Fluctuation specific heat in multilayered superconductors: Bilayered Gaussian-Ginzburg-Landau scenario for the thermal fluctuations of Cooper pairs around T_c in $YBa_2Cu_3O_{7-\delta}$ single crystals

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The effects around the superconducting transition of thermal fluctuations of Cooper pairs on the heat capacity in zero applied magnetic field ($H=0$) are explicitly calculated in bilayered superconductors, with two superconducting layers and tunneling couplings per layer periodicity length. The calculations are performed on the grounds of a generalization to multilayered superconductors of the Lawrence-Doniach Ginzburg-Landau functional, and assuming Gaussian fluctuations. In addition to the fluctuation heat capacity $c_{\rm fl}$, we also obtain various useful relationships between $c_{\rm fl}$ and other fluctuation-induced observables experimentally accessible in multilayered copper oxide superconductors. It is then shown that if the effects of the multilaminarity are taken into account, the mean-field-like Gaussian-Ginzburg-Landau approach may explain simultaneously and at a quantitative level the available experimental data, both the amplitude and the ϵ behavior, of the fluctuation specific heat, the in-plane paraconductivity $\Delta\sigma_{ab}$, and the fluctuation-induced diamagnetism $\Delta\chi_{ab}$, in $YBa_2Cu_3O_{7-\delta}$(Y-123) single crystals under zero or weak magnetic fields ($H\rightarrow 0$), up to reduced temperatures of the order of $\epsilon\equiv|T-T_c|/T_c\sim 10^{-2}$. The corresponding coherence length amplitudes (at $T=0$ K) are $\xi_{ab}(0)=1.1$ nm and $\xi_c(0)=0.12$ nm for the in-plane (ab) and transversal (c) directions, respectively. In contrast, the same data cannot be explained, in the same ϵ region, in terms of the 3DXY theory for full-critical fluctuations with a value of the dynamic critical exponent of $z=\frac{3}{2}$, which corresponds to the same universality class as the superfluid-normal λ transition of ^{4}He liquid, although these analyses do not exclude the applicability of such a scenario for $\epsilon\lesssim 10^{-2}$, as suggested by previous measurements of $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}$ in the same Y-123 single crystals. However, another dynamic universality class, with $z=2$, makes the 3DXY full-critical behavior compatible with the experimental data for $2\times 10^{-2}\lesssim\epsilon\lesssim 10^{-1}$. [S0163-1829(99)06705-3]

I. INTRODUCTION

The reduced temperature extension and location of the so-called full-critical and mean-field-like regions for the thermal fluctuations of Cooper pairs around T_c, and the value of the so-called dynamic critical exponent z controlling the dynamic universality class in the full-critical region, are two important and still controversial issues of the phenomenological descriptions of the high-temperature cooper oxide superconductors (HTSC's).[1–16] The uncertainty of the location of these critical regions is particularly well illustrated by the heat capacity C_p. Some groups, dealing mainly with scaling analyses of C_p measured around the superconducting transition in $YBa_2Cu_3O_{7-\delta}$(Y-123) crystals, have in the last years published numerous works with considerable impact in which it was claimed that the full-critical region extends in quite a wide temperature region around the transition, for $\epsilon\equiv|T-T_{c0}|/T_{c0}$ of the order of 10^{-1} or more, where T_{c0} is the mean-field critical temperature in a zero applied magnetic field.[1] These conclusions are mainly based on the good agreement between the scaling predictions of the 3DXY theory for full-critical fluctuations and the C_p experimental results, although results on other observables[10–12] also suggest such a full-critical behavior. This very strongly contradicts, however, the analyses published by different groups of other observables as the in-plane paraconductivity[1,5–7] $\Delta\sigma_{ab}$, the fluctuation-induced in-plane magneto-conductivity[1,8,9] $\Delta\tilde{\tilde{\sigma}}_{ab}$, and the fluctuation-induced diamagnetism for the magnetic field H applied perpendicularly to the CuO_2 layers,[1,13,14] $\Delta\chi_{ab}$, under weak magnetic fields. These analyses show that both the amplitude and the temperature behavior of these different observables may be explained up to temperatures as close to T_{c0} as $\epsilon\lesssim 10^{-2}$ in terms of the mean-field Gaussian-Ginzburg-Landau (GGL) approach. Moreover, simultaneous analyses of $\Delta\chi_{ab}$ and $\Delta\sigma_{ab}$ suggest that it is only for $\epsilon\lesssim 10^{-2}$ that the 3DXY approach, with a value of the dynamic critical exponent of $z=\frac{3}{2}$ (as for the full-critical region around the normal-superfluid λ transition in the ^{4}He liquid[17]) seems to apply,[6,8,14] in excellent agreement with the recent estimations in *multilayered* HTSC's of the Levanyuk-Ginzburg reduced temperature $\epsilon_{\rm LG}$ for the crossover between the full-critical and mean-field-like regions.[15] Let us already stress here that the controversy of the location and extension of the different critical regions is in part motivated by the weakness of the 3DXY scaling predictions for the full-critical region, even when the magnetic-field dependence is considered,[4] and also by the fact that up to now most of the analyses have been concentrated in each of the observables separately.[1]

A further important open problem, strongly related to the ϵ location of the full-critical region, is the value of the dynamic full-critical exponent z in the normal-superconducting transition in the HTSC. Such a z value controls the dynamic universality class and, hence, drastically affects the entire critical behavior of the transport properties in the full-critical regime.[17] At present, the theoretical analyses are not very conclusive, in part due to the unknown magnitude of the effects of the so-called plasma fluctuations in the HTSC's.[2,18] In addition, the comparisons with the experimental measurements of different transport properties of HTSC's in terms of the 3DXY models lead to different values for z.[1,5–9,11,16] These previous analyses are consistent either with the so-called E-model dynamics (the one of a lambda transition in an uncharged superfluid as ^{4}He liquid, leading to $z=\frac{3}{2}$) or with the A-model dynamics (leading to $z=2$).[17]

In this paper, we propose a different way to further clarify those important and long-standing problems, at least for reduced temperatures of the order of $\epsilon \gtrsim 10^{-2}$ and in weak magnetic fields. Our analysis is based on two main ingredients: First, when analyzing the fluctuation effects on C_p within the GGL approach, we will take into account the presence of various superconducting CuO_2 planes per periodicity length, s, with different tunneling couplings between adjacent planes. For that, we calculate the corresponding C_p explicit expressions within the multilayered GGL framework, for $T>T_{c0}$ and $T<T_{c0}$, in both cases for $H=0$. As first pointed out by Maki and Thompson[19] and by Klemm and co-workers,[13,20] such a multilaminarity may strongly affect the thermal fluctuations in HTSC's. The importance of these effects is now well established in the case of $\Delta\sigma_{ab}$, $\Delta\chi_{ab}$, and $\Delta\tilde{\tilde{\sigma}}_{ab}$.[6,8,14,21] Second, by using these theoretical results, we will make a systematic comparison between different fluctuation-affected observables rather than focusing only on the heat capacity. In particular, here we are going to consider, in addition to C_p at $H=0$, also $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}$ for $H\rightarrow 0$. We emphasize here that focusing on the $H\rightarrow 0$ limit will imply several advantages: (i) The full-critical region is expected to be wider than for $H\neq 0$. (ii) The temperature dependences are less ambiguous than the scaling predictions for different H's. (iii) This will allow us to exclude some values of z for $H=0$ and $2\times 10^{-2}\leqslant\epsilon\leqslant 10^{-1}$. (iv) New hypotheses, like the so-called lowest-Landau-level approximation, customarily added to the mean-field-like theory, become unnecessary. (v) It is possible to check the predictions for the quotients among the above different observables. This latter aspect will prove to be a very discriminating and parameter-reducing test, mainly because, as our theoretical calculations also reveal, in the GGL framework such quotients are ϵ independent and almost (or even completely) parameter free, while they have a rather different ϵ behavior in the 3DXY scenario. All these advantages compensate by far for the shortcoming of having to deal with the ambiguities related with the $H=0$ nonfluctuating background contribution to C_p. Let us stress here that our analysis is going to be concentrated in the ϵ region bounded by $2\times 10^{-2}\leqslant\epsilon\leqslant 10^{-1}$, where at present there exist very well-established experimental results obtained in untwinned $YBa_2Cu_3O_{7-\delta}$(Y-123) single crystals.[6,7,13,14,16,22,23] All the experimental data on Y-123 single crystals used in the present analysis have been confirmed at a quantitative level, both in amplitude and ϵ behavior, by various independent groups, and so we can consider them as very well-established intrinsic results. In fact, though we have chosen the measurements of Refs. 6, 14, and 22, we have also performed the same analyses below using the data published in Refs. 7, 13, 16, and 23, obtaining similar results (see also below).

