Vortex lattice transition in *d*-wave superconductors

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Making use of the extended Ginzburg-Landau theory, which includes the fourth-order derivative term, we study the vortex state in a magnetic field parallel to the *c* axis. The vortex core structure is distorted due to the higher-order term, which reveals the fourfold symmetry. Further, this distortion gives rise to the core interaction energy which favors a square lattice tilted by 45° from the *a* axis. The triangular vortex lattice in small field region transforms into the rhombic vortex lattice (i.e., the square vortex lattice tilted 45° from the *a* axis) at $B=H_{\rm cr}\sim\kappa^{-1}H_{c2}(t)$, where κ is the Ginzburg-Landau parameter and $H_{c2}(t)$ is the upper critical field. Therefore, in most of the *B*-*T* phase diagram the vortex lattice is rhombic. The transition is of the second order and the associated jump in the specific heat should be accessible experimentally. [S0163-1829(99)08605-1]

I. INTRODUCTION

After a few years of controversy, *d*-wave superconductivity in the hole-doped high-T_c cuprates appears to be finally established.[1,2] However, the electron-doped high-T_c cuprates appear to be described by *s*-wave superconductivity.[3,4]

d-wave superconductivity manifests itself as fourfold symmetry of the vortex state when a magnetic field is applied either parallel to the *c* axis or within the *a*-*b* plane.[5] In particular the study of the vortex lattice in the vicinity of the upper critical field[6] and the quasiparticle spectrum around a single vortex[7,8] in a magnetic field parallel to the *c* axis indicate that the square vortex lattice tilted by 45° from the *a* axis should be most stable except in the immediate vicinity of the superconducting transition temperature T_c. Indeed such a square lattice, though elongated in the *a* direction has been seen in Y-Ba-Cu-O monocrystals by small-angle neutron scattering[9] (SANS) and scanning tunneling microscopy[10] (STM) at low temperature and in a low magnetic field. On the other hand, the fourfold symmetry predicted for the density of states near the vortex core appeared not to have been seen by STM (Ref. 10) in Y-Ba-Cu-O monocrystals. This, we believe, indicates the failure of the quasiclassical approximation used in these theoretical analysis. Indeed, recent studies[11] of the Bogoliubov–de Gennes equation clearly indicate not only the breakdown of the quasiclassical approximation for Y-Ba-Cu-O, but also the presence of the extended states with small energies (say $|E|<0.1\Delta$) which exhibits clearly the fourfold symmetry anticipated from the square vortex lattice.

More recently a very similar square vortex lattice has been seen in $ErNi_2B_2C$, YNi_2B_2C, and $LuNi_2B_2C$ by SANS (Refs. 12,13) and in $YuNi_2B_2C$ by STM imaging.[14] Although superconductivity in borocarbides is believed to be conventional *s* wave,[15] the above square lattice together with the presence of antiferromagnetic phase in closely related borocarbides suggest that superconductivity in borocarbides may be of *d* wave as well.[16] Incidentally the square vortex lattice and related vortex lattice transition have recently been studied using the generalized London equation.[14,17,18] The phenomenological free energy used by these authors resembles the one for *d*-wave superconductivity.

The object of this paper is twofold. (i) Making use of the extended Ginzburg-Landau (GL) equation, we first study a single vortex line in a magnetic field parallel to the *c* axis. Unlike Refs. 17,18 we believe that the modification of the vortex core structure is of prime importance. Indeed, the vortex exhibits the fourfold symmetry which will have a number of consequences. For example it will modify the quasiparticle spectrum around a vortex. One more significant fact is that this will generate vortex core interaction energy, which favors the alignment of two vortices either parallel to (1,1,0) or (1,−1,0). Indeed a similar vortex solution has been found numerically previously by Enomoto *et al.*[19] But our analytical result is of prime importance in the following. (ii) From a study of the two-vortex problem, we consider the vortex lattice for a class of isosceles. We find in the low-field limit [i.e., $B\simeq H_{c1}(t)$] the vortices form a triangular lattice as in a conventional *s*-wave superconductor. When the magnetic field increases, the triangular lattice transforms first gradually and then suddenly to the square lattice at $B=H_{\rm cr}$. In the temperature range not very far from T_c (i.e., $\frac{1}{2}T_c<T<T_c$) we predict

$$H_{\rm cr}=0.524(-\ln t)^{-1/2}\kappa^{-1}H_{c2}(t), \tag{1.1}$$

where $t=T/T_c$ and κ is the Ginzburg-Landau parameter. Though the *B* dependence of the apex angle θ we obtained is rather similar to the ones obtained in Refs. 17,18, the detail is quite different. For example, we find the change of the apex angle is much faster though the transition is of the second order as in Refs. 17,18. This θ dependence on *B* is more consistent with the SANS result[12] than that of Ref. 18 which may suggest that the core interaction between two vortices is much more crucial than the term arising from the anisotropy of the magnetic interaction considered in Refs. 17,18. Unfortunately the related SANS study for high-T_c cuprates is not available at the time of this writing. With help

