Transverse magnetoresistance of the two-dimensional chiral metal

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We consider the two-dimensional chiral metal, which exists at the surface of a layered, three-dimensional sample exhibiting the integer quantum Hall effect. We calculate its magnetoresistance in response to a component of magnetic field perpendicular to the sample surface, in the low temperature, but macroscopic, regime where inelastic scattering may be neglected. The magnetoresistance is positive, following a Drude form with a field scale $B_0=\Phi_0/al_{\rm el}$ given by the transverse field strength at which one quantum of flux Φ_0 passes through a rectangle with sides set by the layer-spacing a and the elastic mean free path $l_{\rm el}$. Experimental measurement of this magnetoresistance may, therefore, provide a direct determination of the elastic mean free path in the chiral metal. [S0163-1829(99)05807-5]

I. INTRODUCTION

The existence of a new electronic phase, the two-dimensional chiral metal, formed at the surface of a layered, three-dimensional quantum Hall conductor has recently been predicted theoretically,[1,2] and confirmed experimentally.[3] This chiral metal arises from hybridization of the edge states associated with each layer of the quantum Hall conductor. The resulting surface phase is decoupled from states in the bulk of the conductor, which at the Fermi energy must be localized by disorder for the Hall conductance to be quantized. The distinguishing characteristics of the phase arise from the fact that electron motion is chiral along the edge of each layer, but diffusive in the direction perpendicular to the layers. In particular, drift in the chiral direction prevents repeated, phase coherent scattering of an electron from any given impurity, suppressing localization effects completely, outside the mesoscopic regime. By contrast, interference effects in the mesoscopic regime have a variety of consequences, which have been investigated theoretically in some detail.[1,2,4–12]

Transport measurements are, of course, the obvious experimental probe of the chiral metal. A surface-response current flowing in the chiral direction is associated with the quantized Hall conductance of the three-dimensional system. In contrast, surface conductivity in the nonchiral, diffusive direction depends on microscopic parameters of the system: the elastic mean free path $l_{\rm el}$, the interlayer coupling energy t, the chiral drift velocity v, and the inelastic scattering rate $\tau_{\rm in}^{-1}$; specifically, the mean conductance per square σ is given[2] in units of e^2/h by $\sigma=at^2l_{\rm el}/(\hbar v)^2$. Knowledge of the amplitude of mesoscopic conductance fluctuations, together with sample dimensions, should allow determination of the inelastic scattering length.[9] The first detailed studies of this kind[3] involved vertical transport in multiquantum well semiconductor samples; conduction by surface states also provides an interpretation of earlier[13] and more recent[14] experiments on the bulk quantum Hall effect in semiconductor samples, as well as phenomena in organic metals in strong magnetic fields.[15]

In this paper we suggest that measurements of the transverse magnetoresistance of the chiral metal—the dependence of surface resistance on a magnetic-field component $B_\perp$, perpendicular to the surface—may be a useful source of additional information. As anticipated by Balents and Fisher,[2] this magnetoresistance is positive. We calculate it in the (macrosccopic) low-temperature regime, in which the inelastic scattering rate is much smaller than that for elastic scattering, obtaining a Drude form

$$\sigma(B_\perp)=\frac{\sigma(B_\perp=0)}{1+(B_\perp/B_0)^2}, \tag{1}$$

with $B_0=\Phi_0/al_{\rm el}$. This result is exact for the model we study. Its simplicity is a direct consequence of the elimination of multiple-scattering processes by chiral motion. Moreover, while the value of $\sigma(B_\perp=0)$ depends on two unknown microscopic quantities, the elastic mean free path and the chiral velocity, the magnetoresistance field scale B_0 involves only the first of these. Studies of magnetoresistance are hence potentially both a test of our understanding of the chiral metal, and a way to determine separately the values of $l_{\rm el}$ and v.

We give qualitative arguments that lead to our results in Sec. II, present a detailed calculation in Sec. III, and add remarks bearing on experiment in Sec. IV.