The contents of the present paper are as follows: In Sec. II we present our GGL calculations. In Sec. III, we summarize the equivalent results of the 3DXY theory, paying attention to the possibility of using values for the dynamic critical exponent z, different from the one valid for the normal-superfluid transition of ^{4}He liquid. Then, in Sec. IV A, we compare our bilayered GGL predictions with the existing experimental data on $c_{\rm fl}$, $\Delta\sigma_{ab}$, and $\Delta\chi_{ab}$ in Y-123 crystals. In Secs. IV B, IV C, and IV D, we perform the same analyses but using different theories proposed by other authors: the single-layered GGL scenario, the 3DXY scenario with $z=\frac{3}{2}$ as the dynamic critical exponent in the full-critical region, and the 3DXY scenario with $z=2$. Only the first and fourth of these approaches considered in Sec. IV are not definitively ruled out, always for $2\times 10^{-2}\lesssim\epsilon\lesssim 10^{-1}$ and $H\rightarrow 0$, by our analysis. In Sec. V we summarize our conclusions.

II. GAUSSIAN-GINZBURG-LANDAU FLUCTUATION SPECIFIC HEAT IN MULTILAYERED SUPERCONDUCTORS

We will first calculate the GGL fluctuation specific heat $c_{\rm fl}$ in a multilayered superconductor with N superconducting layers in the layer periodicity length s and in absence of an applied magnetic field. Then, we will obtain some relationships between $c_{\rm fl}$ and other fluctuation-induced observables. Our starting point is the expression for the Ginzburg-Landau (GL) free energy of such superconductors as first introduced by Klemm[20] (see also Ref. 24 for previous related treatments):

$$F=F_n+\sum_{n=-\infty}^{\infty}\sum_{j=1}^{N}\int d^2\mathbf{r}\bigg\{a_0\tau|\Psi_{jn}|^2+\frac{b}{2}|\Psi_{jn}|^4+\frac{\hbar^2}{2m_{ab}}|\nabla_{ab}\Psi_{jn}|^2+a_0\gamma_j|\Psi_{jn}-\Psi_{j+1,n}|^2\bigg\}. \qquad (1)$$

In this equation, F and F_n are, respectively, the total and normal-state free energies, Ψ_{jn} is the two-dimensional (2D) superconducting wave function corresponding to each $j=1,...,N$ plane of the nth cell of length s [we use also the values $(j,n)=(N+1,n)$ for the $(1,n+1)$ layer]; $\tau\equiv(T-T_{c0})/T_{c0}$ and $\epsilon\equiv|\tau|$ are the signed and unsigned reduced temperatures, respectively; T_{c0} is the mean-field critical temperature at zero magnetic field; γ_j is the tunneling coupling constant between the (j,n) and $(j+1,n)$ planes; a_0 and b are the GL constants of each plane; and m_{ab} is the in-plane effective mass of the superconducting pairs (we neglect the possible in-plane anisotropy), which is related to the corresponding correlation length through $\xi_{ab}(\epsilon)=\xi_{ab}(0)\epsilon^{-1/2}=\hbar/\sqrt{(2m_{ab}a_0\epsilon)}$. The c-direction correlation length resulting from Eq. (1) is $\xi_c(\epsilon)=\xi_c(0)\epsilon^{-1/2}$, with $\xi_c(0)=s\sqrt{\gamma_1}$ for single-layered ($N=1$) superconductors and $\xi_c(0)=s/\sqrt{2(\gamma_1^{-1}+\gamma_2^{-1})}$ for the bilayered ($N=2$) superconductors.[20,21] Finally, note that if $N=1$, Eq. (1) recovers the well-known Lawrence-Doniach functional for single-layered superconductors.[25]

To calculate the Gaussian fluctuation specific heat resulting from the above functional, we have to reexpress it, both at $T>T_{c0}$ and $T<T_{c0}$, in an explicitly Gaussian (i.e., qua-

dratic) form. This is done by expanding each superconducting wave function around its uniform-equilibrium value, and retaining terms up to second order in the corresponding expansion of F. We are led then to a matrix functional that has to be diagonalized to obtain the Gaussian fluctuation spectrum, which results to be composed of N different branches. For temperatures above the critical, we already presented the basics of that program in Ref. 21. For $T<T_{c0}$, however, in addition to the above it is also necessary to deal properly with the different subtleties caused by the symmetry breaking.[26,27] For that, we apply an external-source-coupling method (for a similar treatment in the simple 3D case see, e.g., Ref. 27). We stress here that in doing this calculation it is particularly important to apply the Gaussian approximation *before* the diagonalization procedure (this avoids the shortcomings that affect Klemm's paper; see our comment in Ref. 28 and also below). For the superconducting contribution to the heat capacity per unit volume at constant pressure at $T>T_{c0}$ and $T<T_{c0}$, c_{sc}^{+} and, c_{sc}^{-}, respectively, we obtain $c_{sc}^{\pm}=c_0^{\pm}+c_{\rm fl}^{\pm}$ where $c_0^{\pm}$ is the usual equilibrium GL contribution,[29]

$$c_0^{+}=0 \quad \text{and} \quad c_0^{-}=\left(\frac{c_{\rm jump}}{T_{c0}}\right)T, \tag{2}$$

and $c_{\rm fl}^{\pm}$ are the GGL fluctuation contributions. For $c_{\rm fl}$ at $T>T_{c0}$ we obtain a useful intermediate result given by the integral expression

$$c_{\rm fl}^{+}(\epsilon)=\frac{k_B}{8\pi^2\xi_{ab}^2(0)}\sum_{j=1}^{N}\int_{-\pi/s}^{\pi/s}\frac{dk_z}{\epsilon+\omega_{jk_z}}, \tag{3}$$

where k_B is Boltzmann's constant and ω_{jk_z} are the different branches of the fluctuation spectrum, that, for single-layered $(N=1)$ and bilayered $(N=2)$ superconductors, are[21]

$$\omega_{k_z}^{N=1}=2\gamma_1(1-\cos k_z s) \tag{4}$$

and,

$$\omega_{jk_z}^{N=2}=\gamma_1+\gamma_2+(-1)^{j+1}\sqrt{\gamma_1^2+\gamma_2^2+2\gamma_1\gamma_2\cos k_z s}. \tag{5}$$