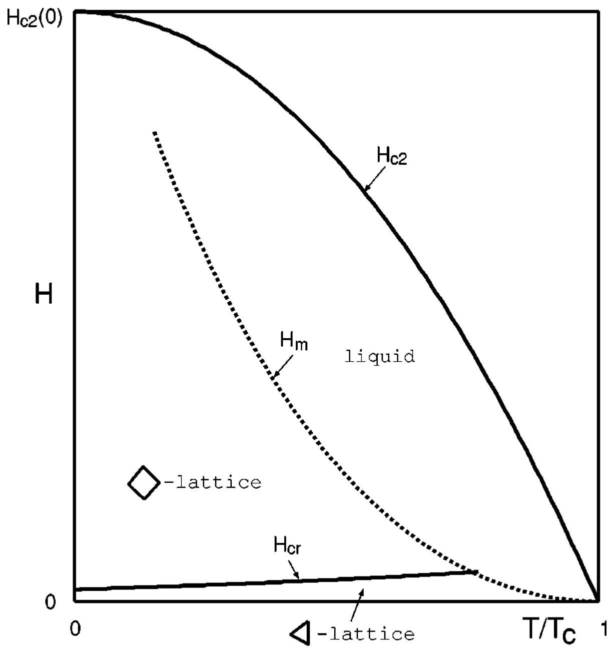

FIG. 1. The *B-T* phase diagram.

of this we constructed the phase diagram of the vortex state as shown in Fig. 1. A preliminary result on this will be published in the proceeding of NATO ASI workshop at Yalta April, 1998.[20]

II. EXTENDED GINZBURG-LANDAU EQUATION AND SINGLE VORTEX PROBLEM

We consider a weak-coupling model for *d*-wave superconductors.[21] Extending the procedure used by Ren *et al.*,[22] we obtain

$$\left(-\ln t+\frac{7\zeta(3)}{2(4\pi T)^2}v^2(\partial_x^2+\partial_y^2)+\frac{31\zeta(5)}{16(4\pi T)^4}v^4[5(\partial_x^2+\partial_y^2)^2+2(\partial_x^2-\partial_y^2)^2]\right)\Delta(\mathbf{r})=\frac{21\zeta(3)}{(4\pi T)^2}|\Delta(\mathbf{r})|^2\Delta(\mathbf{r}), \tag{2.1}$$

which is converted into the dimensionless form

$$(1+(\partial_x^2+\partial_y^2)+\epsilon[5(\partial_x^2+\partial_y^2)^2+2(\partial_x^2-\partial_y^2)^2])\Delta(\mathbf{r})=|\Delta(\mathbf{r})|^2\Delta(\mathbf{r}), \tag{2.2}$$

where we have introduced

$$\xi(T)^2=\frac{7\zeta(3)v^2}{2(4\pi T)^2(-\ln t)},\quad \Delta(T)^2=\frac{(4\pi T)^2(-\ln t)}{21\zeta(3)},$$

$t=T/T_c$, and rescaled $\mathbf{r}\to\xi(T)\mathbf{r}$, $\Delta(\mathbf{r})\to\Delta(T)\Delta(\mathbf{r})$. Here ∂_x and ∂_y are gauge-invariant differential operators and we define the small parameter $\epsilon\equiv 31\zeta(5)(-\ln t)/196\zeta(3)^2 \sim 0.114(-\ln t)$.

Equation (2.1) is written down basically in Ref. 19, though we ignore a few terms of the order of $(-\ln t)^2$ since they are of secondary importance in what follows. Here we concentrate on the effect of the ϵ term, which is the basic symmetry breaking term.