II. MODEL AND QUALITATIVE DISCUSSION

Consider first the edge of a single, two-dimensional layer that has unit-quantized Hall conductance. Under the combined influence of the magnetic-field component normal to the layer and the confining potential at the edge of the sample, electrons at the Fermi energy will drift along the edge, acquiring at most a phase shift from impurity scattering or randomness in the position of the edge. Within a

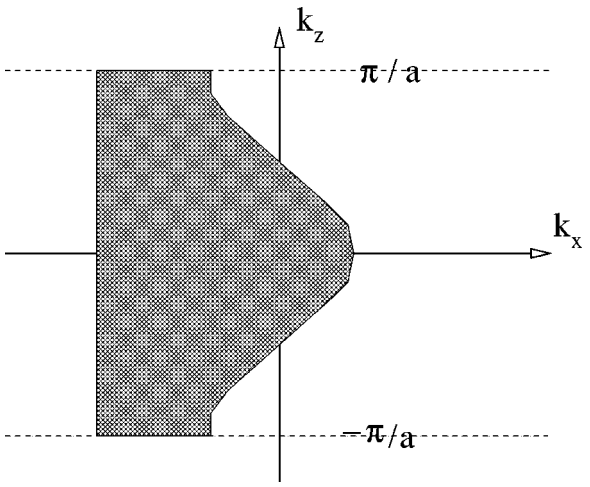

FIG. 1. Fermi sea of chiral metal, with wave vectors k_x in the chiral direction and k_z in the interlayer direction.

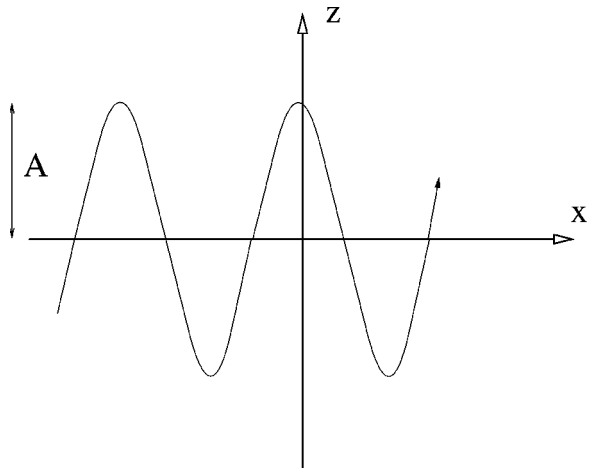

FIG. 2. Trajectory of electrons in real space, in the presence of a transverse component to the magnetic field, with coordinates x in the chiral direction and z in the interlayer direction, and oscillation amplitude A.

single particle description, this may be represented using the Hamiltonian

$$H = v p_x + V(x), \tag{2}$$

where p_x is the momentum operator in the direction parallel to the edge. The potential $V(x)$ includes impurity contributions, and also generates the Aharonov-Bohm phase that electrons accumulate if their path wanders to enclose magnetic flux because of surface roughness.

By extension,[2] the Schrödinger equation for a many-layer sample with a surface in the x-z plane, hopping energy t between neighboring edges (labeled by integer n and separated with spacing a), and transverse magnetic field $B_\perp$, represented by the vector potential $\mathbf{A}=(B_\perp an,0,0)$, is

$$(H\psi)_n(x) = v(-i\hbar\partial_x + eB_\perp an)\psi_n(x) - t[\psi_{n+1}(x)+\psi_{n-1}(x)] + V_n(x)\psi_n(x). \tag{3}$$

This is the model that we study in this paper.

There are two limitations of this model that are worth noting. Both arise from restricting the single-edge problem to one dimension. In doing so we have to worry about the fact that, in semiclassical terms, the guiding center trajectories of particles with different energies will lie at different distances from the bulk of the sample, and will, therefore, experience different impurity potentials. This has the consequence that the scattering phase shift will depend upon the energy of the state—such an effect is not representable in our model as we will see below. A second, potentially more serious effect, is the introduction of a random spatial (x) dependence into the z-axis hopping on account of the different wanderings of the uncoupled edges in neigboring layers; this is representable, but not included in our model. An extreme case would be the directed network model of Ref. 1, which will lead to very different results as we discuss in Sec. IV.

