and hole masses; $\mathbf{A}_e$, $\mathbf{A}_h$ are the vector potentials in electron and hole location, respectively; $\mathbf{r}_e$, $\mathbf{r}_h$ are electron and hole locations along quantum wells (we use units $c = \hbar = 1$).

A conserved quantity for isolated exciton in magnetic field (the exciton *magnetic* momentum) is (see Ref. 14)

$$\hat{\mathbf{P}} = -i\nabla_e - i\nabla_h + e(\mathbf{A}_e - \mathbf{A}_h) - e[\mathbf{H}, \mathbf{r}_e - \mathbf{r}_h]. \tag{2}$$

The conservation of this quantity is connected with the invariance of the system upon a simultaneous translation of e and h and gauge transformation.

Let us consider the coordinates of the center of mass $\mathbf{R} = (m_e\mathbf{r}_e + m_h\mathbf{r}_h)/(m_e + m_h)$ and the internal exciton coordinates $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$. The cylindrical gauge for vector potential is used: $\mathbf{A}_{e,h} = \frac{1}{2}[\mathbf{H}, \mathbf{r}_{e,h}]$.

Eigenfunctions of Hamiltonian Eq. (1) (which are also the eigenfunctions of the magnetic momentum $\mathbf{P}$) are (see Refs. 12 and 14)

$$\Psi_{Pk}(\mathbf{R},\mathbf{r}) = \exp\left\{i\mathbf{R}\left(\mathbf{P} + \frac{e}{2}[\mathbf{H},\mathbf{r}]\right) + i\gamma\frac{\mathbf{Pr}}{2}\right\}\Phi_k(\mathbf{P},\mathbf{r}), \tag{3}$$

where $\Phi_k(\mathbf{P},\mathbf{r})$ is the function of internal coordinates $\mathbf{r}$; $\mathbf{P}$ is the eigenvalue of magnetic momentum; k are quantum numbers of exciton internal motion. In high magnetic fields $k = (n,m)$, where $n = min(n_1,n_2)$, $m = |n_1 - n_2|$, $n_{1,2}$ are Landau quantum numbers for e and h;[12,10] $\gamma = (m_h - m_e)/(m_h + m_e)$.

The effective Hamiltonian H_P has the form

$$\hat{H}_P = -\frac{1}{2\mu^*}\Delta_r - \frac{ie\gamma}{2\mu^*}[\mathbf{H},\mathbf{r}]\nabla_r + \frac{e^2}{8\mu^*}[\mathbf{H},\mathbf{r}]^2$$
$$+ \frac{e}{M}[\mathbf{H},\mathbf{r}]\mathbf{P} + \frac{\mathbf{P}^2}{2M} - \frac{e^2}{\sqrt{\mathbf{r}^2 + \mathbf{D}^2}}, \tag{4}$$

where $\mu^* = m_e m_h(m_e + m_h)$.

Using the Feynman theorem (we denote $\langle Pk|\cdots|Pk\rangle$ as $\langle\cdots\rangle$) one can obtain for isolated magnetoexciton current (see Ref. 14):

$$\mathbf{J}_k(\mathbf{P}) = M\left\langle\frac{\partial\hat{H}_P}{\partial\mathbf{P}}\right\rangle = M\frac{\partial\varepsilon_k(P)}{\partial\mathbf{P}} = M\frac{\mathbf{P}}{P}\frac{\partial\varepsilon_k(P)}{\partial P}, \tag{5}$$

where $M = m_e + m_h$; $\varepsilon_k(P)$ is the magnetoexciton dispersion law [for indirect excitons $\varepsilon_k(P)$ in dependence on H and interwell separations D was analyzed in detail[10]].

The dispersion relation $\varepsilon_k(P)$ of isolated magnetoexciton is the quadratic function at small magnetic momenta:

$$\varepsilon_k(\mathbf{P}) \approx \frac{P^2}{2m_{Hk}}, \tag{6}$$

where m_{Hk} is the effective *magnetic* mass, dependent on H and the distance D between e and h layers and quantum number k (see Ref. 10).

The quadratic dispersion relation Eq. (6) is true for small P at arbitrary magnetic fields H and follows from the fact that $P = 0$ is an extremal point of the dispersion law $\varepsilon_k(\mathbf{P})$. The last statement may be proved by taking into account the regularity of the effective Hamiltonian H_P as a function of the parameter $\mathbf{P}$ at $\mathbf{P} = 0$ and also the invariance of H_P upon simultaneous rotation of r and P in the CQW plane.[15] For magnetoexciton ground state $m_H > 0$.

For high magnetic fields $r_H \ll a_0^*$ and at $D \lesssim r_H$, the quadratic dispersion relation is valid at $P \ll 1/r_H$, but for $D \gg r_H$ it holds over a wider region — at least at $P \ll (1/r_H)(D/r_H)$ (Ref. 10) [$a_0^* = 1/2\mu e^2$ is the radius of a 2D exciton at $H = 0$; $\mu = m_e m_h/(m_e + m_h)$; $m_{e,h}$ are the effective masses of e and h].

Using the quadratic dispersion relation for magnetoexcitons, one has at any H an expression for the magnetoexciton velocity analogous to that for the ordinary momentum $\dot{\mathbf{R}} = \partial\varepsilon_k(\mathbf{P})/\partial\mathbf{P} = \mathbf{P}/m_{Hk}$. So the mass current of an isolated magnetoexciton for small P is

$$\mathbf{J}_k(\mathbf{P}) = \frac{M}{m_{Hk}}\mathbf{P}. \tag{7}$$

III. SPECTRUM OF COLLECTIVE EXCITATIONS

Due to interlayer separation D, indirect magnetoexcitons both in ground state and in excited states have electrical dipole moments. We suppose, that indirect excitons interact as *parallel* dipoles. This is valid, when D is larger than the mean separation $\langle r\rangle$ between electron and hole along quantum wells $D \gg \langle r\rangle$. We take into account that at high magnetic fields $\langle r\rangle \approx Pr_H^2$ ($\langle\mathbf{r}\rangle$ is normal to $\mathbf{P}$) and that the typical value of magnetic momenta (with exactness to logarithm of the exciton density [$\ln(n_{ex})$, see below]) is $P \sim \sqrt{n_{ex}}$ (if the dispersion relation $\varepsilon_k(P) = P^2/2m_{Hk}$ is true). So the inequality $D \gg \langle r\rangle$ is valid at $D \gg \sqrt{n}r_H^2$.