For $T<T_{c0}$, we obtain a similar integral expression for $c_{\rm fl}^{-}$ which we summarize by the relationship

$$c_{\rm fl}^{-}(\epsilon)=2c_{\rm fl}^{+}(2\epsilon), \tag{6}$$

that holds for any value of N and indeed also in the 2D and 3D limits [$\xi_c(\epsilon)\ll s$ and, $\xi_c(\epsilon)\gg s$, respectively]. In the case of single-layered and bilayered superconductors, we have also integrated Eqs. (3) and (6) to obtain the explicit results for $c_{\rm fl}^{+}$ and $c_{\rm fl}^{-}$. For a bilayered superconductor $(N=2)$, the result is

$$c_{\rm fl}^{\pm,N=2}=N_e^{\pm}(\epsilon)\frac{A_{\rm TF}}{\epsilon}(1+\beta^{\pm})^{-1/2}. \tag{7}$$

In this equation, $A_{\rm TF}\equiv k_B/[4\pi\xi_{ab}^2(0)s]$ is the Thouless-Ferrell amplitude,[30] $\beta^{+}\equiv B_{\rm LD}/\epsilon$ and $\beta^{-}\equiv B_{\rm LD}/(2\epsilon)$, $B_{\rm LD}$ is the Lawrence-Doniach parameter[25] $B_{\rm LD}\equiv(2\xi_c(0)/s)^2$, and

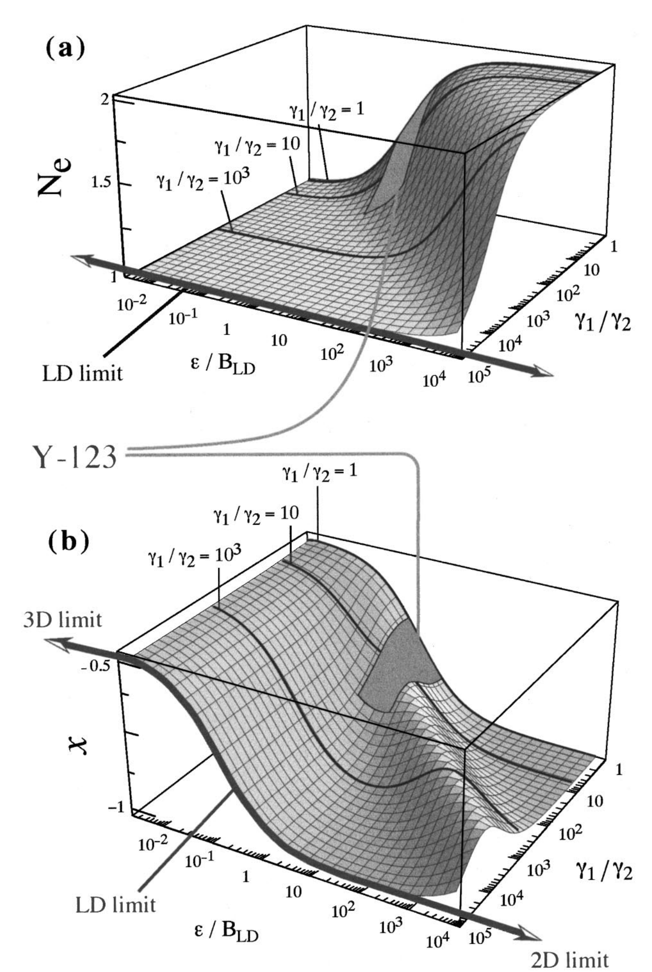

FIG. 1. The effective number N_e^{+} of fluctuating superconducting CuO_2 layers per periodicity length above T_c, and the mean-field critical exponent x as a function of the relative strength of the Josephson coupling between neighbor layers, γ_1/γ_2, and of the reduced temperature relative to the Lawrence-Doniach dimensionality parameter, $\epsilon/B_{\rm LD}$. The shadowed zones are the parts of the $N_e^{+}(\gamma_1/\gamma_2,\epsilon/B_{\rm LD})$ and of the $x(\gamma_1/\gamma_2,\epsilon/B_{\rm LD})$ surfaces which are expected to correspond to the Y-123 crystals, for $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$. The bold lines are the Lawrence-Doniach limit ($\gamma_1/\gamma_2\to\infty$) which correspond to a single layered $(N=1)$ superconductor. The 3D and 2D limits correspond, respectively, to $\epsilon/B_{\rm LD}\to0[\xi_c(\epsilon)\gg s]$ or $\epsilon/B_{\rm LD}\to\infty[\xi_c(\epsilon)\ll s]$.

$$N_e^{\pm}(\epsilon)\equiv\left(\frac{\frac{1}{4}+c_1\beta^{\pm}+c_2{\beta^{\pm}}^2+c_1^2{\beta^{\pm}}^3}{1+\tilde{c}_1\beta^{\pm}+\tilde{c}_2{\beta^{\pm}}^2+c_1^2{\beta^{\pm}}^3}\right)^{-1/2}, \tag{8}$$

where c_1, c_2, $\tilde{c}_1$, and $\tilde{c}_2$ are coefficients depending only on γ_1/γ_2, the ratio of the two coupling strengths between adjacent layers, as $c_1\equiv(\gamma_1/\gamma_2+1)^2/(2\gamma_1/\gamma_2)$, $c_2\equiv c_1^2+c_1/2$, $\tilde{c}_1\equiv2c_1+1$, and $\tilde{c}_2\equiv c_1^2+2c_1$. The $N_e^{\pm}(\epsilon)$ function, bounded by $1\leqslant N_e^{\pm}(\epsilon)\leqslant2$, may be physically seen as an effective number of independent fluctuating planes per layer periodicity length s at zero applied magnetic field, as discussed for $T>T_{c0}$ in Ref. 21. The behavior of N_e^{+} is represented as a function of γ_1/γ_2 and of $\epsilon/B_{\rm LD}$ in Fig. 1(a). In Fig. 1(b), the same representation is also made for x, the $T>T_c$ mean-field critical exponent of the heat capacity, defined as the slope of $c_{\rm fl}^{+}$ in a log-log representation, i.e., by $x\equiv(\partial \log c_{\rm fl}^{+}/\partial \log \epsilon)_{\gamma 1/\gamma 2}$. These $N_e^{+}(\gamma_1/\gamma_2,\epsilon/B_{\rm LD})$ and $x(\gamma_1/\gamma_2,\epsilon/B_{\rm LD})$ surfaces are similar to the ones already shown in Ref. 21, but now we include the parts which are expected to correspond to the Y-123 crystals in the mean-field-like region (for $10^{-2}\lesssim\epsilon\lesssim 10^{-1}$; see also below). For comparison, the single-layered Lawrence-Doniach limit is also indicated. Let us also note that the expressions valid for $N=1$ may be directly obtained by just imposing $N_e^{\pm}=1$ in the above results, and so recovering the known results for single-layered superconductors.[31] Equation (7) provides, for the first time to our knowledge, an explicit expression of the GGL fluctuation heat capacity in a bilayered ($N=2$) superconductor. For $T>T_{c0}$, this equation includes the nonexplicit integral expression proposed by Klemm in the so-called "static limit," whereas for $T<T_{c0}$ the corresponding integral expression due to Klemm is incorrect due to the inadequacy of the Gaussian approximation used by this author (see our comment in Ref. 28). Note also that from the above results it can be seen that the quantity $c_{\rm fl}^{+}(\epsilon)/c_{\rm fl}^{-}(\epsilon)$ is not T independent except in the 2D and 3D limits.

