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Quantum energy flow in mesoscopic dielectric structures

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We investigate the phononic energy-transport properties of mesoscopic, suspended dielectric wires. The Landauer formula for the thermal conductance is derived and its universal aspects discussed. We then determine the variance of the energy current in the presence of a steady-state current flow. In the final part, some initial results are presented concerning the nature of the temperature fluctuations of a mesoscopic electron-gas thermometer due to the absorption and emission of wire phonons. [S0163-1829(99)02707-1]

I. INTRODUCTION

Mesoscopic physics might be defined as the study of certain quantum electronic phenomena, normally belonging to the atomic domain, which through the use of special microfabrication techniques are realized in structures having dimensions ranging from tens of nanometers up to micrometers. One consequence of our improving ability to directly probe quantum phenomena at these scales is the increasing relevance of the more nontrivial aspects of quantum mechanics for the proper explanation of the phenomena, such as the need to include in the description the measurement process. With further advances in fabrication techniques, this trend will continue and we can look forward to mesoscopic structures, which display the counterintuitive aspects of quantum mechanics becoming commonplace.

It should also be possible to fabricate mesoscopic structures in which the *lattice* degrees of freedom behave in a manifestly nonclassical way. Phononic analogues of various mesoscopic electron phenomena are an obvious possibility to consider. For example, we might ask whether the thermal conductance of a dielectric wire with sufficiently small cross section will exhibit steps of universal magnitude (i.e., expressed, apart from a numerical factor, solely in terms of Boltzmann's and Planck's constants) analogous to the electronic conductance steps observed in quantum wires.[1,2] Phononic analogues of various quantum optical phenomena can also be considered, such as squeezed phonon states.[3,4] Phonons may be particularly suited for the study of time-dependent phenomena in the mesoscopic domain. The weakness of the phonon-phonon interaction at low temperatures and also the ability to fabricate mesoscopic structures having only a few defects, may allow for the possibility to track the evolution of nonequilibrium phonon distributions as they approach thermal equilibrium distributions. Such an investigation might provide new insights into the longstanding fundamental problem concerning the recovery of macroscopic irreversibility from the microscopic reversible laws (for a discussion of this problem in the context of mesoscopic systems, see Ref. 5).

Phonon-confining mesoscopic structures are not as straightforward to realize as electron quantum wells, wires, etc. For acoustic phonons, there are no perfect thermal insulators; although confined modes may exist in a heterostructure consisting of layers of material with different elastic properties, there will always be unconfined bulk modes with the same energies. The only solution is to use *suspended* structures, i.e., structures that are physically separated from the substrate for most of their extent. An additional challenge is the problem of probing the phonon dynamics in the suspended structures. For example, in order to measure the thermal conductance of a suspended nanowire, a way must be found in which to heat one end of the wire while keeping the other end at a fixed temperature and also to measure the temperature difference between the two ends. As can be appreciated, it is rather more difficult to fabricate suspended nanostructures integrated with ultrasensitive probes than it is to fabricate conventional heterostructures. Several groups have been involved in related work during the past few years, with pioneering studies carried out by Potts *et al*.[6] and by Seyler and Wybourne.[7] The recent successful experiments of Roukes and co-workers[8] demonstrate their mastery of the fabrication techniques and have opened up for exploration the field of mesoscopic phonon physics.

In this paper we investigate several phonon phenomena which can in principle be observed using devices similar to those considered by Tighe *et al*.[8] In Sec. II we calculate the mean of the energy current flowing in a suspended dielectric wire connected at each end to equilibrium phonon reservoirs at different temperatures. The Landauer formula for the thermal conductance is recovered from the mean-energy current expression and the conditions on the phonon-energy spectrum for the observation of conductance steps determined. In actual dielectric wires the energy spectrum fails to satisfy the conditions, and thus the steps cannot be resolved. The temperature dependence of the conductance is then solved numerically for the special case of a GaAs wire with uniform rectangular cross section. The main results of this section have also been obtained by Angelescu *et al.*[9] and by Rego and Kirczenow.[10]

In Sec. III we calculate the variance of the energy current in the presence of a steady-state current flow. When the temperatures of the two reservoirs coincide, so that the average current flow is zero, we recover the Johnson-Nyquist noise formula for the phonon-energy current.

