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Two-dimensional hopping conductivity in a δ-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure

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We present zero magnetic-field resistivity measurements of the variable-range hopping (VRH) in a two-dimensional electron gas created at a δ-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure. When either the temperature or carrier density are reduced the longitudinal resistivity shows a crossover from two-dimensional Mott VRH, $\rho(T)=\rho_M \exp(T_M/T)^{1/3}$ to Efros-Shklovskii (ES) Coulomb-gap behavior $\rho(T)=\rho_{\rm ES}\exp(T_{\rm ES}/T)^{1/2}$. In both regimes, near the crossover, the data collapse onto universal curves with a temperature independent prefactor $\rho_M \approx (1/2)(h/e^2)$ or $\rho_{\rm ES}\approx h/e^2$. The latter result is in close agreement with measurements of Si metal-oxide-semiconductor field-effect transistors. We discuss the possible implications of our data on the theory of phonon-assisted VRH. [S0163-1829(99)01108-X]

The temperature dependence of the longitudinal resistivity in the variable-range hopping (VRH) regime was shown by Mott[1,2] to be of the form

$$\rho(T)=\rho_0 \exp(T_0/T)^p, \tag{1}$$

where ρ_0 is a prefactor, T_0 is a characteristic temperature, and the exponent p depends on the shape of the density of states (DOS) at the Fermi level. In two-dimensional (2D) systems, $p=1/3$ for a constant DOS $g(\epsilon) = g_F$ at the Fermi level. In this case, called Mott hopping, $T_0 \equiv T_M = C_M(g_F\xi^2)^{-1}$, where ξ is the localization length and $C_M = 13.8$.[1,3] Pollak and co-workers[4,5] and Efros and Shklovskii[6] (ES) showed that electron-electron interactions cause a suppression of the single-particle DOS about the Fermi energy E_F. In two-dimensions the resulting Coulomb gap has the linear form $g(\epsilon)=g_0|\epsilon|$, where the energy ϵ is measured from E_F, $g_0=(2/\pi)(e^2/\kappa)^{-2}$, and κ is the dielectric constant. This linear DOS gives an exponent $p=1/2$ and

$$T_0 \equiv T_{\rm ES}=C_{\rm ES}(e^2/\kappa\xi), \tag{2}$$

where $C_{\rm ES}=6.2$.[7]

Experimental evidence for a Coulomb gap in two-dimensions has been demonstrated by several authors. Previous zero-magnetic-field measurements[8] of GaAs/Al$_x$Ga$_{1-x}$As heterostructures show that the resistivity (in logarithmic scale) follows a $T^{-1/2}$ temperature dependence characteristic of ES behavior, with a temperature-independent prefactor. In contrast, other studies reveal $T^{-1/2}$ behavior with a temperature-dependent prefactor of the form $\rho_0=AT^m$, where $m=1$ in the quantum-Hall regime in GaAs/Al$_x$Ga$_{1-x}$As (Ref. 9) and In$_x$Ga$_{1-x}$As/InP (Ref. 11) heterostructures, or $m=0.8$ (Ref. 10) in GaAs/Al$_x$Ga$_{1-x}$As for all magnetic fields including $B=0$. Hence, there is some uncertainty in the temperature dependence of the prefactor ρ_0.

Recently, it has been shown[12] that the zero-field 2D VRH resistivity in Si metal-oxide-semiconductor field-effect transistors (MOSFET's) follows $T^{-1/2}$ behavior, and for electron densities near the 2D metal-insulator transition the data collapse onto a universal curve with a constant prefactor h/e^2. Aleiner *et al.*[13] suggested that the VRH conductivity may have a prefactor of the form $\sigma_0=(e^2/h)f(T/T_0)$, where $f(T/T_0)\sim 1$ for $T\sim T_0$. There is, however, no theoretical justification for this choice of prefactor; this is in contrast to conventional phonon-assisted hopping where the prefactor $\sigma_0 \ll (e^2/h)$ and would be expected to depend upon parameters such as the wave-function decay constant. To resolve this discrepancy it was suggested[13] that rather than the usual electron-phonon assisted mechanism, the 2D VRH is assisted by electron-electron interactions as discussed by Fleishman *et al.*[14]

In this paper we show that the VRH in a delta-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure in zero magnetic field and at intermediate carrier density $9.18-10.85\times 10^{10}$ cm^{-2} has a temperature-independent prefactor ρ_0, which in the Mott VRH regime is equal to $\rho_0 \equiv \rho_M=(1/2)(h/e^2)$, and in the ES regime $\rho_0 \equiv \rho_{ES}=(h/e^2)$. The latter result is in agreement with measurements of Si-MOSFET's.[12] The material independence of the prefactor suggests that hopping is assisted by some mechanism other than phonons.