From the above integral expressions for $c_{\rm fl}$, we can obtain, in addition to the explicit expressions for $c_{\rm fl}^{N=2}$, also temperature-independent relationships, valid for any value of N, between $c_{\rm fl}^{+}$, $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}$, which will be further crucially discriminating and parameter-reducing tests in the data analysis: By comparing Eq. (3) with similar integral expressions previously obtained for $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}$ in Ref. 21, we obtain (in MKSA units)

$$\frac{c_{\rm fl}^{+}}{\Delta\chi_{ab}/T}=\frac{3\phi_0^2}{4\pi^2\mu_0}\,\xi_{ab}^{-4}(0) \tag{9}$$

and

$$\frac{c_{\rm fl}^{+}}{\Delta\sigma_{ab}}=\frac{4k_B\hbar}{\pi e^2}\,\xi_{ab}^{-2}(0), \tag{10}$$

which are consistent with

$$\frac{\Delta\chi_{ab}/T}{\Delta\sigma_{ab}}=\frac{16\mu_0 k_B}{3\pi\hbar}\,\xi_{ab}^{2}(0). \tag{11}$$

In these equations, $\phi_0\equiv\pi\hbar/e$, $\hbar$ is the Planck constant, e is the electron charge, and μ_0 is the vacuum permeability. The last relationship was first proposed in Ref. 32 for $N=1$. Note that Eqs. (9)–(11) have only $\xi_{ab}(0)$ as material-dependent parameter. In addition, they may be combined to obtain a universal, parameter-free GGL value

$$\frac{c_{\rm fl}^{+}}{\Delta\sigma_{ab}}\,\frac{\Delta\chi_{ab}/T}{\Delta\sigma_{ab}}=\frac{64\mu_0 k_B^2}{3\pi^2 e^2}\simeq 2\times 10^{-14}\ \ \Omega^2\ \mathrm{J/mK^2}. \tag{12}$$

Let us note here that Eqs. (10)–(12) assume that the in-plane paraconductivity arises only from the GL (also called Aslamazov-Larkin) contribution (the case in the HTSC compounds, as first shown in Ref. 33; see also Refs. 6, 8, and 21). Note also that Eqs. (7)–(12) easily lead, when combined, to the explicit expressions for $\Delta\sigma_{ab}^{N=2}$ and $\Delta\chi_{ab}^{N=2}$ already calculated in Ref. 21.

Finally, we must stress here that in obtaining Eqs. (10)–(12), we have used the in-plane paraconductivity expression for the bilayered superconductors which was calculated in Ref. 21 by using the time-dependent GGL approximation and by assuming a lifetime, $\tau_0=\tau_0(0)\epsilon^{-1}$, of the Cooper pairs' fluctuations with wave vector $\mathbf{k}=0$, equal to the one predicted by the conventional (with s-wave pairing) BCS approach (see, e.g., M. Tinkham in Ref. 1): $\tau_0^{\rm BCS}(0)=\pi\hbar/(8k_BT_{c0})$, which for overdoped Y-123 crystals, with $T_c\sim 90$ K, leads to $\sim 3\times 10^{-14}$ s. To our knowledge, $\tau_0^{\rm BCS}(0)$ was used in all the different microscopic and time-dependent GL-like calculations and analyses of $\Delta\sigma_{ab}$ published until now. Note also that the amplitude dependence of $\Delta\sigma_{ab}$ on both $\xi_c(\epsilon)$ and $N_e(\epsilon,\gamma_1/\gamma_2)$ prevents any quantitative estimate of $\tau_0(0)$ from the in-plane paraconductivity amplitude *alone*, even when these data were obtained in high-quality untwinned single crystals. However, Eq. (12) allows a very direct estimate of $\tau_0(0)$ and its comparison with the $\tau_0^{\rm BCS}(0)$ value: In terms of $\tau_0(0)/\tau_0^{\rm BCS}(0)$, the Aslamazov-Larkin amplitude $A_{\rm AL}$ for the in-plane paraconductivity defined as in Ref. 21 may be rewritten as $A_{\rm AL}=(e^2/16\hbar s)\times[\tau_0(0)/\tau_0^{\rm BCS}(0)]$, and, therefore, in turn Eq. (12) may be rewritten as (in MKSA units)

$$\frac{c_{\rm fl}^{+}}{\Delta\sigma_{ab}}\,\frac{\Delta\chi_{ab}/T}{\Delta\sigma_{ab}}=\frac{64\mu_0 k_B^2}{3\pi^2 e^2}\left(\frac{\tau_0^{\rm BCS}(0)}{\tau_0(0)}\right)^2 \simeq 2\times 10^{-14}\left(\frac{\tau_0^{\rm BCS}(0)}{\tau_0(0)}\right)^2. \tag{12$'$}$$

III. SUMMARY OF THE 3DXY RESULTS

The above results are markedly different from those resulting from the 3DXY model for full-critical fluctuations. The prediction of such a theory for $c_{\rm fl}$ is a logarithmic divergence,[27]

$$c_{\rm fl}^{\pm}\propto\ln(\epsilon), \tag{13}$$

while for $\Delta\chi_{ab}$ and $\Delta\sigma_{ab}$ it leads to[2,17,21,34,35]

$$\Delta\chi_{ab}/T\propto\epsilon^{-2/3}, \tag{14}$$

$$\Delta\sigma_{ab}\propto\epsilon^{-(2/3)(z-1)}. \tag{15}$$

In the last expression, z is the so-called dynamic critical exponent. Its value for a superconductor is controversial: Some authors argue that its value has to be the same as for the uncharged superfluids, i.e., $z\simeq\frac{3}{2}$, as predicted by the so-called E-model dynamics and experimentally verified in the superfluid-normal λ transition in ^{4}He liquid (see, e.g., Refs. 17 and 34). However, there are also some arguments by Halperin, according to which the effects of the so-called plasma fluctuations could become strong enough for the z of a superconductor take the value of the A-model dynamics, $z=2$.[2,16,18] The reduced temperature dependence of $\Delta\sigma_{ab}$ in the 3DXY model is, then:

$$\Delta\sigma_{ab}\propto\begin{cases}\epsilon^{-1/3} & \text{for } z=\frac{3}{2}\\ \epsilon^{-2/3} & \text{for } z=2.\end{cases} \tag{16}$$

Note that either of those options lead to ϵ-dependent predictions of the 3DXY theory for the quotients given by Eqs. (9)–(12) (except that $\Delta\chi_{ab}/T\Delta\sigma_{ab}$ will be ϵ independent if