The distinction between excitons and bosons manifests itself in exchange effects (see, e.g., Refs. 16 and 5). These effects for excitons with spatially separated e and h in a rare system $n_{ex}a^2(H,D) \ll 1$ are suppressed due to the negligible overlapping of wave functions of two excitons on account of the potential barrier, associated with the dipole-dipole repulsion of indirect excitons[5] [here n_{ex}, $a(D,H)$ are respectively density and magnetoexciton radius along quantum wells, respectively). Small tunneling parameter connected with this barrier is

$$\exp\left[-\frac{1}{\hbar}\int_{a(H,D)}^{r_0}\sqrt{2m_{Hk}\left(\frac{e^2D^2}{R^3} - \frac{\kappa^2}{2m_{Hk}}\right)}dR\right],$$

where

$$\kappa^2 \sim n/\ln\left(\frac{1}{8\pi nm_{Hk}^2e^4D^4}\right)$$

is the characteristic momentum of the system (see below); $r_0 = (2m_{Hk}e^2D^2/\kappa^2)^{1/3}$ is the classical turning point for the
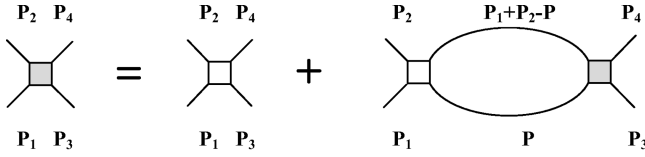
FIG. 1. The equation for the vertex Γ in the representation of magnetic momenta $\mathbf{P}$ and quantum numbers m and n.

dipole-dipole interaction. In high magnetic fields, the small parameter mentioned above has the form $\exp[-2\hbar^{-1}(m_{Hk})^{1/2}eDa^{-1/2}(H,D)]$. Therefore, the system of indirect magnetoexcitons can be treated by the diagram technique for a boson system.

But in contrast with a 2D boson system in the absence of magnetic field (see Ref. 4), some problems arise due to non-separation of the relative motion of e and h and exciton center of mass motion in magnetic fields.[12,10] Due to the non-separation of internal and center of mass motions the Green functions depend on both the external coordinates $\mathbf{R},\mathbf{R}'$ and the internal coordinates $\mathbf{r},\mathbf{r}'$.

For the rare two-dimensional magnetoexciton system [at $n_{ex}a^2(D,H)\ll 1$], the summation of ladder diagrams is adequate. The integral equation for vertex Γ in the ladder approximation is represented on Fig. 1. In the strong magnetic fields, the representation using as a basis of isolated magnetoexciton wave functions $\Psi_{\mathbf{P},m}(\mathbf{r},\mathbf{R})$ is convenient.

We use the following approximation for the interaction between two magnetoexcitons $U(P)=U_0$ at $P<a^{-1}(H,D)$ and $U(P)=0$ at $P>a^{-1}(H,D)$. After exciton-exciton scattering their total magnetic momentum is conserved, but magnetic momentum of each exciton can be changed. Since the mean distance between e and h along quantum wells is proportial to the magnetic momentum, the scattering is accompanied by the exciton polarization. We consider sufficiently low temperatures when magnetoexciton states with only small magnetic momenta $P\ll 1/r_H$ are filled. The change of these magnetic momenta due to exciton-exciton scattering is also negligible due to the conservation of the total magnetic momentum. But these small magnetic momenta correspond to small separation between electrons and holes along quantum wells $\rho\ll r_H$. So, magnetoexciton polarization due to scattering is negligible and the magnetoexciton dipole moment keeps to be almost normal to quantum wells $d=eD$, i.e., the interexciton interaction law is not changed due to the scattering.

The equation for Γ can be solved in the strong magnetic fields $\omega_c=eH/\mu^*\gg e^2/r_H$, when the characteristic value of e-h separation in the magnetoexciton $|\langle\mathbf{r}\rangle|$ has the order of the magnetic length $r_H=1/\sqrt{eH}$. The functions $\Phi_k(\mathbf{P},\mathbf{r})$ [see Eq. (3)] are dependent on the difference $(\mathbf{r}-\rho)$, where $\rho=(r_H^2/H)[\mathbf{H},\mathbf{P}]$.[9,14] At small magnetic momenta $P\ll 1/r_H$ we have $\rho\ll r_H$, and, therefore, in functions $\Phi_k(\mathbf{r}-\rho)$ we can ignore the variable ρ relatively to $\mathbf{r}$. In the strong magnetic-field quantum numbers k correspond to the quantum numbers (m,n) (see above). For the lowest Landau level we denote $\varepsilon_{00}(\mathbf{P})=\varepsilon(\mathbf{P})$ and $\mathbf{J}_{00}(\mathbf{P})=\mathbf{J}(\mathbf{P})$. Using the orthonormality of functions $\Phi_{mn}(\mathbf{0},\mathbf{r})$ we obtain approximate equation for the vertex Γ in strong magnetic fields. In high magnetic field, when the typical interexciton interaction $D^2n^{-3/2}\ll\omega_c$, one can ignore transitions between Landau

levels and consider only the states on the lowest Landau level $m=n=0$. Since typical value of r is r_H, and $P\ll 1/r_H$ in this approximation the equation for the vertex in the magnetic momentum representation P (see Fig. 1) on the lowest Landau level $m=n=0$ has the same form (compare with Ref. 17) as for two-dimensional boson system without a magnetic field, but with the magnetoexciton magnetic mass m_H (which depends on H and D) instead of the exciton mass $(L=m_e+m_h)$ and magnetic momenta instead of ordinary momenta:

$$\Gamma(k,q;L)=U_F(\mathbf{k}-\mathbf{q})$$
$$+\int\frac{d\mathbf{l}}{(2\pi)^2}\frac{U_F(\mathbf{k}-\mathbf{l})\Gamma(l,q;L)}{\dfrac{\kappa^2}{m_H}+\Omega-\dfrac{L^2}{4m_H}-\dfrac{l^2}{m_H}+i\delta},$$
$$\tag{8}$$
$$\mu=\frac{\kappa^2}{2m_H}=n_{ex}\Gamma_0.$$

Here μ is the chemical potential of the system.