Practically no mention is made in Secs. II and III about the ways in which the conductance or variance of the current might actually be measured. This is partly remedied in Sec. IV, where we consider a model thermometer consisting of an electron gas confined to a thin cross sectional slab of the wire. We investigate the temperature fluctuations occuring in this electron gas caused by the absorption and emission of phonons. The remarkable possibility of detecting *single* phonons through the temperature fluctuations is a consequence of the very small volume, and hence heat capacity of the electron gas. From the magnitude of a given temperature fluctuation the energy of the absorbed or emitted phonon is known and, thus, there is the possibility for high-resolution phonon spectroscopy. In particular, the energy dependence of the phonon-transmission probability for a suspended wire can be determined. We develop some of the necessary theory for describing the statistics of the fluctuations and, on the basis of the derived expressions, make some initial observations concerning the extraction of the transmission probability energy dependence from the fluctuation statistics.

In our calculations we use the second quantization method. This formalism arises quite naturally when quantizing the lattice degrees of freedom and also enables a systematic derivation of the thermal conductance, current noise, and temperature fluctuation formulas. Although we do not do so in the present paper, it is important to try to rederive these formulas (particularly the current noise and temperature-fluctuation formulas) using a different approach in which the phonons are described by propagating, spatially localized wave packets.[11,12] Such an approach might provide a clearer picture of what the phonons are ''actually doing'' in the mesoscopic wires.

II. THE THERMAL CONDUCTANCE

The model wire structure that we shall consider is shown in Fig. 1. Two very long, perfect leads (i.e., crystalline and with uniform cross section) join a central segment in which the phonon scattering occurs. The scattering may be caused by any combination of the following: a changing cross section, surface roughness, or various internal defects. The only restriction we place on the scattering is that it be elastic. Phonon-phonon interactions are also neglected. The other ends of the two leads are connected to reservoirs where the phonon distributions are Bose-Einstein distributions. No scattering occurs at the reservoir-lead connections.

Our point of departure is the classical equations of motion for the lattice dynamics of a perfect wire (i.e., no scattering) and also the expression for the classical energy current flowing in the wire. At Kelvin or lower reservoir temperatures, phonon wavelengths typically exceed several hundred angstroms, and thus the continuum approximation can be used for the equations of motion:

$$\rho\partial_t^2 u_i - c_{ijkl}\partial_j\partial_k u_l = 0, \tag{1}$$

where u_i denotes the ith component of the displacement field, ρ is the mass density, and c_{ijkl} is the elastic modulus tensor. The displacement field satisfies the following boundary condition at the wire surface:

$$c_{ijkl} n_j \partial_k u_l|_S = 0, \tag{2}$$

where n_j is the jth component of the unit vector normal to the wire surface S. In terms of the displacement field and elastic modulus tensor, the energy current at a given location x is (the x coordinate runs along the length of the wire):

$$I(x,t) = -c_{xjkl}\int_A dy\,dz\,\partial_t u_j \partial_k u_l, \tag{3}$$

where the integral is over the cross sectional surface A at x.

In order to quantize the equations of motion (1), we require a complete set of normal mode solutions. For a perfect, infinitely long wire, these solutions can be written in the following form:

$$u_{n,q,i}(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} e^{-i(\omega_{n,q}t - qx)} \chi_{n,q,i}(y,z), \tag{4}$$

where q is the longitudinal wave vector along the wire axis and n is the subband label. It follows from the equations of motion that these solutions can be chosen to satisfy the orthonormality condition

$$\int d\mathbf{r}\, u^*_{n,q,i} u_{n',q',i} = \delta_{nn'}\delta(q-q'). \tag{5}$$

In the presence of scattering, we can still construct solutions in the leads using the perfect wire solutions (4) as follows:

$$u^1_{n,q,i} = \begin{cases} u_{n,q,i} + \sum_{n'} u_{n',-q',i} t^{11}_{n'n}(\omega) & \text{lead 1} \\ \sum_{n'} u_{n',q',i} t^{21}_{n'n}(\omega) & \text{lead 2} \end{cases} \tag{6}$$

and

$$u^2_{n,q,i} = \begin{cases} \sum_{n'} u_{n',-q',i} t^{12}_{n'n}(\omega) & \text{lead 1} \\ u_{n,-q,i} + \sum_{n'} u_{n',q',i} t^{22}_{n'n}(\omega) & \text{lead 2,} \end{cases} \tag{7}$$

where $q,q'>0$. The solutions $u^1_{n,q,i}$ describe waves propagating from lead 1 to lead 2, while solutions $u^2_{n,q,i}$ propagate from lead 2 to lead 1. The absolute value of the scattering matrix element $t^{ba}_{n'n}(\omega)$ gives the fraction of the incident wave in lead a, with frequency ω and subband label n, which is transmitted/reflected into lead b and subband n'. In the sum over n', the frequency ω is kept fixed, while q' is treated as a function of n' and ω through the condition $\omega_{n',q'} = \omega_{n,q} = \omega$.