The samples investigated here were fabricated from the wafer used in a previous study,[8] where full details of the layer compositions and doping are given. An essential feature of its structure is the presence of a δ-doped layer on the Al$_x$Ga$_{1-x}$As side of the electron gas, 0.6 nm away from the heterojunction. The samples were patterned into 80×720 μm Hall bars, and the as-grown 2D carrier density

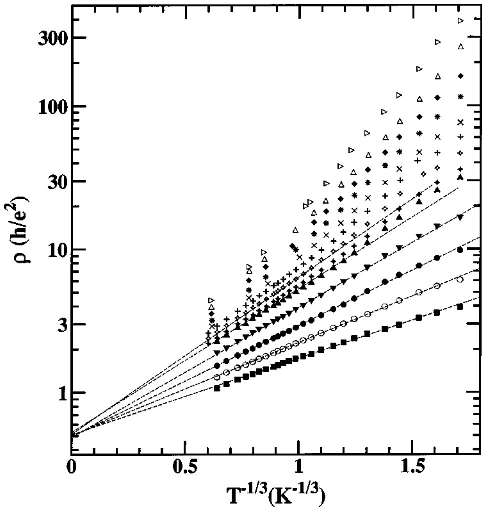

FIG. 1. The resistivity $\rho(T)$ versus $T^{-1/3}$ for different carrier densities n. The carrier densities in units of 10^{10} cm^{-2} from top to bottom are 8.47, 8.6, 8.73, 8.86, 9.0, 9.18, 9.3, 9.4, 9.52, 9.84, 10.2, 10.54, and 10.85. The dashed lines are least-squares fits to Mott behavior.

and low-temperature mobility were $n = 4.65\times10^{11}$ cm^{-2} and $\mu = 4.5\times10^{4}$ cm^2/Vs. The carrier density n was varied by application of a negative gate voltage V_g to a surface gate that is $d = 90$ nm above the heterojunction, and at low temperatures the electron gas is fully depleted at $V_g = -0.70$ V. The resistivity (the resistance per square) at different temperatures and carrier densities was measured from the Ohmic part of the DC four-terminal $I-V$ characteristics. At low-carrier concentrations the $I-V$ characteristics were strongly nonlinear and the resistance was obtained from dV/dI in the zero current limit. Four-terminal AC measurements (with a 0.1 nA current at a frequency of 2.1 Hz) were also performed at some carrier densities, and were found to be in agreement with the DC measurements. In the carrier-density regime investigated, the ratio of Coulomb interaction energy to the Fermi energy is $r_s = 1.65-1.87$.

Figure 1 shows the resistivity ρ in units of h/e^2 at several carrier densities plotted versus $T^{-1/3}$; the symbols are the experimental points and the dashed lines are the least-squares fits to the $T^{-1/3}$ behavior with a temperature-independent prefactor.[15] At the highest carrier densities, $n = 9.84-10.85\times10^{10}$ cm^{-2}, the traces follow $T^{-1/3}$ behavior for $T = 0.2$–4 K. The characteristic temperature T_M for these curves varies from 1.82 to 12.25 K. At carrier densities $9.18-9.52\times10^{10}$ cm^{-2} the data follow $T^{-1/3}$ behavior only above 1 K and the deviations to higher resistance at low temperatures (below 1 K) are due to a crossover to the ES Coulomb-gap regime. This crossover with lowering temperatures occurs when the hopping energy becomes smaller than the Coulomb-gap energy. In order to show the temperature-induced crossover we have measured $\rho(T)$ up to 27 K at $n = 9.2\times10^{10}$cm^{-2}, and in Fig. 2 the data is plotted on a log-log scale. The Fig. 2 inset shows $\ln W$ versus $\ln T$, where $W = -\partial(\ln R)/\partial(\ln T) = p(T_0/T)^p$. The exponent p is measured from the slope of the curve; this method of analysis was used in the investigation of the crossover phenomenon in three-dimensional samples.[18] Around 1 K there is a change of slope of $\ln W$ as p changes from 0.55 ± 0.05 to 0.34 ± 0.03.