We find the solution of Eq. (8) by using the approximation for the effective interaction:

$$\Gamma(P)=\begin{cases}\Gamma_0, & P<a^{-1}(H,D)\\ 0, & P>a^{-1}(H,D).\end{cases}\tag{9}$$

The integral Eq. (8) for the vertex can be solved analytically in the approximation $\kappa\ll\sqrt{n}$. This inequality must be fulfilled simultaneously with the condition of low density $na^2(H,D)\ll 1$, which is necessary for the applicability of the ladder approximation. The solution of the integral equation for the vertex Γ of this system can be expressed through the solution of the equation for the amplitude of scattering $f_0(\kappa)$ of isolated pair of interacting particles (with a mass equal to the magnetic mass m_H of magnetoexciton) in the two-dimensional system *without magnetic field* with the repulsing potential $U(R)=e^2D^2/R^3$:

$$f_0(\kappa)=\frac{\left(\dfrac{\pi i}{2\kappa}\right)^{1/2}}{\ln(\kappa m_He^2D^2)}.\tag{10}$$

Here the characteristic magnetic momentum κ contrary to three-dimensional system is not equal to zero and is determined from the relation

$$\kappa^2=-4nf_0(\kappa)\left(\frac{2\pi\kappa}{i}\right)^{1/2}.\tag{11}$$

This is specific feature of two-dimensional Bose system connected with logarithmic divergence of two-dimensional scattering amplitude at zero energy. A simple analytical solution for the chemical potential can be obtained if $\kappa m_He^2D^2\ll 1$. In strong magnetic fields at $D\gg r_H$ the exciton magnetic mass is $m_H\approx D^3/e^2r_H^4$.[10] So the inequality $\kappa m_He^2D^2\ll 1$ is valid if $D\ll(r_H^4/n^{1/2})^{1/5}$. In result the chemical potential μ is obtained in the form

$$\mu = \frac{\kappa^2}{2m_H} = \frac{8\pi n}{2m_H \ln\left(\frac{1}{8\pi n m_H^2 e^4 D^4}\right)}. \tag{12}$$

The spectrum of collective excitations following from the solution of Eq. (8) at small magnetic momenta is the sound $\varepsilon(P) = c_s P$ with the sound velocity $c_s = \sqrt{n\Gamma/m_H} = \sqrt{\mu/m_H}$, where μ is defined by Eq. (12).

IV. THE DENSITY OF THE SUPERFLUID COMPONENT

The temperature of the Kosterlitz-Thouless transition T_c (Ref. 13) to the superfluid state in a two-dimensional magnetoexciton system is determined by the equation:

$$T_c = \frac{\pi\hbar^2 n_s(T_c)}{2k_B m_H}, \tag{13}$$

where $n_s(T)$ is the superfluid density of the magnetoexciton system as a function of temperature T, magnetic field H, and interlayer distance D; and k_B is Boltzmann constant.

The function $n_s(T)$ (13) can be found from the relation $n_s = n_{ex} - n_n$ (n_{ex} is the total density, n_n is the normal component density). We determine the normal component density by the usial procedure.[18] Suppose that the magnetoexciton system moves with a velocity u. At nonzero temperatures dissipating quasiparticles will appear in this system. Since their density is small at low temperatures, one can assume that the gas of quasiparticles is an ideal Bose gas.

To calculate the superfluid component density we find the total current of quasiparticles in a frame in which the superfluid component is at rest. Then using Eq. (5), we obtain the mean total current of 2D magnetoexcitons in the coordinate system, moving with a velocity $\mathbf{u}$:

$$\langle \mathbf{J}' \rangle = \frac{M}{m_H} \langle \mathbf{P}' \rangle = \int \frac{d\mathbf{P}'}{(2\pi)^2} \mathbf{P}' f_{\varepsilon(P') - \mathbf{P}'\mathbf{u}}. \tag{14}$$

In the first order by $\mathbf{P}'\mathbf{u}/T$ we have

$$\langle \mathbf{J}' \rangle = \mathbf{u}\frac{M}{m_H} \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4}. \tag{15}$$

Using Eq. (15), we see that the total current of the system is proportional to the total magnetic momentum $\mathbf{P}$. Then, we determine the normal component density n_n:

$$\langle \mathbf{J}' \rangle = M n_n \mathbf{u}. \tag{16}$$

Comparing Eqs. (16) and (15) we obtain the expression for the normal density n_n. As a result, we have for the superfluid density:

$$n_s = n_{ex} - n_n = n_{ex} - \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4 m_H}. \tag{17}$$

It occurs that the expression for the superfluid density n_s in the strong magnetic field for the magnetoexciton rare system differs from analogous expression in the absence of

magnetic field (compare with Ref. 5) by replacing of exciton mass $M = m_e + m_h$ with the exciton magnetic mass m_H.

V. SUPERFLUID STATE TRANSITION

A. Kosterlitz-Thouless temperature

The superfluidity of magnetoexcitons appears below the Kosterlitz-Thouless temperature T_c [Eq. (13)], where only bound vortexes are present. Substituting the expression for the superfluid component density n_s from Eq. (17) into Eq. (13), we obtain the equation for the Kosterlitz-Thouless temperature T_c. The solution is

$$T_c = \left[\left\{ 1 + \sqrt{\frac{16}{(6\times0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1} \right\}^{1/3} + \left\{ 1 - \sqrt{\frac{16}{(6\times0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1} \right\}^{1/3} \right] \frac{T_c^0}{(4\pi)^{1/3}}. \tag{18}$$

Here T_c^0 is the auxiliary quantity equal to the temperature of vanishing of superfluid density in the mean-field approximation $n_s(T_c^0) = 0$:

$$T_c^0 = \left(\frac{2\pi n_{ex} c_s^4 m_H}{3\zeta(3)} \right)^{1/3}$$

$$= \left(\frac{32}{3\zeta(3) ln^2\left(\frac{1}{8\pi n m_H^2 D^4}\right)} \right)^{1/3} \frac{\pi n_{ex}}{m_H}. \tag{19}$$

In high fields H and at small $\mathbf{P}$ for the lowest Landau level ($n=0$) and at quantum number $m=0$ the exciton effective magnetic mass is $m_H = 2^{3/2}/e^2 r_H \sqrt{\pi}$ at $D \ll r_H$ and $m_H \approx D^3/e^2 r_H^4$ at $D \gg r_H$. At large D, i.e., for $D \gg a_0^*$ in weak fields ($r_H \gg a_0^*$) or $D \gg r_H$ in high fields ($r_H \ll a_0^*$) one has $m_H = M + H^2 D^3/c^2$.[10]