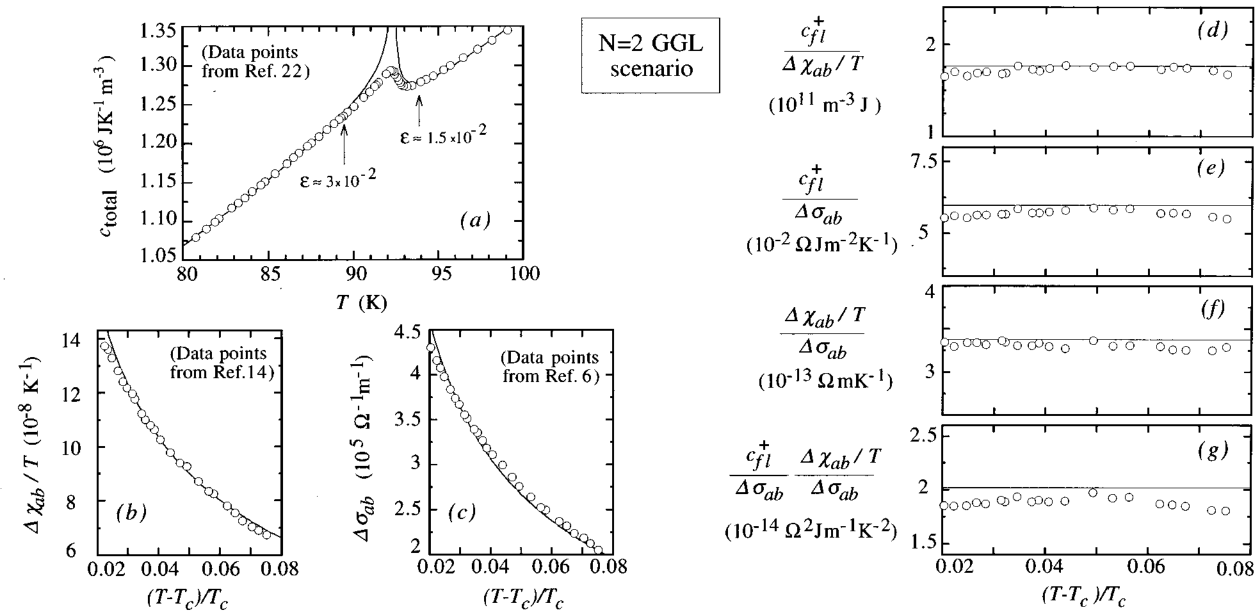

FIG. 2. Comparison between the bilayered GGL theory and the experimental heat capacity, in-plane paraconductivity, and fluctuation-induced diamagnetism in Y-123 crystals in the reduced temperature region bounded by 2×10^{-2} and 8×10^{-2} above T_{c0} and by 3×10^{-2} and 1.3×10^{-1} below T_{c0}. The approximate rms errors are (a) 1% in the ϵ-region bounded by the arrows, as explained in the main text; (b) 3%; (c) 3%; (d) 1%; (e) 3%; (f) 2%; and (g) 7%.

$z=2$). Note also that the proportionality constants in Eqs. (13)–(16) are free in the existing 3DXY theory, except by sign. Moreover, this theory corresponds to the 3D limit, and, therefore, the multilayering effects are irrelevant in this approximation.

IV. ANALYSIS OF THE AVAILABLE DATA IN $YBa_2Cu_3O_{7-\delta}$

A thorough confrontation of the scenarios summarized above with the experimental results may be done by using the available data in $YBa_2Cu_3O_{7-\delta}$ (Y-123) single crystals, by far the most and best studied HTSC compound. This HTSC has two superconducting CuO_2 planes per periodicity length $s=1.17$ nm. We are going to compare the experimental data first with the bilayered GGL scenario, then with the single-layered GGL scenario, and finally with the full-critical 3DXY one, with $z=\frac{3}{2}$ and 2.

For $\Delta\sigma_{ab}$, $\Delta\chi_{ab}$, and C_p, we use the data of Refs. 6, 14, and 22, respectively. We have checked, however, that similar data of the same observables also obtained by other groups in high-quality Y-123 crystals do not substantially change our results.[7,13,16,23] This conclusion holds at least in the ϵ region bounded by $10^{-2}\lesssim\epsilon\lesssim10^{-1}$: Closer to the transition, the uncertainties in T_{c0} and the presence of small inhomogeneities may deeply affect the data, even in the case of good single crystals.[5,6,36] For $\epsilon>10^{-1}$, the intrinsic fluctuation effects may become even smaller than the uncertainties due to the background or again due to the presence of small stoichiometric or structural inhomogeneities.[5,6,14] So our present analyses are going to be concentrated on the ϵ-region bounded by $10^{-2}\lesssim\epsilon\lesssim10^{-1}$, where, as stated above, the data of the three observables studied here, obtained in high-

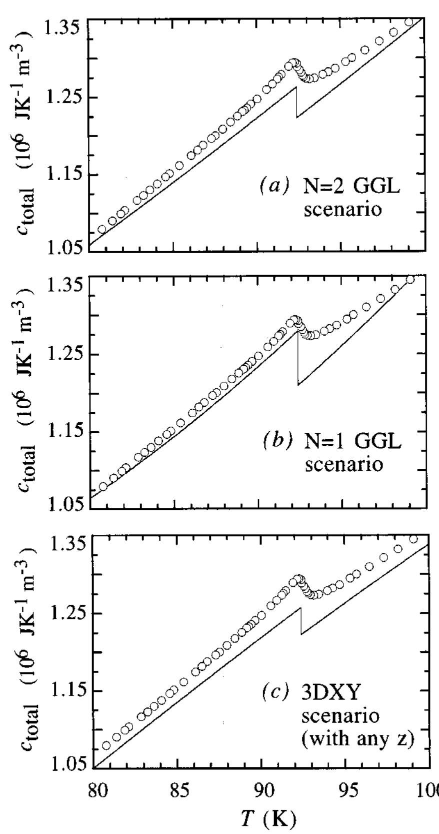

FIG. 3. Comparison between the experimental heat capacity and the nonfluctuating contributions (i.e., background plus c_0) arising in the different critical fluctuation scenarios analyzed in this work and summarized in Figs. 2, 4, 5, and 6.

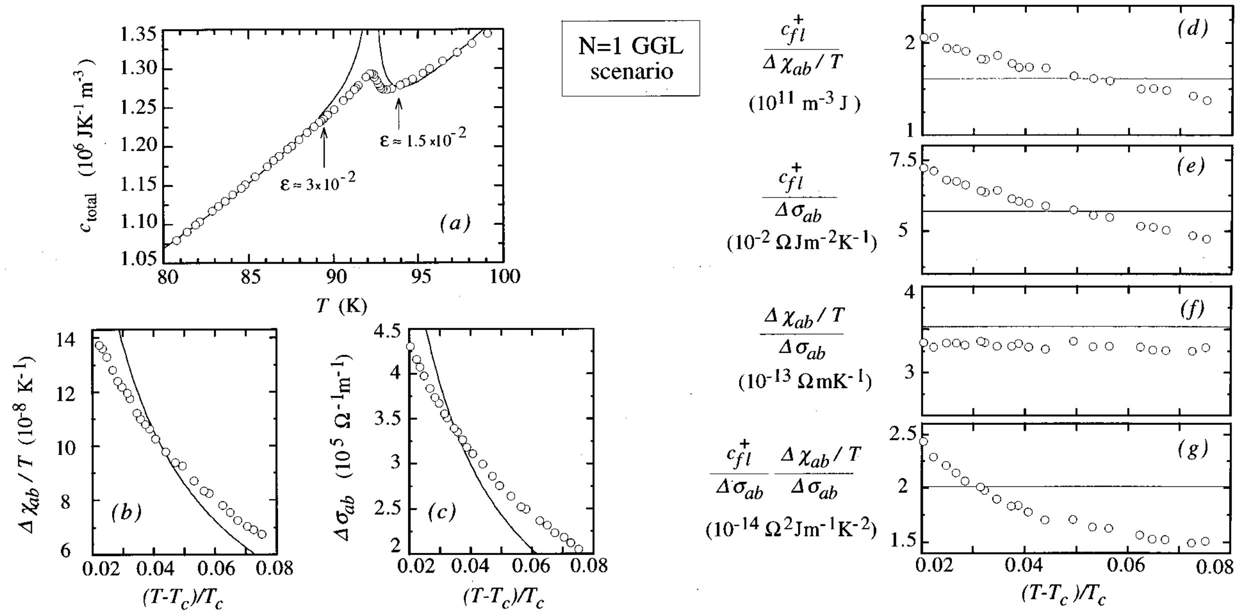

FIG. 4. Comparison between the single-layered GGL theory and the same raw experimental data used in Fig. 2 [in (d)–(g), the data curves are different from those in 2(d)–2(g) because of the different background subtractions to $c_{\rm total}$, see the main text]. This fit leads to a heat capacity jump $\simeq 7\times10^4\,{\rm J\,K^{-1}\,m^{-3}}$. The approximate rms errors are (a) 2% in the ϵ-region bounded by the arrows, as explained in the main text; (b) 30%; (c) 30%; (d) 30%; (e) 30%; (f) 10%, and (g) >100%.

quality Y-123 single crystals, have been reproduced by different experimental groups at a quantitative level and, therefore, they may be considered as intrinsic and definitively not severely affected by extrinsic inhomogeneity effects or by uncertainties in the T_{c0} and in the background (see, in particular, Refs. 5, 6, 8, and 36).