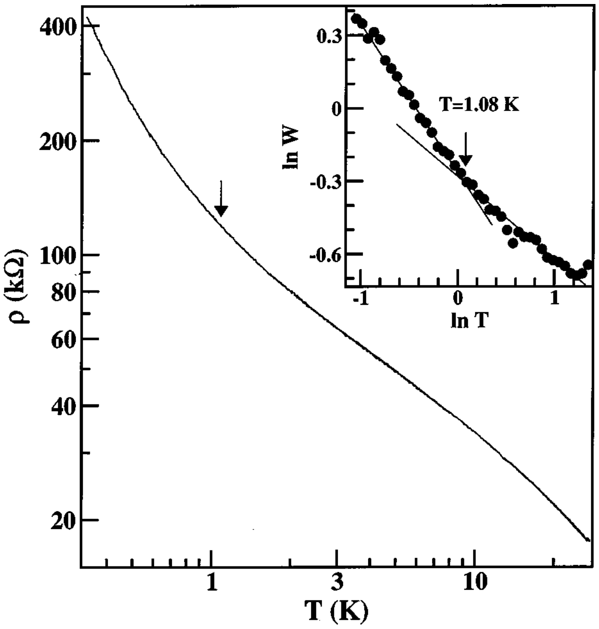

FIG. 2. The resistivity $\rho(T)$ at 9.2×10^{10} cm^{-2}. The inset shows the same data plotted as $\ln W$ versus $\ln T$; the slope gives the exponent p. Around 1 K there is a change of slope of $\ln W$ as p changes from 0.55 ± 0.05 to 0.34 ± 0.03. The arrows indicate the crossover temperature.

The crossover to the ES regime with decreasing T shows that the observed $T^{-1/3}$ behavior is a high-temperature noninteracting Mott regime, rather than the low temperature screened Mott regime reported in Ref. 10.

Figure 3 shows the resistivity plotted versus $T^{-1/2}$ for the same carrier densities presented in Fig. 1. At carrier densities $9.18-9.52\times10^{10}$cm^{-2} the data follow $T^{-1/2}$ behavior only from 0.28 to 1 K, and small deviations can be seen above 1 K due to a crossover to $T^{-1/3}$ behavior as discussed above. For carrier densities between $8.47-9.0\times10^{10}$ cm^{-2}, $T^{-1/2}$ behavior is observed from 0.28 to 4 K.

For temperatures below 0.28 K, the resistivity in the ES regime deviates from $T^{-1/2}$ behavior towards a lower value.

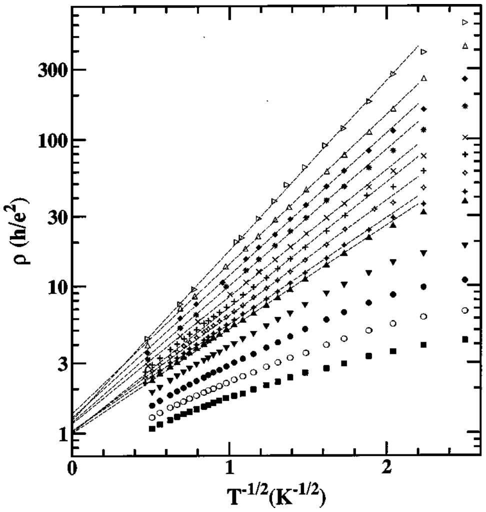

FIG. 3. The resistivity versus $T^{-1/2}$ for the same carrier densities presented in Fig. 1. The dashed lines are least-squares fits to ES behavior, with parameters given in Table I.

TABLE I. Fitting parameters for the data in the ES regime in Fig. 3. The localization lengths ξ are calculated using Eq. (2). According to the findings of Ref. 17, the localization length is expected to be an order of magnitude smaller.

n (10^{10}cm^{-2})	ρ_0 (h/e^2)	T_{ES} (K)	ξ (μm)
8.47	1.22	7.07	1.11
8.60	1.35	5.49	1.44
8.73	1.27	4.98	1.58
8.86	1.18	4.57	1.72
9.0	1.16	3.96	1.99
9.18	1.01	3.86	2.04
9.3	1.03	3.27	2.41
9.4	1.04	2.78	2.83
9.52	1.0	2.65	2.97