The temperature $T_c^0 = T_c^0(D,H)$ may be used as a crude estimate of the crossover region where local superfluid density appears for rare magnetoexciton system on the scales smaller or of the order of mean intervortex separation in the system. The local superfluid density can manifest itself in local optical properties or local transport properties (see below). In the rare two-dimensional system in the ladder approximation [i.e., at $\ln(8\pi n_{ex} m_H^2 e^4 D^4)^{-1} \gg 1$] the Kosterlitz-Thouless temperature is

$$T_c = \frac{T_c^0}{(2\pi)^{1/3}}. \tag{20}$$

At maximal temperature of superfluid (the Kosterlitz-Thouless temperature) the normal density approximately is

$$n_n(T_c) = \frac{3\zeta(3)}{2\pi} \frac{T_c^3}{c_s^4 m_H}. \tag{21}$$

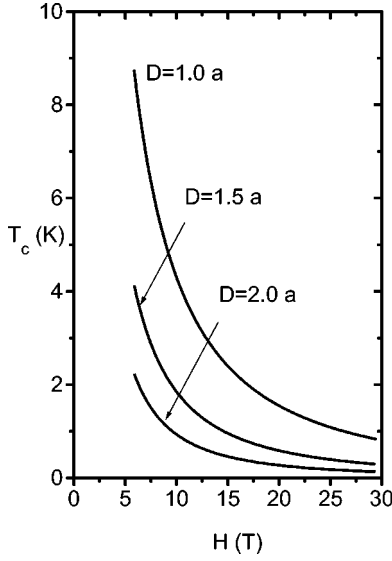This estimate does not take into account small contribution of vortexes. Substituting Eq. (18) to Eq. (21), we obtain

FIG. 2. Dependence of temperature of Kosterlitz-Thouless transition T_c on magnetic field H at different interwell separations D.

$$n_n(T_c) = \frac{n_{ex}}{4\pi}\left\{\left[1 + \sqrt{\frac{16}{(6\times 0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1}\right]^{1/3}\right.$$
$$\left. + \left[1 - \sqrt{\frac{16}{(6\times 0.45)^3\pi^4}\left(\frac{m_H T_c^0}{n_{ex}}\right)^3 + 1}\right]^{1/3}\right\}^3.$$

$$(22)$$

In rare two-dimensional system in the ladder approximation we have

$$\frac{n_n(T_c)}{n_{ex}} = \frac{1}{2\pi}. \tag{23}$$

Note that Eqs. (20) and (23) take place for *any* two-dimensional rare Bose gas. The dimensionless value $n_n(T_c)/n_{ex}$ can be considered as the small parameter. So the approximation of the ideal Bose gas of quasiparticles is valid. Note that for the dense electron-hole system without magnetic field at $n_{ex}\to\infty$ an opposite case $n_n(T_c)/n_{ex}\to 1$ takes place due to exponential vanishing of the order parameter Δ (see, e.g., Refs. 4 and 5).

According to Eqs. (18) and (19), the temperature of the onset of superfluidity due to the Kosterlitz-Thouless transition at *a fixed magnetoexciton density* decreases as a function of magnetic field due to the increase in m_H as a function of H and D, while T_c decreases as $H^{-1/2}$ at $D\ll r_H$ or as H^{-2} at $D\gg r_H$, and n_s is a slowly decreasing function of D. The dependencies of T_c on H are shown in Fig. 2.

From Eqs. (18) and (19) one can see that the Kosterlitz-Thouless temperature of a rare magnetoexciton system is proportional to the magnetoexciton density n_{ex}. At high magnetic fields the symmetry $\nu\to 1-\nu$, $e\leftrightarrow h$ takes place for the Landau level (see Ref. 12). Thus unoccupied states on Landau levels for spatially separated electrons and holes can bind to "antiexcitons" and superfluidity of "antiexcitons" can also take place at $1-\nu\ll 1$. The Kosterlitz-Thouless temperature for superfluidity of antiexcitons as function of H, D for strong H is symmetrical to that for excitons. The

top Kosterlitz-Thouless temperature at high magnetic fields corresponds to the "maximal" density n_{max} of stable magnetoexciton system at the Landau level $n_{max}=\nu_{max}1/4\pi r_H^2 \sim H$, where $\nu_{max}(D)$ is the maximal filling of Landau level for magnetoexcitons—see below [for "antiexcitons" the corresponding critical value is $1-\nu_{max}(D)$].

B. The problem of large magnetic momenta

At large magnetic momenta P, the isolated magnetoexciton spectrum $\varepsilon(P)$ contrary to the case $H=0$ has a constant limit (being equal to Landau level $\hbar\omega_c/2$ for reduced effective mass, see Refs. 10 and 12). As a result the spectrum of *interacting* magnetoexciton system also have a plateau at great momenta. So, Landau criterium of superfluidity is not valid at large P for the interacting magnetoexciton system. However, the probability of excitation of quasiparticles with magnetic momenta $P\gg 1/r_H$ by moving magnetoexciton system is negligibly small at small superfluid velocities. In this sense, the superfluidity of 2D magnetoexcitons keeps to be almost metastable one. This can be shown by the estimation of the probability dW of the excitation of the quasiparticle on the plateau with magnetic momenta $P\gg 1/r_H$; the energy of quasiparticles on the plateau $\varepsilon(P)$ equals to the magnetoexciton binding energy. At high magnetic fields we have

$$\varepsilon(P)\sim\sqrt{\frac{\pi}{2}}\frac{e^2}{r_H} - \frac{e^2}{Pr_H^2}, \quad D\ll a(H,D), \quad Pr_H^2\gg D, \tag{24}$$

$$\varepsilon(P)\sim\frac{e^2}{D} - \frac{e^2}{Pr_H^2}, \quad D\gg a(H,D), \quad Pr_H^2\gg D. $$

At the motion of magnetoexciton liquid in a lattice with the small velocity **u**, which is smaller than the sound velocity c_s, according to the Landau criterium[18] creation of the quasiparticles in the region of plateau at great momenta with the magnetic momentum $P\gg 1/r_H$ and the energy $\sim E_0$ is possible due to the friction between liquid and impurities, defects in the lattice or roughness of boundaries of quantum wells. So when one quasiparticle appears the liquid gets the magnetic momentum P. The appearance of the large magnetic momentum in the liquid is equivalent to the great mean separation between electron and hole along one layer $\rho = (r_H^2/H)[\mathbf{H},\mathbf{P}]$ (see Sec. II) So magnetoexcitons with *very* large P does not exist due to the interaction of electron and hole with impurities etc.