Without impurity scattering or a transverse magnetic field, the surface states fill a one-sided Fermi sea as sketched in Fig. 1. Impurity scattering generates a finite conductance,[1,2] which can be estimated as follows. First, note that eigenfunctions $\psi(x)$ of the single-edge Hamiltonian with energy E have the form

$$\psi(x) = \psi(0)\exp\left(\frac{i}{\hbar v}\int_0^x [E - V(x')]dx'\right). \tag{4}$$

The phase acquired as a result of the disordered potential (which is independent of energy as promised), $\alpha(x) = (1/\hbar v)\int_0^x V(x')dx'$ is of no importance for a particle propagating along an isolated edge, but sets the mean free path $l_{\rm el}$ for a system of coupled edges, this being the distance an electron must propagate to accumulate a random phase of unit magnitude. Suppose for definiteness that the potential on the nth edge $V_n(x)$ is Gaussian distributed with short-range correlations, so that $\langle V_n(x)\rangle = 0$ and $\langle V_n(x)V_m(x')\rangle = \Delta\delta_{nm}\delta(x-x')$. Then the condition $\langle\alpha^2(x=l_{\rm el})\rangle = 1$ results in $l_{\rm el} = (\hbar v)^2/\Delta$.

Given the mean free path, it is easy to estimate the diffusion constant D and surface conductivity σ in the transverse direction. Typical velocities in this direction have magnitude $at/\hbar$, while the scattering rate is $v/l_{\rm el}$, so that

$$D \sim \left(\frac{at}{\hbar}\right)^2 \frac{l_{\rm el}}{v}. \tag{5}$$

The density of states in energy is $(hv)^{-1}$ per unit length for a single edge, and $(hva)^{-1}$ per unit area for the surface, and hence the Einstein relation gives

$$\sigma \sim \frac{e^2}{h}\frac{at^2 l_{\rm el}}{(\hbar v)^2}. \tag{6}$$

Consider now the effect of a transverse magnetic field. In the absence of impurities, the Lorentz force arising from the chiral motion will sweep electrons across the Brillouin zone in the nonchiral direction, in a time $(h/a)/(eB_\perp v)$. Correspondingly, electrons follow a snaking path in real space (Fig. 2) with an amplitude A for oscillations of their coordinate in the interplane direction, which decreases with increasing $B_\perp$, having the dependence $A \sim t/eB_\perp v$. The field scale for the magnetoresistance is the field strength at which the period of these oscillations is comparable to the elastic scattering time $l_{\rm el}/v$ from which follows the value B_0

$\sim h/eal_{\rm el}$. At field strengths much smaller than this, the electron scatters too frequently from impurities for its path to be influenced by the transverse magnetic field, while at much larger-field strengths its interplane coordinate follows a random walk, with step length A and step rate $v/l_{\rm el}$. The resulting diffusion coefficient $D(B_\perp)\sim t^2/(e^2B_\perp^2 v l_{\rm el})$ is reduced from its zero-field value by a factor $(B_0/B_\perp)^2$, as is, therefore, also the conductivity, producing the large-field behavior of Eq. (1).

III. CALCULATION

The disorder-averaged one- and two-particle Green's functions can be calculated exactly for the chiral metal, provided disorder correlations are shortrange, because motion in the chiral direction suppresses repeated scattering from any given impurity. As a result, modeling the impurity potential with Gaussian white noise, the one-particle Green's function is given *exactly* by the Born approximation, and the two-particle function by a sum of ladders. We show below that corrections to this behavior in a potential with a finite correlation length λ are small in powers of $\lambda/l_{\rm el}$; they are also strongly irrelevant (in the renormalization-group sense) to long-distance properties, as shown by Balents and Fisher.[2]

We simplify notation by introducing the dimensionless field strength $b=eB_\perp v a/t$, and choosing units in which $t=\hbar v=1$, so that the model [Eq. (3)] takes the form

$$\begin{aligned}(H\psi)_n(x)&=-i\partial_x\psi_n(x)-[\psi_{n+1}(x)+\psi_{n-1}(x)]\\&\quad+bn\psi_n(x)+V_n(x)\psi_n(x)\\&\equiv(H_0\psi)_n(x)+V_n(x)\psi_n(x).\end{aligned}\tag{7}$$