Assume that $\Delta(\mathbf{r})$ is given by

$$\Delta(\mathbf{r})=g(r)e^{i\phi}+\epsilon[e^{4i\phi}\alpha(r)+e^{-4i\phi}\beta(r)+\gamma(r)]e^{i\phi}. \tag{2.3}$$

Substituting this in Eq. (2.2) we find $g(r)$ for $r\gg 1$;

$$g(r)=1-\frac{1}{2}r^{-2}-\frac{9}{8}r^{-4}-\frac{161}{16}r^{-6}\cdots, \tag{2.4}$$

and equations for $\alpha(r),\beta(r)$, and $\gamma(r)$ for $r\gg 1$;

$$A(r)+\left[1+\left(\partial_r^2+\frac{1}{r}\partial_r-\frac{25}{r^2}\right)\right]\alpha(r)=g(r)^2[2\alpha(r)+\beta(r)], \tag{2.5}$$

$$B(r)+\left[1+\left(\partial_r^2+\frac{1}{r}\partial_r-\frac{9}{r^2}\right)\right]\beta(r)=g(r)^2[\alpha(r)+2\beta(r)], \tag{2.6}$$

$$C(r)+\left[1+\left(\partial_r^2+\frac{1}{r}\partial_r-\frac{1}{r^2}\right)\right]\gamma(r)=g(r)^2 3\gamma(r), \tag{2.7}$$

where

$$A(r)=\frac{105}{2}r^{-4}-\frac{945}{4}r^{-6}-\frac{31\,185}{16}r^{-8}-\frac{1\,450\,449}{32}r^{-10}\cdots, \tag{2.8}$$

$$B(r)=-\frac{15}{2}r^{-4}-\frac{105}{4}r^{-6}-\frac{8505}{16}r^{-8}-\frac{557\,865}{32}r^{-10}\cdots, \tag{2.9}$$

$$C(r)=-18r^{-4}-135r^{-6}-\frac{14\,175}{4}r^{-8}-\frac{1\,065\,015}{8}r^{-10}\cdots. \tag{2.10}$$

Then we find

$$\alpha(r)=\frac{5}{2}r^{-2}+\left(c-\frac{55}{4}\ln r\right)r^{-4}+\left(\frac{-2873-456c}{80}+\frac{627}{8}\ln r\right)r^{-6}\cdots, \tag{2.11}$$

$$\beta(r)=-\frac{5}{2}r^{-2}+\left(\frac{5-2c}{2}+\frac{55}{4}\ln r\right)r^{-4}+\left(\frac{-6627-184c}{80}+\frac{253}{8}\ln r\right)r^{-6}\cdots, \tag{2.12}$$

and

$$\gamma(r)=-9r^{-4}-\frac{297}{2}r^{-6}-\frac{5313}{8}r^{-8}\cdots. \tag{2.13}$$

In this solution we find a free parameter c, which fortunately does not show up in the core interaction term which we are going to discuss in the following section. Note that the choice $c=5/4$ makes the first few terms symmetric; $\alpha(r)=5/2r^{-2}+(1-11\ln r)5/4r^{-4}\cdots$, $\beta(r)=-5/2r^{-2}+(1+11\ln r)5/4r^{-4}\cdots$. We will also discuss in the next paragraph that the choice $c\sim 5/4$ is necessary to have approximate solutions.

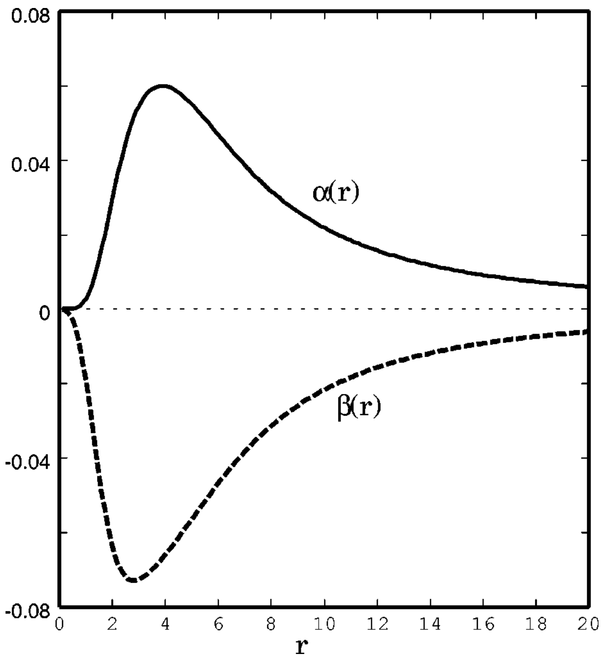

FIG. 2. Plots of $\alpha(r)$ and $\beta(r)$.