Let us estimate the probability dW_P of the transition of the superfluid system from the initial state with the magnetic momentum $P=0$ without quasiparticles to the final state with one quasiparticle with the large magnetic momentum $P\gg 1/r_H$ by using Fermi golden rule taking into account the "friction" interaction V. We have for the probability per unit of time $dW(P)$:

$$dW(P) = \frac{2\pi}{\hbar}|\langle 0|\hat{V}\hat{\alpha}^\dagger|0\rangle|^2\delta[\Delta E_k + \varepsilon(P) + \mathbf{Pu}]d\nu_\varepsilon, \tag{25}$$

where ν_ε is the density of final states of the system; ΔE_k is the change in the kinetic energy of superfluid liquid; $\hat{V}$ is the

''friction'' interaction (see below); $|0\rangle$ is a ground state of magnetoexciton superfluid; $\alpha_{\mathbf{P}}^{\dagger}$ is the quasiparticle creation operator. After quasiparticle creation total magnetic momentum of the system is conserved.

At large momentum $P \gg 1/r_H$, the wave function of quasiparticle is almost the same as wave function of the isolated magnetoexciton. It means that the quasiparticle annihilation operator $\alpha_{\mathbf{P}}$ is almost the same as the ordinary particle annihilation operator $a_{\mathbf{P}}$.

In second quantified representation the ''friction'' interaction operator $\hat{V}$ can be represented as

$$\hat{V} = \sum_{\mathbf{P'P''}} V_{\mathbf{P'P''}} a_{\mathbf{P'}}^{\dagger} a_{\mathbf{P''}}, \qquad (26)$$

where $V_{\mathbf{P'P''}}$ is the matrix element of ''friction'' interaction calculated with the use of isolated magnetoexciton eigenfunctions Eq. (3).

We find

$$\langle 0|\hat{V} a_P^{\dagger}|0\rangle = V_F(\mathbf{P}) e^{-[(1-2/\gamma)8]P^2 r_H^2}, \qquad (27)$$

where $V_F(\mathbf{P})$ is the Fourier-transform of V. Then the probability dW_P of the creation of the quasiparticle per unit of time with the large magnetic momentum P and the energy $\varepsilon(P)$ is

$$dW(P) = \frac{1}{(2\pi)^2 \hbar^3} e^{-[(1+2\gamma^2)/4]r_H^2 P^2} |V_F(\mathbf{0},\mathbf{P})|^2$$
$$\times \delta[\Delta E + \varepsilon(P) + \mathbf{Pu}] P dP. \qquad (28)$$

Thus, the probability dW_P of the creation of the quasiparticle with the large magnetic momenta $P \gg 1/r_H$ is negligibly small as $dW_P \sim e^{-[(1+2\gamma^2)/4]r_H^2 P^2} \ll 1$. So the superfluidity of 2D magnetoexcitons keeps to be almost metastable one. Note that at small magnetic momenta $P \ll 1/r_H$ in the region of the sound spectrum of interacting magnetoexcitons Landau criterium of superfluidity is valid and the probability dW_P of the creation of the quasiparticle in the region of the sound spectrum at $u < c_s$ is zero due to $\delta[\Delta E + \varepsilon(P) + \mathbf{Pu}] = 0$ in Eq. (25).

At low temperatures, $T < T_c \ll E_0$ states with large magnetic momenta are negligibly filled ($\exp[-\varepsilon(P)/T] \ll 1$, where $\varepsilon(P)$ is the magnetoexciton energy that has the same order as magnetoexciton binding energy E_0; at high magnetic fields $E_0 = \sqrt{\pi/2} e^2/r_H$ at $D \ll a(H,D)$ and $E_0 = e^2/D$ at $D \gg a(H,D)$). So quasiparticles at large magnetic momenta P give a small contribution to the densities of the normal component n_n and superfluid component n_s [see Eq. (17)]. Hence, the expressions given above for the temperature of Kosterlitz-Thouless transition are valid.

VI. THERMODYNAMICS AND EQUATION OF THE STATE OF THE SYSTEM AT HIGH TEMPERATURES

Now, we estimate correction terms in the chemical potential and the equation of the state for slightly nonideal gas of 2D magnetoexcitons at high temperatures due to exchange effects and dipole-dipole interaction between magnetoexcitons.[19] We show these correction terms are small at high magnetic fields and high temperatures, and so

their contributions to the chemical potential and the equation of the state are additive. So, we can consider these effects separately.

One has for the free energy F of ideal gas of bosons:[20]

$$F = F_{Bol}\left(1 - \frac{1}{4} e^{\mu/T}\right). \qquad (29)$$

The chemical potential μ^0 of ideal gas of magnetoexcitons can be obtained from the normalization condition for the number of magnetoexcitons:

$$\mu^0 = -T \ln\left(\frac{m_H T}{2\pi\hbar^2 n_{ex}}\right). \qquad (30)$$

At high temperatures and high magnetic fields the inequality $e^{\mu/T} \ll 1$ is true for a rare system. Using the relation for the pressure $P = -(\partial F/\partial S)_{T,N}$ and Eq. (29), we have for the equation of the state

$$P = \frac{NT}{S}\left(1 - \frac{\pi\hbar^2 n_{ex}}{2m_H T}\right). \qquad (31)$$

Using the relation $\mu = (\partial F/\partial N)_{T,S}$ for the chemical potential μ, we obtain the contribution of exchange interactions to the chemical potential μ [with exactness to $O(n^2)$]:

$$\mu = -T \ln\left(\frac{m_H T}{2\pi\hbar^2 n_{ex}}\right) + 2T \frac{\pi\hbar^2 n_{ex}}{2m_H T}. \qquad (32)$$

Now we analyze the contribution of interaction. We estimate for rare 2D magnetoexciton system the second virial coefficient $B(T)$ in expansion of 2D pressure on $1/S$ (S is the area of the system; Boltzmann constant $k_B = 1$)

$$P = \frac{NT}{S}\left[1 + \frac{NB(T)}{S} + \cdots\right]. \qquad (33)$$

At high temperatures the virial coefficient is

$$B(T) = \frac{1}{2} \int (1 - e^{-U(R)/T}) dS$$
$$\approx \pi T^{-2/3} (eD)^{4/3} \ln\left(\frac{2\pi\hbar^2 n_{ex}}{m_H T}\right), \qquad (34)$$

where $U(R) = e^2 D^2/R^3$ is the pair interaction between particles. We integrate Eq. (34) by coordinate from the classical turning point for the dipole-dipole interaction $R_0 = (e^2 D^2/\mu)^{1/3}$, substituting the chemical potential μ Eq. (30). At high temperatures, $U(R_0)/T \ll 1$ [where $R_0 \sim (\pi n)^{-1/2}$].