A. Comparison with the bilayered GGL approach

Figure 2 shows a comparison between the bilayered GGL expressions and the experimental data. In doing such an analysis, we have first compared the $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}/T$ data with the corresponding theoretical expressions for multilayered superconductors published in Ref. 21. The fitting region was in both cases the reduced temperature window above T_{c0} given by $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$, that corresponds to the ϵ region where the experimental uncertainties on $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}/T$ do not exceed 15%.[6,8,14] The $\Delta\sigma_{ab}$ fit (in which we have already neglected the so-called indirect terms[6,21,33]) involves as free parameters γ_1/γ_2 and $\xi_c(0)$, for which we obtain $1\lesssim\gamma_1/\gamma_2\lesssim50$ and $\xi_c(0)\simeq0.12\,{\rm nm}$ (see Fig. 1 and Ref. 37). By using these values in the $\Delta\chi_{ab}/T$ fit, we obtain $\xi_{ab}(0)\simeq1.1\,{\rm nm}$. Then, we have fitted the heat capacity both above and below T_{c0}, now in a wider ϵ range in which it is possible to keep the rms error of the fit below 1%. That criterion leads to an ϵ region bounded approximately by $1.5\times10^{-2}\lesssim\epsilon\lesssim10^{-1}$ above T_{c0} and $3\times10^{-2}\lesssim\epsilon\lesssim10^{-1}$ below T_{c0} (which includes the fitting region used for $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}$). As the parameters entering in the fluctuation contribution to the heat capacity are already fixed from the $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}/T$ fits, the heat capacity fit includes as free parameters only $c_{\rm jump}$ and the background contribution, which is supposed to have the general form $a+bT+cT^2$. As may be seen in Fig. 2, the agreement, for both the amplitude and the ϵ dependence, obtained with the multilayered GGL scenario is excellent in all the region indicated before. The disagreement in the very close vicinity of T_{c0} may correspond, as will be commented upon in Sec IV C, to the entering in a different fluctuation regime (GGL-to-3DXY crossover) or to extrinsic effects associated with T_{c0} inhomogeneities.[36] The heat-capacity jump resulting from the above fits is $c_{\rm jump}\simeq4.0\times10^4\,{\rm J\,K^{-1}\,m^{-3}}$. The nonfluctuating contribution to C_p (i.e., the background plus $c_0^{\pm}$) is represented in Fig. 3(a).

As an additional test of the above analysis, the same parameter values may also be introduced in the multilayered-GGL expressions for the in-plane magnetoconductivity under weak magnetic fields,[21] $\Delta\tilde{\tilde{\sigma}}_{ab}$. These analyses, similar to those already done in Y-123 single crystals by Pomar *et al.*,[8] confirm the adequacy of the scenario summarized above also to explain the fluctuation-induced magnetoconductivity in the same ϵ region above T_{c0}. Finally, let us note as well that the N_e values resulting from the above values of $\xi_c(0)$ and γ_1/γ_2 are bounded, for $10^{-2}\leqslant\epsilon\leqslant10^{-1}$, by $1.2\lesssim N_e^+\lesssim1.6$, which provides a direct indication of the relevance of multilayering effects on thermal fluctuations of Cooper pairs in Y-123 crystals.

Finally, note that when the results shown in Fig. 2(g) are compared with Eq. (12′) we obtain $\tau_0(0)/\tau_0^{\rm BCS}(0)\sim1$. This value, obtained by Eq. (12′) solely, may be very appreciably affected by the indeterminations associated with the background of C_p. A much more detailed analysis of the values of $\tau_0(0)$ obtained from thermal fluctuation measurements in different observables and in different HTSC's will be presented elsewhere. This analysis confirms that for the compound studied in the present article $\tau_0(0)/\tau_0^{\rm BCS}(0)\sim1$, but this time with 15% accuracy. Note, however, that to dis-

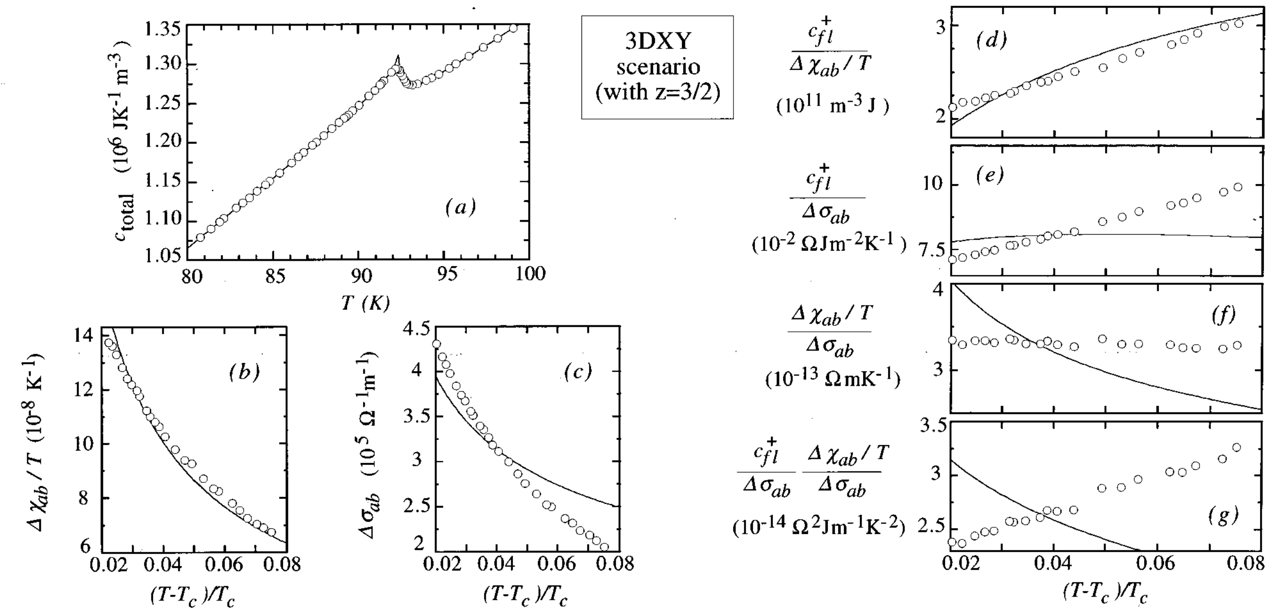

FIG. 5. Comparison between the 3DXY theory with dynamic critical exponent $z=\frac{3}{2}$ and the same raw experimental data used in Fig. 2 [in (d)–(g) the data curves are different from those in Figs. 2(d)–2(g) because of the different background subtractions to c_{total}; see the main text]. The approximate rms errors are (a) 1%, (b) 5%, (c) 40%, (d) 5%, (e) 40%, (f) >100%, and (g) >100%.

criminate through $\tau_0(0)$ different pairing mechanisms or symmetries it will be necessary to know also the expressions for $\tau_0(0)$ corresponding to such different theoretical frameworks.