From energy conservation, the time average of the energy current $I(x,t)$ should be independent of the position x. Substituting into the definition for the energy current (3) an ar-

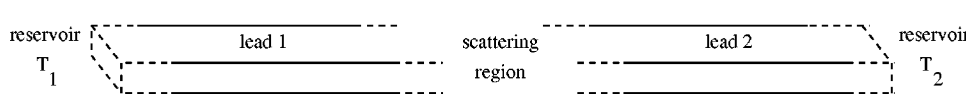

FIG. 1. Schematic diagram of the model wire. The left and right reservoirs are at temperatures T_1 and T_2, respectively.

bitrary linear combination of solutions (6) and (7) and demanding that the time-averaged energy currents in leads 1 and 2 be the same, we obtain the following conditions on the scattering matrix elements:

$$\sum_{n''} v_{n'',q''} t^{11}_{n''n}(\omega) t^{11*}_{n''n'}(\omega) + \sum_{n''} v_{n'',q''} t^{21}_{n''n}(\omega) t^{21*}_{n''n'}(\omega) = v_{n,q}\delta_{nn'}, \tag{8}$$

$$\sum_{n''} v_{n'',q''} t^{22}_{n''n}(\omega) t^{22*}_{n''n'}(\omega) + \sum_{n''} v_{n'',q''} t^{12}_{n''n}(\omega) t^{12*}_{n''n'}(\omega) = v_{n,q}\delta_{nn'}, \tag{9}$$

and

$$\sum_{n''} v_{n'',q''} t^{11}_{n''n}(\omega) t^{12*}_{n''n'}(\omega) + \sum_{n''} v_{n'',q''} t^{21}_{n''n}(\omega) t^{22*}_{n''n'}(\omega) = 0, \tag{10}$$

where $v_{n,q} = \partial\omega_{n,q}/\partial q$ is the group velocity. In the derivation of these conditions, we require the following very useful orthogonality condition:

$$i\pi c_{xijl}\int_A dy\,dz\,(u_{n,q,i}\partial_j u^*_{n',q',l} - u^*_{n',q',i}\partial_j u_{n,q,l}) = \rho\omega_{n,q} v_{n,q}\delta_{nn'}, \tag{11}$$

where $\omega_{n',q'} = \omega_{n,q}$. This relation is obtained from the equations of motion (1). Note that, using Eqs. (5) and (8)–(10), one can also show that the wire scattering-mode solutions satisfy the following orthonormality condition:

$$\int d\mathbf{r}\, u^{\sigma *}_{n,q,i} u^{\sigma'}_{n',q',i} = \delta_{\sigma\sigma'}\delta_{nn'}\delta(q-q'), \quad \sigma,\sigma' = 1,2, \tag{12}$$

where, in the integral over the x coordinate, leads 1 and 2 have been given fictitious extensions so that they are described by the coordinate ranges $x<0$ and $x>0$, respectively (see Sec. V of Ref. 13 and also Appendix A of Ref. 14 for a discussion of such orthonormality conditions in the case of electron scattering-wave states). Only orthogonality condition (11) will be required in the subsequent analysis, however.