Using additivity of small exchange and dipole-dipole interaction corrections, we have the equation of the state with both corrections included

$$PS = NT\left[1 - \frac{\pi\hbar^2 n_{ex}}{2m_H T} + \pi T^{-2/3}(eD)^{4/3} n_{ex} \ln\left(\frac{2\pi\hbar^2 n_{ex}}{m_H T}\right)\right]. \qquad (35)$$

For the chemical potential μ, we obtain with exactness to $O(n^2)$ the chemical potential with both terms included

$$\mu = -T \ln\left(\frac{m_H T}{2\pi\hbar^2 n_{ex}}\right) + 2\pi T^{1/3}(eD)^{4/3} n_{ex} \ln\left(\frac{2\pi\hbar^2 n_{ex}}{m_H T}\right)$$

$$+ 2T\frac{\pi\hbar^2 n_{ex}}{2m_H T}. \tag{36}$$

The virial coefficient $B(T)$ decreases vs H due to increase of magnetic mass m_H. Hence, in high magnetic fields, the system of indirect magnetoexcitons is almost ideal gas due to $B(T) \ll 1$. Decrease of interexciton interaction can be investigated experimentally. In noninteracting system shape of spectral lines of magnetoexciton photoluminescence is determined by Doppler effect. For an interacting system it was demonstrated[21] that the shape of lines of photoluminescence of semiconductor quantum wells is dominated by many-body interactions, and it is essentially different from isolated particles because of exciton-exciton interactions. Comparing lineshapes at the different magnetic fields, one can demonstrate transition to an ideal gas in the exciton system in high magnetic fields.

VII. REGION OF EXISTENCE OF MAGNETOEXCITON PHASE

A. Ionization of magnetoexcitons in the quasiclassical region

In magnetoexciton system in coupled quantum wells at essentially higher temperatures than the transition to the superfluid state the exciton thermal ionization takes place. Magnetoexcitons existence line $T_i(H,n)$ (more strictly, crossover region) can be obtained in quasiclassical regime from the ionization equilibrium condition analogous to Saha relation[20] from condition of equality of exciton chemical potential to the sum of chemical potentials of electrons and holes.

We neglect transitions between Landau levels in high magnetic fields $\hbar\omega_c \gg T$ ($\omega_c = eH/m_e c$ is the cyclotron energy) and are measured the chemical potentials of electrons μ_e, holes μ_h and magnetoexcitons μ_{ex} from the lowest Landau level. Then, we have for the chemical potentials of electrons and holes at $2\pi r_H^2 n = \nu \ll 1$ (n is the density of magnetoexcitons at $T=0$) (see Ref. 25)

$$\mu_e = \mu_h = E_0\nu \tag{37}$$

and the chemical potential of magnetoexcitons is

$$\mu_{ex} = -T\ln\left(\frac{m_H T}{2\pi\hbar^2 n_{ex}}\right) + 2T\left[\frac{\pi\hbar^2 n_{ex}}{m_H}\right.$$

$$\left. + \pi T^{-2/3}(eD)^{4/3} n_{ex} \ln\left(\frac{2\pi\hbar^2 n_{ex}}{m_H T}\right)\right] + E_0, \tag{38}$$

where ν is the filling factor and E_0 is the magnetoexciton energy; at high magnetic fields $E_0 = \sqrt{\pi/2}e^2/r_H$ at $D \ll a(H,D)$ and $E_0 = e^2/D$ at $D \gg a(H,D)$.[10]

We obtain the equation for the characteristic temperature of ionization $T_i(n_{ex},D,H)$ from the condition of the ionization equilibrium:[20]
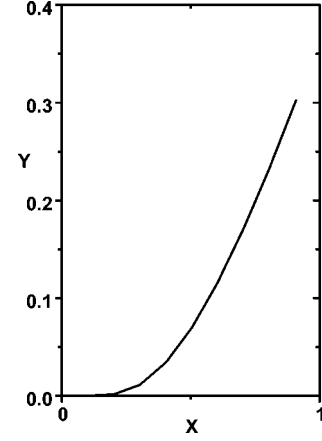

FIG. 3. Magnetoexcitons existence line $T_i(H,n)$; $y = E_{deg}/E_0$, $E_{deg} = 2\pi\hbar^2 n_{ex}/m_H$ is the energy of degeneration and $x = T/E_0$.

$$0 = \mu_{ex} - \mu_e - \mu_h = -T\ln\left(\frac{m_H T}{2\pi\hbar^2 n_{ex}}\right) + 2T\left(\frac{\pi\hbar^2 n_{ex}}{2m_H T}\right)$$

$$+ \pi T^{-2/3}(eD)^{4/3} n_{ex} \ln\left(\frac{2\pi\hbar^2 n_{ex}}{m_H T}\right) + E_0 - E_0\nu. \tag{39}$$

As $n_{ex} \to 0$ the characteristic temperature of ionization $T_i(n_{ex},D,H) \to 0$ (and the same for antiexcitons). The dependence of maximal $T_i(H)$ (corresponding to the maximal magnetoexciton density) is defined by the magnetoexciton binding energy E_0 (so $T_i \sim \sqrt{H}$ at small D) and rises vs magnetic field and decreases vs the interlayer separation increase.

If we introduce $y = E_{deg}/E_0$ ($E_{deg} = 2\pi\hbar^2 n_{ex}/m_H$ is the energy of degeneration) and $x = T/E_0$, this dependence is shown on Fig. 3.