For later purposes, it is convenient to introduce the interpolation expressions which give the correct asymptotics for $r\to 0$. We find

$$g(r)=\tanh\frac{r}{c_0}-\frac{1}{2r^2}\left(1-c_1\,\mathrm{sech}\frac{r}{c_0}\right)\tanh^5\frac{r}{c_0}-\frac{9}{8r^4}\left(1-c_2\,\mathrm{sech}\frac{r}{c_0}\right)\tanh^9\frac{r}{c_0}\cdots, \tag{2.14}$$

$$\alpha(r)=\frac{5}{2}r^{-2}\tanh^7\frac{r}{c_3}+\left(\frac{5}{4}-\frac{55}{4}\ln r\right)r^{-4}\tanh^{11}\frac{r}{c_3}\cdots, \tag{2.15}$$

$$\beta(r)=-\frac{5}{2}r^{-2}\tanh^5\frac{r}{c_3}+\left(\frac{5}{4}+\frac{55}{4}\ln r\right)r^{-4}\tanh^{9}\frac{r}{c_3}\cdots, \tag{2.16}$$

where $c_0=1.71$, $c_1=0.80$, $c_2=1.35$. The way to fix these constants is the following. Using the GL equation (2.2), we can express all the constants $c_1,c_2,\ldots$ by c_1. The constant c_1 can be obtained by performing numerical integration of the GL equation with the boundary conditions $g(0)=0$, $\lim_{r\to\infty}g(r)=1$. In principle, we can apply the same procedure to $\alpha(r)$ and $\beta(r)$. However, we simply start from the ansatz (2.15) and (2.16) which are given from Eq. (2.11) and (2.12) by introducing suitable powers of $\tanh r/c_3$, and observe that these with $c_3=2.5$ and $c=5/4$ agree very nicely with the numerical results obtained by Enomoto *et al.*[19] In Fig. 2 $\alpha(r)$ and $\beta(r)$ are plotted as function of r. These are compared with $8f_1^{(1)}(r)$ and $8f_{-1}^{(1)}(r)$ in Enomoto *et al.* It can be seen that our analytic expressions are very close to the numerical ones from Ref. 19. We have not shown $\gamma(r)$ as this term is somewhat different from the one in Enomoto *et al.* since our starting equation is different.

III. INTERACTION BETWEEN TWO VORTICES

Before studying the regular vortex lattice, let us consider the two-vortex problem. We assume that two vortices are placed at (0,0) and ($d\cos\theta, d\sin\theta$) and ($\kappa\gg d\gg 1$). The free energy in dimensionless units is given by

$$\Omega=\int d^2r\left(-|\Delta|^2+|\partial_x\Delta|^2+|\partial_y\Delta|^2-\epsilon|[5(\partial_x^2+\partial_y^2)+2(\partial_x^2-\partial_y^2)]\Delta|^2+\frac{1}{2}|\Delta|^4+\frac{1}{8\pi}b^2\right)=\int d^2r\left(-\frac{1}{2}|\Delta|^4+\frac{1}{8\pi}b^2\right), \tag{3.1}$$

where $b=b(\mathbf{r})$ is the local magnetic field. Making use of the usual approximation

$$\Delta(\mathbf{r})=\Delta\prod_i f(\mathbf{r}-\mathbf{r}_i), \tag{3.2}$$

where

$$f(\mathbf{r})=[g(r)+\epsilon(\alpha(r)e^{4i\phi}+\beta(r)e^{-4i\phi}+\gamma(r))]e^{i\phi}, \tag{3.3}$$

is the single vortex solution, $g(r)\sim\tanh r$, and neglecting $\gamma(r)$ which is irrelevant for the fourfold symmetry, we obtain

$$\Omega_{\text{two-vortex}}\simeq-\frac{1}{2}\int d^2r\{\tanh r+\epsilon\cos 4\phi[\alpha(r)+\beta(r)]\}^4\times\{\tanh r'+\epsilon\cos 4\phi'[\alpha(r')+\beta(r')]\}^4\simeq-\frac{1}{2}\{A-2a_1-2a_1\epsilon[\alpha(d)+\beta(d)]\cos 4\theta\}, \tag{3.4}$$

where A is the area and

$$a_1=\int d^2r(2\,\mathrm{sech}^2 r-\mathrm{sech}^4 r)=\frac{8\pi}{3}\left(\ln 2+\frac{1}{8}\right)\simeq 6.854. \tag{3.5}$$

On the other hand, the magnetic interaction between two vortices is given by $(2\pi/\kappa^2)K_0(d/\kappa)$ (the London formula) where $K_0(z)$ is the modified Bessel function. Strictly speaking the magnetic interaction is also modified due to the higher-order term (see, for example, Ref. 18). Indeed the correction term decays like d^{-2} with d, but this term does not contain extra κ dependence. Therefore, the correction term to the magnetic interaction is completely negligible when $\kappa\gg 1$ as in high-T_c cuprates. Therefore, the core interaction gives a strongly directional energy $\sim d^{-4}\cos 4\theta$, while the magnetic energy is isotropic as in the conventional s-wave superconductor.