We are now ready to quantize. In the wire leads, the displacement field operator has the solution

$$\hat{u}_i(\mathbf{r},t) = \sum_{n,\sigma}\int_0^\infty dq\,\sqrt{\frac{\hbar}{2\rho\omega_{n,q}}} \times [\hat{a}^\sigma_{n,q} u^\sigma_{n,q,i}(\mathbf{r},t) + \hat{a}^{\sigma\dagger}_{n,q} u^{\sigma *}_{n,q,i}(\mathbf{r},t)], \tag{13}$$

where the phonon creation and annihilation operators satisfy the commutation relations

$$[\hat{a}^\sigma_{n,q}, \hat{a}^{\sigma'\dagger}_{n',q'}] = \delta_{\sigma\sigma'}\delta_{nn'}\delta(q-q'). \tag{14}$$

Substituting the field operator solution (13) into the energy-current operator

$$\hat{I} = -\frac{1}{2}c_{xjkl}\int_A dy\,dz\,(\partial_t\hat{u}_j\partial_k\hat{u}_l + \partial_k\hat{u}_l\partial_t\hat{u}_j) \tag{15}$$

and then taking the expectation value of $\hat{I}$ at any location x in leads 1 or 2, we obtain

$$\langle\hat{I}\rangle = \frac{1}{2\pi}\sum_{n,n'}\int_{\omega_{n,0}}^\infty d\omega\;\hbar\omega\; v^{-1}_{n,q} v_{n',q'} t^{21}_{n'n}(\omega) t^{21*}_{n'n}(\omega) \times [n_1(\omega) - n_2(\omega)], \tag{16}$$

where

$$n_\sigma(\omega) = \frac{1}{e^{\hbar\omega/k_BT_\sigma} - 1}, \tag{17}$$

with T_σ the temperature of the reservoir at the end of lead σ. In the derivation of Eq. (16), use is made of relation (11) and conditions (8)–(10). We also use the following creation/annihilation operator expectation values:

$$\langle\hat{a}^{\sigma\dagger}_{n,q}\hat{a}^{\sigma'}_{n',q'}\rangle = n_\sigma(\omega_{n,q})\delta_{\sigma\sigma'}\delta_{nn'}\delta(q-q'). \tag{18}$$

Defining

$$T^{21}_{n'n}(E) = v^{-1}_{n,q} v_{n',q'} t^{21}_{n'n}(\omega) t^{21*}_{n'n}(\omega), \tag{19}$$

where $E = \hbar\omega$, we can rewrite Eq. (16) as follows:

$$\langle\hat{I}\rangle = \frac{1}{2\pi\hbar}\sum_{n,n'}\int_{E_{n,0}}^\infty dE\,E\,T^{21}_{n'n}(E)[n_1(E) - n_2(E)]. \tag{20}$$

This is our key expression for the mean energy current. From the form of this expression and condition (8), we see that the matrix $T^{21}_{n'n}(E)$ is naturally interpreted as the probability for a phonon with energy E in subband n of lead 1 to be transmitted into subband n' of lead 2. Equation (20) is the starting point for the investigations in Refs. 9 and 10.

When the temperature difference between the reservoirs is small, i.e., $|T_1 - T_2| \ll T_1, T_2$, we can expand Eq. (20) to obtain the wire thermal conductance:

$$\kappa = \frac{\langle\hat{I}\rangle}{|T_1 - T_2|} = \frac{\pi k_B^2 T}{6\hbar}\sum_{n,n'}\int_{E_{n,0}/k_BT}^\infty d\epsilon\, g(\epsilon) T^{21}_{n'n}(\epsilon k_B T), \tag{21}$$

where T is the average temperature and

$$g(\epsilon) = \frac{3\epsilon e^\epsilon}{\pi^2(e^\epsilon - 1)^2}. \tag{22}$$

Equation (21) relates the thermal conductance to the single-phonon transmission probability, and thus we call this the Landauer expression for the phonon thermal conductance.

The function $g(\epsilon)$ satisfies $\int_0^\infty d\epsilon\, g(\epsilon) = 1$. Therefore, in the absence of scattering a given subband n contributes to the reduced conductance κ/T the *universal* quantum $\pi k_B^2/6\hbar \approx 9.465\times 10^{-13}\ \mathrm{W\,K^{-2}}$ in the limit $E_{n,0}/k_BT \to 0$.[10] Whether or not steps can be resolved in the temperature dependence of the reduced conductance depends on the separation of the subband edges $E_{n+1,0} - E_{n,0}$ and also on the size of the temperature interval over which the integral