B. Quantum transition to two-layer incompressible liquid state

The rare excitonic system is stable at $D < D_{cr}(H)$ and $T = 0$ when the magnetoexciton energy $E_{exc}(D,H)$ (calculated in Ref. 10) is larger than the sum of activation energies $E_L = ke^2/\epsilon r_H$ for incompressible Laughlin liquids of electrons or holes; $k = 0.06$ for $\nu = \frac{1}{3}$, etc.[22] (compare Ref. 23 for stability of *dense* excitonic phase—see below). Since $k \ll 1$, the critical value $D_{cr} \gg r_H$. In this case one has $E_{exc} = (e^2/\epsilon D)(1 - r_H^2/D^2)$ for a magnetoexciton with quantum numbers $n = m = 0$ (see Ref. 10). As a result, we have from the stability condition (see above) $D_{cr} = r_H(1/2k - 2k)$. For greater ν it gives an upper bound on D_{cr}. In the rare system in high magnetic fields, $\mu \ll E_{ex}$. The coefficient k in the activation energy E_L may be represented as $k = k_0\sqrt{\nu}$. So, from the relation between D_{cr} and r_H one has: $\nu_{cr} = (1/k_0^2)(r_H^2/D^2)(1 - r_H^2/8D^2)$. Thus, maximal density for stable magnetoexciton phase is $n_{max} = \nu_{cr}/2\pi r_H^2$ (see below). Hence, the *maximum* Kosterlitz-Thouless temperature, at which superfluidity appears in the system is $T_c^{max} \sim n^{max}(H,D)/m_H \sim \sqrt{H}$ at $D \lesssim r_H$ or $T_c^{max} \sim H^{-1}$ at $D \gg r_H$ in high magnetic fields. It would be interesting to check this fact in experiments on magnetoexciton systems. Note that if at a given density of e and h and a given magnetic field H

several Landau levels are filled (but high-field limit $r_H \ll a_0^*$ is true) the superfluid phase can exist for magnetoexcitons on the highest nonfilled Landau level. At $\nu = \frac{1}{2}$ the elecron-hole phase can be unstable due to the pairing of electron and hole composite fermions (which form the Fermi surface of composite fermions in the mean-field approximation[24]).

VIII. PHASE TRANSITIONS IN THE ''DENSE'' SYSTEM

In the ''dense'' system (at $\nu \sim 1/2$ contrary to rare system at $\nu \ll 1$) the ionization of magnetoexcitons can be estimated in the Gor'kov approximation.[25] It takes place at the temperature T_i^{dense},

$$T_i^{dense} = \frac{J}{2} \frac{1-2\nu}{\ln(\nu^{-1}-1)}, \qquad (40)$$

where

$$J = \int \frac{d^2q}{(2\pi)^2} V_{12}(q) \exp(-q^2 r_H^2/2) \qquad (41)$$

and

$$V_{12}(q) = \frac{2\pi e^2}{q} \exp(-qD). \qquad (42)$$

So the temperature of the ionization crossover in the Gor'kov approximation increases with rise of magnetic field as $\sqrt{H}$ and decreases with the interlayer separation (it is consist and with the estimation for the rare system—see above).

Let us calculate now the density of the normal component n_n. The contribution of the one-particle overgap excitations to the density of the normal component is determined by the transitions between Landau levels.[12] So, in high magnetic fields this contribution is negligible as $e^2/r_H \omega_c \ll 1$. We need also to take account of the contribution of collective excitations to the density of the normal component. In contrast to superconductors, where, as a consequence of the charge of the Cooper pairs, instead of an acoustic spectrum of collective oscillations high-frequent plasma mode arises, in the exciton phase e-h pairs are neutral and zero-gap collective excitations exist. At low temperatures, the contribution of the elementary excitations in thermodynamic equilibrium can be described in the approximation of an ideal Bose gas. So, the density of the superfluid component n_s can be estimated by Eq. (17). Now, one need to estimate the sound velocity c_s of collective mode in the system.

It can be shown easily that the equations of the hydrodynamics of magnetoexcitons contain $\mathbf{P}' = (M/m_H)\mathbf{P}$ instead of $\mathbf{P}$. So, we have for the sound velocity

$$c_s^2 = \frac{\partial P'}{\partial \rho} = \frac{M}{m_H} \frac{\partial P}{\partial \rho}. \qquad (43)$$

In result we obtain the expression for sound velocity with magnetic mass m_H instead of M (compare Ref. 18)

$$c_s^2 = \frac{n}{m_H} \frac{\partial \mu}{\partial n}. \qquad (44)$$

Using the relation $\nu = 2\pi r_H^2 n$ we have

$$c_s = \sqrt{\frac{\nu}{m_H} \frac{\partial \mu}{\partial \nu}}. \qquad (45)$$

The chemical potential of dense system of electron-hole pairs with spatially separated e and h in high magnetic field is[25]

$$\mu = -J + 2\nu(I-J). \qquad (46)$$

Here, $I = \int [d^2q/(2\pi)^2] V_{11}(q) \exp(-q^2 r_H^2/2)$, $V_{11}(q) = 2\pi e^2/q$, and J is defined by Eq. (41).

From Eqs. (46) and (45), one has for the sound velocity

$$c_s = \sqrt{\frac{2\nu(I-J)}{m_H}}. \qquad (47)$$

The temperature of the Kosterlitz-Thouless transition to the superfluid state can be calculated by using Eq. (18) (see Sec. V), where T_c^0 is the auxiliary quantity equal to the temperature of vanishing of superfluid density in the mean-field approximation $n_s(T_c^0) = 0$

$$T_c^0 = \left(\frac{\nu c_s^4 m_H}{3 r_H^2 \zeta(3)}\right)^{1/3} = \left(\frac{32 r_H^4 (I-J)^2}{3\zeta(3)m_H}\right)^{1/3} \frac{\nu}{2r_H^2}. \qquad (48)$$

In high magnetic fields for the dense system, the Kosterlitz-Thouless temperature is obtained by substituting of T_c^0 from Eq. (48) to Eq. (18). Kosterlitz-Thouless temperature decreases vs magnetic field analogously to the rare system (see above).