B. Comparison with the single-layered GGL approach

For completeness, here we are going to compare the same experimental data briefly with the single-layered GGL theory. We proceed again by comparing the $\Delta\sigma_{ab}$ and $\Delta\chi_{ab}/T$ data, in the ϵ region $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$ above T_{c0}, with the corresponding theoretical expressions, published, e.g., in Ref. 21 (see also Refs. 25 and 38). The obtained values for the free parameters are $\xi_c(0)\simeq0.05\,\mathrm{nm}$, and $\xi_{ab}(0)\simeq1.13\,\mathrm{nm}$, but, as it may be seen in Figs. 4(b) and 4(c), even with such best-fit values the disagreement between experiments and theory is rather evident. We note that such a disagreement may be strongly mitigated by adding to $\Delta\sigma_{ab}$ the so-called indirect fluctuation contributions (e.g., Maki-Thompson terms).[21] However, the use of non-negligible indirect terms is inconsistent with the in-plane fluctuation-induced magnetoconductivity measurements, as shown, e.g., in Refs. 8 and 21. Then, we fit the heat-capacity data in the same ϵ window as for the bilayered GGL scenario, by imposing $N=1$ in Eq. (7) (or, equivalently, $N_e=1$) and the above values of $\xi_{ab}(0)$ and $\xi_c(0)$. As Fig. 4(a) shows, the resulting heat-capacity fit, with the three background coefficients and c_{jump} as free parameters, is not as good as the one obtained with $N=2$, but it is still satisfactory, with a rms error of 2%, though with a wrong tendency at high temperatures. That tendency may be more clearly seen in Fig. 3(b), where we compare the experimental heat capacity with the non-fluctuating contributions resulting from the above fit. Also, as can be seen in Fig. 3(b), in this scenario the fluctuation effects on C_p are rather asymmetrical with respect to T_{c0}, in contrast with the situation in the bilayered-GGL and 3DXY approaches [see Figs. 3(a) and 3(c)]. The origin of such a difference is the too small value in this approach of $\xi_c(0)/s$, that would imply quasi-2D fluctuations in this compound in all the studied reduced temperature range. Another origin of such a difference is the value obtained for c_{jump} ($\simeq7\times10^4\,\mathrm{J\,K^{-1}\,m^{-3}}$, around twice that obtained from the bilayered-GGL and 3DXY analyses), implying deviations between experimental data and background rather different at each side of the transition. The disagreement between the experimental data and the $N=1$ theory is also well demonstrated by the quotients shown in Figs. 4(d)–4(g). These results show that the single-layered GGL theory is unable to explain the available experimental data of the Cooper pairs fluctuation effects in the Y-123 compound in zero or weak magnetic fields.

C. Comparison with the 3DXY approach with $z=\frac{3}{2}$

Let us analyze now the same experimental data in terms of the 3DXY theory using the value $z=\frac{3}{2}$ (the same that is observed in the full-critical region around the normal-superfluid λ transition in liquid ^{4}He). We follow the same procedure as the one used in the precedent subsections for the GGL scenario: We first fit the 3DXY predictions for $\Delta\chi_{ab}/T$ and $\Delta\sigma_{ab}$ [Eqs. (14) and (15)] to the experimental data, again in the ϵ region $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$. Then we fit the heat capacity, using the logarithmic divergence predicted by the 3DXY theory [Eq. (13)], in a wider ϵ range in which it is possible to keep the rms error of the fit below 1%. Note here that the fits of $\Delta\sigma_{ab}$, $\Delta\chi_{ab}/T$, and of the heat capacity are now independent, due to the unrelated proportionality constants in the theoretical predictions. However, we may see quite clearly in Fig. 5 that only the heat capacity (with c_{jump}, the c_{fl} proportionality constant, and the background

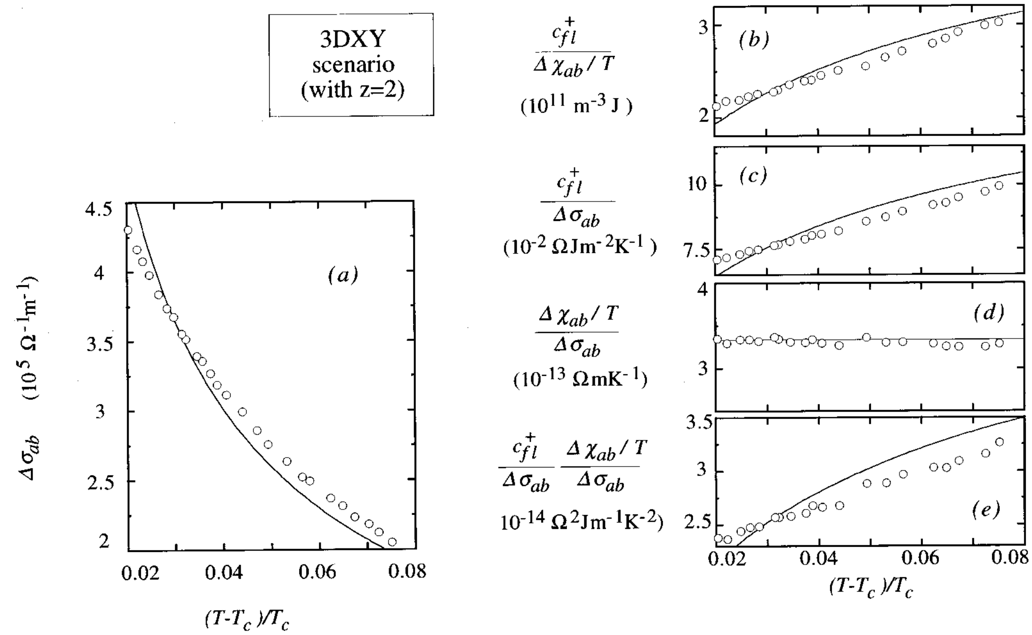

FIG. 6. Comparison between the 3DXY theory with dynamic critical exponent $z=2$ and the same raw experimental data used in Fig. 2 [in (b)–(e) the data curves are different from those in Figs. 2(d)–2(g) because of the different background subtractions to $c_{\rm total}$; see the main text]. In this figure we do not plot the fits of the heat capacity and fluctuation-induced diamagnetism because they are the same as Figs. 5(a) and 5(b). The approximate rms errors of the fits here are (a) 5%, (b) 5%, (c) 5%, (d) 1%, and (e) 7%.

coefficients as free parameters) may be adequately fitted. As shown in Figs. 5(d)–5(f), the disagreement is still more important for the quotients among the three different observables. This has to be compared with Figs. 2(d)–2(f), where the same quotients agree with the GGL theory, and in this case without any free parameter [$\xi_{ab}(0)$ was extracted from the $\Delta\chi_{ab}/T$ fit]. The heat-capacity jump that would result from this scenario is $c_{\rm jump}\simeq 3.5\times 10^4\,{\rm J\,K^{-1}\,m^{-3}}$. The corresponding nonfluctuating contribution to C_p is represented in Fig. 3(c). As visible in that figure, and because of the small divergence of the 3DXY prediction for $c_{\rm fl}$, in this scenario the fluctuation effects are appreciable in the heat capacity even for $|\epsilon|$ well above 10^{-1}, in contrast with the absence of appreciable fluctuation effects on χ_{ab} and σ_{ab} observed in this same ϵ-region.