This could be due to the screening of the Coulomb interaction by the metallic gate and has been experimentally studied in Refs. 10 and 12. It was theoretically shown[16] that when the average hopping length $r \sim \frac{1}{4}\,\xi(T_{\text{ES}}/T)^{1/2}$ is greater than $2d$, where d is the distance between the metallic gate and the 2D electron gas (2DEG), then the Coulomb interaction will be screened by the gate and $T^{-1/3}$ rather than $T^{-1/2}$ behavior will be observed. Assuming κ is not a function of carrier density n,[10] the localization length ξ can be calculated from Eq. (2) using T_{ES} measured from the slope of the traces in Fig. 1. Table I lists T_{ES} and ξ for the different carrier densities in the ES regime shown in Fig. 1. For these traces the maximum hopping length obtained at 0.28 K was $r = 2.2$ μm, which is approximately 12 times larger than $2d$. This discrepancy in the different length scales is similar to that observed by Mason *et al.*,[12] suggesting that the value $C_{\text{ES}} = 6.2$ for single-electron hopping is too large. Recent numerical calculations[17] of a Coulomb glass with multielectron hopping have given a lower value of $C_{\text{ES}} \approx 0.62$. We note that an experimental lower limit of $C_{\text{ES}} = 0.64$ was also obtained in Ref. 10.

We now discuss the measured hopping prefactor ρ_0. In the Mott hopping regime, all the traces collapse onto a single curve when $\rho(T)$ is plotted against the dimensionless parameter $(T/T_M)^{-1/3}$, as shown in Fig. 4. A least-squares fit to the universal curve gives a temperature independent prefactor $\rho_M \approx h/2e^2$. In the ES hopping regime, the least-squares fits to the data from 1 to 0.28 K for $n = 9.18 - 9.52 \times 10^{10}$ cm^{-2} in Fig. 3 have a common intercept $\rho_{\text{ES}} \approx h/e^2$. This is more clearly seen in the plot of $\rho(T)/(h/e^2)$ versus $(T/T_{\text{ES}})^{-1/2}$ shown in Fig. 5, where all the traces collapse onto a universal curve. This observation is similar to that found[12] for Si-MOSFET's. As the sample becomes more insulating, the prefactor ρ_0 in our sample becomes larger than h/e^2, as can be seen from Fig. 3. A constant conductivity preexponential factor $\sigma_{\text{min}} \approx 0.6e^2/h$ in the activated hopping regime was previously measured[19] in n-type GaAs, where two dimensionality was achieved by decreasing the thickness of the conducting layer.

Recent measurements[10] of GaAs/Al$_x$Ga$_{1-x}$As heterostructures have shown that to fit to ES and Mott behavior it is necessary to divide the data by $T^{0.8}$. This result is consistent with phonon-assisted hopping, and may be different to our results because of the type of disorder. The samples in Ref. 10 are a conventional heterostructure without a spacer layer where the donors are randomly distributed over a large distance. This amounts to disorder in the 2DEG due to the impurity distribution and the depth of the potential. In our sample, there is a δ-doped layer very close to the interface and the depth of the impurity potential in the 2DEG is uniform, and disorder comes from the randomness in the lateral distribution of the dopants.

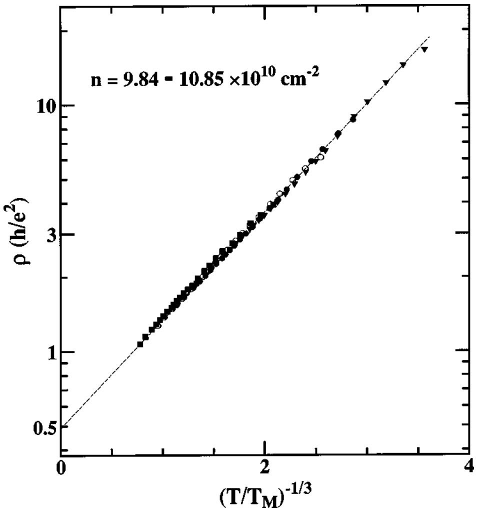

FIG. 4. The resistivity $\rho(T)$ versus $(T/T_M)^{-1/3}$ in the Mott regime, at densities $n = 9.84 - 10.85 \times 10^{10}$ cm^{-2} taken from Fig. 1. The dashed line is a least-squares fit to the data with an intercept $\rho_M = (0.49 \pm 0.02)h/e^2$.

In phonon-assisted VRH the resistivity prefactor $\rho_0 \gg h/e^2$, and is expected to depend upon temperature and material properties. The universality of the prefactor in both δ-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure and Si-MOSFET in the not strongly insulating regime suggests that the VRH in this regime is not assisted by phonons. Other mechanisms such as electron-electron interaction-assisted hopping are possible.