Then we have

$$I-J = e^2 \int_0^\infty (1 - e^{-qD}) e^{-q^2 r_H^2/2} dq. \qquad (49)$$

Let us consider for estimate small interlayer distances $D \ll 1/q_0$, where q_0 is the characteristic wave vector $q_0 \sim 1/r_0$, where $r_0 = 1/\sqrt{\pi n_{ex}}$ is the mean distance between particles. Maximal density of particles at $\nu = \frac{1}{2}$ is $n_{ex} = 1/4\pi r_H^2$ (at $\nu > 1/2$ we deal with antiexcitons—see below). If $D \lesssim r_H$, using Eq. (49), we have

$$I-J = e^2 D \int_0^\infty q e^{-\frac{q^2 r_H^2}{2}} dq = \frac{e^2 D}{r_H^2}. \qquad (50)$$

Then, we have for the temperature of the phase transition to the superfluid state in the mean-field approximation at small D

$$T_c^0 = \left(\frac{2\pi n_{ex} c_s^4 m_H}{3\zeta(3)}\right)^{1/3} = \left(\frac{32 e^4 D^2}{3\zeta(3)m_H}\right)^{1/3} \pi n_{ex}. \qquad (51)$$

As we mentioned above in high magnetic fields the symmetry $\nu \to 1 - \nu$, $e \leftrightarrow h$ takes place at the Landau level and unoccupied states on Landau levels for spatially separated electrons and holes can bind to form ''antiexcitons'' and superfluidity of ''antiexcitons'' can take place. The Kosterlitz-Thouless temperature for superfluidity of antiexcitons as function of H, D is symmetrical to that for excitons. Hence, in expressions for temperatures of phase transitions in Eqs. (48), and (51) we can use $\nu(1-\nu)$ instead of ν.

We can see that the temperature of the Kosterlitz-Thouless transition to the superfluid state at fixed density decrease with rise of magnetic field as $H^{-1/6}$ (for the rare system as $H^{-1/2}$—see above). T_c decreases also vs interlayer separation. At values of D greater than some critical one D_{cr} the dense superfluid system of magnetoexitons must transform into the system of two incompressible liquids.[23] The dense excitonic system is stable at $D<D_{cr}(H)$ and $T=0$ when the Hartree-Fock energy is larger than the sum of activation energies $E_L=ke^2/\epsilon r_H$ for incompressible Laughlin liquids of electrons or holes (it is consistent with the results for rare system—see above).

IX. MAGNETOEXCITONS IN UNBALANCED TWO-LAYER ELECTRON SYSTEM

The system of indirect magnetoexcitons can appear also in unbalanced two-layer *electron* system in CQW in strong magnetic fields near the filling factor $\nu=1$. An external electric voltage between quantum wells change the filling, so say in the first quantum well the filling factor will be $\nu_1=\Delta\nu \leqslant\frac{1}{2}$ and in another one it will be $\nu_2=1-\Delta\nu$. Unbalanced filling factors $\nu_1=1+\Delta\nu$, $\nu_2=1-\Delta\nu$ is also possible. Thus, in the first quantum well (QW) there are rare electrons on the second Landau level, and in the second QW there are rare empty places ("holes") on the first Landau level. "Excess" electrons in the first QW and "holes" in the second QW can bound to indirect magnetoexcitons with the density $n_{ex}=eH\Delta\nu/2\pi$. Superfluidity in a two-layer e-e system in high magnetic fields in the cases mentioned is analogous to the superfluidity of two-layer e-h system.

The expressions for critical values ν_{cr}, D_{cr} for Kosterlitz-Thouless temperature T_c calculated above are applicable also for two-layer electron system under consideration. So, in this approximation the phase diagram for unbalanced two-layer electron system is analogous to the two-layer electron-hole system. In condition for the stability of superfluid magnetoexciton liquid relating to quantum transition to two incompressible Laughlin liquids (see Sec. VII) one must use $\Delta\nu$ instead of ν. Due to Jain's mapping of fractional Landau-level fillings to integer Landau-level fillings[26] analogous results take place also for slightly unbalanced fractional fillings.

X. POSSIBLE EXPERIMENTAL MANIFESTATIONS OF MAGNETOEXCITON SUPERFLUIDITY

The appearance of local superfluid density above T_c can be manifested, for example, in observations of temperature dependence of the exciton diffusion on intermediate distances (with the help of local measurements of exciton photoluminescence at two points using optical fibers or pinholes in experiments like those in Ref. 1).

The superfluid state at $T<T_c$ can manifest itself in the existence of persistent ("superconducting") oppositely di-

rected electric currents in each layer. The interlayer tunneling in an *equilibrium* spatially separated electron-hole system leads to interesting Josephson phenomena in the system: to a transverse Josephson current, inhomogeneous (many sin-Gordon soliton) longitudinal currents,[8] diamagnetism in a magnetic field H parallel to the junction (when H is less than some critical value H_{c1}, depending on the tunneling coefficient), and a mixed state with Josephson vortices for $H>H_{C1}$ (Ref. 15) (in addition, taking tunneling into account leads to a loss of symmetry of the order parameter and to a change in the character of the phase transition).

XI. CONCLUSIONS

We have shown that at fixed exciton density n_{ex} the Kosterlitz-Thouless temperature T_c for the onset of superfluidity of magnetoexcitons decreases as a function of magnetic field as $H^{-1/2}$ (at $D\lesssim r_H$). But *the maximal T_c* (corresponding to the maximal magnetoexciton densities) increases with H in high magnetic fields as $T_c^{max}(H,D)\sim\sqrt{H}$ (at $D\lesssim r_H$). This fact needs to be compared in detail with the results of experimental studies of the collective properties of magnetoexcitons. The excitonic phase is more stable than the Laughlin states of electrons and holes at a given Landau filling ν if $D<D_{cr}=r_H(1/2k-2k)$, where k is the coefficient in the Laughlin activation energy. Below the Kosterlitz-Thouless temperature one can observe the appearance of persistent currents in separate quantum wells. We have shown, that in extremely high magnetic fields the system of indirect magnetoexcitons at fixed T has the statistical properties of almost ideal gas. At small interlayer distances $D\lesssim r_H$ the temperature of the Kosterlitz-Thouless transition to the superfluid state in the dense system decreases with rise of magnetic field as $H^{-1/6}$ due to rise of the magnetic mass of indirect magnetoexciton.[10] We discuss also the quantum transition to the incompressible liquid state. We calculated the characteristic temperature of insulator-metal transition and established that it rises with magnetic field and decreases with interlayer separation. Note that in some region of Landau filling inside $(0,\nu_{cr})$ (see above) crystal phase of indirect magnetoexcitons must exist (see Ref. 27). Note that its melting curve is analogous to one for electron crystal near metal gate because due to image forces effective interaction in the system is also dipole-dipole interaction[28,29] (the difference in results for these two systems is due to their different statistics).

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