Let us also briefly comment in this subsection on the temperature region closer to T_{c0} than $\epsilon\simeq 2\times 10^{-2}$ above T_{c0}, and closer to T_{c0} than $\epsilon\simeq 3\times 10^{-2}$ below T_{c0}. In that region, as shown above, the GGL predictions disagree with the experimental data, while the 3DXY logarithmic law for $c_{\rm fl}$ produces good-quality fittings. Also, the measurements of the in-plane paraconductivity in untwinned Y-123 single crystals, with very sharp transitions, suggest a crossover of the critical exponent, at $\epsilon\lesssim 2\times 10^{-2}$, from the 3D GGL value ($x=-\frac{1}{2}$) to precisely the value $x=-\frac{1}{3}$ given by the 3DXY theory with $z=3/2$.[6,7,16] At around the same reduced temperatures, a crossover from $x=-\frac{1}{2}$ to $-\frac{2}{3}$ has been also observed in the case of the in-plane fluctuation-induced diamagnetism measured in the same Y-123 crystals.[6,14] However, as stressed above, so close to T_{c0} it is difficult to discriminate definitively if such features are really associated with the penetration into the true full-critical region or if they are due to extrinsic effects associated with small sample inhomogeneities, which may be present even in apparently very good single crystals.[5,36] Indeed, any analysis of the thermal fluctuation effects for $\epsilon<10^{-2}$ is not reliable when the samples are not excellent single crystals.[36,39]

D. Comparison with the 3DXY approach with $z=2$

Let us now analyze the same experimental data, always for $2\times 10^{-2}\lesssim\epsilon\lesssim 10^{-1}$, using the 3DXY theory with the value $z=2$ for the dynamic critical exponent. We perform just the same fits as in Sec. IV C with the only change that now for $\Delta\sigma_{ab}$ we use the exponent $-\frac{2}{3}$ instead of $-\frac{1}{3}$. The results are represented in Fig. 6 (in that figure we do not plot the $c_{\rm fl}$ and $\Delta\chi_{ab}$ fits provided those are the same as in Fig. 5). As seen in that figure, the agreement obtained in the new fits is not so good as those obtained with the bilayered GGL approach, despite the fact that in this 3DXY scenario the amplitudes are free (in contrast with the GGL approaches), but it is still rather reasonable and it is compatible with the experimental uncertainties. So this theoretical scenario cannot be ruled out on the basis of the present analysis, and more work is needed to elucidate, in the ϵ region bounded by $10^{-2}\lesssim\epsilon\lesssim 10^{-1}$, between it and the bilayered-GGL one. A way to discriminate between both possibilities may be to analyze the fluctuation-induced in-plane magnetoconductivity, $\Delta\tilde{\tilde{\sigma}}_{ab}$, with a weak magnetic field applied perpendicular to the superconducting planes, measured in high-quality single crystals of Y-123 by various groups.[8,9,16] The recent analysis by Kim *et al.* of experimental data[16] for $\Delta\sigma_{ab}$ and $\Delta\tilde{\tilde{\sigma}}_{ab}$, that was made considering only the possibility of a 3DXY behavior over the entire studied temperature range ($\epsilon\lesssim 10^{-1}$), suggests that $z=2$ is probably not compatible, at least in the presence of a nonweak applied magnetic field, with the experimental $\Delta\tilde{\tilde{\sigma}}_{ab}$. However, these analyses, based on scaling predictions, are in our opinion not very discriminating, and further work is needed to confirm such a conclusion. We are currently focusing our future work in that direction.

V. CONCLUSIONS

In conclusion, we have obtained explicit expressions of the GGL fluctuation specific heat in a bilayered superconductor, in zero applied magnetic field, both above and below T_{c0}. Also, parameter-reducing relationships have been obtained among such a fluctuation specific heat and the in-plane paraconductivity and the fluctuation-induced diamagnetism under weak magnetic fields, valid for any number of superconducting layers per layer periodicity length. By using these theoretical results, it has been shown that if multilayering effects are taken into account the GGL approach is able to explain, simultaneously and at a quantitative level, the available experimental data on the thermal fluctuation effects on C_p, σ_{ab}, and χ_{ab} in Y-123 crystals in the reduced temperature region bounded by $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$. These results are in agreement with recent estimates[15] of the Levanyuk-Ginzburg criterion in bilayered HTSC's, that predicts a mean-field-like to full-critical crossover at $\epsilon\simeq10^{-2}$. Note that such an upper limit for the full-critical region would hold for weak magnetic fields. In the presence of a nonweak magnetic field, the full-critical region would be expected to be still smaller in reduced temperature, or eventually it may even disappear.[3,4]

As a further attempt to discriminate between the GGL and 3DXY scenarios, the same data were also analyzed in terms of the full-critical 3DXY approach, by considering the two possibilities for the value of the dynamic critical exponent z proposed in the literature ($z=\frac{3}{2}$ and 2). These analyses allow us to show, to our knowledge for the first time unambiguously, that the experimental data, always in the $2\times10^{-2}\leqslant\epsilon\leqslant10^{-1}$ reduced temperature region and for $H\to0$, cannot be explained in terms of the 3DXY approach with $z=\frac{3}{2}$ (as for the full-critical fluctuations in the λ transition of ^{4}He). However, if $z=2$ is taken instead, the agreement with the experiments is compatible with the error bands of the considered experiments, but certainly not as good as with the bilayered-GGL approach, even though in the 3DXY approaches the amplitudes are free (in contrast with the GGL approaches). These ambiguities could be resolved by further measurements of the in-plane fluctuation magnetoconductivity in high-quality Y-123 samples in the low-magnetic-field regime.

Let us also stress here that our present analysis does not exclude the adequacy of the 3DXY approach, with $z=\frac{3}{2}$, closer to the superconducting transition, for $\epsilon\lesssim10^{-2}$, as first suggested experimentally in Ref. 5 and then observed at a more quantitative level in the experiments described in Refs. 6, 7, and 14. However, as also already stressed in these papers, in this ϵ region, so close to the transition, the experimental data may be appreciably affected by small inhomogeneities (see also Ref. 36), and they are at present less well established than for $\epsilon\gtrsim10^{-2}$ (mainly in the case of the excess diamagnetism). Therefore, this ϵ region has been excluded from our present analysis. Various other related important aspects of the effects of the thermal fluctuations of Cooper pairs around T_{c0} in multilayered HTSC's in general and in Y-123 crystals in particular remain still open. This is the case of the critical behavior of other observables not analyzed here (e.g., the penetration depth,[10] the microwave conductivity,[11] or the thermal expansion[12]), which have been not confirmed by independent groups (or even that have been measured by different groups obtaining different results, as it is the case of the penetration depth[39]). Further measurements in high-quality single crystals of these observables and also of different observables for $\epsilon\leqslant10^{-2}$ will be very useful. Also, the presence or not of the so-called dynamic and high-temperature effects,[20,40] clearly absent for $\epsilon\leqslant10^{-1}$, need to be checked at higher reduced temperatures. Finally, let us note here that our present results strongly confirm the irrelevance of the so-called indirect fluctuation effects (Maki-Thompson and density of states) in the in-plane paraconductivity and in the in-plane fluctuation-induced magnetoconductivity. However, various groups are claiming that these indirect contributions are crucial for an understanding of the transversal (in the c direction) transport properties around T_{c0} of the Y-123 crystals.[41] Thus detailed measurements around T_{c0} in good single crystals of these transversal properties, as well as reliable comparisons with the theoretical approaches, will also be very useful at present. Let us finally note that the self-consistency of the GGL theory in the different temperature ranges above T_c in extreme type-II superconductors with very small superconducting correlation length amplitudes is still a general open problem which will deserve further theoretical and experimental work. In the case of the Y-123 crystals, our present results suggest such a self-consistency for $10^{-2}\lesssim\epsilon\lesssim10^{-1}$.

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