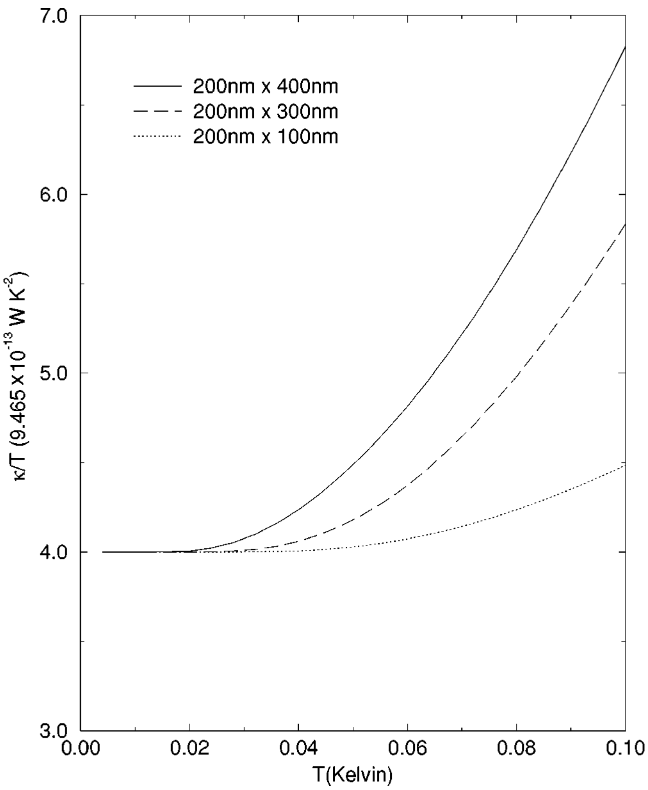

FIG. 2. Reduced thermal conductance vs temperature for perfect GaAs wires with uniform rectangular cross section 200 nm $\times 400$ nm (solid line), 200 nm$\times$300 nm (dashed line), and 200 nm$\times$100 nm (dotted line). The reduced conductance is given in units $\pi k_B^2/6\hbar \approx 9.465\times 10^{-13}$ W K^{-2}.

$\int_{E_{n,0}/k_BT}^{\infty} d\epsilon g(\epsilon)$ goes from being much less than one to close to one. A rough criterion can be arrived at by requiring that the temperature at which the nth subband contributes 90% of a universal quantum be less than the temperature at which the $n+1$th subband contributes 10% of a universal quantum. This yields the following condition on the subband edge separation:

$$E_{n+1,0} > 14 E_{n,0}. \tag{23}$$

Therefore, in order to resolve the steps, the subband separation would have to increase by an order of magnitude from one subband to the next. In an actual wire, the separation typically goes like $E_{n+1,0}/E_{n,0} \sim (n+1)/n$, and thus the steps cannot be resolved. The same conclusion is reached in Ref. 9, where the possibility of using nonequilibrium, narrow-band phonon distributions to observe the steps is also considered.

In Fig. 2, we show the temperature dependence of the reduced thermal conductance for perfect GaAs wires with uniform, rectangular cross sections of various dimensions comparable to those used in the experiment of Ref. 8. The only GaAs wire characteristics that are needed in order to determine the conductance are the zone-center frequencies $\omega_{n,0}$. These can be calculated using the elegant numerical method developed in Ref. 15. As expected, there are no step-like features. There is, however, a plateau for $T \to 0$ where only phonons in the lowest subband with $E_{n,0}=0$ contribute (see also Ref. 10). The plateau has the value of 4 in universal quantum units, a consequence of there being four basic mode types: dilatational, torsional and, two types of flexural mode.[15]

Of course, in actual wires phonon scattering will occur. For example, the reservoirs can be much larger than the wires, with a sharp decrease in cross section where they join. Reservoir phonons approaching the wire with transverse wavelength component exceeding the cross sectional dimensions of the wire will be backscattered with high probability and the resulting suppression in the dielectric wire thermal conductance at low temperatures may conceal the plateau described above. An initial investigation of the consequences for the thermal conductance of a nonuniform cross section can be found in Refs. 9 and 10. In classical wave optics and acoustics, the same strong-reflection phenomenon occurs for waves traveling in narrowing waveguides and is called ''diffractional blocking.'' This phenomenon is also somewhat analogous to the situation in an electronic quantum wire when the Fermi level lies below the lowest subband edge, so that electrons can only tunnel from one contact region to the other, resulting in an exponential suppression in the conductance.

Some closely related work to that described in this section is in the area of dielectric point contact spectroscopy.[16–18] In fact, diffractional blocking has already been observed in thermal conductance measurements of point contacts;[18] by measuring the temperature at which the thermal conductance dropped sharply, it was possible to estimate the contact diameters that were found to be in the region of tens of nanometers.