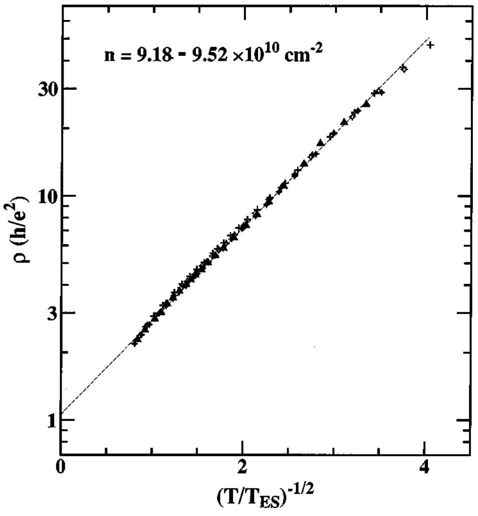

FIG. 5. Plots of the resistivity $\rho(T)$ versus $(T/T_{\text{ES}})^{-1/2}$ for carrier densities $9.18 - 9.52 \times 10^{10}$ cm^{-2} taken from Fig. 3. The dashed line is a least-squares fit to the data from 0.28 to 1 K, with an intercept $\rho_{\text{ES}} = (1.04 \pm 0.05)h/e^2$.

Significantly, the prefactor in the ES regime is twice that in the Mott regime. In the conventional theory, the conductivity prefactor is proportional to the density of states at the Fermi energy, and this change by a factor of 2 in the prefactor suggests that the density of states has altered by a lifting of the spin degeneracy. This conclusion is backed up by the fact that in the regime in which interactions dominate the transport, the spin degeneracy of the states should be removed by the interactions as, for example, the spin degeneracy is lifted in a quantum dot when interactions dominate.

In conclusion, we have presented zero-field VRH resistivity measurements of a disordered two-dimensional electron gas created in a δ-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure. We show that when either the carrier density or the temperature is reduced the longitudinal resistivity shows a crossover from 2D Mott VRH to ES VRH. At intermediate values of $n=9.18-10.85\times10^{10}$ cm^{-2} near the crossover, the data in the ES regime collapses onto a universal curve with a prefactor $\rho_{ES}\approx h/e^2$, while all traces in the Mott-hopping regime collapse onto a universal curve with a prefactor $\rho_M \approx 1/2(h/e^2)$. The universality of the prefactor and its material independence provide evidence of possible phononless hopping in 2D at zero magnetic field.

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[1] N. F. Mott, J. Non-Cryst. Solids **1**, 1 (1968).

[2] N. Mott *et al.*, Proc. R. Soc. London, Ser. A **345**, 169 (1975).

[3] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, Berlin, 1984).

[4] M. Pollak, Discuss. Faraday Soc. **50**, 13 (1970).

[5] M. L. Knotek and M. Pollak, J. Non-Cryst. Solids **8-10**, 505 (1972).

[6] A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).

[7] V. L. Nguen, Sov. Phys. Semicond. **18**, 207 (1984).

[8] F. Tremblay *et al.*, J. Phys.: Condens. Matter **2**, 7367 (1990).

[9] G. Ebert *et al.*, Solid State Commun. **45**, 625 (1983).

[10] F. W. Van Keuls, X. L. Hu, H. W. Jiang, and A. J. Dahm, Phys. Rev. B **56**, 1161 (1997).

[11] A. Briggs *et al.*, Phys. Rev. B **27**, 6549 (1983).

[12] W. Mason, S. V. Kravchenko, G. E. Bowker, and J. E. Furneaux, Phys. Rev. B **52**, 7857 (1995).

[13] I. L. Aleiner, D. G. Polyakov, and B. I. Shklovskii, in *Proceedings of the 22nd International Conference on the Physics of Semiconductors, Vancouver, 1994*, edited by D. J. Lockwood (World Scientific, Singapore, 1994), p. 787.

[14] L. Fleishman, D. C. Licciardello, and P. W. Anderson, Phys. Rev. Lett. **40**, 1340 (1978).

[15] This result contradicts Ref. 10, where $T^{-1/2}$ and $T^{-1/3}$ behavior has been observed only after introducing a prefactor of the form $\rho_0\propto T^{0.8}$. If our data is plotted with a temperature-dependent prefactor of the form $\rho_0\propto T^m$, with either $m=0.8$ or $m=1.0$, $\ln \rho_{xx}(T)/T^m$ does not exhibit $T^{-1/2}$ or $T^{-1/3}$ behavior.

[16] I. L. Aleiner and B. I. Shklovskii, Phys. Rev. B **49**, 13 721 (1994).

[17] A. Perez-Garrido *et al.*, Phys. Rev. B **55**, 8630 (1997).

[18] Y. Zhang, P. Dai, M. Levy, and M. P. Sarachik, Phys. Rev. Lett. **64**, 2687 (1990).

[19] M. Pepper, Philos. Mag. B **37**, 187 (1978).