We use a position vector $\mathbf{r}=(x,n)$ with one continuous and one discrete component. Denoting the Green's function by $g\equiv(z-H)^{-1}$, we require its disorder average,

$$G(z;\mathbf{r}_1,\mathbf{r}_2)\equiv\langle g(z;\mathbf{r}_1,\mathbf{r}_2)\rangle,\tag{8}$$

the diffusion propagator,

$$K(\omega;\mathbf{r}_1,\mathbf{r}_2)\equiv\langle g(\omega+i0;\mathbf{r}_1,\mathbf{r}_2)g(-i0;\mathbf{r}_2,\mathbf{r}_1)\rangle,\tag{9}$$

and in particular the behavior at small wave vectors of its Fourier transform,

$$K(\omega;\mathbf{k})\equiv\int_{-\infty}^{\infty}dx\sum_n e^{i\mathbf{k}\cdot\mathbf{r}}K(\omega;\mathbf{0},\mathbf{r}).\tag{10}$$

A first step is to treat the problem without disorder. The eigenfunctions of H_0 satisfy

$$(H_0\psi)_n(x)=(k+\epsilon_\alpha)\psi_n(x),\tag{11}$$

with

$$\psi_n(x)=\frac{1}{\sqrt{2\pi}}e^{ikx}\phi_\alpha(n),\tag{12}$$

where

$$\epsilon_\alpha\phi_\alpha(n)=bn\phi_\alpha(n)-\phi_\alpha(n+1)-\phi_\alpha(n-1).\tag{13}$$

This equation has the solution

$$\phi_\alpha(n)=J_{n-\alpha}(2/b),\tag{14}$$

with $\epsilon_\alpha=\alpha b$ and α integer, where $J_l(x)$ is the Bessel function of order l.

The single-particle Green's function in the absence of disorder $g_0(z)\equiv(z-H_0)^{-1}$ is given by

$$g_0(z;\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk\sum_\alpha\frac{e^{ik(x_2-x_1)}\phi_\alpha(n)\phi_\alpha(m)}{z-(k+\epsilon_\alpha)}.\tag{15}$$

In particular, for $\mathcal{I}m(z)>0$, $g_0(z;\mathbf{r}_1,\mathbf{r}_2)=0$ if $x_2<x_1$, while for $x_2=x_1+0^+$, $g_0(z;\mathbf{r}_1,\mathbf{r}_2)=-i\delta_{nm}$.

Disorder generates a self-energy after averaging, which is diagonal in real space and given in the Born approximation by

$$\Sigma(z)=-i\frac{\Delta}{2}\mathrm{sgn}[\mathcal{I}m(z)].\tag{16}$$

In order to examine corrections to the Born approximation, suppose temporarily that the Gaussian distributed potential $V_n(x)$ is piecewise constant on segments of length λ, so that the second cumulant is $\langle V_n(x)V_m(x')\rangle=(\Delta/\lambda)\delta_{n,m}$, if there is an integer N for which $N\lambda<x,x'<(N+1)\lambda$, and zero otherwise. Self-energy contributions at the pth order are proportional to $(\Delta/\lambda)^p\lambda^{2p}(g_0)^{2p-1}$, where the factor of λ^{2p} arises from integration over internal position coordinates, restricted to an interval of range λ. Restoring dimensionful units, the pth order contribution is proportional to $(\lambda/l_{\rm el})^p\cdot(g_0)^{-1}$, so that only the Born term ($p=1$) need be retained as $(\lambda/l_{\rm el})\to 0$. Hence, we have

$$G(E+i0;\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk\sum_\alpha\frac{e^{ik(x_2-x_1)}\phi_\alpha(n)\phi_\alpha(m)}{E+i\Delta/2-(k+\epsilon_\alpha)}.\tag{17}$$