III. ENERGY CURRENT NOISE

Using the methods developed in the preceding section, it is possible to calculate more nontrivial quantities characterizing the energy flow in the wire, such as the variance of the energy current $(\Delta I)^2 = \langle \hat{I}^2 \rangle - \langle \hat{I} \rangle^2$. If we take the expectation value of $[\hat{I}(x,t)]^2$, we obtain a meaningless divergent result, however. Given that we cannot measure the current at a precise instant, a more realistic quantity to consider is the following (see, e.g., Ref. 19):

$$I_m(x,t) = \int_{-\infty}^{\infty} d\tau H(t-\tau) I(x,\tau), \tag{24}$$

where $H(t)$ is a causal filter function satisfying $H(t)=0$ for $t<0$ and $\int_{-\infty}^{\infty} dt H(t) = 1$. We call I_m the measured current. The expectation value of $[\hat{I}_m(x,t)]^2$ is now finite and well defined. The variance of the measured current is calculated using a similar procedure to that outlined in the previous section for the mean. Omitting the details and going directly to the final result, we find

$$(\Delta I_m)^2 \approx \frac{B}{2\pi\hbar} \sum_{n,n'} \int_{E_{n,0}}^{\infty} dE E^2 \{ \mathcal{T}^{21}_{n'n} [n_1 - n_2]^2 + T^{21}_{n'n} [n_1(n_2+1) + n_2(n_1+1)] \}, \tag{25}$$

where the transmission probability $T^{21}_{n'n}$ is defined in Eq. (19) and the transmission matrix $\mathcal{T}^{21}_{n'n}$ is defined as follows:

$$\mathcal{T}^{21}_{n'n}=\sum_{m,m'}(v_n v_{n'})^{-1}v_m v_{m'}t^{21}_{m'n'}t^{21*}_{m'n}t^{21}_{mn}t^{21*}_{mn'}. \quad (26)$$

The constant B is the filter bandwidth:

$$B=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\tilde{H}^2(\omega), \quad (27)$$

where $\tilde{H}(\omega)=\int_{-\infty}^{\infty}dte^{-i\omega t}H(t)$. Approximation (25) is a good one provided that the energy scale $E=\hbar\omega$ over which $\tilde{H}(\omega)$ is nonzero is small as compared with the energy scales over which the transmission matrices and phonon distributions vary (see, e.g., Ref. 19).

Formula (25) resembles the electron current variance formula.[20,21,12] (The correspondence is even more direct if the electron *energy* current variance is used for comparison rather than the more commonly considered charge-current variance.) Just as for the electron case, we see that phonon current noise in the presence of a nonzero steady-state current (i.e., $T_1\neq T_2$) contains more information concerning the transmission characteristics of the wire than the thermal conductance.

For the special case where the reservoir temperatures are the same, the $\mathcal{T}$ matrix term drops out and Eq. (25) can be written as follows:

$$(\Delta I_m)^2\approx 2Bk_BT^2\kappa, \quad (28)$$

where κ is the thermal conductance (21). Thus, the equilibrium phonon noise gives the same information concerning the wire transmission characteristics as the thermal conductance. We call Eq. (28) the Johnson-Nyquist noise formula for the phonon-energy current. Again, this formula bears a close resemblance to the electron-current Johnson-Nyquist noise formula.[11,21] When the temperature difference is nonzero but small, we see from the form of the phonon distribution terms in Eq. (25) that corrections to the Johnson-Nyquist equilibrium noise are of second order in the temperature difference.

IV. A MESOSCOPIC THERMOMETER

In order to probe the phonon dynamics of a wire, some kind of measuring apparatus is obviously required. It is important to understand the behavior of that part of the apparatus that interacts directly with the wire phonons, so that we can know just what properties of the phonon system are in fact being measured.