IV. VORTEX LATTICE

Let us consider a vortex lattice where lattice points are given by

$$\mathbf{r}_{l,m}=r_{l,m}(\cos\theta_{l,m},\sin\theta_{l,m}) = ld(\cos\theta,\sin\theta)+md(\cos\theta,-\sin\theta), \qquad (4.1)$$

where l,m are integers $d=\sqrt{\phi_0/\sin(2\theta)B}$, and ϕ_0 is the flux quantum. For later convenience, we separate the lattice into even and odd lattices as

$$\mathbf{r}^{(e)}_{l,m}=r^{(e)}_{l,m}(\cos\theta^{(e)}_{l,m},\sin\theta^{(e)}_{l,m})=(2ld\cos\theta,2lm\sin\theta), \qquad (4.2)$$

$$\mathbf{r}^{(o)}_{l,m}=r^{(o)}_{l,m}(\cos\theta^{(o)}_{l,m},\sin\theta^{(o)}_{l,m}) = [(2l+1)d\cos\theta,(2m+1)d\sin\theta]. \qquad (4.3)$$

Note that in Eqs. (4.2) and(4.3) l and m run over all *integers*. Then the free energy of the vortex lattice is given by

$$\Omega=-\frac{1}{2}\left(A-a_1\xi^2 n_\phi-\epsilon 10 a_1\xi^2 n_\phi {\sum_{l,m}}'\frac{\xi^4}{r^4_{l,m}}\cos 4\theta_{l,m}\right) +\frac{2\pi}{\kappa^2}n_\phi\xi^2{\sum_{l,m}}'K_0\left(\frac{r_{l,m}}{\lambda}\right), \qquad (4.4)$$

where $n_\phi=B/\phi_0$ is the vortex density per unit area. Here we consider only the vortex core interaction between two vortices, since the three vortex interaction is exponentially small when $d/\xi\gg 1$. Further, we have neglected the fourfold symmetric term in the magnetic interaction term since it is proportional to ϵ/κ^2. So except for the condensation energy $(-\frac{1}{2}A)$, the second term and the last term are proportional to B, while the core interaction energy (the third term) is proportional to B^3. As the magnetic field increases from $B=H_{c1}(t)$, the third term becomes more dominant and for $B\geq H_{\rm cr}$ the square vortex lattice will be established. The last term in Eq. (4.4) contains the sum

$$\sum_{\substack{l,m\in\mathbf{Z}\\ p=e,o}}{}'K_0\left(\frac{r^{(p)}_{l,m}}{\lambda}\right)={\sum_{l,m}}'K_0[(l^2\mu^2+m^2\mu'^2)^{1/2}] +\sum_{l,m}K_0\{[(l-1/2)^2\mu^2 +(m-1/2)^2\mu'^2]^{1/2}\},$$

where $\mu=2d\sin\theta/\lambda$, $\mu'=2d\cos\theta/\lambda$. Following the argument by Fetter *et al.*,[23] namely, using the integral representation of the function $K_0(x)$ and two Poisson summation formulas (see Appendix), we can rewrite these infinite summations. Then the last term in Eq. (4.4) becomes (for $\lambda\gg d$)

$$\frac{2\pi}{\kappa^2}n_\phi\xi^2{\sum_{l,m}}'K_0\left(\frac{r_{l,m}}{\lambda}\right) \simeq\frac{2\pi}{\kappa^2}n_\phi\xi^2\Bigg\{\frac{4\pi}{\mu\mu'}+\frac{1}{2}\ln\frac{\mu\mu'}{4\pi}-\frac{1}{2}(1-\gamma) +\frac{1}{2}{\sum_{l,m}}'\Bigg[E_1\left(\pi\left(l^2\frac{\mu}{\mu'}+m^2\frac{\mu'}{\mu}\right)\right) +\frac{(-1)^{l+m}+\exp\{-\pi[l^2(\mu'/\mu)+m^2(\mu/\mu')]\}}{\pi[l^2(\mu'/\mu)+m^2(\mu/\mu')]}\Bigg]\Bigg\}. \qquad (4.5)$$