Analogous arguments show that the diffusion propagator is given in the same limit by a sum of ladder diagrams, with the result that

$$K(\omega;\mathbf{k})=\frac{\Lambda(\omega;\mathbf{k})}{1-\Delta\Lambda(\omega;\mathbf{k})},\tag{18}$$

where

$$\Lambda(\omega;\mathbf{k})=\int_{-\infty}^{\infty}dx\sum_n e^{i\mathbf{k}\cdot\mathbf{r}}G(\omega+i0;\mathbf{0},\mathbf{r})G(-i0;\mathbf{r},\mathbf{0}).\tag{19}$$

Using Eqs. (13) and (17), we obtain

$$\Lambda(\omega;\mathbf{k})=\sum_l\frac{J_l^2[4\sin(k_y/2)/b]}{\Delta-i(k_x+\omega+lb)}.\tag{20}$$

At small wave vectors this gives the behavior

$$K(\omega;\mathbf{k})=\frac{1}{i(\omega+k_x)+Dk_y^2},\tag{21}$$

with

$$D = \frac{2\Delta}{\Delta^2 + b^2}. \tag{22}$$

Using the Einstein relation to obtain the conductivity, we arrive at our main result: Eq. (1).[16]

IV. DISCUSSION

The observability of transverse magnetoresistance depends on a number of factors. The inelastic scattering rate $\tau_{\rm in}^{-1}$ must be smaller than the interchain hopping rate $t/\hbar$ for transport not simply to be incoherent; and for the theory we have described to apply $\tau_{\rm in}^{-1}$ should also be smaller than the elastic scattering rate $v/l_{\rm el}$. The fact that mesoscopic conductance fluctuations have been observed[3] in transport by surface states in semiconductor samples seems a good indication that both these conditions can be met experimentally at low temperature. If this is the case, the remaining condition is that the field scale B_0 should not be too large. In order that B_0 is much smaller than the magnetic field component parallel to the sample surface, which is responsible for the bulk quantum Hall effect, we require a clean, flat surface, so that the elastic mean free path satisfies $l_{\rm el} \gg l_{\rm B}^2/a$, where $l_{\rm B}$ is the magnetic length within the layers of the sample, and a is the interlayer spacing.

We should remark that the intrinsic magnetoresistance we have discussed is distinct from the simple reduction of the interplane matrix element by an in-plane magnetic field, analyzed for example in Ref. 17. The dependence of the tunneling energy t on $B_\perp$ will make a contribution to the magnetoresistance that is additional to the one we calculate, and with a field scale that will be much larger than B_0 if $l_{\rm el} \gg l_{\rm B}$. The two contributions should be distinguishable by comparing surface magnetoresistances in two configurations. In one, the magnetic-field component that lies within the plane of the sample layers is directed normal to the sample surface; in the other, it lies within the surface plane. A magnetic field normal to the surface should generate a magnetoresistance via both mechanisms, whereas a magnetic field within the surface plane will affect only the interlayer tunneling energy. A rectangular sample with a large aspect ratio would be ideal for such a comparison.

Finally, as noted in the discussion following Eq. (3), substantial roughness at the surface may imply that something like the directed network model of Ref. 1 is a better description of the details of transport at the edge. This would lead to a second type of chiral metal, which would exhibit *no* intrinsic magnetoresistance, even in the absence of inelastic scattering. In such a system, interlayer tunneling takes place only at discrete points, and not continuously in the chiral direction as in Eq. (3). If a large random-scattering phase is accumulated between tunneling points, then a transverse magnetic-field component can have no effect. Consequently, a null result for the magnetoresistance would plausibly indicate that strong spatial randomness in the z-axis tunneling is essential for a proper description of the transport in experimental systems.

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[16] More generally, we obtain $\sigma_{zz}(k_x=0,k_y,\omega)$ in the diffusive regime. Due to the lack of rotational invariance of the system and its chirality in the x direction, a full specification of the linear transport of the chiral metal requires the full set of $\sigma_{ij}(k_x,k_y,\omega)(ij=x,z)$ which we expect to discuss in a future publication. Here we will limit ourselves to noting that there is no dc Hall conductance for the chiral metal, i.e., both σ_{xz} and σ_{zx} vanish in the transport limit in which **k** is taken to zero before ω.
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