As our model measuring system, we consider an electron gas confined to a thin cross sectional slab of one of the wire leads. The gas density and slab thickness are small enough so that the wire phonon current is hardly affected by its presence. In other words, a phonon traversing the gas layer has only a very small probability to be absorbed. The gas density should also be large enough so that the time scale for the electron gas to reach internal thermal equilibrium due to electron-electron scattering is much less than the time scale separating consecutive phonon absorption or emission events. This latter assumption allows us to assign a temperature to the electron gas, which fluctuates in time due to the absorption and emission of phonons. The remaining part of the apparatus, which we do not describe, measures the electron-gas temperature with negligible disturbance to the gas. This measuring system is in fact a closely related idealization of that employed by Tighe *et al.* in their experiments.[8]

Measuring the wire thermal conductance presents no problem. A known constant power source is supplied to reservoir 1, say, while reservoir 2 acts as heat sink with known temperature. The electron-gas thermometer is located at the reservoir end of lead 1 and its average temperature measured. The conductance is then just the power divided by the difference between the gas thermometer temperature and the temperature of reservoir 2 [see Eq. (21)].

The fluctuations of the electron-gas temperature give much more information concerning the wire-phonon dynamics than the average temperature. The real possibility to detect temperature fluctuations is a consequence, as the following estimates show, of the very small electron-gas volume that can be achieved. For a nearly degenerate electron gas the specific heat is approximately

$$\frac{\partial E}{\partial T}\approx\frac{\pi^2 nVk_B^2T}{2E_F}, \quad (29)$$

where n and V are the electron-gas number density and volume, respectively. Using the relation between E_F and n for free electrons to eliminate E_F, Eq. (29) becomes

$$\frac{\partial E}{\partial T}\approx\frac{\pi^{2/3}mn^{1/3}Vk_B^2T}{3^{2/3}\hbar^2}. \quad (30)$$

If the electron-gas absorbs or emits a thermal phonon with energy $3k_BT$, then from Eq. (30) we get an approximate temperature change

$$\delta T\approx\frac{3^{5/3}\hbar^2}{\pi^{2/3}mn^{1/3}Vk_B}. \quad (31)$$

For GaAs with, e.g., $n=10^{18}\ \mathrm{cm}^{-3}$, this gives

$$\delta T\approx 0.4V^{-1}\ \mathrm{mK}, \quad (32)$$

where the volume V is given in units $\mu\mathrm{m}^3$. Thus, for an electron-gas thermometer with submicron dimensions (which can be achieved with present fabrication techniques[8]), absorption or emission of a thermal phonon will produce a temperature fluctuation in excess of a milliKelvin.

Note, however, that it is not possible to measure the energy-current fluctuations using the electron-gas thermometer. Although the energy of an absorbed phonon can be determined from the size of the temperature fluctuation, all information is lost concerning the direction in which the wire phonon was travelling. To gain an initial idea about what information can be obtained concerning the phonon dynamics, we shall now examine more closely the temperature fluctuations.

Much of the theory of photoelectric light detection in quantum optics (see, e.g., Chap. 14 of Ref. 22) can be adapted to our present problem. As a phonon detector, however, the electron-gas thermometer behaves in a more nontrivial manner than the photoelectric detector. Unlike a conventional photoelectric detector, the gas thermometer can not only detect phonons, but measure their energy as well. Furthermore, the gas thermometer can emit phonons. When successive phonon detections are correlated, these properties can make the calculation of various detection probabilities more difficult. In the following, we shall neglect the correlations. This then allows us to recover all statistical properties of the temperature fluctuations from the detection probability for very short time intervals (i.e., short enough so that the probability is much less than one). It should be borne in mind, however, that many of the expected interesting quantum properties will be correlation effects and, thus, it is important to try to include the correlations in future improvements of the theory.

The quantity of interest, then, is the probability $R(E,E')\delta t$ that the electron gas, initially with total energy E, has energy $E'\neq E$ after a short time interval δt, due to the absorption or emission of a phonon with energy $\Delta=|E'-E|$. Recall that we are assuming the electron gas to be in internal thermal equilibrium between absorption/emission events. For a large number of electrons, the electron-gas temperature T can be determined to good approximation from the total energy E of the electron gas by using the relation $E=2\Sigma_\alpha \varepsilon_\alpha f(\varepsilon_\alpha)$, where ε_α is a single electron-energy eigenvalue and $f(\varepsilon_\alpha)$ is the Fermi-Dirac distribution. Neglecting correlations, an energy probability distribution $P(E)$ will evolve in time according to the following equation:

$$\frac{\partial P}{\partial t}(E,t)=\sum_{E'} P(E',t)R(E',E)-\sum_{E'} P(E,t)R(E,E'). \tag{33}$$

Thus, knowing the rate $R(E,E')$ allows us to in principle determine how a probability distribution evolves.