The angle $\theta_{\min}$ which minimizes the free energy is obtained by studying the function

$$f(\theta)=\left(\frac{B}{H^*(t)}\right)^2{\sum_{l,m}}'\frac{\sin^2 2\theta\cos 4\theta_{l,m}}{[(l+m)^2\sin^2\theta+(l-m)^2\cos^2\theta]^2} +{\sum_{l,m}}'\left(E_1[\pi(l^2\tan\theta+m^2\cot\theta)] +\frac{(-1)^{l+m}+\exp[-\pi(l^2\cot\theta+m^2\tan\theta)]}{\pi(l^2\cot\theta+m^2\tan\theta)}\right),$$

where

$$H^*(t)=\left(\frac{98\zeta(3)^2(2\pi)^3}{155a_1\zeta(5)(-\ln t)}\right)^{1/2}\frac{H_{c2}(t)}{\kappa} \sim 5.64667(-\ln t)^{-1/2}\frac{H_{c2}(t)}{\kappa}.$$

Then the minimization of $f(\theta)$ gives Fig. 3 where the apex angle $\theta_{\min}$ is shown as a function of $B/H_{\rm cr}$ where

$$H_{\rm cr}=0.524(-\ln t)^{-1/2}\kappa^{-1}H_{c2}(t). \qquad (4.6)$$

For $B\geq H_{\rm cr}$ the square lattice is fully established. Note also that $d\theta/dB$ diverges at $B=H_{\rm cr}$ indicating a possible phase transition. Earlier a similar θ-B curve was obtained within the generalized London equation.[17,18] However, the present result appears to be more consistent with the observed B dependence of θ by SANS from $ErNi_2B_2C$ at $T=3.5$ K.[12] Inserting θ determined thus into Eq. (4.4), we find the free energy

$$\Omega=\Omega_0+\frac{2\pi\xi^2H_{\rm cr}}{\kappa^2\phi_0}\psi\left(\frac{B}{H_{\rm cr}}\right), \qquad (4.7)$$

where the first term

$$\Omega_0=-\frac{A}{2}+\frac{a_1\xi^2}{2\phi_0}B+\frac{2\pi\xi^2}{\kappa^2\phi_0}B \times\left[\frac{2\pi\lambda^2}{\phi_0}B+\frac{1}{2}\ln\frac{\phi_0}{2\pi\lambda^2B}-\frac{1}{2}(1-\gamma)\right], \qquad (4.8)$$

depends on B in a nonsingular way, and the second term is

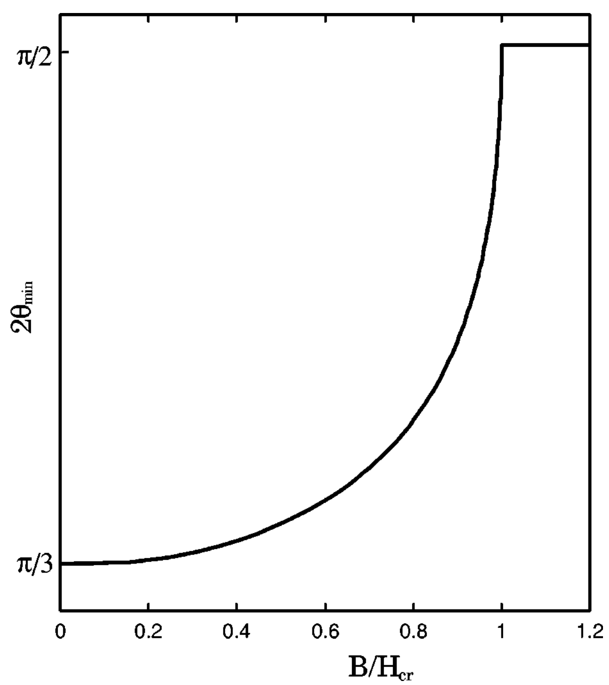

FIG. 3. Apex angle $2\theta_{\rm min}$ as a function of $B/H_{\rm cr}$ where $2\theta_{\rm min}=90°$ and 120° correspond to the square lattice and the triangular lattice with hexagonal symmetry, respectively.

$$\psi\left(\frac{B}{H_{\rm cr}}\right)=\frac{B}{H_{\rm cr}}f\left(\theta_{\rm min}\left(\frac{B}{H_{\rm cr}}\right)\right). \tag{4.9}$$

In Fig. 4 $\psi(B/H_{\rm cr})$ as a function of $B/H_{\rm cr}$ for $0\leqslant B/H_{\rm cr}\leqslant 1.2$ is plotted and it is seen that ψ is continuous at $B=H_{\rm cr}$. The magnetization $-M=\partial\Omega/\partial B$ has a kink at $B=H_{\rm cr}$. Figure 5 shows the part of the magnetization $\psi'(B/H_{\rm cr})=\partial\psi(B/H_{\rm cr})/\partial(B/H_{\rm cr})$ for $0\leqslant B/H_{\rm cr}\leqslant 1.2$. Figures 4 and 5 show clearly that this phase transition is of the

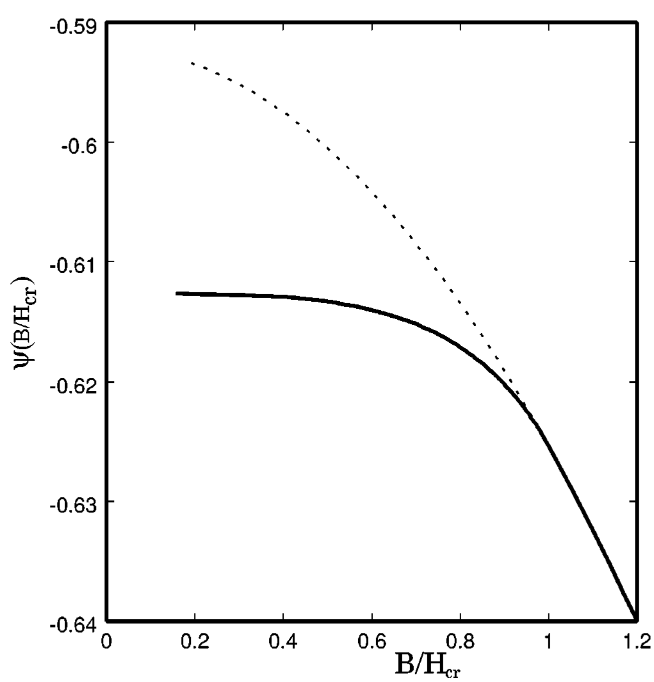

FIG. 4. Singular part of the free energy ψ as a function of $B/H_{\rm cr}$.

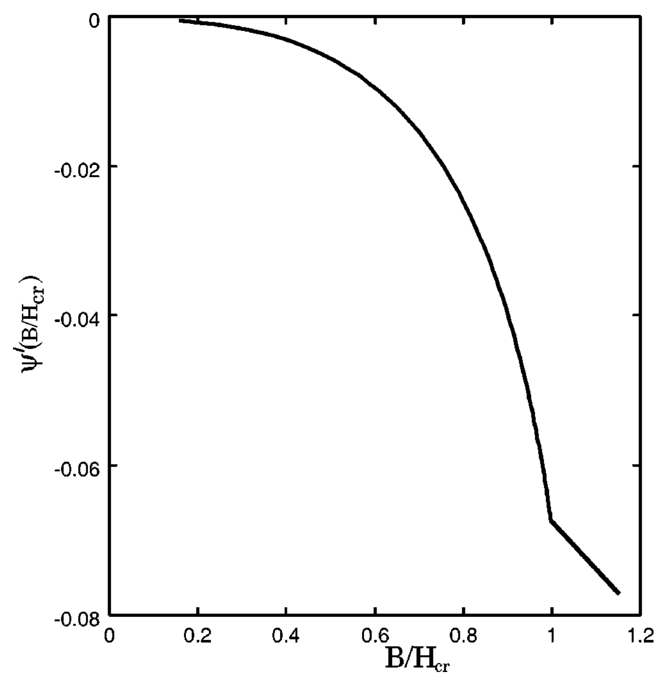

FIG. 5. Singular part of the magnetization ψ' as a function of $B/H_{\rm cr}$.

second order with a jump in the specific heat, which should be accessible experimentally. If we choose $\chi=\theta-90°$ as the order parameter, the behavior of χ and ψ are exactly what expected for the second-order transition. Naturally this transition line terminates at $B=H_m$ the melting transition line where the vortex lattice melts into the vortex liquid.