Using the methods of, e.g., Chap. 14 of Ref. 22, we obtain the following expression for the rate:

$$\begin{aligned}R(E,E\pm\Delta) &= \frac{\pi}{\rho\Delta}\sum_{\substack{\alpha,\beta\\ \varepsilon_\beta-\varepsilon_\alpha=\pm\Delta}} f_\alpha(1-f_\beta)\Bigg[\sum_{n,n'} v_{n,q}^{-1}\big(|\lambda_{n,q}^{\alpha\beta}|^2\delta_{n'n}\\ &\quad+|\lambda_{n,-q}^{\alpha\beta}|^2\delta_{n'n}+\lambda_{n,q}^{\alpha\beta *}\lambda_{n',-q'}^{\alpha\beta}t_{n'n}^{11}+\lambda_{n,q}^{\alpha\beta}\lambda_{n',-q'}^{\alpha\beta *}t_{n'n}^{11*}\big)\\ &\quad\times(n_1+\tfrac{1}{2}\mp\tfrac{1}{2})+\sum_{n,n',n''} v_{n,q}^{-1}\lambda_{n',-q'}^{\alpha\beta}\lambda_{n'',-q''}^{\alpha\beta *}t_{n'n}^{12}t_{n''n}^{12*}\\ &\quad\times(n_2-n_1)\Bigg],\end{aligned} \tag{34}$$

where all the phonon quantities are evaluated at $\omega=\Delta/\hbar$. This rate expression is for the case where the thermometer is located in lead 1. To obtain the corresponding expression when it is located in lead 2, the lead indices ''1'' and ''2'' should be interchanged wherever they appear. The quantity $\lambda_{n,q}^{\alpha\beta}$ is the electron-phonon matrix element:

$$\begin{aligned}\lambda_{n,q}^{\alpha\beta}=\int_V d\mathbf{r}\,\psi_\beta^*(\mathbf{r})\Bigg(&\Xi_d\partial_i u_{n,q,i}(\mathbf{r})\\ &+\frac{\overline{e}e_{14}}{4\pi\epsilon}\int_V d\mathbf{R}\frac{e^{-q_0|\mathbf{r}-\mathbf{R}|}}{|\mathbf{r}-\mathbf{R}|}\partial^2_{(12}u_{n,q,3)}(\mathbf{r})\Bigg)\psi_\alpha(\mathbf{r}),\end{aligned} \tag{35}$$

where the integrals are over the electron-gas volume, ψ_α is the electron-energy eigenstate, and $u_{n,q,i}$ is the phonon-mode solution in the lead [see Eq. (4)]. The first term in the large brackets is the deformation component of the potential and the second term is the piezoelectric component (see, e.g., Chap. 3 of Ref. 23).

The rate expression (34) comprises two terms, of which the second involving the wire phonon-transmission matrix t^{12} is the most interesting. A possible experimental procedure would be to measure the rates for $T_2\neq T_1$ and also for $T_2=T_1$, with T_1 the same in each case. The difference between the two rates would then be given by just the second term in Eq. (34). Because this term is proportional to $t^{12}t^{12*}$ evaluated at $\omega=\Delta/\hbar$, the rate difference, therefore, provides direct information concerning the energy dependence of the phonon-transmission matrix averaged over the various subbands. Of course, knowledge of the electron-phonon matrix elements (35) would be required in order to extract this information.

V. CONCLUSION

We have presented several results concerning the energy-flow properties of mesoscopic, suspended dielectric wires. The mean of the energy current was calculated and a Landauer formula for the thermal conductance obtained. When scattering is absent, each phonon subband contributes a universal quantum $\pi k_B^2/6\hbar$ to the reduced conductance κ/T. Steps are not observed, however, because of the broadness of the Bose-Einstein distribution as compared with the subband edge separation. The temperature dependence of the reduced conductance was solved numerically for the example of a GaAs wire with uniform, rectangular cross section. The variance of the energy current was then calculated and the Johnson-Nyquist equilibrium noise formula obtained as a special case. In the final part, an initial investigation was carried out concerning the nature of the fluctuations of a mesoscopic electron-gas thermometer due to the absorption and emission of wire phonons. It was found that the fluctuations give direct information concerning the energy dependence of the phonon transmission matrix for the wire.

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