V. CONCLUDING REMARKS

By analyzing the extended Ginzburg-Landau equation for *d*-wave superconductor, we discover that the vortex core contains a long-range fourfold term which is proportional to $r^{-4}\cos 4\phi$ when $r\geqslant\xi$. The effect of this term on the quasiparticle spectrum is under current study. This fourfold term gives rise to the vortex core interaction, which favors the orientation of two vortices parallel to the diagonal directions (1,1,0) and (1,−1,0). In the low-field regime we find that the vortex lattice transforms from triangular to square as B increases and that the approach to the square lattices is rather steep. The transition is of the second order. The present result appears to describe very well the vortex transition observed in $ErNi_2B_2C$, though the superconductivity in borocarbides is believed to be *s* wave. Turning to high-T_c cuprates there is no similar measurement available even for Y-Ba-Cu-O monocrystals. On the other hand, if we put $\kappa=100$, $H_{c2}(0)=120$ T for Y-Ba-Cu-O, we estimate $H_{\rm cr}=1$ T, which is consistent with the observation of the square lattice at low temperature and in a magnetic field of a few Tesla. Clearly a parallel measurement of the B dependence of the apex angle θ in high-T_c cuprates is highly desirable.

Coming back to the vortex lattice transformation in the vicinity of $B\simeq H_{c2}(t)$, it is shown that the transition is again continuous in contrast to an earlier analysis.[24] In particular the full transition to the square lattice is completed at $t=0.81$. Therefore it is now possible to draw a vortex lattice phase diagram in the T-B plane as shown in Fig. 1.

We expect also that the directional core potential not only modifies the equilibrium vortex lattice configuration but also the collective mode, the elastic, and dynamic response of the vortex lattice. At this moment we can say only that d-wave superconductivity should bring a profound change in our understanding of the vortex motion.

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APPENDIX

In this appendix, we list some useful formulas for studying the free energy Ω [Eq. (4.4)] of the vortex lattice with the apex angle θ.

We have to treat the lattice sums

$$\Xi^{(e)}(\mu,\mu')=\sum_{l,m}{}' K_0[(l^2\mu^2+m^2\mu'^2)^{1/2}],$$

$$\Xi^{(o)}(\mu,\mu')=\sum_{l,m} K_0\{[(l-1/2)^2\mu^2+(m-1/2)^2\mu'^2]^{1/2}\}.$$

The Poisson sum formulas

$$\sum_l \exp[-l^2\mu^2/4\tau]=\frac{\sqrt{4\pi\tau}}{\mu}\sum_l \exp(-4\pi^2\tau l^2\mu^2),$$

$$\sum_l \exp[-(l-1/2)^2\mu^2/4\tau]=\frac{\sqrt{4\pi\tau}}{\mu}\sum_l (-1)^l\exp(-4\pi^2\tau l^2\mu^2),$$

can be obtained from Jacobi's imaginary transformations for the elliptic θ functions;

$$\vartheta_3(v,\tau)=e^{\pi i/4}\tau^{-1/2}e^{-\pi i v^2/\tau}\vartheta_3(v/\tau,-1/\tau)$$

and

$$\vartheta_4(v,\tau)=e^{\pi i/4}\tau^{-1/2}e^{-\pi i v^2/\tau}\vartheta_2(v/\tau,-1/\tau).$$

Using the argument by Fetter *et al.*,[23] we obtain

$$\Xi^{(e)}(\mu,\mu')=\frac{2\pi}{\mu\mu'}+\frac{1}{2}\ln\frac{\mu\mu'}{4\pi}-\frac{1}{2}(1-\gamma)+\frac{1}{2}\sum_{l,m}{}'\left\{E_1\left[\pi\left(l^2\frac{\mu}{\mu'}+m^2\frac{\mu'}{\mu}\right)\right]+\frac{\exp\{-\pi[l^2(\mu'/\mu)+m^2(\mu/\mu')]\}}{\pi[l^2(\mu'/\mu)+m^2(\mu/\mu')]}\right\}$$
$$-\frac{2\pi}{\mu\mu'}\sum_{l,m}{}'\frac{1}{[1+4\pi^2(l^2/\mu^2+m^2/\mu'^2)][4\pi^2(l^2/\mu^2+m^2/\mu'^2)]},$$

$$\Xi^{(o)}(\mu,\mu')=\frac{2\pi}{\mu\mu'}+\frac{2\pi}{\mu\mu'}\sum_{l,m}{}'\frac{(-1)^{l+m}}{[4\pi^2(l^2/\mu^2+m^2/\mu'^2)]}-\frac{2\pi}{\mu\mu'}\sum_{l,m}{}'\frac{(-1)^{l+m}}{[1+4\pi^2(l^2/\mu^2+m^2/\mu'^2)][4\pi^2(l^2/\mu^2+m^2/\mu'^2)]}.$$

[1] See, for instance, Proceedings of the International Conference on LT21 [Czech. J. Phys. **46**, 56-3151 (1996)]; and those of the International Conference on M^{2}S-HTSC, Beijing, China (1997) [Physica C **282-289**, 4 (1997)].
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